SUMMER INTERNSHIP PROJECT REPORT

ON

RENDEZEVOUS GUIDANCE

TRAJECTORY PLANNING FOR ROBOTIC INTERCEPTION

SUBMITTED BY

Piyush Sudhakar More

Priyadarshini College of Engineering, NAGPUR

GUIDED BY

Prof. Satadal Ghosh & Prof. Joel George

Department of Aerospace Engineering

Indian Institute of Technology, Madras

ACKNOWLEDGEMENT

I am very grateful to the Department of Aerospace Engineering and my project supervisor Dr. Satadal Ghosh for allowing and supporting me to perform this internship. I would also like to thank him for letting me perform the experiments in his lab and providing me a healthy atmosphere for carrying out the research activities.

I would like to thanks Dr. Joel George M, Assistant Professor, Department of Aerospace Engineering, Indian Institute of Technology, Madras for his invaluable guidance, and encouragement for this project and my guide Mr. Rahul Dinkar, for his unconditional help at my work.

……………………………………

Piyush More,

Aeronautical Engineering,

Priyadarshini College of Engineering, Nagpur

CONTENTS

ABSTRACT

This correspondence presents a online trajectory planning method for the autonomous robotic interception of moving targets i.e., position and velocity matching (also referred to as rendezvous).

In the proposed methodology, first, a parallel-navigation rule, originally introduced in the missile-guidance literature, is applied to generate a set of instantaneous task-space velocity commands. Subsequently, a rendezvous-guidance method is utilized to reduce the original command set to one with velocity-matching capability. Finally, the fastest velocity command in the reduced set is chosen such that the dynamic limitations of the actuators of the robot are not violated.

The proposed algorithm results in a fast and robust interception.

INTERCEPTION VIA RG

Let us define a LOS as the relative position vector ***r*** connecting the interceptor/robot to the target, as shown in Fig. 2. The parallel navigation rule states that the relative velocity $\dot{\boldsymbol{r}}$ between the robot and the target should remain parallel to ***r*** at all times [4]. If this rule holds throughout the motion of the interceptor, the distance between the interceptor and the target would decrease until they collide.

The parallel-navigation law is expressed by the following two relationships:

$\boldsymbol{r} \times \dot{\boldsymbol{r}} = 0$ (1)

$\boldsymbol{r} \cdot \dot{\boldsymbol{r}} < 0$ (2)

Equation (1) guarantees that the LOS and the relative velocity remain parallel,
Equation (2) ensures that the interceptor is not receding from the target

The above equations can be solved for $\dot{\boldsymbol{r}}$ in a parametric form to yield

$\dot{\boldsymbol{r}} = -\alpha \, \boldsymbol{r}$ (3)

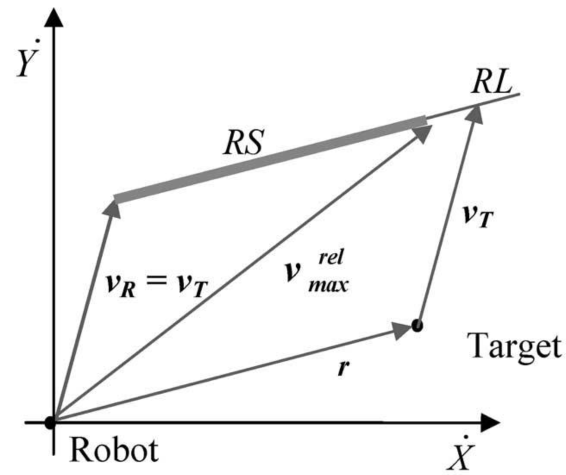
where α is a positive real number. The instantaneous relative velocity,also referred to as the "closing velocity," can then be written in terms of the robot and target velocities, denoted by $\boldsymbol{v}R$ and $\boldsymbol{v}\mathrm{T}$, respectively,as follows:

$\dot{\boldsymbol{r}} = \boldsymbol{v}\mathrm{T} - \boldsymbol{v}R$ (4)

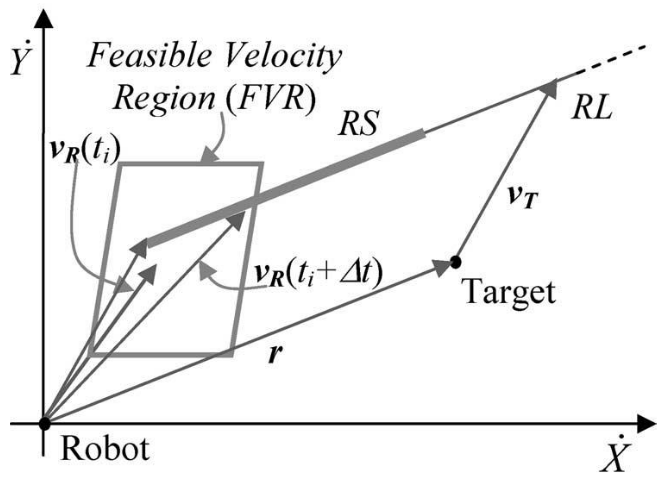
Substituting (3) into (4) and solving for the robot velocity yields

$\boldsymbol{v}R = \boldsymbol{v}\mathrm{T} + \alpha \, \boldsymbol{r}$. (5)

The vectors $\boldsymbol{r}$ and $\boldsymbol{v}\mathrm{T}$ are determined using the data received from a vision module, based on the instantaneous positions of the robot and the target. Substituting these two known vectors into (5) would result in a locus for the robot's velocity vectors $\boldsymbol{v}R$, all lying on a semiline parameterized by α. This semiline, referred to as the rendezvous line (RL), is depicted in Fig. 2. The center of the coordinate frame is located on the robot to show the instantaneous relative position of the target. The endpoints of the velocity vectors show the position of the target or the robot after one sampling period, should they adopt the corresponding velocities. If the robot continually adopts a velocity command that falls on the instantaneous RL, the direction of the LOS would remain constant, and the positional matching between the robot and the manoeuvring target is guaranteed.



(a)



(b)

In order to rendezvous with a target, the velocity of the robot/ interceptor must also match the velocity of the manoeuvring target at the time of the interception. The velocity commands generated based on (5) guarantee the position matching. Thus, the next task is to find an α value such that velocity matching is also assured. Let us assume that, from the current instant until interception, the robot is guided by the velocity commands that lie on the instantaneous RL. This assumption allows us to consider the interception problem only in the direction of the LOS. Let us, furthermore, consider that the acceleration capability of the robot in this direction is given by A. This acceleration would be used to bring the closing velocity down to zero. Assuming a constant acceleration for the rest of the robot motion, the simultaneous reduction of the velocity and position differences in the direction of the LOS for interception may then be written as

$$\dot{\boldsymbol{r}}^{\mathrm{rendmax}} - A(t_r) = 0$$
$$\boldsymbol{r} - \dot{\boldsymbol{r}}^{\mathrm{rendmax}}(t_r) + 1/2\, A(t_r)^2 = 0 \qquad (6)$$

where $\dot{\boldsymbol{r}}^{\mathrm{rendmax}}$ is the magnitude of the maximum allowable closing/ rendezvous velocity (hence, the superscript rend) and t_r is the time remaining to intercept the target from the current instant. The maximum instantaneous allowable closing velocity is then obtained by solving (6)

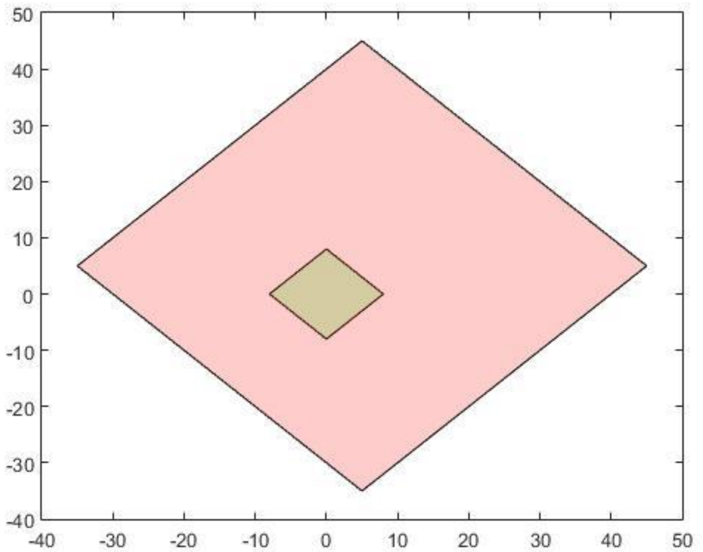
$$\dot{\boldsymbol{r}}^{\mathrm{rendmax}} = \sqrt{2\boldsymbol{r}A}. \qquad (7)$$

$$\boldsymbol{v}^{\mathrm{relmax}} = \dot{\boldsymbol{r}}^{\mathrm{rendmax}}$$

The end points of all the velocity-command vectors on the RL that have a closing-velocity component smaller than $\boldsymbol{v}^{\mathrm{relmax}}$ constitute a line segment extending from $\boldsymbol{v}_R = \boldsymbol{v}_{\mathrm{T}}$ to $\boldsymbol{v}_R = \boldsymbol{v}_{R,\max}(= \boldsymbol{v}_{\mathrm{T}} + \boldsymbol{v}_{\mathrm{relmax}}(\boldsymbol{r}/\ \mathrm{norm}\boldsymbol{r}\ ))$. This set of points is referred to herein as the rendezvous set (RS) [Fig. 3(a)]. The velocity represented by $\boldsymbol{v}^{\mathrm{relmax}}$ [Fig. 3(a)] may not be achievable by the robot within the sampling period Δt. Therefore, we define a feasible velocity region (FVR) representing all the velocities achievable by the robot within Δt, taking into account the kinematic and dynamic constraints on the robot [3]. This region is depicted by the polygon in Fig. 3(b). The velocity selected by the robot for the sampling interval

Δt is the component of the RS within the FVR with the maximum value, which is represented by $\boldsymbol{v}_R(ti + \Delta t)$ in Fig. 3(b). It is, thus, concluded that, if the robot adopts the velocity commands from within the RS with the largest allowable closing-velocity component, then, a time-efficient interception can be achieved.

Making of the Feasible Velocity Region from the Feasible Acceleration Region



Yellow block- The feasible acceleration region (FAR)
Red block- The feasible velocity region (FVR)

Fig 1. FVR created for the corresponding FAR

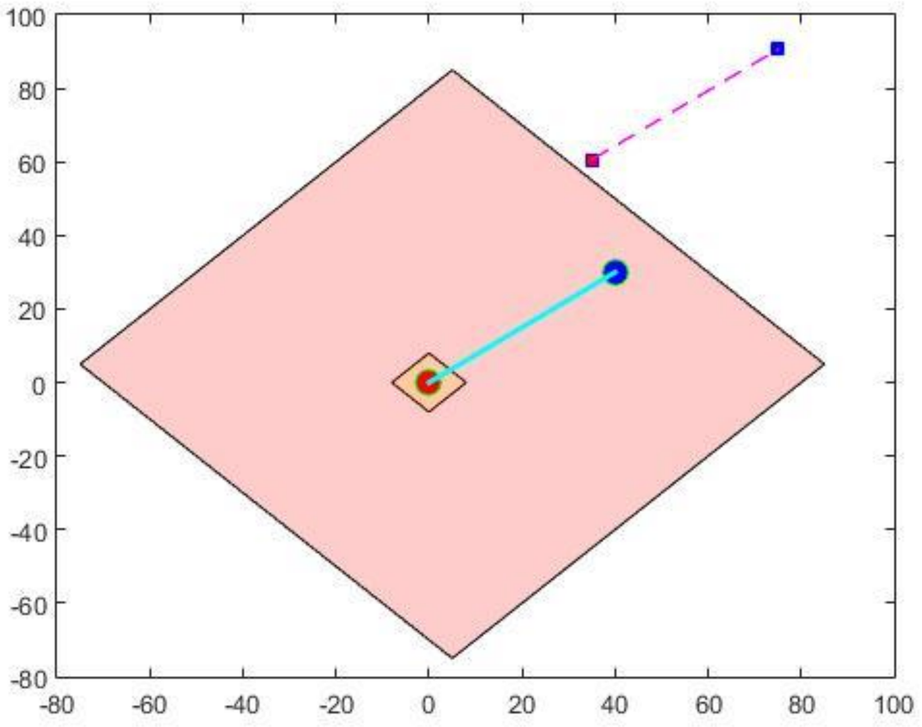
let the velocity of the robot be given by $\mathbf{v}_{R,t}$ at time, t

Furthermore, assume that during the time interval, dt, while two consecutive commands are issued (usually in the order of tens of milliseconds), the acceleration of the end-effector, $\mathbf{a}_{R,t}$, remains constant. Then, the velocity of the end-effector at the end of this period is given by:

$$\mathbf{v}_{R,t+\mathrm{d}t} = \mathbf{v}_{R,t} + \mathbf{a}_{R,t}\,\mathrm{d}t. \qquad (1)$$

If one substitutes $\mathbf{a}_{R,t}$ with each of the points on the boundary of feasible acceleration region, under the above linear approximation, this region can be mapped into a feasible velocity region. The feasible end-effector velocity command region,
which is also in the form of a parallelogram, shown in Figure above, defines the set of robot velocities that can be reached by the robot before the next command instant.

Adapting The Parallel Navigation Law



Red dot-robot position
Blue dot-target position
Cayenne colour line- line of sight (LOS) joining the robot and the target position
Magenta colour dotted line- Rendezvous line (RL) parallel to the line of sight (LOS)

Fig 2. Above is the simulation produced in matlab from the below given algorithm

The parallel navigation law is expressed by the following two relations:

$r \times \dot{r} = 0$ (2)

&

$r \cdot \dot{r} < 0$ (3)

Equation (2) guarantees that the LOS and relative velocity remain parallel, while
Equation (3) ensures that the interceptor is not receding from the target.

The above equations can be solved for $\dot{\mathbf{r}}$ in a parametric form to yield
$\dot{r} = -\alpha r$, (4)

where α is a positive real number. The instantaneous relative velocity, also referred to as "closing velocity," can be, then, written in terms of the robot and Target velocities, denoted by vR and vT, respectively, as follows:

$\dot{r} = vT - vR.$ (5)

Substituting Equation (4) into Equation (5) and solving for the robot velocity yields:

$\mathbf{v}R = \mathbf{v}T + \alpha\mathbf{r}.$ (6)

One may note that, according to Equation (6), even for a spatial scenario, interception is instantaneously planar; i.e., the velocity of the robot is in the same plane as the velocity of the moving target and LOS although this plane might change for the next time instant.

Let us suppose that from the current instant until the achievement of interception tolerances, the robot is guided by velocity commands that lie on the instantaneous RL. This assumption allows us to consider the interception problem only in the direction of LOS. Let us further consider that the acceleration capability of the robot in this direction is given by the value of acceleration radius, A. This acceleration would be used to bring the closing velocity down to zero. Assuming a constant acceleration radius for the rest of the robot motion, the simultaneous reduction of velocity and position differences in the direction of LOS for interception may, then, be written as:

$\dot{r}^{\text{rendmax}} - A*(t_r) = 0,$
$r - \dot{r}^{\text{rendmax}} *(tr) + ½*A*(t_r)^2 = 0,$ (7)

where r is the magnitude of the initial position vector, $\dot{r}^{\text{rend}}$
max is the magnitude of the maximum allowable closing velocity as imposed by the rendezvous-guidance criterion (hence, the superscript *rend*), and t_r is the time remaining to intercept the object from the current instant.

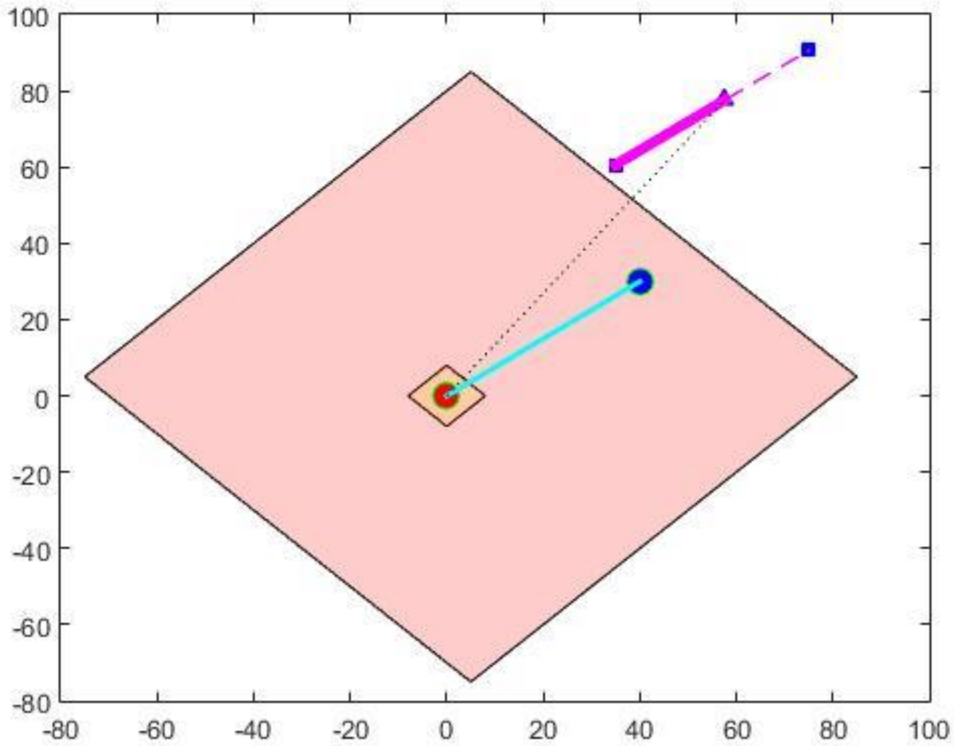
By eliminating t_r in Equation (7), the maximum allowable closing velocity for the current instant would be obtained as:

$$\dot{r}^{rendmax}=\sqrt{2rA}. \quad (8)$$

$$v^{relmax} = \dot{r}^{rendmax} \quad (9)$$

The above algorithm was implemented in matlab and below is the simulation produced

Producing of the Rendezvous Set on the Rendezvous Line



Red dot-robot position
Blue dot-target position
Cayenne colour line- line of sight (LOS) joining the robot and the target position
Magenta colour dotted line- Rendezvous line (RL) parallel to the line of sight (LOS)
Thick Magenta colour line - Rendezvous Set (RS) on the Rendezvous line (RL)

Fig 3. Above is the simulation produced in matlab from the below given algorithm

The end points of all velocity command vectors on RL that have a closing velocity component smaller than v^{relmax} constitute a finite line segment extending from

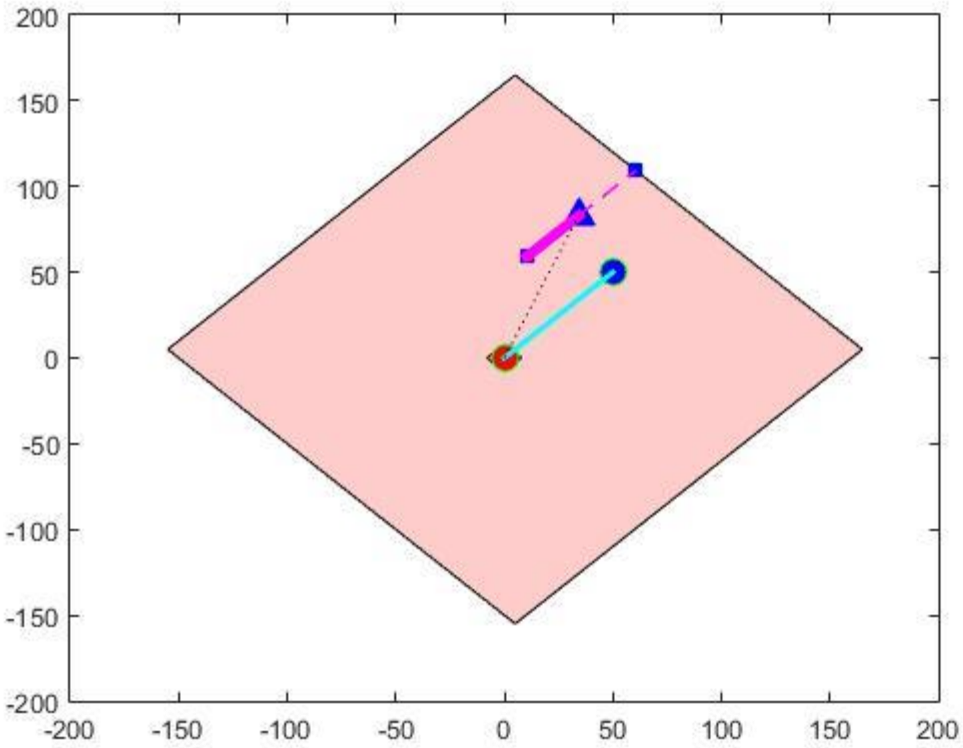
vR = vT to vR,max = vT +$\mathbf{v^{relmax}}$(r/norm(r)).

This set of points, together with the points along **vR = vT** vector, is referred to herein as the Rendezvous Set (RS). Figure above shows the RS for the same instant as shown in Fig 2. and a sample value for maximum closing velocity

By including the points on the Target velocity vector within the RS, the initialization of the robot motion is also made possible: Namely, since the robot has a limited acceleration capability, when starting its motion from a standstill, it might take several sampling times before the velocity of the end-effector can reach one of the points on the RL. By accelerating in a direction parallel to the velocity of the object, the end-effector's velocity approaches the RL, hence, justifying the inclusion of the points on **vR** = **vT** within the RS.

It can, thus, be concluded that if the end-effector adopts velocity commands from within the RS with the largest allowable closing velocity components, then, a time-efficient interception would be achieved

Selecting of the Robots Final Velocity or Generation of Robot Velocity Command

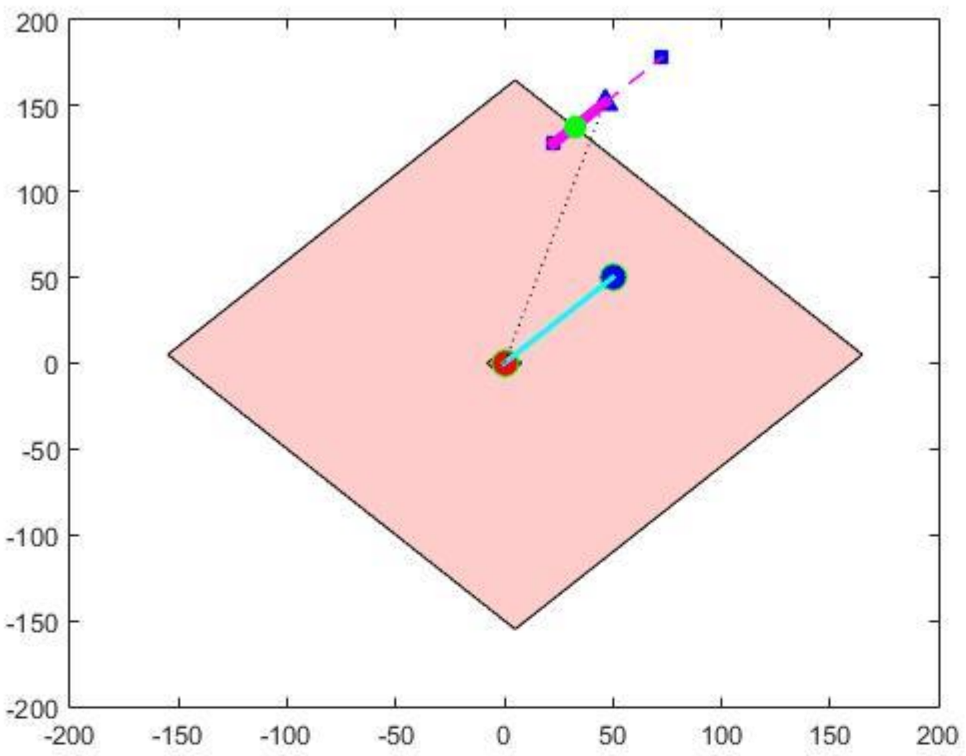


Red dot-robot position
Blue dot-target position
Cayenne colour line- line of sight (LOS) joining the robot and the target position
Magenta colour dotted line- Rendezvous line (RL) parallel to the line of sight (LOS)
Thick Magenta colour line - Rendezvous Set (RS) on the Rendezvous line (RL)
Blue Triangle- vrelmax on the RL

Fig 4a. Case1

<u>Case 1</u>

In this case the RS lies inside the FVR thereby maximum closing velocity component, v^{relmax} is within the FVR so the desired velocity $v_R(ti + \Delta t)$ for the next time interval is set to v^{relmax} and the velocity is obtained from the above explained RG method.

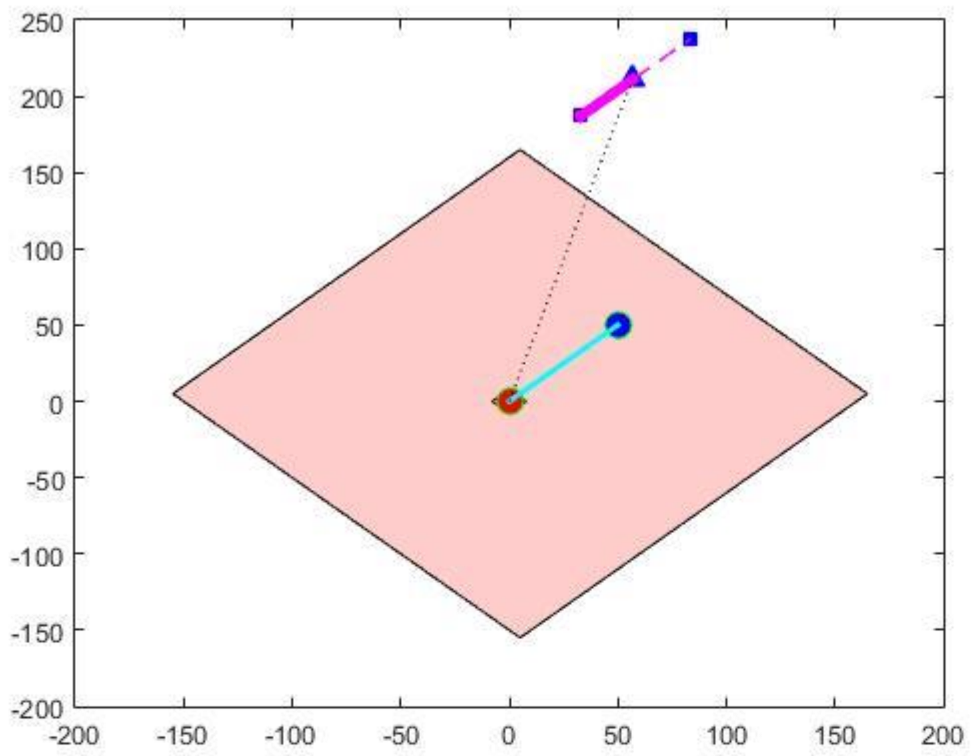


Red dot-robot position
Blue dot-target position
Cayenne colour line- line of sight (LOS) joining the robot and the target position
Magenta colour dotted line- Rendezvous line (RL) parallel to the line of sight (LOS)
Thick Magenta colour line - Rendezvous Set (RS) on the Rendezvous line (RL)
Blue Triangle- vrelmax on the RL
Green dot-intersection of RS with the FVR chosen as the final velocity for this case

Fig 4b. Case2

Case 2

In this case RS intersects the FVR at a distinct point,the maximum closing velocity component, v^{relmax} is outside the FVR so the desired velocity $v_R(ti +\Delta t)$ for the next time interval is set to point of intersection of the RS and the FVR which is represented by the green dot in the above figure



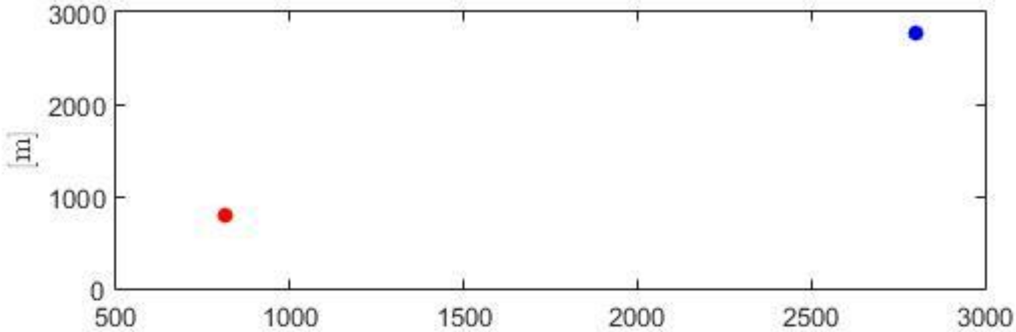
Red dot-robot position
Blue dot-target position
Cayenne colour line- line of sight (LOS) joining the robot and the target position
Magenta colour dotted line- Rendezvous line (RL) parallel to the line of sight (LOS)
Thick Magenta colour line - Rendezvous Set (RS) on the Rendezvous line (RL)
Blue Triangle- vrelmax on the RL

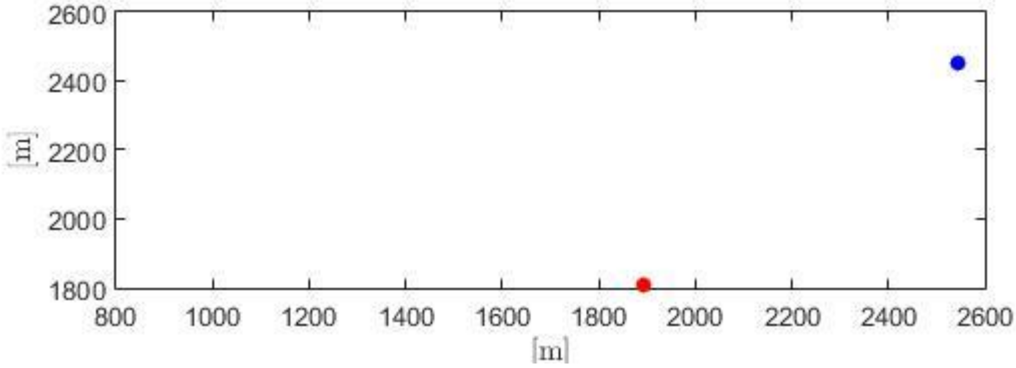
Fig 4c. Case3

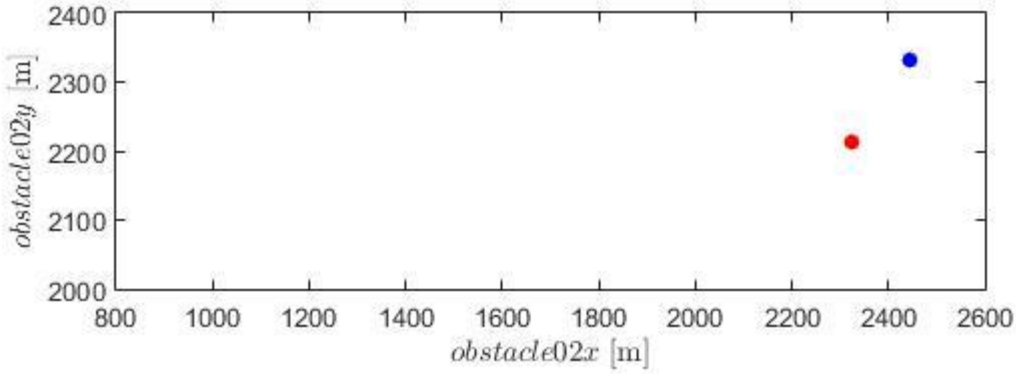
<u>Case 3</u>

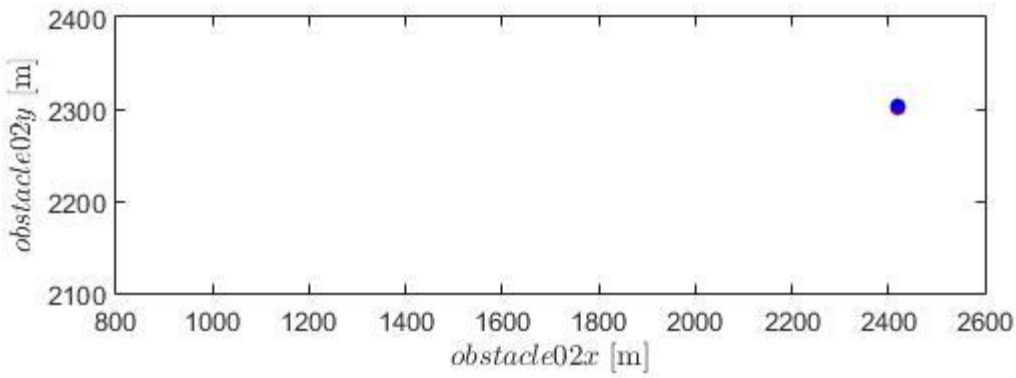
In this case the RS lies outside the FVR thereby the maximum closing velocity component, v^{relmax} is outside the FVR so the desired velocity $v_R(ti + \Delta t)$ for the next time interval is set to v^{relmax} showed by blue triangle as in the above figure

Overall Trajectory-Planning Algorithm









1. Receive the instantaneous state of the Target and the robot.

2. Determine the feasible robot acceleration region and using the dynamic model of the robot and the characteristic equation of its actuators.

3. Construct the set of feasible robot velocity commands by applying Equation (1) on the feasible acceleration region obtained in Step 2.

4. Calculate the maximum allowable closing velocity using Equation (9).

5. Construct the Rendezvous Set (RS) for the current state of the robot and the moving target, using the value of maximum allowable closing velocity.

6. Determine the robot velocity command to be executed by the robot based on the intersection of RS and the set of feasible velocity commands obtained in Step 3.

The above sequence should be executed every time new information is received from the object-motion-prediction module until both the position- and velocity matching errors are reduced to within their specified interception tolerances, respectively.

REFERENCE

Faraz Kunwar and Beno Benhabib “Rendezvous-Guidance Trajectory Planning for Robotic Dynamic Obstacle Avoidance and Interception”

F. Agah,M.Mehrandezh, R. G. Fenton, and B. Benhabib, “On-line robotic interception planning using rendezvous-guidance technique,” *J. Intell. Robot. Syst.: Theory Appl.*, vol. 40, no. 1, pp. 23–44, May 2004.

P. Fiorini and Z. Shiller, “Motion planning in dynamic environments using velocity obstacles,” *Int. J. Robot. Res.*, vol. 17, no. 7, pp. 711–727, 1998.

P. J. Yuan and S. C. Hsu, “*Rendezvous guidance with proportional navigation*,” J. Guid., Control, Dyn., vol. 17, no. 2, pp. 409–411, 1993.

Antonelli, G., Chiaverini, S., and Fusco, G.: Real-time end-effector path following for robot manipulators subject to velocity, acceleration, and jerk joint limits, in: *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Como, Italy, 2001, pp. 452–457.