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CSCI/MATH 3180

Lab Assignment #2

Description:

This program output estimates the derivatives of 4 relatively simple Ordinary Differential Equations (ODEs), using Euler’s, Heun’s, and Runge-Kutta’s Taylor expansions, with the support of an analytical solution, the program can calculate an absolute error, relative error, and can attempt to quantize Loss of Significant Figures (LOS) in terms of it’s Catastrophic Cancellation (CC).

Instructions:

First, the user picks a choice of one of the four ODEs (a,b,c,d (lowercase))

Next, the user manually enters the analytical solution.

The next input is the initial value for x, in the form x(t0).

After that, the next two options are [a,b]: I decided to have b the first argument and a the second.

Lastly, you can choose the step size (sub-intervals), N.

**A**

\*\*\* CHOICE A: N=4 \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

a

For the ODE, a, what is the Analytical Solution? (be as accurate as you can!) 3

Next, choose an initial value for x(t0): 1

Next, choose the interval starting with b, in [a,b]: 2

Next, choose the beginning of the interval with a, in [a,b]: 0

Finally, choose the number of subintervals, N: 4

Euler's:

0 1

0.5 1.5

1 2

1.5 2.5

2 3

Absolute Error : 0

Relative Error : 0

Catastrophic Cancellation : 0

Heun's:

0 1

0.5 1.5

1 2

1.5 2.5

2 3

Absolute Error : 0

Relative Error : 0

Catastrophic Cancellation : 0

Runge-Kutta's:

0 1

0.5 1.5

1 2

1.5 2.5

2 3

Absolute Error : 0

Relative Error : 0

Catastrophic Cancellation : 0

\*\*\* CHOICE A: N=100 FIRST 3 AND LAST \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

a

For the ODE, a, what is the Analytical Solution? (be as accurate as you can!) 3

Next, choose an initial value for x(t0): 1

Next, choose the interval starting with b, in [a,b]: 2

Next, choose the beginning of the interval with a, in [a,b]: 0

Finally, choose the number of subintervals, N: 100

Euler's:

0 1

0.02 1.02

0.04 1.04

0.06 1.06

2 3

Absolute Error : 1.77636e-15

Relative Error : 5.92119e-16

Catastrophic Cancellation : 1.18424e-15

Heun's:

0 1

0.02 1.02

0.04 1.04

0.06 1.06

2 3

Absolute Error : 1.77636e-15

Relative Error : 5.92119e-16

Catastrophic Cancellation : 1.18424e-15

Runge-Kutta's:

0 1

0.02 1.02

0.04 1.04

0.06 1.06

2 3

Absolute Error : 1.77636e-15

Relative Error : 5.92119e-16

Catastrophic Cancellation : 1.18424e-15

**B**

\*\*\* CHOICE B: N=4 \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

b

For the ODE, b, what is the Analytical Solution? (be as accurate as you can!) 11.77811220

Next, choose an initial value for x(t0): 1

Next, choose the interval starting with b, in [a,b]: 2

Next, choose the beginning of the interval with a, in [a,b]: 0

Finally, choose the number of subintervals, N: 4

Euler's:

0 1

0.5 1.5

1 2.5

1.5 4.25

2 7.125

Absolute Error : 4.65311

Relative Error : 0.395064

Catastrophic Cancellation : 4.25805

Heun's:

0 1

0.5 1.75

1 3.28125

1.5 6.08203

2 10.9458

Absolute Error : 0.832311

Relative Error : 0.0706659

Catastrophic Cancellation : 0.761645

Runge-Kutta's:

0 1

0.5 1.79688

1 3.43469

1.5 6.45875

2 11.7679

Absolute Error : 0.0101716

Relative Error : 0.000863598

Catastrophic Cancellation : 0.00930795

\*\*\* CHOICE B: N=100 FIRST 3 AND LAST \*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

b

For the ODE, b, what is the Analytical Solution? (be as accurate as you can!) 11.77811220

Next, choose an initial value for x(t0): 1

Next, choose the interval starting with b, in [a,b]: 2

Next, choose the beginning of the interval with a, in [a,b]: 0

Finally, choose the number of subintervals, N: 100

Euler's:

0 1

0.02 1.02

0.04 1.0408

0.06 1.06242

2 11.4893

Absolute Error : 0.28882

Relative Error : 0.0245218

Catastrophic Cancellation : 0.264298

Heun's:

0 1

0.02 1.0204

0.04 1.04162

0.06 1.06366

2 11.7762

Absolute Error : 0.00194097

Relative Error : 0.000164795

Catastrophic Cancellation : 0.00177618

Runge-Kutta's:

0 1

0.02 1.0204

0.04 1.04162

0.06 1.06367

2 11.7781

Absolute Error : 4.08958e-08

Relative Error : 3.47219e-09

Catastrophic Cancellation : 3.74236e-08

**C**

\*\*\* CHOICE C: N=4 \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

c

For the ODE, c, what is the Analytical Solution? (be as accurate as you can!) 2.270670566

Next, choose an initial value for x(t0): 2

Next, choose the interval starting with b, in [a,b]: 3

Next, choose the beginning of the interval with a, in [a,b]: 1

Finally, choose the number of subintervals, N: 4

Euler's:

1 2

1.5 1.5

2 1.5

2.5 1.75

3 2.125

Absolute Error : 0.145671

Relative Error : 0.0641531

Catastrophic Cancellation : 0.0815175

Heun's:

1 2

1.5 1.75

2 1.78125

2.5 1.98828

3 2.30518

Absolute Error : 0.0345052

Relative Error : 0.015196

Catastrophic Cancellation : 0.0193092

Runge-Kutta's:

1 2

1.5 1.71354

2 1.73634

2.5 1.94679

3 2.2711

Absolute Error : 0.000428975

Relative Error : 0.00018892

Catastrophic Cancellation : 0.000240055

\*\*\* CHOICE C: N=100 FIRST 3 AND LAST \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

c

For the ODE, c, what is the Analytical Solution? (be as accurate as you can!) 2.270670566

Next, choose an initial value for x(t0): 2

Next, choose the interval starting with b, in [a,b]: 3

Next, choose the beginning of the interval with a, in [a,b]: 1

Finally, choose the number of subintervals, N: 100

Euler's:

1 2

1.02 1.98

1.04 1.9608

1.06 1.94238

3 2.26524

Absolute Error : 0.00543145

Relative Error : 0.002392

Catastrophic Cancellation : 0.00303945

Heun's:

1 2

1.02 1.9804

1.04 1.96158

1.06 1.94354

3 2.27071

Absolute Error : 3.6638e-05

Relative Error : 1.61353e-05

Catastrophic Cancellation : 2.05027e-05

Runge-Kutta's:

1 2

1.02 1.9804

1.04 1.96158

1.06 1.94353

3 2.27067

Absolute Error : 1.20715e-09

Relative Error : 5.31626e-10

Catastrophic Cancellation : 6.75522e-10

**D**

\*\*\* CHOICE D: N=4 \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

d

For the ODE, d, what is the Analytical Solution? (be as accurate as you can!) 1.744562647

Next, choose an initial value for x(t0): 1

Next, choose the interval starting with b, in [a,b]: 4

Next, choose the beginning of the interval with a, in [a,b]: 2

Finally, choose the number of subintervals, N: 4

Euler's:

2 1

2.5 1.16667

3 1.34848

3.5 1.53838

4 1.73305

Absolute Error : 0.0115146

Relative Error : 0.0066003

Catastrophic Cancellation : 0.00491434

Heun's:

2 1

2.5 1.17424

3 1.35891

3.5 1.54976

4 1.74457

Absolute Error : 7.41443e-06

Relative Error : 4.25002e-06

Catastrophic Cancellation : 3.16441e-06

Runge-Kutta's:

2 1

2.5 1.17423

3 1.3589

3.5 1.54975

4 1.74456

Absolute Error : 2.52685e-07

Relative Error : 1.44842e-07

Catastrophic Cancellation : 1.07844e-07

\*\*\* CHOICE D: N=100 FIRST 3 AND LAST \*\*\*\*

Please choose an Ordinary Differential Equation:

a: x'= x / (1+t) Analytical Solution: x= t+1

b: x'= t+x Analytical Solution: x= -t+2e^t-1

c: x'= t-x Analytical Solution: x= t+2e^(1-t)-1

d: x'= (t-x) / (t+x) Analytical Solution: x= sqrt(2t^2+1)-t

d

For the ODE, d, what is the Analytical Solution? (be as accurate as you can!) 1.744562647

Next, choose an initial value for x(t0): 1

Next, choose the interval starting with b, in [a,b]: 4

Next, choose the beginning of the interval with a, in [a,b]: 2

Finally, choose the number of subintervals, N: 100

Euler's:

2 1

2.02 1.00667

2.04 1.01336

2.06 1.02009

4 1.74415

Absolute Error : 0.000407818

Relative Error : 0.000233765

Catastrophic Cancellation : 0.000174053

Heun's:

2 1

2.02 1.00668

2.04 1.01339

2.06 1.02013

4 1.74456

Absolute Error : 5.23279e-11

Relative Error : 2.99949e-11

Catastrophic Cancellation : 2.23331e-11

Runge-Kutta's:

2 1

2.02 1.00668

2.04 1.01339

2.06 1.02013

4 1.74456

Absolute Error : 4.61445e-10

Relative Error : 2.64505e-10

Catastrophic Cancellation : 1.9694e-10

Conclusions / Findings:

a.) dx = x/(1+t) N=4: No loss of significance at all.

N=100: (1.18424e-15 CC) for all of the methods, which is very small.

Loss of significance was in the 1 \* 10^15 range!

b.) dx = t+x N=4: 4.25805 > 0.761645 > 0.00930795 Runge-Kutta’s accuracy can be seen in this one.

N=100: WAY more accurate overall with more steps.

c.) dx = t-x The accuracy on this one was actually better than b, but acted almost the same in terms of accuracy and loss of significance.

d.) dx = (t-x) / (t+x) This one was interseting. Heun’s beat Runge-Kutta in N=100.

In terms of Catastrophic Cancellation, the winners of the Runge-Kutta’s N=100 relay race:

A 1.18424e-15 The most accurate/least LOS

D 1.9694e-10

C 6.75522e-10

B 3.74236e-08 The least accurate/Most LOS