Adam Mischke

CSCI/MATH 3180 – Numerical Analysis – Lab Assignment 5 – Root Finding

#1 Program Output:

1. Enter the input file name: data.txt
2. f(x) = 1.000000000x^6 + -1.000000000x^1 + -1.000000000
3. f'(x) = 6.000000000x^5 + -1.000000000
4. -----------------------------------------------------------
5. Newton's Method
6. -----------------------------------------------------------
7. Iteration(i) xi f(xi) f'(x)
8. 0 1.000000000 -1.000000000 5.000000000
9. 1 1.200000000 0.785984000 13.929920000
10. 2 1.143575843 0.093031957 10.734811370
11. 3 1.134909462 0.001907397 10.296849293
12. Time taken to complete (in seconds): 0.005000000
13. Secant Method
14. --------------------------------------------------------------------------
15. Iteration(i) xi xi+1 f(xi) f(xi+1)
16. 0 1.000000000 3.000000000 -1.000000000 725.000000000
17. 1 3.000000000 1.002754821 725.000000000 -0.986111641
18. 2 1.002754821 1.005467692 -0.986111641 -0.972209822
19. 3 1.005467692 1.195189630 -0.972209822 0.719691831
20. 4 1.195189630 1.114486754 0.719691831 -0.198248670
21. 5 1.114486754 1.131916247 -0.198248670 -0.028691114
22. 6 1.131916247 1.134865521 -0.028691114 0.001454987
23. 7 1.134865521 1.134723176 0.001454987 -0.000009905
24. Time taken to complete (in seconds): 0.014000000
25. Bisection Method
26. -------------------------------------------------------------------------
27. Iteration(i) a b f(a) f(b)
28. 0 1.000000000 3.000000000 -1.000000000 725.000000000
29. 1 1.000000000 2.000000000 -1.000000000 61.000000000
30. 2 1.000000000 1.500000000 -1.000000000 8.890625000
31. 3 1.000000000 1.250000000 -1.000000000 1.564697266
32. 4 1.125000000 1.250000000 -0.097713470 1.564697266
33. 5 1.125000000 1.187500000 -0.097713470 0.616653025
34. 6 1.125000000 1.156250000 -0.097713470 0.233268925
35. 7 1.125000000 1.140625000 -0.097713470 0.061577832
36. 8 1.132812500 1.140625000 -0.019575551 0.061577832
37. 9 1.132812500 1.136718750 -0.019575551 0.020618995
38. 10 1.132812500 1.134765625 -0.019575551 0.000426842
39. 11 1.133789063 1.134765625 -0.009597993 0.000426842
40. 12 1.134277344 1.134765625 -0.004591496 0.000426842
41. 13 1.134521484 1.134765625 -0.002083808 0.000426842
42. 14 1.134643555 1.134765625 -0.000828854 0.000426842
43. 15 1.134704590 1.134765625 -0.000201099 0.000426842
44. Time taken to complete (in seconds): 0.035000000
45. Please select two more intervals for the domain of x, [x0,x1] -> enter like x0 x1: 1.1346 1.1348
46. Bisection Method
47. -------------------------------------------------------------------------
48. Iteration(i) a b f(a) f(b)
49. 0 1.134600000 1.134800000 -0.001276707 0.000780579
50. 1 1.134700000 1.134800000 -0.000248312 0.000780579
51. 2 1.134700000 1.134750000 -0.000248312 0.000266071
52. 3 1.134700000 1.134725000 -0.000248312 0.000008864
53. 4 1.134712500 1.134725000 -0.000119728 0.000008864
54. 5 1.134718750 1.134725000 -0.000055433 0.000008864
55. 6 1.134721875 1.134725000 -0.000023285 0.000008864
56. 7 1.134723437 1.134725000 -0.000007211 0.000008864
57. Time taken to complete (in seconds): 0.014000000
58. f(x) = 1.000000000x^5 + -9.000000000x^4 + -1.000000000x^3 + 17.000000000x^2 + -8.000000000x^1 + -8.000000000
59. f'(x) = 5.000000000x^4 + -36.000000000x^3 + -3.000000000x^2 + 34.000000000x^1 + -8.000000000
60. -----------------------------------------------------------
61. Newton's Method
62. -----------------------------------------------------------
63. Iteration(i) xi f(xi) f'(x)
64. 0 0.000000000 -8.000000000 -8.000000000
65. 1 -1.000000000 8.000000000 -4.000000000
66. 2 1.000000000 -8.000000000 -8.000000000
67. 3 0.000000000 -8.000000000 -8.000000000
68. 4 -1.000000000 8.000000000 -4.000000000
69. 5 1.000000000 -8.000000000 -8.000000000
70. 6 0.000000000 -8.000000000 -8.000000000
71. 7 -1.000000000 8.000000000 -4.000000000
72. 8 1.000000000 -8.000000000 -8.000000000
73. 9 0.000000000 -8.000000000 -8.000000000
74. 10 -1.000000000 8.000000000 -4.000000000
75. 11 1.000000000 -8.000000000 -8.000000000
76. 12 0.000000000 -8.000000000 -8.000000000
77. 13 -1.000000000 8.000000000 -4.000000000
78. 14 1.000000000 -8.000000000 -8.000000000
79. 15 0.000000000 -8.000000000 -8.000000000
80. Time taken to complete (in seconds): 0.025000000
81. Secant Method
82. --------------------------------------------------------------------------
83. Iteration(i) xi xi+1 f(xi) f(xi+1)
84. 0 0.000000000 2.000000000 -8.000000000 -76.000000000
85. 1 2.000000000 -0.235294118 -76.000000000 -5.191751000
86. 2 -0.235294118 -0.399188729 -5.191751000 -2.272573968
87. 3 -0.399188729 -0.526780384 -2.272573968 0.344273291
88. 4 -0.526780384 -0.509994384 0.344273291 -0.009137594
89. 5 -0.509994384 -0.510428393 -0.009137594 -0.000019952
90. Time taken to complete (in seconds): 0.011000000
91. Bisection Method
92. -------------------------------------------------------------------------
93. Iteration(i) a b f(a) f(b)
94. Error: Evaluation of x0 and x1 must have opposite signs.
95. Please select two more intervals for the domain of x, [x0,x1] -> enter like x0 x1: -.5103 -.5105
96. 0 -0.510300000 -0.510500000 -0.002717350 0.001484474
97. 1 -0.510400000 -0.510500000 -0.000616470 0.001484474
98. 2 -0.510400000 -0.510450000 -0.000616470 0.000433994
99. 3 -0.510425000 -0.510450000 -0.000091240 0.000433994
100. 4 -0.510425000 -0.510437500 -0.000091240 0.000171377
101. 5 -0.510425000 -0.510431250 -0.000091240 0.000040069
102. 6 -0.510428125 -0.510431250 -0.000025586 0.000040069
103. 7 -0.510428125 -0.510429687 -0.000025586 0.000007242
104. 8 -0.510428906 -0.510429687 -0.000009172 0.000007242
105. Time taken to complete (in seconds): 0.016000001
106. f(x) = 1.000000000x^3 + -3.000000000x^1 + 2.000000000
107. f'(x) = 3.000000000x^2 + -3.000000000
108. -----------------------------------------------------------
109. Newton's Method
110. -----------------------------------------------------------
111. Iteration(i) xi f(xi) f'(x)
112. 0 3.000000000 20.000000000 24.000000000
113. 1 2.166666667 5.671296296 11.083333333
114. 2 1.654970760 1.567933833 5.216784652
115. 3 1.354415162 0.421348447 2.503321291
116. 4 1.186099393 0.110344130 1.220495312
117. 5 1.095690091 0.028345976 0.601610325
118. 6 1.048573254 0.007192685 0.298517605
119. 7 1.024478578 0.001812270 0.148669073
120. 8 1.012288619 0.000454886 0.074184743
121. 9 1.006156817 0.000113953 0.037054620
122. 10 1.003081558 0.000028517 0.018517833
123. 11 1.001541569 0.000007133 0.009256543
124. 12 1.000770982 0.000001784 0.004627677
125. Time taken to complete (in seconds): 0.020000000
126. Secant Method
127. --------------------------------------------------------------------------
128. Iteration(i) xi xi+1 f(xi) f(xi+1)
129. 0 3.000000000 5.000000000 20.000000000 112.000000000
130. 1 5.000000000 2.565217391 112.000000000 11.184351114
131. 2 2.565217391 2.295105917 11.184351114 7.204178300
132. 3 2.295105917 1.806199702 7.204178300 2.473869792
133. 4 1.806199702 1.550510181 2.473869792 1.076022798
134. 5 1.550510181 1.353687672 1.076022798 0.419529455
135. 6 1.353687672 1.227909031 0.419529455 0.167665750
136. 7 1.227909031 1.144178149 0.167665750 0.065359096
137. 8 1.144178149 1.090686274 0.065359096 0.025417805
138. 9 1.090686274 1.056645160 0.025417805 0.009807778
139. 10 1.056645160 1.035257131 0.009807778 0.003773023
140. 11 1.035257131 1.021885004 0.003773023 0.001447342
141. 12 1.021885004 1.013563121 0.001447342 0.000554370
142. 13 1.013563121 1.008396779 0.000554370 0.000212110
143. 14 1.008396779 1.005195029 0.000212110 0.000081105
144. 15 1.005195029 1.003212819 0.000081105 0.000031000
145. Time taken to complete (in seconds): 0.028000001
146. Bisection Method
147. -------------------------------------------------------------------------
148. Iteration(i) a b f(a) f(b)
149. Error: Evaluation of x0 and x1 must have opposite signs
150. Please select two more intervals for the domain of x, [x0,x1] -> enter like x0 x1: -2.0001 -1.9800
151. Bisection Method
152. -------------------------------------------------------------------------
153. Iteration(i) a b f(a) f(b)
154. 0 -2.000100000 -1.980000000 -0.000900060 0.177608000
155. 1 -2.000100000 -1.990050000 -0.000900060 0.088956970
156. 2 -2.000100000 -1.995075000 -0.000900060 0.044179586
157. 3 -2.000100000 -1.997587500 -0.000900060 0.021677593
158. 4 -2.000100000 -1.998843750 -0.000900060 0.010398230
159. 5 -2.000100000 -1.999471875 -0.000900060 0.004751452
160. 6 -2.000100000 -1.999785938 -0.000900060 0.001926288
161. 7 -2.000100000 -1.999942969 -0.000900060 0.000513262
162. 8 -2.000021484 -1.999942969 -0.000193362 0.000513262
163. 9 -2.000021484 -1.999982227 -0.000193362 0.000159959
164. 10 -2.000001855 -1.999982227 -0.000016699 0.000159959
165. 11 -2.000001855 -1.999992041 -0.000016699 0.000071630
166. 12 -2.000001855 -1.999996948 -0.000016699 0.000027466
167. 13 -2.000001855 -1.999999402 -0.000016699 0.000005383
168. 14 -2.000000629 -1.999999402 -0.000005658 0.000005383
169. Time taken to complete (in seconds): 0.029999999
170. f(x) = 1.000000000x^3 + -1.000000000x^1 + 1.000000000
171. f'(x) = 3.000000000x^2 + -1.000000000
172. -----------------------------------------------------------
173. Newton's Method
174. -----------------------------------------------------------
175. Iteration(i) xi f(xi) f'(x)
176. 0 1.000000000 1.000000000 2.000000000
177. 1 0.500000000 0.625000000 -0.250000000
178. 2 3.000000000 25.000000000 26.000000000
179. 3 2.038461538 7.432009558 11.465976331
180. 4 1.390282147 2.296972595 4.798653347
181. 5 0.911611898 0.845970637 1.493108756
182. 6 0.345028497 0.696045305 -0.642866009
183. 7 1.427750704 2.482679233 5.115416219
184. 8 0.942417913 0.894591991 1.664454565
185. 9 0.404949357 0.661455851 -0.508048054
186. 10 1.706904645 4.266202096 7.740570403
187. 11 1.155756361 1.388071510 3.007318298
188. 12 0.694191813 0.640340800 0.445706821
189. 13 -0.742494299 1.333158837 0.653893351
190. 14 -2.781295941 -17.733716712 22.206821329
191. 15 -1.982725247 -4.811763065 10.793598216
192. Time taken to complete (in seconds): 0.037000000
193. Secant Method
194. --------------------------------------------------------------------------
195. Iteration(i) xi xi+1 f(xi) f(xi+1)
196. 0 1.000000000 3.000000000 1.000000000 25.000000000
197. 1 3.000000000 0.916666667 25.000000000 0.853587963
198. 2 0.916666667 0.843019772 0.853587963 0.756099489
199. 3 0.843019772 0.271830410 0.756099489 0.748255621
200. 4 0.271830410 -54.216036625 0.748255621-159306.243280455
201. 5 -54.216036625 0.271574483-159306.243280455 0.748454868
202. 6 0.271574483 0.271318490 0.748454868 0.748654274
203. 7 0.271318490 1.232424968 0.748654274 1.639469949
204. 8 1.232424968 -0.536409184 1.639469949 1.382065588
205. 9 -0.536409184 -10.033702869 1.382065588 -999.111272745
206. 10 -10.033702869 -0.549528594 -999.111272745 1.383581028
207. 11 -0.549528594 -0.562644228 1.383581028 1.384528773
208. 12 -0.562644228 18.597544862 1.384528773 6414.710653036
209. 13 18.597544862 -0.566780589 6414.710653036 1.384707859
210. 14 -0.566780589 -0.570918378 1.384707859 1.384828792
211. 15 -0.570918378 46.811700154 1.384828792102534.317959338
212. Time taken to complete (in seconds): 0.039999999
213. Bisection Method
214. -------------------------------------------------------------------------
215. Iteration(i) a b f(a) f(b)
216. Error: Evaluation of x0 and x1 must have opposite signs.
217. Please select two more intervals for the domain of x, [x0,x1] -> enter like x0 x1: -1.3246 -1.3248
218. 0 -1.324600000 -1.324800000 0.000502989 -0.000349909
219. 1 -1.324700000 -1.324800000 0.000076580 -0.000349909
220. 2 -1.324700000 -1.324750000 0.000076580 -0.000136655
221. 3 -1.324700000 -1.324725000 0.000076580 -0.000030035
222. 4 -1.324712500 -1.324725000 0.000023273 -0.000030035
223. 5 -1.324712500 -1.324718750 0.000023273 -0.000003381
224. 6 -1.324715625 -1.324718750 0.000009946 -0.000003381
225. 7 -1.324717188 -1.324718750 0.000003283 -0.000003381
226. Time taken to complete (in seconds): 0.014000000
227. f(x) = -6.000000000x^5 + 15.000000000x^4 + 10.000000000x^3 + -30.000000000x^2 + 5.000000000
228. f'(x) = -30.000000000x^4 + 60.000000000x^3 + 30.000000000x^2 + -60.000000000x^1 + 0.000000000
229. -----------------------------------------------------------
230. Newton's Method
231. -----------------------------------------------------------
232. Iteration(i) xi f(xi) f'(x)
233. 0 1.000000000 -6.000000000 0.000000000
234. 1 inf -nan(ind) -nan(ind)
235. 2 -nan(ind) -nan(ind) -nan(ind)
236. 3 -nan(ind) -nan(ind) -nan(ind)
237. 4 -nan(ind) -nan(ind) -nan(ind)
238. 5 -nan(ind) -nan(ind) -nan(ind)
239. 6 -nan(ind) -nan(ind) -nan(ind)
240. 7 -nan(ind) -nan(ind) -nan(ind)
241. 8 -nan(ind) -nan(ind) -nan(ind)
242. 9 -nan(ind) -nan(ind) -nan(ind)
243. 10 -nan(ind) -nan(ind) -nan(ind)
244. 11 -nan(ind) -nan(ind) -nan(ind)
245. 12 -nan(ind) -nan(ind) -nan(ind)
246. 13 -nan(ind) -nan(ind) -nan(ind)
247. 14 -nan(ind) -nan(ind) -nan(ind)
248. 15 -nan(ind) -nan(ind) -nan(ind)
249. Time taken to complete (in seconds): 0.021000000
250. Secant Method
251. --------------------------------------------------------------------------
252. Iteration(i) xi xi+1 f(xi) f(xi+1)
253. 0 1.000000000 3.000000000 -6.000000000 -238.000000000
254. 1 3.000000000 0.948275862 -238.000000000 -5.921227371
255. 2 0.948275862 0.895928441 -5.921227371 -5.688031568
256. 3 0.895928441 -0.380911780 -5.688031568 0.458404445
257. 4 -0.380911780 -0.285684362 0.458404445 2.429704795
258. 5 -0.285684362 -0.403055880 2.429704795 -0.068709059
259. 6 -0.403055880 -0.399828038 -0.068709059 0.009599410
260. 7 -0.399828038 -0.400223721 0.009599410 0.000026544
261. Time taken to complete (in seconds): 0.013000000
262. Bisection Method
263. -------------------------------------------------------------------------
264. Iteration(i) a b f(a) f(b)
265. Error: Evaluation of x0 and x1 must have opposite signs.
266. Please select two more intervals for the domain of x, [x0,x1] -> enter like x0 x1: .47032 .47034
267. 0 0.470320000 0.470340000 0.000196509 -0.000139671
268. 1 0.470330000 0.470340000 0.000028419 -0.000139671
269. 2 0.470330000 0.470335000 0.000028419 -0.000055626
270. 3 0.470330000 0.470332500 0.000028419 -0.000013604
271. 4 0.470331250 0.470332500 0.000007408 -0.000013604
272. 5 0.470331250 0.470331875 0.000007408 -0.000003098
273. 6 0.470331563 0.470331875 0.000002155 -0.000003098
274. Time taken to complete (in seconds): 0.012000000
275. Press any key to continue . . .

#2 Does Maple match this? Yes, mostly, but there are some rounding differences I’m guessing that are in Maple’s algorithm (looked like sometimes they used ceiling(). Some loss of significance could be attributed to this.

#3 Find the absolute error in the x values.

|  |  |
| --- | --- |
| EQ 1 |  |
| EQ 2 |  |
| EQ 3 |  |
| EQ 4 |  |
| EQ 5 |  |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Newton | Secant | Bisection | Numeric | Newton | Secant | Bisection |
| EQ 1 | 1.134909462 | 1.134723176 | 1.1347242185 | 1.134724138 | 0.000185324 | 0.000000962 | 0.0000000805 |
| EQ 2 | Diverge | -0.510428393 | -0.510428906 | -0.5104293428 | - | 0.0000009498 | 0.0000004368 |
| EQ 3 | 1.000770982 | 1.003212819 | -2.0000000155 | 1 & -2 | 0.000770982 | 0.003212819 | 0.0000000155 |
| EQ 4 | Diverge | Diverge | -1.324717969 | -1.324717957 | - | - | 0.000000012 |
| EQ 5 | Divide by 0 | -0.400223721 | 0.470331719 | -.4002248181 & .4703316907 | - | 0.0000010971 | 0.0000000283 |

Some loss of significance could be attributed to this.

#4 Pros and Cons

Iterations: When Newton's converges, it takes the least amount of iterations. When Secant converges, it takes a couple more iterations than Newtons. Bisection takes way more iterations than either of them, especially when the error needs to be < .000001.

Domain Picking: I would say pretty much all of these methods have some disadvantage on this. Newton’s and Secant’s can diverge, Bisection cannot. But if you happen to pick the wrong domain (one without a positive and negative y or even if you pick a root!) then you must re-pick.

Speed: Newtons is the fastest. Even though it evaluates two functions instead of one, it generally has less iterations than Secant, making it the winner generally. Bisection is generally the slowest to converge, but it does get there 100% of the time unless the domain is wrong.

Problems: Newtons and Secant will always have the problem of never converging. Newtons and Secant will also always have a chance to divide by 0 (very bad! But can be handled if programmed defensively) Causing the root finding to fail. Bisection, especially for the requirements of this lab (such low iterations and such high error accuracy) is useless here, you basically must reenter until you're 3 or 4 digits close considering bisection's accuracy takes some 3 or 4 iterations to get one tenths more accuracy.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Newton | Secant | Bisection |
| Convergence | Quadratic (2) | Golden ratio (~1.618) | Linear (1) |
| Can diverge? | Yes, 2/5 times in this case. | Yes, 1/5 times in this case. | No |
| Speed | Fastest | Fast | Slow |
| Divide by 0? | Yes, 1/5 times in this case. | Yes, it could happen. | No |