ERASMUS UNIVERSITY ROTTERDAM
MSC SUPPLY CHAIN MANAGEMENT

AIRFARE LOCK-IN PRODUCTS

Option Valuation in the Aviation Industry

Rotterdam, August 13, 2013

Author
P.N. van Kempen
322165

Supervisor
dr. ig. N.A.H. Agatz

Advisor
dr. N. Mishra

Executive summary

Airlines have to operate on low profit margins while dealing with extreme risks and uncertain demand. To cope with these challenges, the aviation industry has implemented revenue management models. The models allowed companies to better segment their customers and make more accurate estimates of a passenger's maximum willingness to pay. While this has led to large increases in revenues for the airliners, customers have to deal with much higher levels of uncertainty. Airfares might jump hundreds of dollars overnight, and insuring yourself for possible cancellation is many times more expensive than the cheapest airfare.

In this master's thesis, I want to study the concept of *airfare lock-in products* and test whether it is viable for an external company without capacity information to sell this new type of goods. Airfare lock-in products allow the customer to fixate the price of a seat for a predetermined period at a certain premium. The customer can thus decide at a later moment in time whether to actually buy the ticket or not, while covering himself from price increases. Furthermore, when an airfare drops a large amount, the customer can still choose to not execute the product and buy the ticket at the lower rate. An airfare lock-in product therefore also allows the customer to gain from price decreases. This concept is thus comparable to call options in financial markets.

I will research the viability of offering such options by training and testing historical data. Data on 22 routes is collected for flights departing in a period from August 19^{th} till September 30^{th}. Each flight's fare is checked 4 times per day for a period of 6 weeks before departure. The first 4 weeks of the census (i.e., August 19^{th} – September 16^{th}) will be used to train two option valuation models. The first method will base its predictions using the Black–Scholes method, while the other approximates the best option price using Monte Carlo. After the creation of the models a simulation will be run based upon the last two weeks of the historical data. This simulation tests the profitability of the two models and compares them with the most optimal situation. It will analyse whether it

is theoretically and practically possible for an external company that sells the options to sustain. Furthermore, the simulation will also conclude which of the option valuation models approximates the optimal option price best.

After the simulation a sensitivity analysis will be performed as well. In this analysis the three parameters of a passenger — *(i)* risk-utility, *(ii)* forecasting technique, and *(iii)* likelihood of travelling — will be altered to see what the effect of these properties is on the profitability of the company that offers airfare lock-in products.

Contents

List of Figures

List of Tables

1

Introduction

The fastest way to become a millionaire is to start off with a billion dollars and invest in an airline

It might not come as a surprise that companies in the aviation sector have to operate on profit margins close to none[1], and that they were struck hard by the financial crisis. To cope with these challenges, airliners are known to implement many revue management concepts (e.g. market segmentation, protection levels). Robert Crandall, former CEO of American Airlines, even said stated that yield management is "*the single most important technical development in transportation management since we entered deregulation*" (Cross, 1997, p. 30).

The implementation of such revenue management models has caused major changes in the aviation sector. For instance, the application of booking protection levels has created many fluctuations in airfares within a single flight. Early bookers usually can purchase a ticket at a low rate while passengers later in the booking process can only acquire the same seat at a much higher fare. Early booking of a flight, however, brings along much more uncertainty, as there might occur many situations in which the buyer is unable to utilize the ticket. For example, when a passenger wants to go skiing, but breaks his leg prior to departure. A person can cover himself from any losses caused by these kind of situations by buying insurance. However, insurance does not always provide a solution for no-shows. By way of illustration, consider you

[1] `http://on.wsj.com/MftvqH`

would like to go on holiday with a group of your friends, but your friends are undecided over whether they can go on the trip or not. Now you have two options to choose from: *(i)* buy the flight now and risk not using this ticket because your friends can't come along, or *(ii)* wait to buy the flight until you know for sure your friends will go and risk sudden price increases in airfare or even sell-outs. This extra level of uncertainty is undesired by customers, and makes it more difficult to make rational decisions.

A recent trend in the aviation industry might provide a solution to this problem of uncertainty. Airliners offering *airfare lock-in products* allow their customers to cover themselves for certain risks. An airfare lock-in product allows a customer who is unsure whether he wants to directly buy a ticket to buy an *option* on that flight instead. The option gives him the opportunity to buy the actual ticket at a later time at a predetermined price. Some airliners (e.g., United Airlines, Air France-KLM, Estonian Air) already offer this new type of service and allow the customer to reserve a seat for 2 to 14 days. New external companies like OptionIt, OptionsAway, SteadyFare, also arise, though all still in their initial stage[2].

The offering of lock-in products might be viable to passengers and external companies that write these options, as well as the actual airlines. However, research on this topic is scarce and results of successful applications of the strategy are not yet public. Further theoretical and empirical research might provide answers to the actual viability, and could offer support for companies that would like to implement this type of product.

In this thesis I would like to contribute to this topic and study whether external companies can offer these kind of airfare lock-in products in a sustainable way. Furthermore, I want to test different practical option valuation models, and see which model performs best in a realistic setting. This research

[2]OptionIt currently offers only options for sporting events, but plans to offer options for tickets of major airlines. OptionsAway just made their service available to the public for flights departing from Chicago, and SteadyFare is not available for public use

could therefore be useful to companies that write such options, as well the airline carrier of the underlying flights. On a more general perspective, concepts studied in this thesis might also be applicable to other organizations that have to deal with perishable services. This paper could offer insights to these kinds of industries as well.

2

Literature Review

2.1 Yield management in the aviation industry

According to Anderson, Davison, and Rasmussen (2004, pp. 2) the term yield management can be described as "*[the] process of managing perishable inventories to maximize the total revenue from these inventories*". The aviation sector was the first industry to apply this concept of revenue management. The costs of the services airlines sells are mostly fixed (e.g., fuel, maintenance) and consist of negligible variable costs (e.g., meals) (O'Connor, 2001). In such a scenario, unsold seats become obsolete upon departure and would result in lost revenues to the company. A more airline-related definition of yield management is given by Akgunduz, Turkmen, and Bulgak (2007, pp. 136), who state that "*revenue management is [. . .] the practice of managing the booking requests with an objective of increasing the sales revenues*".

Smith, Leimkuhler, and Darrow (1992) defined three major changes in the aviation industry which contributed to the development of yield management. These changes, which all occurred in the 1960's and 1970's, had to do with:

Deregulation Arguably the most important factor in the creation of yield management in the aviation industry is its deregulation. The deregulation was formed by the approval of the Airline Deregulation Act (1978), and removed price, route and entry restrictions on airlines. After removal of these restrictions, airliners were allowed to set their own prices. The ability to change prices for different segments of customers is one of the core concepts of revenue management.

Automated reservation systems Computerized reservation systems allowed airlines to centrally control reservation activity, and process and store huge amounts of data. With the development and implementation of the Semi-Automated Business Research Environment (SABRE) in 1966, American Airlines pioneered in the field of online reservation systems for the travel industry (Voneche, 2005). The data acquired via the system could be analysed and used to forecast the booking process, and automatically adjust airfares of flights according to their demand.

Discount fares The idea behind the concept of discount fares is to offer reduced prices to specific segments of customers to sell seats which would otherwise be empty (Belobaba, 1987). This concept added a whole new dimension to the airline industry. For instance, by offering low prices to low-grossing customers early in the booking process, there would be fewer seats to sell to high-grossing travellers. The concept of determining the right amount of tickets with discount at specific times is called *Discount allocation*.

Discount allocation has its origin in the article by Littlewood (1972). In his paper, for which he would receive the INFORMS Revenue Management and Pricing Section Prize for Historical Works (2013) in 2004, Littlewood described a number of approaches to handle the allocation of discount fares in the airline industry. By predicting future demand and analysing consumer sell-up behaviour, one would be able to create a better allocation of price discounts which resulted in a higher load factor and revenues for the airlines. More specifically, Littlewood's rule states that a "*request for a seat should be fulfilled only if its revenue exceeds the expected future value of the seat in question*" (Wright, Groenevelt, & Shumsky, 2010, pp. 17).

For the concept of discount allocation, historical pricing and demand acquired by the implementation of computerized reservation systems proved to be very useful to create more accurate predictions (Littlewood, 1972).

In their paper, Smith et al. (1992) describe the case of revenue management at American Airlines. Of all the airliners, American Airlines was the first to actually implement the theory of yield management in the 1970s. After years of research in the 1960s, the company developed an application using the data acquired via their computerized reservation system, SABRE. The system, which was later named DINAMO, was based upon the concepts described by Littlewood (1972), and implemented the options of overbooking, discount allocation and traffic management [1]. The system decreased the spoilage of unsold seats from 15 percent to 3 percent (Smith et al., 1992). Furthermore, in 1992, the former CEO of American Airlines, Robert Crandall, showed that yield management contributed to an increase of $ 1.4 billion in revenues and an estimated $ 500 million of extra profits in the first three years after implementation of DINAMO.

While the introduction of discount fares increased utilization of aircrafts and significant expanded revenues of airliners, it also resulted in higher uncertainty for consumers. Where the customers first could base their decisions on linear prices, airfares have become very dynamic and follow unintuitive paths. As an example, Groves and Gini (2013) state in their paper that, in contrast to popular believe, the principle to buy tickets early in the booking process to avoid an increase in price is suboptimal. Their research shows that this strategy almost always results in higher prices and could be easily outperformed by other models (see for a complete description of their article).

To overcome the problem of uncertainty created by yield management, new theories of pricing have been developed. One such example is the theory of *real options* as described by Amram and Kulatilaka (1999). These specific

[1] traffic management is the optimal reservation of single-legged flights to provide for higher-profit multi-legged flights, while preventing empty seats

kind of options use the framework developed by modern financial instruments to cope with uncertainty of underlying assets. In accordance, Dixit, Pindyck, and Davis (1996) argue that in the presence of uncertainty and managerial flexibility in operating strategies, real option may provide a solution.

The practice of yield management is not only restricted to the aviation industry, but applies to many other sectors. Kimes (1989) supplies a list containing common characteristics of industries that successfully applied revenue management:

- relatively fixed capacity;
- ability to segment markets;
- perishable inventory;
- products sold in advance;
- fluctuating demand;
- low marginal sales cost/high marginal capacity change cost.

Harris and Peacock (1995) give in their paper a few examples of industries that all share these characteristics, namely: hotels, car rental agencies, railroads, cruise ships, printing and publishing. Dynamic pricing due to revenue management practices creates uncertainty on the customer-side in these sectors as well. So, the theory of real options could also be applied to these industries.

In the next section I will analyse current research on this topic and give an overview of option-based models applied to these kind of industries.

2.2 Real options in current industries

An option gives its holder the right, not the obligation, to buy or sell (respectively named *call*- and *put*-options) a specific underlying asset on or before a certain date (*maturity*) for a predetermined price (*strike price*) (J. C. Hull, 1999). A real option differs from a financial option, in that the underlying asset is a tangible object, rather than a derivative. Furthermore, real options are not normally not tradable like financial options are. Therefore they are often referred to as *deferral options* (Jain & Cox, 2011).

A real option is most commonly used as an alternative to the *net present value*-method for managing capital budgeting. However, this type of could also be applied to set up contracts for other tangible assets. This last category of options can be offered to consumers to sell for example tickets for games, energy or insurance. In their paper, Sainam, Balasubramanian, and Bayus (2010) provide another type of option even more consumer related. The option, for which they coined the term *consumer option*, is like a real option, but the quality of the outcome is different per consumer segment. The authors give the example of the final of a sports tournament, where a customer only wants to see the final when his team is playing. When the customer's team did not make the finals, he or she will not exercise the option.

The term *consumer option* is not often used in literature, and authors usually use *real options* to also refer to consumer options. To avoid confusion, I will use the term *real option* to refer to all options with tangible underlying assets which are provided to consumers.

An important distinction between options is the time at which holders are allowed to execute the contract (J. C. Hull, 1999). On the one extreme, there are *European* options, which allow the customer to only execute its option *on* the specified maturity date. At the other end of the spectrum are *American* options, which allow the holder to execute their right *before* the determined maturity date. In between these two types of options there are many alternatives, like Bermudan or switch options which allow execution of options a certain number of times before maturity date. An example implementation of such an option can be found in utility markets.

Utility industry

An industry in which a real-option model is already applied, is the utility industry (Keppo, 2004). The options, called *swing options*, are traded over-thecounter by energy suppliers to their customers. A consumer buys a forward contract which obliges him to buy a predetermined amount of electricity for a certain period at a specific price. To allow for flexibility, a customer can bundle this forward contract together with the swing part, which gives him the right to change the amount a number of times during this period (Jaillet, Ronn, & Tompaidis, 2004). The swing option thus gives the holder the opportunity to have a set price, while he is still able to respond to uncertainty by changing the amount of energy when circumstances change.

Thompson (1995) was the first researcher to determine the optimal price of such a swing option. He created a tree-based path-dependent model based upon previous research by J. Hull and White (1990), and used dynamic programming to numerically approximate the best value of these contracts. The author admits that solving path-dependent is difficult, because - in the case of swing options - prior exercise decisions influence the outcome of the model. This makes it very hard to determine the optimal price of the swing option analytically.

Jaillet et al. (2004) present in their paper a valuation framework for swing options. The authors based their model on multilayered trinomal trees. The concept of trinominal trees for the use of option valuation was first used by Boyle (1986), and allows to evaluate the contract on multiple decision variables. Jaillet et al. (2004) also state in their paper that exact valuation of options in path-dependent models is difficult, and their proposed model faces the same difficulties as in other real option frameworks.

Both of the previously described techniques use numerical techniques that use trees to evaluate the optimal price of a swing contract as a whole. (Keppo, 2004) on the other hand, replicates the swing contact by a future contract and a call option. The author estimates price the call part by looking at the commodity exchange market by using the Black–Scholes method (Black, 1976). He also concludes that due to the swing option's path dependency and Bermudan style, it is not possible to solve the problem algebraically.

Swing options have some similarities with airfare lock-in products, but there are also large differences. For example, costs when switching from energy providers are high and often impossible due to contracts. Customers are 'stuck' at their current energy provider, while in the airline industry, one can easily switch between airliners without additional costs. Furthermore, a swing option can be seen as multiple European and/or American options, which creates path dependency. Options for airline tickets consist of a single European or American option. Lastly, Jaillet et al. (2004) state that the uncertainty of a swing option is determined by the unpredictability of prices (future part) combined with the quantity which is actually needed (swing part). The uncertainty of airline tickets, however, is primarily based upon the unpredictability of prices.

Car rental industry

Car rental agencies continuously face the problem of allowing a car to be booked for a future time right now, or rather wait and sell the car at a higher rate in the future. When the agency decides to rent out the car right now there might be missed revenues due to not being able to offer high-end customers a product. On the other hand, when agencies refuse the current booking, they face the probability that the car is not booked at all. This would result in even higher loss of revenues as the car has 'perished' and can never be rented out for that passed date again. According to Anderson et al. (2004), this classic revenue management problem is similar to the swing option method described in previous section. They present the case of real options applied to the car rental industry by trying to apply the swing option model currently available in electricity and gas industries to the operating agent of such a company. In the paper the authors were able to solve the operator's optimal rental strategy by considering the operator as the holder of a swing option on car rentals.

While Anderson et al. (2004) state in their article to have found an optimal solution to this problem which is extendible to other industries, they also indicate some difference among the car rental and airline industry.

First, the authors state that airliners offer airfares by opening and closing discrete pricing classes. While the car rental industry also has different classes (size, AC, ...), they also differ prices within each class. This form of dynamic pricing is more difficult to predict, and hence creates more uncertainty. Secondly, car rental customers are less prone to changes in fares, while this is not the case in the airliner industry. And lastly, this model might not be applicable to this research as it defines the concept the other way around. The framework presents the idea in which the operators use swing-like options to determine the best car rental strategy. However, this paper focusses on the ability of *customers* to buy real options to lock in airfares.

Tournament industry

A more recent adoption of real options handling uncertainty, is the category associated with the sports market. Tickets for games usually sell weeks before the actual game. Fans of a team are willing to pay a large sum for a ticket when their team actually plays in the finals, but less or even nothing when this is not the case. However, the outcome of the actual teams that play in the finals is often not known in advance. Furthermore, customers need to buy tickets early in advance to prevent sell-outs. This puts the fans at risk, because there is a probability that their favourite team does not play in the finals. According to Sainam et al. (2010), this uncertainty creates a suboptimal situation where ticket agencies cannot charge the best prices due to customers taking into account the possibility that they buy a ticket for a final in which their favourite team does not play. The authors propose a new concept in which, instead of the ticket, customers buy real options. This real option gives its holder the right to buy the actual ticket when the outcomes are known.

To test this concept, Sainam et al. (2010) empirically compare three pricing mechanisms, knowing:

Selling in advance This is the current situation. Customers buy tickets well in advance and hope that their team plays the actual finals;

Pricing on full information Customers are sold tickets after the outcome of teams in the final is known;

Providing real options In this concept the customer first buys an option well in advance which gives him the right to attend the game. When the outcome is known, the customer can decide whether to exercise the option for an extra amount.

The authors studied the concept using these three pricing mechanisms, and showed that in many cases revenues where higher when applying the *real options*-model, rather than *advance* or *full information* selling. This was due to the fact that agencies can charge higher prices, because the *willingness to pay* of customers is not driven down by uncertainty. The concept also generates extra revenue from selling real options, and expands the existing market due to customers willing to buy tickets at a low rate of uncertainty. The profit advantage for option selling was highest when uncertainty was highest, which clarifies the purpose of options. The paper also found that *risk-averse* customers are willing to pay more for their ticket under uncertainty than their *non-risk averse* counterpart.

Berkowitz and Rotthoff (2012) even go further into this concept and propose a concept in which customers can *only* exercise their options when their favourite team reaches a given round. Furthermore, the authors suggest that a combination of advanced selling and real options might increase a ticket agency's revenue even further due to segmentation of game- and team-based fans. In their paper, the authors test this proposition and come to the conclusion that not only the revenue increases for the organizer, but also the utility of fans due to the fact that they are sure to have a ticket when their team plays in the finals. Lastly, more fans from the playing teams are present, which could increase the experience of the spectators.

Balseiro, Gallego, Gocmen, and Phillips (2011), who apply a combination of the models proposed by Berkowitz and Rotthoff (2012) and Sainam et al. (2010), also show these increases in revenue. They argue that this is due to the ability to better segment customers and thus better determine their *willingness to pay*. Furthermore, they argue that the extra certainty increased social welfare of the customer.

Berkowitz and Rotthoff (2012) state in their paper that their model could be used in many other industries as well. The authors give examples of which airline tickets is amongst them. While similarities can be found on a very abstract level (like the *binary option approach*), there seem to be major differences among the industries. For example, finals for sport events have close to no fluctuations in price and a very high sell-out probability closer to the date of the final. In the aviation industry, however, high fluctuation in price is much more common. Furthermore, flights do sell-out, but often this is not the case [2]. Also, the few number of alternatives in sporting events creates less price elasticity of customers. This is not applied to the aviation sector where a small change in price might cause a customer to switch to another vendor.

Aviation industry

The first occurrence of option pricing in the aviation industry in literature is provided by Walker, Case, Jorasch, and Sparico (1998). In their patent, the authors provide a system which determines the price of an option on a specific flight. This option gives the holder the right to buy the ticket at a moment in the future at the specified price, without actually holding the risk of sell-outs and increases in airfares. According to the authors, real options sold in this kind of setting have three major differences compared to traditional financial options:

1. the product is only supplied by a number of airlines;

[2] currently the average load factor is 83.6 percent. See `http://www.transtats.bts.gov/`

2. the product is not interchangeable, because customer might only want to fly with a specific airline at a specific time and date;
3. there are only a finite number of seats available (capacity constrained).

To overcome these differences, they describe a system which evaluates options with a method other than the traditional systems. The method determines the price of an option by multiplying a base price for the option by several relevant factors. These factors include: *number of days before departure*, *expected load of aircraft*, *desirability of the customer* (low-grossing versus high-grossing), *flexibility of the customer's flight plans*, and *historic volatility of ticket prices*.

Jain and Cox (2011) argue in their paper that traditional option valuation methods like the *Black–Scholes* model (Black, 1976) can actually be applied to this type of real options. The followed prices of 14 flights for 79 days during a period from November 2005 till February 2006. After collection of the data, the authors used the Black–Scholes model to evaluate the theoretically best price for a call- and put-option with a maturity date of 79 days (i.e., ticket bought on 79 day before departure) on these specific flights. The authors calculated the affordability of each option, which they defined as price of an option divided by the average ticket price of the flight ($p_o/\bar{p_f}$). Their results show that the affordability of lock-in products on their analysed flight range from 2.46 percent to 11.28 percent. The average affordability across all 14 flights is 7.2 percent. Put options to lock-in the price range from 1.55 to 10.34 with a mean of 6.37 percent of the ticket's price.

One major limitation according to Jain and Cox (2011), is that their analysis is based on *ex post data*. This means that the option price, including the volatility, was based upon data that was only available to the customer in the future. While the authors recognize this as a major flaw in their research, they argue that this data is available at the airliners.

Other types of real options are more prevalent in the aviation industry. For example, Akgunduz et al. (2007), Gallego, Kou, and Phillips (2008), Ching, Li, Siu, and Wu (2010), all describe the use of options in the booking process of an airliner. They illustrate the concept in which airliners sell tickets with a very high discount to customers early in the booking process. Coupled with these low-fare tickets the airliner buys a call option, which gives them the right to recall the ticket when capacity is needed for higher-grossing customers. This gives the airliner the opportunity to sell tickets at high-fares when demand is high, while still having the backup of the low-fare customer to prevent empty seats. A put option is also described, in which a travel agent agrees upon buying tickets from the airline at a very low-fare when these are still available close before departure of the flight. This allows the airliner to still sell tickets which would have been empty seats otherwise.

The previously mentioned authors all use numerical methods to determine the optimal mix of call/put and normal seats. Their shared conclusion is that this type of yield management is superior to the current overbooking methods, because it generates more revenues and also higher social welfare for customers. The model is also applicable to other concepts. Graf and Kimms (2011) and Graf and Kimms (2013) show for example that this concept of capacity based option pricing can be implemented at airline alliances. In this situation, the operating airline can sell options to the ticketing airline. When the ticketing party sells a seat of higher value than the strike price of the option, it exercises the option. The operating carrier can also buy a put option on the sold call option. These kind of option on options, also defined as *compound options* (Trigeorgis, 1996), give the operator the right to recall options from the ticketing company when demand is high. In a strategic alliance this only happens when the revenue gained from accepting a seat request is greater than the strike price plus the option price Graf and Kimms (2011). The authors use simulation-based evaluation of booking limits (i.e., how many calls and puts to sell) to determine the best scenario of such an options.

2.3 Option valuation methods

Theoretical model: Black–Scholes

The most used options valuation method for pricing options in financial markets is the Black–Scholes model (J. C. Hull, 1999). It was first proposed in a paper by Black and Scholes (1973), and was later expanded mathematically by Merton (1973). The model calculates the value of an option for the underlying stock. The article shows that the expected rate of return of the underlying asset and risk preference of the investor do not matter. The formula rather uses other variables (e.g., volatility of the underlying asset, maturity, risk-return rate) to estimate the theoretical option price. The Black–Scholes formula is defined as follows:

$$
\begin{aligned}
C(S,t) &= N(d_1)S - N(d_2)Ke^{-r(T)} \\
d_1 &= \frac{\ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \\
d_2 &= d_1 - \sigma\sqrt{T-t}
\end{aligned}
$$

Black and Scholes (1973) make a few assumptions of the underlying stock, including:

- The option style is European;
- The stock price follows a geometric Brownian motion (i.e., log-normal distribution) with constant drift and volatility;
- The stock pays no intermediary dividends.

These assumptions might also be applicable to airfares. For instance, a rational customer would only exercise its airfare lock-in product at maturity (see section *Option style* in chapter 4 for a detailed explanation). Furthermore, the numerous factors and entities that influence the price of a ticket can be very unpredictable. This implies a random walk where price increases and decreases follow a certain log-normal distribution with an upward trend.

The model proved itself to be very useful on the stock-market and was quickly implemented. While implementers of the tool use it often with adjustments, the equation approaches the actual observed option prices on the exchanges (Bodie, Kane, & Marcus, 2008). Next to option valuation in financial markets, the Black–Scholes-model proved itself to also be applicable to other assets. For example, Finch, Becherer, and Casavant (1998) give the example of the valuation of the options held by a firm which allows it to abandon the capital investment of a project prior to completion. Another more recent — but theoretical — implementation of the method is shown by Jain and Cox (2011), who showed that the theory might also be applicable in the airline industry. A full description of this paper can be found in section *Aviation industry*.

Numerical approximation: Monte Carlo

Rather than an analytical approach, the *Monte Carlo* method uses simulation to approximate the outcome of a model. It does so by generating large quantities of pseudo-random numbers that follow certain distributions, and use these as input for the variables used in the model. By running the model many times, the outcomes of the model will converge to a particular result. This convergence is caused by the *law of large numbers*. That the simulation method can be quite accurate after a certain amount of trials is shown by Richardson (2009). In his article, the author compared option prices yielded by the theoretical Black–Scholes model with the ones produced by a Monte Carlo model. Both approaches assumed a distribution with log-normal returns. He showed that after a small number of trials (i.e., 5 000), the Monte Carlo method differed from the theoretical model by a few thousands of a Dollar.

The first to use Monte Carlo model for the valuation of financial options was Boyle (1977). In his paper, the author shows that Monte Carlo simulation can be used as a different approach to option valuation. For instance, the author gives the example of a model in which also the price of European options on dividend-paying derivatives can be estimated.

The approach of simulation in option valuation techniques thus has a few advantages compared to more exact methods like Black–Scholes. Boyle (1977), for example, state that Monte Carlo can make use of different distributions. As described in previous section, the Black–Scholes formula assumes a geometric Brownian motion. When the underlying asset does not follow such a log-normal distribution, this method will yield unreliable results. A Monte Carlo model, however, can be used to exactly represent the probability density function of underlying returns. This method might therefore be more applicable, as the section *Predictability of airfares* shows that airfares might be predictable to some extend, and do thus not follow these Brownian properties.

Maidanov (2010) state in their article that the assumptions that the Black–Scholes model makes can never all be satisfied in a realistic setting. The authors argue that Monte Carlo simulation can even evaluate European options more optimally in a realistic world than theoretical models. This is due to the fact that the Monte Carlo method is extendible to any possible situation, and its only restrained by the computational capacity of the underlying hardware.

Valuation of American options proved to be much more difficult. J. C. Hull (1999, p. 408) even stated that these types of options "*[...] cannot easily handle situations where there are early exercise opportunities*". This is due to the nature of the underlying decision model for executing the option. This path-dependency makes it very hard to evaluate the price of options, because holders can exercise their option an infinite number of times. Thompson (1995) states in his article that simulation approaches like Monte Carlo are best not employed when exercise decisions are present. However, many studies are available in which the researchers have developed reliable simulation models for this type of options (e.g., Garcıa (2003), Longstaff and Schwartz (2001), Rogers (2002), ...).

2.4 Predictability of airfares

While there is much debate whether stocks follow a random distribution, ticket prices for flights might be predictable to a certain extend. Since 2000 the Trading Agent Competition (TAC) has submitted challenges to develop Artificial Intelligence-models allowing travel-related predictions Stone and Greenwald (2005). While some of the models created in this competition seem to be fruitful (e.g., Wellman, Reeves, and Lochner (2003)), the simulation's circumstances are somewhat unrealistic. For example, the competition assumes an infinite capacity, and the demand distribution is stochastic with an upward trend.

The first to successfully apply an airfare prediction model on real data were Etzioni, Tuchinda, Knoblock, and Yates (2003). During a 41 day period, the authors scraped pricing data on two flights (i.e., LAX–BOS and SEA–IAD) every 3 hours. They then converted the acquired data into a time-series model and compared five different prediction methods on these data: *(1)* Time Series Forecasting; *(2)* Rule-based selection by hand; *(3)* Rule-based selection by an automated program; *(4)* Q-learning; *(5)* Hamlet.

The authors do not describe the *Hamlet* approach in full, but a patent filed by Etzioni, Yates, Knoblock, and Tuchinda (2008) suggests that the model was build out of the best features of each of the four other approaches.

The models predicted whether future fares of a ticket were likely to rise or decline at a certain point in time. This prediction was then used to conclude if it was best to buy the ticket at instantly, or rather wait and hope for a better price. To compare the models with each other, Etzioni et al. (2003) ran a simulation and analysed the saving generated by the method with the optimal savings for every flight. The authors showed that the best approach to this 'buy-or-wait'–decision was the Hamlet model. The simulation stated that Hamlet could generate savings up to 60 percent of the optimal situation. The worstperforming method, the one based upon time-series forecasting, resulted in more than 80 percent extra costs compared to buying instantly.

Groves and Gini (2013) later altered this model to predict the lowest price at a certain time in the future for *any* flight and *any* airline. They argue that this is more difficult due to the fact that aggregation of the data results in less variance. The authors also compare their different approaches. They concluded that Partial Least Square regression (PLS) with automated feature selection is the most optimal savings-method resulting in a savings of 75 percent. Groves and Gini (2013) based this automated feature selection upon a model build by the same authors (Groves & Gini, 2011), which rationally and iteratively compared all different feature sets and computed the optimal solution.

3

Research Questions

In this paper, I would like to theoretically and empirically analyse whether it is viable for an external company to offer airfare lock-in products. In this thesis, I refer to *viable* as the availability of an option price that both satisfies the option buyer (i.e., customer) and the option writer (i.e., the external company). The external company's minimum price at which he is still willing to sell the option (i.e., *willingness to accept* or *WTA*) is equal to the expected costs of the risk associated with this product. This risk can be expressed as the expected increase in ticket price combined with the probability of the costumer not exercising the option[1]. The customer will only accept the offer when his willingness to pay for the option (i.e., *WTP*) is higher than this minimal price. In this thesis I will therefore test what the minimum price of the offer should be according to the seller, and compare this with the maximum price that the customer's *WTP*. The main research question in this paper can be defined as

> *Is it sustainable for an external company without capacity information or seat reservation capabilities to set the price of airfare lock-in products at a level that the customer accepts?*

The maximum price a passenger is willing to pay depends upon many internal factors. For the second research question I therefore want to consider

[1]This only applies in a risk-neutral, fully predictable setting with rational customers that have equal information. See subsection 4.1 for a more detailed description of the theoretical scenario's

the influence of some of these variables on the customer's *WTP* and the relation with the option seller's *WTA*. The three variables that will be considered are

Risk-utility This is the customer's preference of risk. A customer can be categorized as *(i)* risk-averse, *(ii)* risk-neutral, or *(iii)* risk-seeking. In the setting of airfare lock-in products, customers that can be labeled as risk-averse are more likely to buy this kind of option, as they are inclined to pay higher prices to cover their uncertainty;

Forecasting technique This variable represents the techniques a customer uses to make its own forecast. In my models I will consider three types of techniques, namely predictions based upon: *(i)* a log-normal distribution with historical volatility, *(ii)* an empirical distribution acquired from historical data, or *(iii)* the actual future price (i.e., foreknowledge);

Likelihood of travelling This factor represents the probability P^f that a customer actually wants to make use of the flight. The allowed values are $0 \leq P^f \leq 1$ where 0 represents the state '*certainly not flying*' and 1 the case '*certainly flying*'.

My second research question can thus be defined as

> *What is the influence of the variables risk-utility, accuracy of forecast, and likelihood of travelling on the gap between the option buyer's WTP and the option writer's WTA?*

The accuracy of forecasts depends heavily on the underlying mechanism. In this study I will compare two different valuation strategies empirically, and test which method approximates the optimal model most. The first method uses a theoretical approach and evaluates the options using the *Black–Scholes* model. The second method uses numerical approximation to determine the best option price. This method makes use of *Monte Carlo* simulation.

Both methods will base their forecasts upon aggregated data. The different categories upon which the flights will be divided are *(i)* per *specific departure*

airport (e.g., AMS), *(ii)* per *specific arrival airport* (e.g., JFK), *(iii)* per *operating carrier* (e.g., American Airlines), or *(iv)* a combination of these. The model will therefore make predictions on a set of flights that share the same properties, rather than for each individual flight separately. This will not only make the models easier to implement on smaller datasets, but might also prevent the customer from *gaming the system*. For example, when a customer sees that an option for a specific flight is more expensive than other options for the same route, he can conclude that the prices are likely to go up and act accordingly. Making prediction on aggregated data will make this gaming more difficult because the option pricing data will not disclose information on expectations for individual flights.
The third research question in this study is defined as

> *What feasible underlying option valuation strategy — theoretical or numerical — approaches the optimal model in option prices best?*

Scope

This study will analyse a set of 22 different single-legged routes covering flights within the United States and Europe, as well as intercontinental legs (see appendix B for a the selected routes). Results of this research will therefore only be generalizable to flights falling within the boundaries of this particular set. However, the findings might be of use to other industries where similar constructions are applicable. Think of other travel-related companies like hotels or car-rental services, or in industries where uncertainty might withhold consumers to buy products like ticket sales for basketball games.

Furthermore, this research will only build models based upon theoretical frameworks and empirical data that is available to the public. I will thus consider the third party as an independent option writer that does not have any inside information or capabilities other than a customer. For instance, the model will assume that the external company has no possibility of reserving capacity on a flight, and therefore has to deal with the risk of sell-outs. In a practical situation, however, the company might make an arrangement with the operating carrier to reserve seats for a certain premium. This would cut uncertainty for the option writer, and thus decrease option prices. The model I use in this study might therefore be harsher than realistically would be the case.

4

Methodology

4.1 Entities

Throughout this research, four main entities will be used. These concepts are *(i)* the flight ticket, *(ii)* the airfare lock-in product, *(iii)* the external company which sells the airfare lock-in products, and *(iv)* the passenger.

Flight ticket

A flight ticket gives its holder the right to fly on a particular date and time from and to a particular airport using a specified airline. Flights can be distinct by their unique *call-sign*, which consists of three different components: *(i)* the IATA airline code, *(ii)* the IATA flight number, and *(iii)* the date of departure. However, a quick analysis of the data shows that some flights use the same call sign for every leg in a multi-legged route. In this research I have therefore added the departure airport to this sign to make it possible to distinct the underlying legs.

A ticket for a particular flight can be bought many weeks prior to departure. At a certain time, tickets are either available or *sold-out* — in which an alternate flight with compensation is necessary. In the aviation industry, '*sold-out*' does not always mean that there are no tickets available at a future moment in time. As a result of revenue management and discount allocation, airliners are known to open and close certain *discount buckets* (McGill & Van Ryzin, 1999). The opening and closing of these buckets prevents missed revenues of high-grossing customers due to actual sell-outs. It is therefore possible that a flight seems to be sold-out weeks before departure, but reappears again closer to the deadline.

The revenue management concept of discount buckets has lead to different prices throughout the booking period. Customers that want to buy a seat for a certain flight earlier to the departure date are likely to be offered other fares than early bookers . Flight tickets are thus dynamically priced.

Airfare lock-in product

The airfare lock-in products gives its holder the right — but not the obligation — to buy the underlying flight on or before a certain maturity date for a predetermined strike price. The holder is thereby covered from sudden increases in price or potential sell-outs, but is still able to not fly at all or gain from large price decreases.

External company

An external company is the entity that offers the airfare lock-in products to passengers. The seller sets its minimal option prices based upon the expected loss due to increase or sell-out of the underlying flight. The company therefore only accepts to sell the product when the customer will offer a price that is equal or higher than this expected costs.

When the company writes an option, it is thereby obliged to offer the underlying ticket at the specified strike price if the holder exercises it. Flight tickets are capacity constrained, and therefore not unlimited. This means that sometimes the situation emerges in which the option holder wants to exercise its right, but the flight is sold-out. The option writer than has to offer the customer a ticket for an alternative flight plus an extra compensation. The level of compensation could be calculated according to European Legislation[1]. A sell-out thus creates much increased costs. Therefore, the option price will dramatically increase when the probability of a sell-out is high. In such situations the selling company might refrain itself from offer the option for that particular flight in full.

Passenger

The most central entity in the simulation is the *passenger*. In this research, a passenger is defined as a customer who considers to purchase a ticket or option for a particular flight on a particular number of days before actual departure of this flight. The distribution of the arrival process for the simulation of passengers will be derived from the article by Weatherford, Bodily, and Pfeifer (1993). In their paper, the authors simulate the arrival according to a inhomogeneous Poisson process. Bertsimas and De Boer (2005) defined a model more specific for the aviation industry based on the PARM-model.

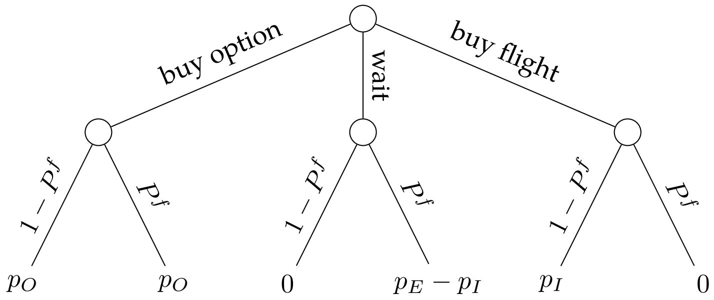
The simulation model upon which this research is based will only consider the arrival of economy class passengers. This is because the data provide no reliable method of distinguishing different classes, and persons that travel in economy are more price elastic than their business class counterparts. They are therefore more likely to consider an airfare lock-in product to cover their risk.

Passenger's decision tree

When a doubting passenger arrives at the booking process, there is a series of events that might occur. The passenger's decision model is illustrated in Figure 4.1.

First, the customer has to decide whether to *buy the flight* immediately, *wait* a certain amount of time or *buy an option* on the flight and postpone the decision.

[1] `http://europa.eu/youreurope/citizens/travel/passenger-rights/air/`



P^f	Probability of flying	p_I	Initial airfare
p_E	Expected airfare	p_O	Option price

Figure 4.1: Passenger's decision tree

Next, after a certain period of time, the customer will know whether he will actually take the flight. The probability of flying or not flying is respectively P^f and $1 - P^f$, and is based upon uncertain events that might occur. For instance, the probability of not flying might consist of the probability that the passenger is not able to take a week vacation from work, or even that the weather is bad when the customer wants to go on a sunny holiday.

The action that follows this outcome depends upon the decision made in the first phase. The outcome is calculated by comparing the extra costs incurred by the decision with the '*optimal*' decision that would have been made by a passenger that knew whether he would fly or not (P^f would respectively be 1 or 0). The difference of these options is the monetary value of the passenger's *regret* of making his first decision. So, when the passenger decides to actually fly, the costs of his chosen path are compared with the costs of directly buying the flight in the first phase (i.e., p_I). On the other hand, when the customer decides not to go, the outcome is calculated by comparing the path with *not buying the ticket in the first place* (i.e., 0).

The different paths that can be followed are

buy flight → fly in this scenario, the passenger has already bought the ticket and no further action has to be undertaken. The customer has no regret of making his first decision (0);

buy flight → don't fly the passenger will not fly, and the bought ticket is thus rendered useless. The regret of this path is equal to the initial ticket price in the first phase (p_I);

wait → fly when the passenger decides to fly after waiting, it will have to buy a ticket. The customer's regret of waiting is equal to the expected price in the second phase minus the initial ticket price ($p_E - p_I$)

wait → don't fly the passenger has to undertake no action, and has not lost any money to a ticket. The customer has therefore no regret of making his first decision (0);

buy option → fly the passenger will buy the ticket at the agreed strike price. The extra costs relative to buying the ticket in the primary phase are equal to the option price plus the strike price (p_S) minus the initial ticket price. In this research, the strike price will be equal to the initial price, so the customer's regret of making this decision will in this case be the same as the price of the option ($p_O + p_S - p_I = p_O$)

buy option → don't fly the passenger will not exercise the option and will not buy the ticket; The customer's regret is therefore equal to the option price (p_O).

The expected outcome of each decision made in the first phase can be calculated by multiplying the decision variables P^f and $1 - P^f$ with the result of each branch. The passenger's regret of buying the flight immediately is therefore:

$$P^f \times 0 + (1 - P^f) \times p_I = (1 - P^f) \times p_I$$

The cost of waiting can be defined as:

$$P^f \times (p_E - p_I) + (1 - P^f) \times 0 = P^f \times (p_E - p_I)$$

Lastly, the regret of buying an option is:

$$P^f \times p_O + (1 - P^f) \times p_O = p_O$$

Passenger's WTP and the third party's WTA

In a risk neutral setting with equal shared information between option seller and buyer, the price a passenger is willing to pay for the option equals:

$$\min(P^f \times (p_E - p_I), (1 - P^f) \times p_I) \tag{4.1}$$

From the third party's perspective the minimum option price the company is willing to accept is equal to expected incurred costs of selling the product. For a set $\{E(p_1), E(p_2), \ldots, E(p_n)\}$ where $E(p_i)$ is the expected result (i.e., $(p_i - p_I) \times P^{p_i}$) of price p_i occurring at maturity. The value of p_O can thus be defined as:

$$p_O = \sum_{i=1}^{n} \begin{cases} E(p_i), & \text{if } E(p_i) > 0 \\ 0, & \text{if } E(p_i) \leq 0 \end{cases} \tag{4.2}$$

This minimum price is related to the costs an option writer expects from selling the airfare lock-in product. The formula yields the prices at which the company anticipates to lose as much as it gains from selling the option (i.e., the resulting profit from selling this option is 0).

The equations of the customer's *WTP* and the seller's *WTA* imply that — in a risk neutral setting — there will arise an equilibrium between the maximum a customer wants to pay, and the minimum a providing company wants to accepts. This stationary formula for this equilibrium can be defined as:

$$\frac{1}{p_E/p_I} = P^f \tag{4.3}$$

The stationary points are thus dependent on the ratio $\frac{p_E}{p_I}$. This is illustrated by Figure 4.2

For combinations of P^f and $\frac{p_E}{p_I}$ that result in an outcome left of the stationary line there is an call option price that both satisfies the passenger and the provider. The combinations to the right of this function do not result in such an option price, and the customer will rather directly buy the flight ticket. An option seller can still target passengers in this area by providing them with a

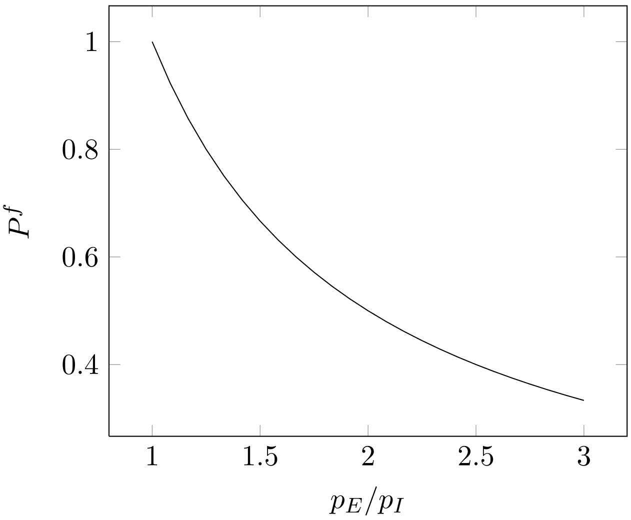


Figure 4.2: Stationary points

ticket and an option that allows the customers to return the ticket's without additional costs.

Option style

While customers might be allowed to execute their option during the whole period up till maturity (i.e., American style), I will assume that all options are exercised on the date of maturity (i.e., European option). This is in accordance with the theory of Merton (1973), who states that an American option that does not return dividends should rationally only be exercised at maturity. This assumption can easily be illustrated with the use of an example. If a customer who has bought a real option has decided to exercise the option, it has nothing to lose by waiting till the option matures. By waiting this extra time, the customer only gains an advantage which allows him to still withhold himself from actually buying the ticket. In this way, when there occurs an unforeseen situation after his decision, he still has not bought the ticket and can waive the flight. Therefore it would be unrealistic of the customer to exercise the option before the date of maturity.

4.2 Phases

This research will be divided into three consecutive phases.

The first part, section *Data Collection and Parsing*, will focus on retrieving the pricing-data on a predetermined set of routes. This phase will gather data during a period of 6 weeks and parse it into a new interlinked format. The information retrieved and parsed during this period will be used as the census for the whole study.

The second phase, section *Option valuation models*, will deal with the analysis of the data and build models around it. To prevent issues like in Jain and Cox (2011), the training of the data and testing of the data will be performed on two separate parts: the first 4 weeks of the retrieved data will be used to perform the analysis of the two different valuation mechanisms, while the lasts 2 weeks is only used to test the models upon. This concept is quite common in machine learning, and prevents the analysis being based on data that is not yet available when the analysis would be performed in real-life.

The third and last phase, section *Simulation and sensitivity analysis*, actually tests the previously built models and an optimal one with the use of simulation. This analysis will result in answering the first and third research question. Next, this phase also consists of sensitivity analysis, which will provide the conclusion for the second research question.

Data Collection and Parsing

In the first phase information on flights will be collected from the Internet. This data will be used as the census for this study. The price setting systems, simulation models and prediction system will all be based upon this dataset.

The website of Google Flights will be used to gather this information. I have selected this resource, because contrary to intermediary sites like Expedia.com, Google Flights does show the lowest available fare and merely redirect users to the airliner's site. Furthermore, direct calls to Google's RPC-server enable users to request XSS-protected JSON-arrays with flight data. See Appendix A for a detailed list of information included in these arrays.

In this research I will restrict myself to collecting data on 22 different round-trip flights. Of these 22 routes, 10 cover domestic routes within the United States, 6 are domestic routes within the European Union, and the remaining 6 routes are international. The 10 US domestic flights are two routes selected from the 5 busiest domestic airports within the US, as retrieved from the Bureau of Transportation Statistics (2013). The 6 EU domestic flights are a permutation of routes between the airports *AMS*, *CDG* and *LHR*. These three airports are amongst the top 5 busiest airports in Europe EuroStat (2013). Due to Google Flight's limitation of only providing data for flights departing from the US and the EU, I have selected 6 international routes departing from those areas. See Appendix B for the detailed list of selected routes. The routes examined in this study are all round-trip flights with an interval of 7 days between the departure date for the outbound and inbound flight.

Data will be collected on the previously mentioned routes departing during the period from Monday the 19^{th} of August up until Sunday the 29^{th} of

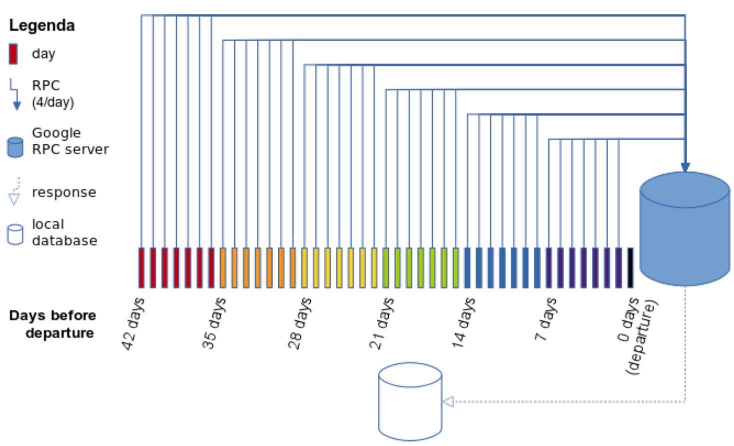


Figure 4.3: Data retrieval process

September. Price data on all these flights will start 6 weeks prior to departure, up until the day before take-off. The data retrieval can best be illustrated with the help of Figure 4.3. Each day (represented by a rectangular box) an automated program will send a *Remote Procedure Call* (blue arrow) to the *RPC-server* of Google Flights (blue cylinder). This request asks the server to send data on a flight that meets certain specifications. For example, the first call the program makes on July the 8^{th} (6 weeks prior to start of collection of specific route) is "*return data on all flights from ATL to MCO departing on August* 19^{th} *and returning on August* 26^{th} *(7 days after departure)*". The call is send to the server as a JSON-formatted request (See Appendix C for an example). In return, Google's server will send data on all the flights that meet those requirements (blue dashed arrow) and the automated program will store the received information in a local database (white cylinder). Because Google Flights does not always send back the same flights, the same RPC is send every 6 hours (at 00:00, 06:00, 12:00 and 18:00) [2]. The next day the same process is repeated by sending the exact same query to the RPC-server again. This procedure is repeated each day up until the day before departure.

The illustrated process only describes one route (i.e., *ATL* to *MCO*) on one specific date (i.e., departure on *August* 19^{th}). The automated program however takes into account every route described in **??**, and all dates in the period from August 19^{th} till September 29^{th}. The total number of requests made to Google's RPC-server is thereby 22,176 calls [3].

After retrieval of the flight information, the data is parsed and stored in the HDF5-format. This data format is often used in big data analysis, and enables quick reading of big sets of data to memory. Following the loading into RAM, the data will be converted to a Pandas object after which analysis can be done.

Distinction between flights is made by hashing flight's properties[4]. Prices of the flight will be stored in the HDF5-file using the hash as the identification

[2] Each blue arrow thus represents 4 requests on that single day
[3] 4 times per day for 7 days per week for 6 weeks for each 22 flights for a period of 6 weeks
[4] The *call sign* of each segment and *ticketing code* of the flight

key. This enables easy extraction of price fluctuations within exact same flights relative to time.

Option valuation models

In this phase the models for option valuation will be built. Currently airliners that offer options on their flights, offer these at a fixed price. This price only differentiates on maturity date, and is equal for all flights and time of purchase. For instance, customers are presented the same price when purchasing the option 1 day or 4 weeks before departure. See Table 4.1 for examples of current prices at which aviation companies offer fare lock-in products.

Airline	2 days	3 days	7 days	14 days
United Airways		\$ 5.99	\$ 8.99	
Air France-KLM				\$ 20
Estonian Air	\$ 15			

Table 4.1: Price of airfare lock-in

In this research I will compare three different types of option pricing methods, namely *(i)* an optimal model based upon foreknowledge, *(ii)* a theory-based approach based upon the Black–Scholes model, and *(iii)* a numerical approximation approach based upon the Monte Carlo method. The first model is is the theoretical optimal method, while the latter are practically feasible.

Theoretically optimal: foreknowledge

The first option valuation model built in this research is based upon the assumption that the option seller knows the exact price of the underlying flight in the future. While this is infeasible in practice, this theoretical model will provide the answer to the first research question. By using this technique one can conclude whether the external company is able to provide options at a price which the passenger accepts. If it is viable to offer such option in this theoretical setting, more realistic models can be built to see whether it is also feasible in the practical world. Furthermore, this optimal method will be used as baseline to compare the other two models.

Theory-based: Black–Scholes model

The Black–Scholes model will be the first practical implementations of a option valuation technique in this research. The model is used to determine the European option price based upon the volatility of the underlying asset. Jain and Cox (2011) demonstrated in their paper that this model can also be used to determine optimal option prices based on the volatility of airfares. In this paper, I calculate the volatility of the airfares by using the first 4 weeks of the census. With this variable, the optimal option price according to the Black–Scholes method can be calculated, and the outcome can be tested in the simulation model. The adapted equation used for this flight specific scenario is derived from Jain and Cox (2011, p. 13):

$$
\begin{aligned}
p_O &= p_I \times N(d_1) - p_S \times e^{-rT} N(d_2) \\
d_1 &= \frac{\ln(p_I/E) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

See appendix D for a full description of the equation.

Numerical approximation: Monte Carlo

The next practical implementation of an option valuation technique is the Monte Carlo method. This model is more flexible and can be used when the underlying returns do not follow a log-normal distribution. Furthermore, as Walker et al. (1998) states, options on flights are quite different from options found in the financial market. The main differences are *non-interchangeability* and *capacity restrictions*. A Monte Carlo can be configured in such a way that it also takes into account such distinctions.

Richardson (2009) gives an example implementation of the Monte Carlo method to set the prices of options. Instead of volatility and random returns, I will implement this model based upon the expected returns seen in the training set of the data. The outcomes will be tested using the designated set of the data, and relatively compared with the optimal model.

Simulation and sensitivity analysis

In the third phase the three pricing models described in section *Option valuation models* will be evaluated using simulation.

This simulation model generates passengers that want to buy tickets of a certain flight as a function of time (6 weeks till one day before departure).

To make sure the models are not overfitted on the train data, the simulation will be run on the separate set of test data.

Sensitivity Analysis

During the simulation a sensitivity analysis will be performed as well. In this analysis the three parameters of a passenger — *(i)* risk-utility, *(ii)* forecasting technique, and *(iii)* likelihood of travelling. — will be altered to see what the effect of these properties is on the price setting model.

5

Planning

Figure 5.1 shows the schedule for this master's thesis in the form of a Gantt chart. The overall process from data collection till the actual submission of the report will take about 5 months to complete. Expected graduation will take place in December.

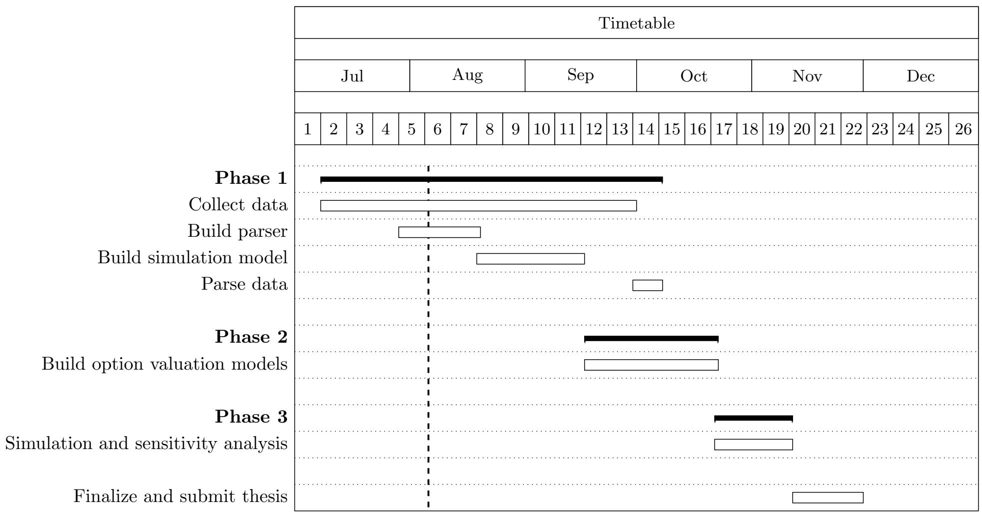


Figure 5.1: Timetable

A

Data extracted from Google Flight's RPC-request

Index	Description
Array[0][0][1][3][0][*]	Array of all possible segments (Segment)
Array[0][0][1][3][1][*]	Array of all possible routes (Route)
Segment[1][0]	Departure airport (IATA code)
Segment[1][1]	Departure date and time
Segment[1][2]	Arrival airport (IATA code)
Segment[1][3]	Arrival date and time
Segment[1][4]	Carrier code (IATA code)
Segment[1][5]	Flight number
Route[0][0]	Index of outbound segments
Route[0][1]	Index of inbound segments
Route[1]	Price of round-trip
Route[3][0][0]	Ticketing code of outbound flight (IATA code)
Route[3][1][0]	Ticketing code of inbound flight (IATA code)

Table A.1: Information in Google Flight's RPC response

B

Selected routes

Type	Outbound airport	Inbound airports	
US Domestic	ATL	MCO	LAX
	DEN	PHX	FLL
	DFW	LAX	SFO
	LAX	SFO	LAS
	ORD	LGA	BOS
EU Domestic	AMS	CDG	LHR
	CDG	AMS	LHR
	LHR	AMS	CDG
International	AMS	DXB	JFK
	JFK	CDG	LHR
	LHR	JFK	LAX

Table B.1: Routes to be examined

C

Sample JSON RPC


```
{’1’: [{’1’: ’fs’, ’2’:
  ’{"1":{"1":[
    {"1":[ATL],          # departure airport
     "2":[MCO],          # arrival airport
     "3":"2013-08-26"},  # departure date
    {"1":[MCO],          # departure airport return
     "2":[ATL},          # arrival airport return
     "3":"2013-08-19"}   # departure date return
  ]}}’, u’4’: 1}],
’2’: {’1’: [{’1’: ’b_am’, ’2’: ’fs’},
            {’1’: ’b_qu’, ’2’: ’2’},
            {’1’: ’b_qc’, ’2’: ’2’}
]}}
```

D

Adapted Black-Scholes formula

$$
\begin{aligned}
p_O &= p_I \times N(d_1) - p_S \times e^{-rT} N(d_2) \\
d_1 &= \frac{\ln(p_I/E) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

p_O is the call value

p_I is the initial price of th flight

T is the time to maturity

$N(\cdot)$ is the cumulative normal probability

σ is the volatility of returns of the underlying flight

p_S is the strike price of the option

r is the risk free rate

References

Airline Deregulation Act. (1978, October 24). *Pub.L. 95–504, 49 U.S.C. §1371 et seq.*

Akgunduz, A., Turkmen, B., & Bulgak, A. A. (2007). Using financial options in airline booking process. *Journal of Revenue & Pricing Management*, *6*(2), 135–150.

Amram, M., & Kulatilaka, N. (1999). *Real options: Managing strategic investment in an uncertain world* (Vol. 4). Harvard Business School Press Boston.

Anderson, C., Davison, M., & Rasmussen, H. (2004). Revenue management: A real options approach. *Naval Research Logistics (NRL)*, *51*(5), 686–703.

Balseiro, S., Gallego, G., Gocmen, C., & Phillips, R. (2011). *Revenue management of consumer options for sporting events* (Tech. Rep.). Tech. rep., IEOR, Columbia University, New York.

Belobaba, P. P. (1987). Survey paperâĂŤairline yield management an overview of seat inventory control. *Transportation Science*, *21*(2), 63–73.

Berkowitz, J., & Rotthoff, K. (2012). Real options applied to consumer goods: A look at option pricing in sports tournaments. *Available at SSRN 2154929*.

Bertsimas, D., & De Boer, S. (2005). Simulation-based booking limits for airline revenue management. *Operations Research*, *53*(1), 90–106.

Black, F. (1976). The pricing of commodity contracts. *Journal of financial economics*, *3*(1), 167–179.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *The journal of political economy*, 637–654.

Bodie, Z., Kane, A., & Marcus, A. J. (2008). *Investments (7th international ed.).* Boston: McGraw-Hill.

Boyle, P. P. (1977). Options: A monte carlo approach. *Journal of financial economics*, *4*(3), 323–338.

Boyle, P. P. (1986). Option valuation using a three-jump process. *International Options Journal*, *3*(7-12), 1.

Bureau of Transportation Statistics. (2013). *Air carriers : T-100 domestic market (u.s. carriers).* `http://www.transtats.bts.gov/Fields.asp?Table_ID=258`. (Accessed: 04-23)

Ching, W.-K., Li, X., Siu, T. K., & Wu, Z. (2010). Improving revenue management: A real option approach. In *Innovative quick response programs in logistics and supply chain management* (pp. 123–139). Springer.

Cross, R. G. (1997). *Revenue management hard-core tactics for market domination*. New York: Broadway Books.

Dixit, A. K., Pindyck, R. S., & Davis, G. A. (1996). Investment under uncertainty. *Resources Policy*, *22*(3), 217.

Etzioni, O., Tuchinda, R., Knoblock, C., & Yates, A. (2003). To buy or not to buy: mining airfare data to minimize ticket purchase price. In *proceedings of the 9th ACM SIGKDD international conference on Knowledge discovery and data mining* (pp. 119–128).

Etzioni, O., Yates, A., Knoblock, C. A., & Tuchinda, R. (2008, March 18). *Performing predictive pricing based on historical data.* Google Patents. (US Patent 7,346,520)

EuroStat. (2013). *avia_par tables.* `http://epp.eurostat.ec.europa.eu/NavTree_prod/everybody/BulkDownloadListing?sort=1&dir=data`. (Accessed: 04-23)

Finch, J. H., Becherer, R. C., & Casavant, R. (1998). An option-based approach for pricing perishable services assets. *Journal of Services Marketing*, *12*(6), 473–483.

Gallego, G., Kou, S., & Phillips, R. (2008). Revenue management of callable products. *Management Science*, *54*(3), 550–564.

Garcıa, D. (2003). Convergence and biases of monte carlo estimates of american option prices using a parametric exercise rule. *Journal of Economic Dynamics and Control*, *27*(10), 1855–1879.

Graf, M., & Kimms, A. (2011). An option-based revenue management procedure for strategic airline alliances. *European Journal of Operational Research*, *215*(2), 459–469.

Graf, M., & Kimms, A. (2013). Transfer price optimization for option-based airline alliance revenue management. *International Journal of Production Economics*.

Groves, W., & Gini, M. (2011). *A regression model for predicting optimal purchase timing for airline tickets* (Tech. Rep.). Technical Report 11-025, U. of Minnesota, Minneapolis, MN.

Groves, W., & Gini, M. (2013). An agent for optimizing airline ticket purchasing. In *Proceedings of the 2013 international conference on autonomous agents and multi-agent systems* (pp. 1341–1342).

Harris, F. H., & Peacock, P. (1995). Hold my place, please. *Marketing Management*, *4*(2), 34–45.

Hull, J., & White, A. (1990). Valuing derivative securities using the explicit finite difference method. *Journal of Financial and Quantitative Analysis*, *25*(1), 87–100.

Hull, J. C. (1999). *Options, futures, and other derivatives*. Pearson Education India.

INFORMS Revenue Management and Pricing Section Prize for Historical Works. (2013). *Historical prize.* `https://www.informs.org/Community/revenue-mgt/Prizes/Historical-Prize`. (Accessed: 07-08)

Jaillet, P., Ronn, E. I., & Tompaidis, S. (2004). Valuation of commodity-based swing options. *Management science*, *50*(7), 909–921.

Jain, A. K., & Cox, R. A. (2011). Airfare price insurance: a real option model. *Journal of Risk Finance, The*, *12*(1), 5–14.

Keppo, J. (2004). Pricing of electricity swing options. *The Journal of Derivatives*, *11*(3), 26–43.

Kimes, S. E. (1989). Yield management: a tool for capacity-considered service firms. *Journal of operations management*, *8*(4), 348–363.

Littlewood, K. (1972). Forecasting and control of passenger bookings, agifors 12th annual symposium proceedings. *October, Nathanya, Israel*, 193–204.

Longstaff, F. A., & Schwartz, E. S. (2001). Valuing american options by simulation: A simple least-squares approach. *Review of Financial studies*, *14*(1), 113–147.

Maidanov, S. A. (2010). Monte carlo european options pricing implementation using various industry library solutions. *Newyork City, USA: INTEL*.

McGill, J. I., & Van Ryzin, G. J. (1999). Revenue management: Research overview and prospects. *Transportation science*, *33*(2), 233–256.

Merton, R. C. (1973). Theory of rational option pricing. *The Bell Journal of Economics and Management Science*, 141–183.

O'Connor, W. E. (2001). *An introduction to airline economics*. Greenwood Publishing Group.

Richardson, M. (2009). Numerical methods for option pricing. *Unpublished notes*.

Rogers, L. C. (2002). Monte carlo valuation of american options. *Mathematical Finance*, *12*(3), 271–286.

Sainam, P., Balasubramanian, S., & Bayus, B. L. (2010). Consumer options: Theory and an empirical application to a sports market. *Journal of Marketing Research*, *47*(3), 401–414.

Smith, B. C., Leimkuhler, J. F., & Darrow, R. M. (1992). Yield management at american airlines. *Interfaces*, *22*(1), 8–31.

Stone, P., & Greenwald, A. (2005). The first international trading agent competition: Autonomous bidding agents. *Electronic Commerce Research*, *5*(2), 229–265.

Thompson, A. C. (1995). Valuation of path-dependent contingent claims with multiple exercise decisions over time: The case of take-or-pay. *Journal of Financial and Quantitative Analysis*, *30*(02), 271–293.

Trigeorgis, L. (1996). *Real options: Managerial flexibility and strategy in resource allocation.* MIT Press (Cambridge, Mass.).

Voneche, F. (2005). Yield management in the airline industry.

Walker, J. S., Case, T. S., Jorasch, J. A., & Sparico, T. M. (1998, August 18). *Method, apparatus, and program for pricing, selling, and exercising options to purchase airline tickets.* Google Patents. (US Patent 5,797,127)

Weatherford, L. R., Bodily, S. E., & Pfeifer, P. E. (1993). Modeling the customer arrival process and comparing decision rules in perishable asset revenue management situations. *Transportation Science*, *27*(3), 239–251.

Wellman, M. P., Reeves, D. M., & Lochner, K. M. (2003). Price prediction in a trading agent competition [extended abstract]. In *Proceedings of the 4th acm conference on electronic commerce* (pp. 216–217).

Wright, C. P., Groenevelt, H., & Shumsky, R. A. (2010). Dynamic revenue management in airline alliances. *Transportation Science*, *44*(1), 15–37.