Exercise 2, Lecture slides p. 45 (Ex. 1 p. 19 cont'd)

1.) Calculate ES_i and EF_i with the LCA

iter / i∈V	Init		1		2		3		4		5		6		7		RESULT		EF_i
	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	d_{oi}	p_{oi}	
0	0	0															0	0	0
1	-∞	-1	0	0													0	0	6
2	-∞	-1			1	1											1	1	5
3	-∞	-1	0	0													0	0	2
4	-∞	-1					2	3	3	5							3	5	7
5	-∞	-1	8	0													8	0	10
6	-∞	-1			6	1			10	5							10	5	10
Q	⟨0⟩		⟨1,3,5⟩		⟨3,5,2,6⟩		⟨5,2,6,4⟩		⟨2,6,4⟩		⟨6,4⟩		⟨4⟩		⟨⟩		⟨⟩		

$ES_i = d_{oi}$

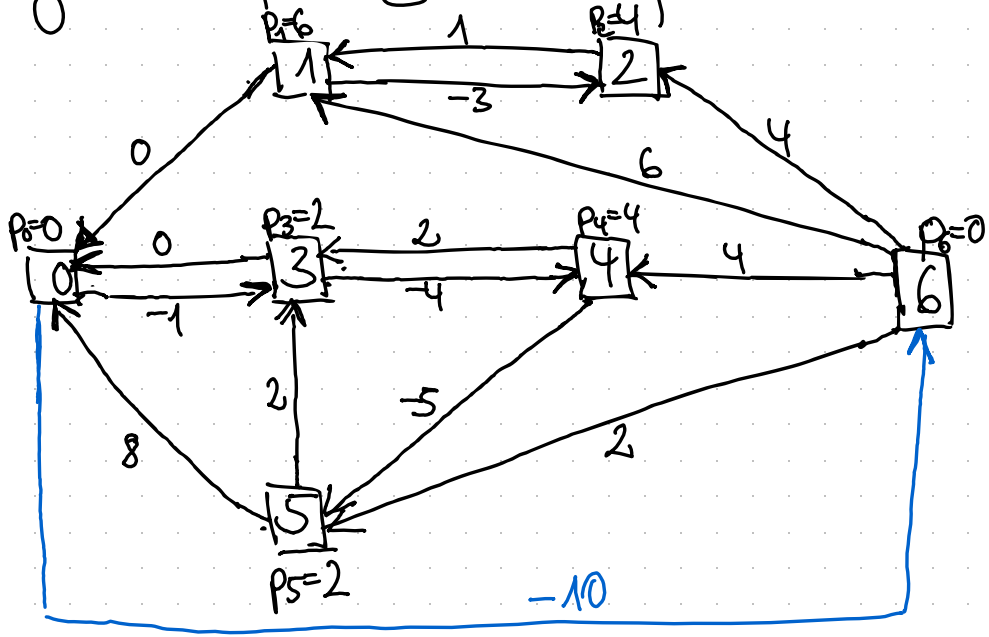
$EF_i = d_{oi} + p_i$

2.) Assume $LF_6 = EF_6 = 10$ and calculate latest start and finish times with the LCA

→ To compute latest start and finish times with the LCA, we first have to modify our network

- add an arc $(6,0)$ with $\delta_{60} = -LF_6 = -10$
- reverse the direction of all arcs in our network

→ This gives the following auxiliary network $G^{\#}$:



→ Now, you apply the standard steps of the LCA to this modified network:

iter / i∈V	Init		1		2		3		4		5		6		7		RESULT		LF_i
	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	
0	0	0															0	0	0
1	-∞	-1			-4	6											-4	6	10
2	-∞	-1			-6	6											-6	6	10
3	-∞	-1	-1	0													-1	0	3
4	-∞	-1			-6	6	-5	3									-5	3	9
5	-∞	-1			-8	6											-8	6	10
6	-∞	-1	-10	0													-10	0	10
Q	⟨0]		⟨6,3]		⟨3,1,2,4,5]		⟨1,2,4,5]		⟨2,4,5]		⟨4,5]		⟨5]		⟨]		⟨]		

next, we pick node 6 and consider its successors (=predecessors in original G) - that's what I meant with "backwards"...

$LS_i := -doi$

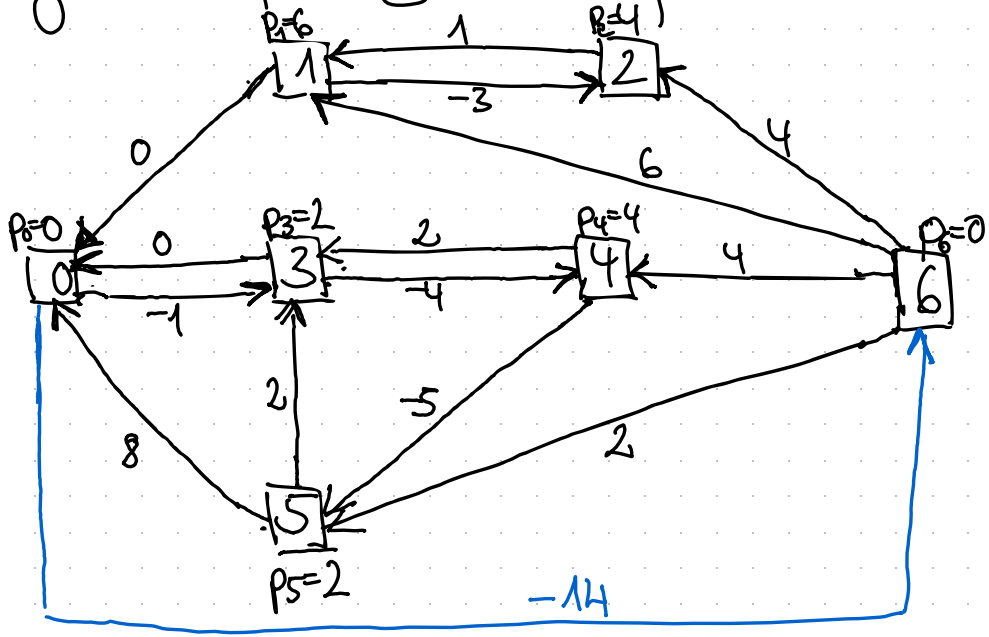
$LF_i := LS_i + p_i$

3.) Now, consider $LF_6 = 14$ and calculate latest start and finish times with the LCA

→ To compute latest start and finish times with the LCA, we first have to modify our network

- add an arc $(6,0)$ with $\delta_{60} = -LF_6 = -14$
- reverse the direction of all arcs in our network

→ This gives the following auxiliary network $G^{\#}$:



→ Now, you apply the standard steps of the LCA to this modified network:

iter / i∈V	Init		1		2		3		4		5		6		7		RESULT		LF_i
	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	doi	poi	
0	0	0															0	0	0
1	-∞	-1			-8	6											-8	6	14
2	-∞	-1			-10	6											-10	6	14
3	-∞	-1	-1	0													-1	0	3
4	-∞	-1			-10	6	-5	3									-5	3	9
5	-∞	-1			-12	6							-10	4			-10	4	12
6	-∞	-1	-14	0													-14	0	14
Q	⟨0]		⟨6,3]		⟨3,1,2,4,5]		⟨1,2,4,5]		⟨2,4,5]		⟨4,5]		⟨5]		⟨]		⟨]		

next, we pick node 6 and consider its successors (=predecessors in original G) - that's what I meant with "backwards" ...

$LS_i := -doi$

$LF_i := LS_i + p_i$

4.) Assume $LF_6 = 14$ and calculate earliest and latest start times with the FWA

Init:

(Notation d_{ij} p_{ij})

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	$-\infty$ -1	0 0	$-\infty$ -1	8 0	$-\infty$ -1
1	$-\infty$ -1	0 1	1 1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	6 1
2	$-\infty$ -1	-3 2	0 2	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	4 2
3	-1 3	$-\infty$ -1	$-\infty$ -1	0 3	2 3	2 3	$-\infty$ -1
4	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	-4 4	0 4	$-\infty$ -1	4 4
5	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	-5 5	0 5	2 5
6	-14 6	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	0 6

Step 1 : $v=0$ (Notation ~~d_{ij} p_{ij}~~)

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	$-\infty$ -1	0 0	$-\infty$ -1	8 0	$-\infty$ -1
1	$-\infty$ -1	0 1	1 1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	6 1
2	$-\infty$ -1	-3 2	0 2	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	4 2
3	-1 3	-1 0	$-\infty$ -1	0 3	2 3	7 0	$-\infty$ -1
4	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	-4 4	0 4	$-\infty$ -1	4 4
5	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	-5 5	0 5	2 5
6	-14 6	-14 0	$-\infty$ -1	-14 0	$-\infty$ -1	-6 0	0 6

Step 2 : v=1 (Notation ~~dij pij~~)

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	1 1	0 0	-∞ -1	8 0	6 1
1	-∞ -1	0 1	1 1	-∞ -1	-∞ -1	-∞ -1	6 1
2	-∞ -1	-3 2	0 2	-∞ -1	-∞ -1	-∞ -1	4 2
3	-1 3	-1 0	0 1	0 3	2 3	7 0	5 1
4	-∞ -1	-∞ -1	-∞ -1	-4 4	0 4	-∞ -1	4 4
5	-∞ -1	-∞ -1	-∞ -1	-∞ -1	-5 5	0 5	2 5
6	-14 6	-14 0	-13 1	-14 0	-∞ -1	-6 0	0 6

Step 3: v=2 (Notation ~~dij pij~~)

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	1 1	0 0	∞ -1	8 0	6 1
1	∞ -1	0 1	1 1	∞ -1	∞ -1	∞ -1	6 1
2	∞ -1	-3 2	0 2	∞ -1	∞ -1	∞ -1	4 2
3	-1 3	-1 0	0 1	0 3	2 3	7 0	5 1
4	∞ -1	∞ -1	∞ -1	-4 4	0 4	∞ -1	4 4
5	∞ -1	∞ -1	∞ -1	∞ -1	-5 5	0 5	2 5
6	-14 6	-14 0	-13 1	-14 0	∞ -1	-6 0	0 6

Step 4 : v=3 (Notation ~~d_{ij} p_{ij}~~)

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	1 1	0 0	2 3	8 0	6 1
1	-∞ -1	0 1	1 1	-∞ -1	-∞ -1	-∞ -1	6 1
2	-∞ -1	-3 2	0 2	-∞ -1	-∞ -1	-∞ -1	4 2
3	-1 3	-1 0	0 1	0 3	2 3	7 0	5 1
4	-5 3	-5 3	-4 3	-4 4	0 4	3 3	4 4
5	-∞ -1	-∞ -1	-∞ -1	-∞ -1	-5 5	0 5	2 5
6	-14 6	-14 0	-13 1	-14 0	-12 3	-6 0	0 6

Step 5: $v = 4$ (Notation ~~d_{ij} p_{ij}~~)

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	1 1	0 0	2 3	8 0	6 1
1	$-\infty$ -1	0 1	1 1	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	6 1
2	$-\infty$ -1	-3 2	0 2	$-\infty$ -1	$-\infty$ -1	$-\infty$ -1	4 2
3	-1 3	-1 0	0 1	0 3	2 3	7 0	6 4
4	-5 3	-5 3	-4 3	-4 4	0 4	3 3	4 4
5	-10 4	-10 4	-9 4	-9 4	-5 5	0 5	2 5
6	-14 6	-14 0	-13 1	-14 0	-12 3	-6 0	0 6

Step 6: v=5 (Notation ~~dij pij~~)

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	1 1	0 0	3 5	8 0	10 5
1	-∞ -1	0 1	1 1	-∞ -1	-∞ -1	-∞ -1	6 1
2	-∞ -1	-3 2	0 2	-∞ -1	-∞ -1	-∞ -1	4 2
3	-1 3	-1 0	0 1	0 3	2 3	7 0	9 5
4	-5 3	-5 3	-4 3	-4 4	0 4	3 3	5 5
5	-10 4	-10 4	-9 4	-9 4	-5 5	0 5	2 5
6	-14 6	-14 0	-13 1	-14 0	-11 5	-6 0	0 6

Step 7 $v=6$ (Notation d_{ij} p_{ij})

i \ j	0	1	2	3	4	5	6
0	0 0	0 0	1 1	0 0	3 5	8 0	10 5
1	-8 6	0 1	1 1	-8 6	-5 6	0 6	6 1
2	-10 6	-3 2	0 2	-10 6	-7 6	-2 6	4 2
3	-1 3	-1 0	0 1	0 3	2 3	7 0	9 5
4	-5 3	-5 3	-4 3	-4 4	0 4	3 3	5 5
5	-10 4	-10 4	-9 4	-9 4	-5 5	0 5	2 5
6	-14 6	-14 0	-13 1	-14 0	-11 5	-6 0	0 6

→ we iterated over all nodes and can now terminate.

→ from this final table we can derive $FS_i = d_{0i}$ and $LS_i = -d_{i,0}$ $\forall i \in V$