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Complex Scheduling in Manufacturing and Services

Solution Worksheet 1

1 Time scheduling

1.1 General time lag relationships

Given is the following project plan as a directed graph $G = (\mathcal{V}, \mathcal{E})$ with a set of activities $\mathcal{V}$ and time lag relationships $\mathcal{E}$. Activities $i \in \mathcal{V}$ have the duration d_i and are connected through relationships $(i, j) \in \mathcal{E}$ with minimum time lags $\underline{\delta}_{ij}^{FS}$ (Finish-to-start), $\underline{\delta}_{ij}^{SS}$ (Start-to-start), $\underline{\delta}_{ij}^{FF}$ (Finish-to-finish) or $\underline{\delta}_{ij}^{SF}$ (Start-to-finish).

i	d_i	Time lag relationships	δ_{ij} (Solution for a))
0	0	$\underline{\delta}_{0,1}^{SS} = 0$, $\underline{\delta}_{0,2}^{SS} = 0$, $\underline{\delta}_{0,3}^{SS} = 0$	$\delta_{0,1} = 0$, $\delta_{0,2} = 0$, $\delta_{0,3} = 0$
1	5	$\underline{\delta}_{1,4}^{SS} = 3$	$\delta_{1,4} = 3$
2	7	$\underline{\delta}_{2,5}^{FS} = 0$, $\underline{\delta}_{2,6}^{FF} = 12$	$\delta_{2,5} = 7$, $\delta_{2,6} = 13$
3	4	$\underline{\delta}_{3,7}^{SF} = 2$	$\delta_{3,7} = -2$
4	6	$\underline{\delta}_{4,8}^{FS} = -4$, $\underline{\delta}_{4,9}^{SF} = 12$	$\delta_{4,8} = 2$, $\delta_{4,9} = 4$
5	9	$\underline{\delta}_{5,8}^{FF} = 10$, $\underline{\delta}_{5,9}^{FS} = 8$	$\delta_{5,8} = 13$, $\delta_{5,9} = 17$
6	6	$\underline{\delta}_{6,10}^{SS} = 6$	$\delta_{6,10} = 6$
7	4	$\underline{\delta}_{7,10}^{FS} = 3$	$\delta_{7,10} = 7$
8	6	$\underline{\delta}_{8,10}^{FF} = 7$	$\delta_{8,10} = 4$
9	8	$\underline{\delta}_{9,11}^{SF} = 20$	$\delta_{9,11} = 10$
10	9	$\underline{\delta}_{10,l2}^{FS} = -1$	$\delta_{10,12} = 8$
11	10	$\underline{\delta}_{11,13}^{FS} = 0$	$\delta_{11,13} = 10$
12	9	$\underline{\delta}_{12,13}^{FS} = 0$	$\delta_{12,13} = 9$
13	0		

a) Transform all time lag relationships into Start-to-start time lags δ_{ij}.

b) Write down the earliest possible start time (ES_i) for every activity $i \in \mathcal{V}$

$$ES_i = \begin{cases} 0 & i = 0 \\ \max_{j:(j,i)\in\mathcal{E}}(ES_j + \delta_{ji}) & i \in \mathcal{V} \setminus \{0\} \end{cases}$$

c) Determine the minimal total duration of the project plan (ES_{13}). For every activity $i \in \mathcal{V}$ write down the latest start time (LS_i) so that the calculated total duration of the project plan will not be exceeded.

$$LS_i = \begin{cases} \min_{j:(i,j)\in\mathcal{E}}(LS_j - \delta_{ij}) & i \in \mathcal{V} \setminus \{13\} \\ ES_i & i = 13 \end{cases}$$

d) For every activity $i \in \mathcal{V}$, calculate

- *total float*: $TF_i = LS_i - ES_i$
- *early free float*: $EFF_i = \min_{j:(i,j)\in\mathcal{E}}(ES_j - \delta_{ij}) - ES_i$
- *late free float*:$LFF_i = LS_i - \max_{j:(j,i)\in\mathcal{E}}(LS_j + \delta_{ji})$

For activities $i = 0$ and $i = 13$, all float times are 0.

e) Research in Zimmermann et al. (2010) or Demeulemeester and Herroelen (2002) the meaning of the float times and the meaning of the *critical path*. Which activities of the project plan can be found on the critical path?

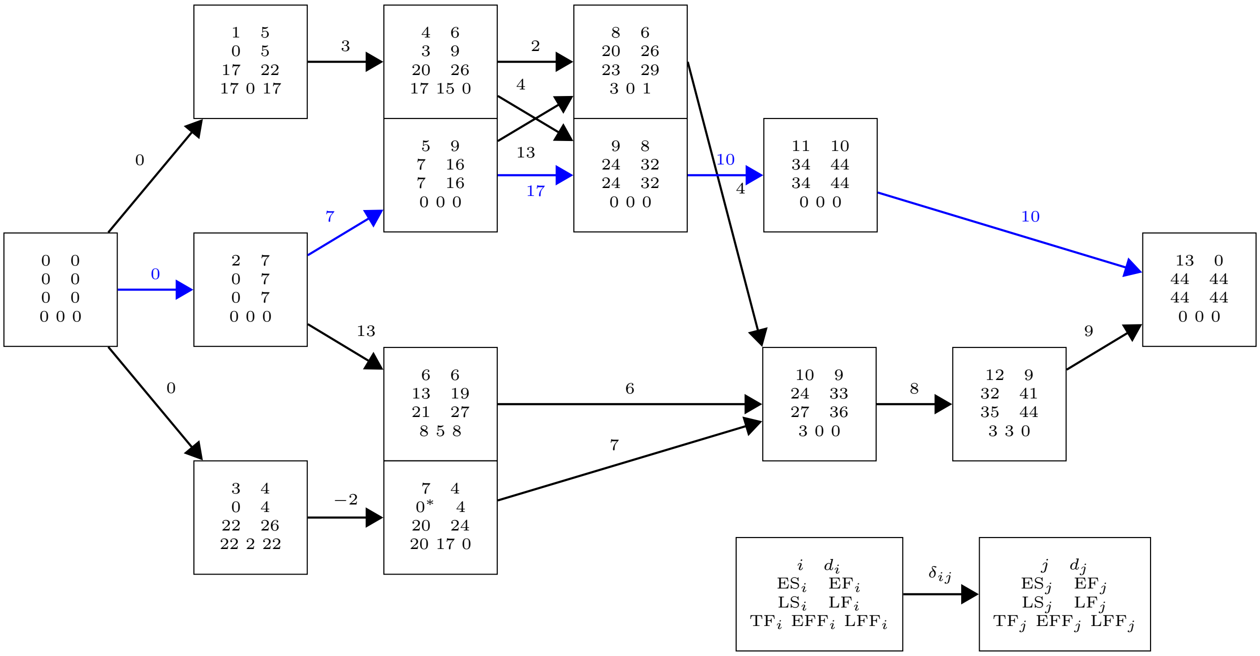
Total float (TF_i) is the maximum amount of time by which the start of activity i can be delayed beyond its earliest start time ES_i without delaying the latest completion of the project.

Early free float (EFF_i) is the maximum amount of time by which the earliest start of activity i (ES_i) can be increased under the requirement that all immediately succeeding activities j can be started at their earliest start times ES_j.

Late free float (LFF_i) is the maximum amount of time by which the latest start of activity i (LS_i) can be decreased under the requirement that all immediately preceeding activities j started at their latest start times LS_j.

Activities i on the critical path are all critical and can not be delayed. They must start at $S_i = ES_i = LS_i$.

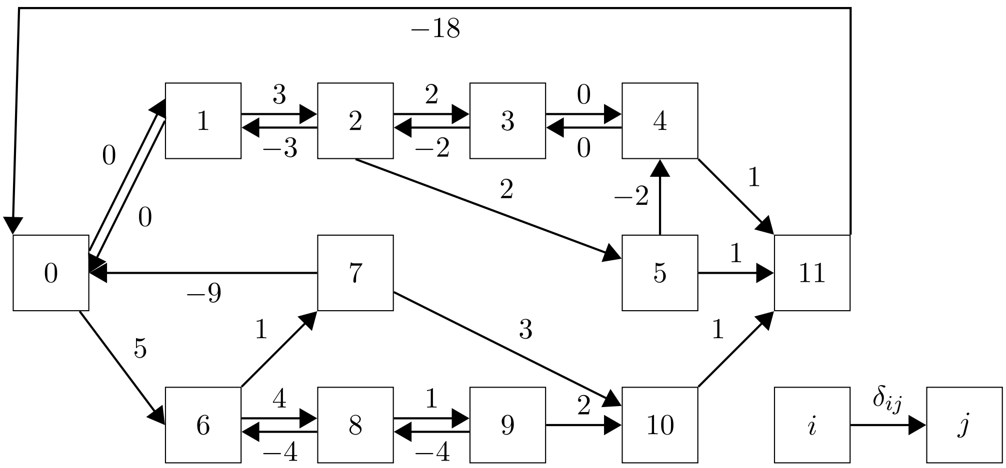
Solution for b) - e):



- * All starting points are assumed to be greater or equal to 0.
- The critical path is highlighted in blue.

1.2 Label-Correcting-Algorithm (LCA)

Given is the following project plan consisting of activities $i \in \mathcal{V}$ and Start-to-start time lags $(i, j) \in \mathcal{E}$ with arc weights δ_{ij}.



a) Determine the longest paths from node $r = 0$ to all other nodes using the *Label-Correcting-Algorithm*. For the initialization and all iterations of the Label-Correcting-Algorithm, write down the distance values d_{ri}, the indices of the predecessor nodes p_{ri} as well as the current queue Q in the following table.

Iteration	Init.	
i	d_{ri}	p_{ri}
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
Q		

b) Reverse all arcs in the project plan and determine the longest paths from node $r = 0$ to all the other nodes.

Iter.	Init.		1		2		3		4		5		6		7		8		9		10		11		12		13		14	
i	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	$-\infty$	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	$-\infty$	-1	$-\infty$	-1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1	3	1
3	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2
4	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	5	3	5	3	5	3	5	3	5	3	5	3	5	3	5	3
5	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2	5	2
6	$-\infty$	-1	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0
7	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
8	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	9	6	9	6	9	6	9	6	9	6	9	6	9	6	9	6	9	6	9	6	9	6	9	6
9	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	10	8	10	8	10	8	10	8	10	8	10	8	10	8	10	8	10	8
10	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	9	7	9	7	9	7	9	7	9	7	12	9	12	9	12	9	12	9	12	9
11	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	6	5	10	10	10	10	10	10	10	10	13	10	13	10
Q	$\{0\}$		$\{1,6\}$		$\{6,2\}$		$\{2,7,8\}$		$\{7,8,3,5\}$		$\{8,3,5,10\}$		$\{3,5,10,9\}$		$\{5,10,9,4\}$		$\{10,9,4,11\}$		$\{9,4,11\}$		$\{4,11,10\}$		$\{11,10\}$		$\{10\}$		$\{11\}$		$\emptyset$	

Iter.	Init.		1		2		3		4		5		6		7		8		9		10		11		12		13		14	
i	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}	d_{ri}	p_{ri}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	$-\infty$	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	$-\infty$	-1	$-\infty$	-1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1	-3	1
3	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-5	2	-5	2	-5	2	-5	2	-5	2	-5	2	-5	2	-5	2	-5	2	-5	2
4	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-5	3	-5	3	-5	3	-5	3	-5	3
5	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-7	4	-7	4
6	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-8	7	-8	7	-8	7	-8	7	-8	7	-8	7	-8	7	-8	7	-8	7	-8	7	-8	7
7	$-\infty$	-1	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0	-9	0
8	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-12	6	-12	6	-12	6	-12	6	-12	6	-12	6
9	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-15	10	-15	10	-15	10	-15	10	-15	10	-15	10	-15	10	-15	10
10	$-\infty$	-1	$-\infty$	-1	$-\infty$	-1	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11	-17	11
11	$-\infty$	-1	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0	-18	0
Q	$\{0\}$		$\{1,11,7\}$		$\{11,7,2\}$		$\{7,2,5,10,4\}$		$\{2,5,10,4,6\}$		$\{5,10,4,6,3\}$		$\{10,4,6,3\}$		$\{4,6,3,9\}$		$\{6,3,9\}$		$\{3,9,8\}$		$\{9,8,4\}$		$\{8,4\}$		$\{4\}$		$\{5\}$		$\emptyset$	

c) Write down the earliest and latest start time for every activity $i \in \mathcal{V}$ as well as the total float time.

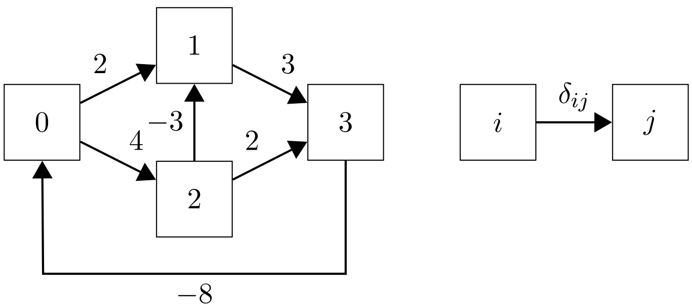
i	ES_i	LS_i	TF_i
0	0	0	0
1	0	0	0
2	3	3	0
3	5	5	0
4	5	5	0
5	5	7	2
6	5	8	3
7	6	9	3
8	9	12	3
9	10	15	5
10	12	17	5
11	13	18	5

1.3 Floyd-Warshall-Algorithm (FWA)

Given is the following project plan consisting of activities and Start-to-start time lags

Activity	Immediate predecessor	Time lag (arc weight)
0	–	
1	0	$\delta_{0,1} = 2$
1	2	$\delta_{2,1} = -3$
2	0	$\delta_{0,2} = 4$
3	1	$\delta_{1,3} = 3$
3	2	$\delta_{2,3} = 2$

a) The maximum project duration is 8 time units. Draw the resulting project network plan.



b) Initialize the distance matrix for the FW-Algorithm.

	0	1	2	3
0	0	2	4	$-\infty$
1	$-\infty$	0	$-\infty$	3
2	$-\infty$	-3	0	2
3	-8	$-\infty$	$-\infty$	0

c) The following matrix shows the distance matrix after the first main step for $v = 0$:

	0	1	2	3
0	0	2	4	$-\infty$
1	$-\infty$	0	$-\infty$	3
2	$-\infty$	-3	0	2
3	-8	-6	-4	0

Perform the second main step of the FW-Algorithm for $v = 1$.

	0	1	2	3
0	0	2	4	5
1	$-\infty$	0	$-\infty$	3
2	$-\infty$	-3	0	2
3	-8	-6	-4	0

d) How do you recognize that there is a valid/feasible solution? Why can a project plan be invalid/infeasible?

- If there is at least one value greater than 0 on the main diagonal, then a cycle of positive length exists. Cycles of positive length indicate infeasible project plans.
- A project plan can be infeasible, if the maximum allowed project duration is shorter than the minimum required project length (makespan).

e) The distance matrix at the end of the FW-algorithm looks like this:

	0	1	2	3
0	0	2	4	6
1	-5	0	-1	3
2	-6	-3	0	2
3	-8	-6	-4	0

For every activity $i \in \mathcal{V}$, write down the earliest and latest start time as well as the total float time.

i	ES_i	LS_i	TF_i
0	0	0	0
1	2	5	3
2	4	6	2
3	6	8	2

References

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