

Theorem. If

$$T(n) = aT\left(\left\lceil \frac{n}{b} \right\rceil\right) + O(n^d)$$

for constants $a > 0$, $b > 1$, $d \geq 0$, then:

$$T(n) = \begin{cases} O(n^d), & \text{if } d > \log_b a, \\ O(n^d \log n), & \text{if } d = \log_b a, \\ O(n^{\log_b a}), & \text{if } d < \log_b a \end{cases}.$$

Example.

$$T(n) = 4T\left(\frac{n}{2}\right) + O(n) \Rightarrow T(n) = O(n^2).$$

$$T(n) = 3T\left(\frac{n}{2}\right) + O(n) \Rightarrow T(n) = O(n^{\log_2 3}).$$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n) \Rightarrow T(n) = O(n \log n).$$

$$T(n) = T\left(\frac{n}{2}\right) + O(1) \Rightarrow T(n) = O(\log n).$$