

# Team notebook

HCMUS-PenguinSpammers

February 3, 2022

<b>Contents</b>			
<b>1 Algorithms</b>	<b>1</b>	<b>5 Geometry</b>	<b>8</b>
1.1 Mo's Algorithm	1	5.1 Closest Pair Problem	8
1.2 Mo's Algorithms on Trees	1	5.2 Convex Diameter	8
1.3 Parallel Binary Search	1	5.3 Pick Theorem	9
		5.4 Square	9
		5.5 Triangle	10
<b>2 Combinatorics</b>	<b>2</b>	<b>6 Graphs</b>	<b>10</b>
2.1 Factorial Approximate	2	6.1 Bridges	10
2.2 Factorial	2	6.2 Dijkstra	10
2.3 Fast Fourier Transform	2	6.3 Directed MST	11
2.4 General purpose numbers	3	6.4 Edge Coloring	11
2.5 Lucas Theorem	3	6.5 Eulerian Path	11
2.6 Multinomial	4	6.6 Floyd - Warshall	12
2.7 Others	4	6.7 Ford - Bellman	12
2.8 Permutation To Int	4	6.8 Gomory Hu	12
2.9 Sigma Function	4	6.9 Karp Min Mean Cycle	12
		6.10 Konig's Theorem	13
<b>3 Data Structures</b>	<b>4</b>	6.11 LCA	13
3.1 Binary Index Tree	4	6.12 Math	13
3.2 Disjoint Set Union (DSU)	5	6.13 Minimum Path Cover in DAG	13
3.3 Fake Update	5	6.14 Planar Graph (Euler)	13
3.4 Fenwick Tree	5	6.15 Push Relabel	13
3.5 Hash Table	5	6.16 SCC Kosaraju	14
3.6 Range Minimum Query	6	6.17 Tarjan SCC	14
3.7 STL Treap	6	6.18 Topological Sort	15
3.8 Segment Tree	6		
3.9 Sparse Table	6	<b>7 Linear Algebra</b>	<b>15</b>
3.10 Trie	7	7.1 Matrix Determinant	15
		7.2 Matrix Inverse	15
<b>4 Dynamic Programming Optimization</b>	<b>7</b>	7.3 PolyRoots	15
4.1 Convex Hull Trick	7	7.4 Polynomial	15
4.2 Divide and Conquer	7		
		<b>8 Misc</b>	<b>15</b>
		8.1 Dates	15
		8.2 Debugging Tricks	16
		8.3 Interval Container	16
		8.4 Optimization Tricks	16
		8.4.1 Bit hacks	16
		8.4.2 Pragmas	16
		8.5 Ternary Search	16
		<b>9 Number Theory</b>	<b>16</b>
		9.1 Chinese Remainder Theorem	16
		9.2 Convolution	17
		9.3 Diophantine Equations	17
		9.4 Discrete Logarithm	17
		9.5 Ext Euclidean	18
		9.6 Fast Eratosthenes	18
		9.7 Highest Exponent Factorial	18
		9.8 Miller - Rabin	18
		9.9 Mod Integer	18
		9.10 Mod Inv	19
		9.11 Mod Mul	19
		9.12 Mod Pow	19
		9.13 Number Theoretic Transform	19
		9.14 Pollard Rho Factorize	19
		9.15 Primes	19
		9.16 Totient Sieve	20
		9.17 Totient	20
		<b>10 Probability and Statistics</b>	<b>20</b>
		10.1 Continuous Distributions	20
		10.1.1 Uniform distribution	20
		10.1.2 Exponential distribution	20
		10.1.3 Normal distribution	20
		10.2 Discrete Distributions	20

10.2.1 Binomial distribution . . . . .	20
10.2.2 First success distribution . . . . .	21
10.2.3 Poisson distribution . . . . .	21
10.3 Probability Theory . . . . .	21

<b>11 Strings</b>	<b>21</b>
11.1 Hashing . . . . .	21
11.2 Incremental Aho Corasick . . . . .	21
11.3 KMP . . . . .	22
11.4 Minimal String Rotation . . . . .	22
11.5 Suffix Array . . . . .	22
11.6 Suffix Automation . . . . .	23
11.7 Suffix Tree . . . . .	23
11.8 Z Algorithm . . . . .	24

## 1 Algorithms

### 1.1 Mo's Algorithm

---

```

/*
  https://www.spoj.com/problems/FREQ2/
*/
vector<int> MoQueries(int n, vector<query> Q){
    block_size = sqrt(n);
    sort(Q.begin(), Q.end(), [](const query &A, const
        query &B){
        return (A.l/block_size != B.l/block_size)?
            (A.l/block_size < B.l/block_size) : (A.r <
                B.r);
    });
    vector<int> res;
    res.resize((int)Q.size());

    int L = 1, R = 0;
    for(query q: Q){
        while (L > q.l) add(--L);
        while (R < q.r) add(++R);

        while (L < q.l) del(L++);
        while (R > q.r) del(R--);

        res[q.pos] = calc(1, R-L+1);
    }
    return res;
}

```

---

### 1.2 Mo's Algorithms on Trees

---

```

/*
  Given a tree with N nodes and Q queries. Each node has
  an integer weight.
  Each query provides two numbers u and v, ask for how
  many different integers weight of nodes
  there are on path from u to v.

```

-----  
 Modify DFS:  
 -----

For each node u, maintain the start and the end DFS  
 time. Let's call them ST(u) and EN(u).  
 => For each query, a node is considered if its  
 occurrence count is one.

-----  
 Query solving:  
 -----

Let's query be (u, v). Assume that ST(u) <= ST(v).  
 Denotes P as LCA(u, v).

Case 1: P = u  
 Our query would be in range [ST(u), ST(v)].

Case 2: P != u  
 Our query would be in range [EN(u), ST(v)] + [ST(p),  
 ST(p)]  
 \*/

```

void update(int &L, int &R, int qL, int qR){
    while (L > qL) add(--L);
    while (R < qR) add(++R);

    while (L < qL) del(L++);
    while (R > qR) del(R--);
}

```

```

vector<int> MoQueries(int n, vector<query> Q){
    block_size = sqrt((int)nodes.size());
    sort(Q.begin(), Q.end(), [](const query &A, const
        query &B){
        return (ST[A.l]/block_size !=
            ST[B.l]/block_size)? (ST[A.l]/block_size <
                ST[B.l]/block_size) : (ST[A.r] < ST[B.r]);
    });
    vector<int> res;
    res.resize((int)Q.size());

```

```

    LCA lca;
    lca.initialize(n);

```

```

int L = 1, R = 0;
for(query q: Q){
    int u = q.l, v = q.r;
    if(ST[u] > ST[v]) swap(u, v); // assume that
        S[u] <= S[v]
    int parent = lca.get(u, v);

    if(parent == u){
        int qL = ST[u], qR = ST[v];
        update(L, R, qL, qR);
    }else{
        int qL = EN[u], qR = ST[v];
        update(L, R, qL, qR);
        if(cnt_val[a[parent]] == 0)
            res[q.pos] += 1;
    }

    res[q.pos] += cur_ans;
}
return res;
}

```

---

### 1.3 Parallel Binary Search

---

```

int lo[N], mid[N], hi[N];
vector<int> vec[N];

void clear() //Reset
{
    memset(bit, 0, sizeof(bit));
}

void apply(int idx) //Apply ith update/query
{
    if(ql[idx] <= qr[idx])
        update(ql[idx], qa[idx]),
        update(qr[idx]+1, -qa[idx]);

    else
    {
        update(1, qa[idx]);
        update(qr[idx]+1, -qa[idx]);
        update(ql[idx], qa[idx]);
    }
}

bool check(int idx) //Check if the condition is
    satisfied
{
    int req=reqd[idx];
    for(auto &it:owns[idx])

```

```

    {
        req-=pref(it);
        if(req<0)
            break;
    }
    if(req<=0)
        return 1;
    return 0;
}

void work()
{
    for(int i=1;i<=q;i++)
        vec[i].clear();
    for(int i=1;i<=n;i++)
        if(mid[i]>0)
            vec[mid[i]].push_back(i);
    clear();
    for(int i=1;i<=q;i++)
    {
        apply(i);
        for(auto &it:vec[i]) //Add appropriate
            check conditions
        {
            if(check(it))
                hi[it]=i;
            else
                lo[it]=i+1;
        }
    }
}

void parallel_binary()
{
    for(int i=1;i<=n;i++)
        lo[i]=1, hi[i]=q+1;
    bool changed = 1;
    while(changed)
    {
        changed=0;
        for(int i=1;i<=n;i++)
        {
            if(lo[i]<hi[i])
            {
                changed=1;
                mid[i]=(lo[i] + hi[i])/2;
            }
            else
                mid[i]=-1;
        }
        work();
    }
}

```

## 2 Combinatorics

### 2.1 Factorial Approximate

Approximate Factorial:

$$n! = \sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n \quad (1)$$

### 2.2 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

### 2.3 Fast Fourier Transform

```

/**
 * Fast Fourier Transform.
 * Useful to compute convolutions.
 * computes:
 *   C(f star g)[n] = sum_m(f[m] * g[n - m])
 * for all n.
 * test: icpc live archive, 6886 - Golf Bot
 */

```

```

using namespace std;
#include <bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'

```

```

const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];

```

```

const double PI = acos(-1.0);

```

```

struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
    cpx(){}
};

```

```

cpx operator + (const cpx &c1, const cpx &c2) {
    return cpx(c1.real + c2.real, c1.image + c2.image);
}

```

```

cpx operator - (const cpx &c1, const cpx &c2) {
    return cpx(c1.real - c2.real, c1.image - c2.image);
}

```

```

cpx operator * (const cpx &c1, const cpx &c2) {
    return cpx(c1.real*c2.real - c1.image*c2.image,
        c1.real*c2.image + c1.image*c2.real);
}

```

```

int rev(int id, int len) {
    int ret = 0;
    for (int i = 0; (1 << i) < len; i++) {
        ret <<= 1;
        if (id & (1 << i)) ret |= 1;
    }
    return ret;
}

```

```

cpx A[1 << 20];

```

```

void FFT(cpx *a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT * 2 *
            PI / m));
        for(int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for(int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
    }
    if (DFT == -1) for (int i = 0; i < len; i++)
        A[i].real /= len, A[i].image /= len;
    for (int i = 0; i < len; i++) a[i] = A[i];
    return;
}

```

```

cpx in[1 << 20];

```

```

void solve(int n) {
    memset(d, 0, sizeof d);
    int t;
}

```

```

for (int i = 0; i < n; ++i) {
    cin >> t;
    d[t] = true;
}
int m;
cin >> m;
vector<int> q(m);
for (int i = 0; i < m; ++i)
    cin >> q[i];

for (int i = 0; i < MN; ++i) {
    if (d[i])
        in[i] = cpx(1, 0);
    else
        in[i] = cpx(0, 0);
}

FFT(in, MN, 1);
for (int i = 0; i < MN; ++i) {
    in[i] = in[i] * in[i];
}
FFT(in, MN, -1);

int ans = 0;
for (int i = 0; i < q.size(); ++i) {
    if (in[q[i]].real > 0.5 || d[q[i]]) {
        ans++;
    }
}
cout << ans << endl;
}

int main() {
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int n;
    while (cin >> n)
        solve(n);
    return 0;
}

```

## 2.4 General purpose numbers

### Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).

$B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

### Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1) \dots (x+n-1)$$

$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$

### Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

### Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$  For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

### Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$

# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$

# with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

### Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$

[noitemsep]sub-diagonal monotone paths in an  $n \times n$  grid. strings with  $n$  pairs of parenthesis, correctly nested. binary trees with  $n+1$  leaves (0 or 2 children). ordered trees with  $n+1$  vertices. ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines. permutations of  $[n]$  with no 3-term increasing subseq.

## 2.5 Lucas Theorem

For non-negative integers  $m$  and  $n$  and a prime  $p$ , the following congruence relation holds:

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where:

$$m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0,$$

and:

$$n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$$

are the base  $p$  expansions of  $m$  and  $n$  respectively. This uses the convention that  $\binom{m}{n} = 0$  if  $m \leq n$ .

## 2.6 Multinomial

```

/**
 * Description: Computes  $\displaystyle \binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(k_1 + \dots + k_n)!}{k_1! k_2! \dots k_n!}$ .
 * Status: Tested on kattis:lexicography
 */
#pragma once

long long multinomial(vector<int>& v) {
    long long c = 1, m = v.empty() ? 1 : v[0];
    for (long long i = 1; i < v.size(); i++) {

```

```

    for (long long j = 0; j < v[i]; j++) {
        c = c * ++m / (j + 1);
    }
    return c;
}

```

---

## 2.7 Others

**Cycles** Let  $g_S(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

**Derangements** Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

**Burnside's lemma** Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = Z_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

## 2.8 Permutation To Int

```

/**
 * Description: Permutation -> integer conversion. (Not
 *              order preserving.)
 * Integer -> permutation can use a lookup table.
 * Time: O(n)
 */
int permToInt(vector<int>& v) {
    int use = 0, i = 0, r = 0;
    for(int x : v) r = r * ++i +
        __builtin_popcount(use & ~(1<<x)),

```

```

        use |= 1 << x;           // (note:
                                minus, not ~!)
    return r;
}

```

---

## 2.9 Sigma Function

The Sigma Function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when  $x = 0$  is called the divisor function, that counts the number of positive divisors of  $n$ .

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

If  $n$  is written as prime factorization:

$$n = \prod_{i=1}^k P_i^{e_k}$$

We can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^k g(e_k + 1)$$

where  $g(x)$  is the sum of the first  $x$  positive numbers:

$$g(x) = (x * (x + 1)) / 2$$

## 3 Data Structures

### 3.1 Binary Index Tree

```

struct BIT {
    int n;
    int t[2 * N];

    void add(int where, long long what) {
        for (where++; where <= n; where += where &
            -where) {
            t[where] += what;
        }
    }
}

```

---

```

}

void add(int from, int to, long long what) {
    add(from, what);
    add(to + 1, -what);
}

long long query(int where) {
    long long sum = t[0];

    for (where++; where > 0; where -= where &
        -where) {
        sum += t[where];
    }

    return sum;
}

};

```

---

### 3.2 Disjoint Set Union (DSU)

```

class DSU{
public:
    vector<int> parent;
    void initialize(int n){
        parent.resize(n+1, -1);
    }

    int findSet(int u){
        while(parent[u] > 0)
            u = parent[u];
        return u;
    }

    void Union(int u, int v){
        int x = parent[u] + parent[v];
        if(parent[u] > parent[v]){
            parent[v] = x;
            parent[u] = v;
        }else{
            parent[u] = x;
            parent[v] = u;
        }
    }
};

```

---

### 3.3 Fake Update

```
vector<int> fake_bit[MAXN];

void fake_update(int x, int y, int limit_x){
    for(int i = x; i < limit_x; i += i&(-i))
        fake_bit[i].pb(y);
}

void fake_get(int x, int y){
    for(int i = x; i >= 1; i -= i&(-i))
        fake_bit[i].pb(y);
}

vector<int> bit[MAXN];

void update(int x, int y, int limit_x, int val){
    for(int i = x; i < limit_x; i += i&(-i)){
        for(int j = lower_bound(fake_bit[i].begin(),
            fake_bit[i].end(), y) -
            fake_bit[i].begin(); j <
            fake_bit[i].size(); j += j&(-j))
            bit[i][j] = max(bit[i][j], val);
        }
    }

int get(int x, int y){
    int ans = 0;
    for(int i = x; i >= 1; i -= i&(-i)){
        for(int j = lower_bound(fake_bit[i].begin(),
            fake_bit[i].end(), y) -
            fake_bit[i].begin(); j >= 1; j -= j&(-j))
            ans = max(ans, bit[i][j]);
        }
    }
    return ans;
}

int main(){
    _io
    int n; cin >> n;
    vector<int> Sx, Sy;
    for(int i = 1; i <= n; i++){
        cin >> a[i].fi >> a[i].se;
        Sx.pb(a[i].fi);
        Sy.pb(a[i].se);
    }
    unique_arr(Sx);
    unique_arr(Sy);
    // unique all value
    for(int i = 1; i <= n; i++){
        a[i].fi = lower_bound(Sx.begin(), Sx.end(),
            a[i].fi) - Sx.begin();
```

```
        a[i].se = lower_bound(Sy.begin(), Sy.end(),
            a[i].se) - Sy.begin();
    }

    // do fake BIT update and get operator
    for(int i = 1; i <= n; i++){
        fake_get(a[i].fi-1, a[i].se-1);
        fake_update(a[i].fi, a[i].se, (int)Sx.size());
    }

    for(int i = 0; i < Sx.size(); i++){
        fake_bit[i].pb(INT_MIN); // avoid zero
        sort(fake_bit[i].begin(), fake_bit[i].end());
        fake_bit[i].resize(unique(fake_bit[i].begin(),
            fake_bit[i].end()) - fake_bit[i].begin());
        bit[i].resize((int)fake_bit[i].size(), 0);
    }

    // real update, get operator
    int res = 0;
    for(int i = 1; i <= n; i++){
        int maxCurLen = get(a[i].fi-1, a[i].se-1) + 1;
        res = max(res, maxCurLen);
        update(a[i].fi, a[i].se, (int)Sx.size(),
            maxCurLen);
    }
}
```

### 3.4 Fenwick Tree

```
template<typename T>
class FenwickTree{
    vector<T> fenw;
    int n;
public:
    void initialize(int _n){
        this->n = _n;
        fenw.resize(n+1);
    }

    void update(int id, T val) {
        while (id <= n) {
            fenw[id] += val;
            id += id&(-id);
        }
    }

    T get(int id){
        T ans{};
        while(id >= 1){
            ans += fenw[id];
```

```
            id -= id&(-id);
        }
        return ans;
    }
};
```

### 3.5 Hash Table

```
/*
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 *
 */

const int MN = 1001;
struct ht {
    int _s[(MN + 10) >> 5];
    int len;
    void set(int id) {
        len++;
        _s[id >> 5] |= (1LL << (id & 31));
    }
    bool is_set(int id) {
        return _s[id >> 5] & (1LL << (id & 31));
    }
};
```

### 3.6 Range Minimum Query

```
/*
    return min(v[a], v[a + 1], ..., v[b - 1]) in
    constant time
*/

template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V);
            pw *= 2, ++k) {
            jmp.emplace_back(sz(V) - pw * 2 +
                1);
            rep(j, 0, sz(jmp[k]))
                jmp[k][j] = min(jmp[k -
                    1][j], jmp[k - 1][j +
                        pw]);
        }
    }
};
```

```

T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1
        << dep)]);
}
};

```

### 3.7 STL Treap

```

struct Node {
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val); each(n->r,
        f); }
}

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {};
    if (cnt(n->l) >= k) { // "n->val >= k" for
        lower_bound(k)
        auto pa = split(n->l, k);
        n->l = pa.second;
        n->recalc();
        return {pa.first, n};
    } else {
        auto pa = split(n->r, k - cnt(n->l) -
            1); // and just "k"
        n->r = pa.first;
        n->recalc();
        return {n, pa.second};
    }
}

Node* merge(Node* l, Node* r) {
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        l->recalc();
        return l;
    } else {
        r->l = merge(l, r->l);

```

```

        r->recalc();
        return r;
    }
}

Node* ins(Node* t, Node* n, int pos) {
    auto pa = split(t, pos);
    return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r] to index k
void move(Node*& t, int l, int r, int k) {
    Node *a, *b, *c;
    tie(a,b) = split(t, l); tie(b,c) = split(b, r -
        l);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
}

```

### 3.8 Segment Tree

```

#include <bits/stdc++.h>
using namespace std;

const int N = 1e5 + 10;

int node[4*N];

void modify(int seg, int l, int r, int p, int val){
    if(l == r){
        node[seg] += val;
        return;
    }
    int mid = (l + r)/2;
    if(p <= mid){
        modify(2*seg + 1, l, mid, p, val);
    }else{
        modify(2*seg + 2, mid + 1, r, p, val);
    }
    node[seg] = node[2*seg + 1] + node[2*seg + 2];
}

int sum(int seg, int l, int r, int a, int b){
    if(l > b || r < a) return 0;
    if(l >= a && r <= b) return node[seg];
    int mid = (l + r)/2;
    return sum(2*seg + 1, l, mid, a, b) + sum(2*seg +
        2, mid + 1, r, a, b);
}

```

### 3.9 Sparse Table

```

template <typename T, typename func = function<T(const
    T, const T)>>
struct SparseTable {
    func calc;
    int n;
    vector<vector<T>> ans;

    SparseTable() {}

    SparseTable(const vector<T>& a, const func& f) :
        n(a.size()), calc(f) {
        int last = trunc(log2(n)) + 1;
        ans.resize(n);
        for (int i = 0; i < n; i++){
            ans[i].resize(last);
        }
        for (int i = 0; i < n; i++){
            ans[i][0] = a[i];
        }
        for (int j = 1; j < last; j++){
            for (int i = 0; i <= n - (1 << j); i++){
                ans[i][j] = calc(ans[i][j - 1], ans[i +
                    (1 << (j - 1))][j - 1]);
            }
        }
    }

    T query(int l, int r){
        assert(0 <= l && l <= r && r < n);
        int k = trunc(log2(r - l + 1));
        return calc(ans[l][k], ans[r - (1 << k) +
            1][k]);
    }
};

```

### 3.10 Trie

```

const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];

```

```

int nodes;

void clear(){
    tree[nodes].c = 0;
    memset(tree[nodes].a, -1, sizeof tree[nodes].a);
    nodes++;
}

void init(){
    nodes = 0;
    clear();
}

int add(const string &s, bool query = 0){
    int cur_node = 0;
    for(int i = 0; i < s.size(); ++i){
        int id = gid(s[i]);
        if(tree[cur_node].a[id] == -1){
            if(query) return 0;
            tree[cur_node].a[id] = nodes;
            clear();
        }
        cur_node = tree[cur_node].a[id];
    }
    if(!query) tree[cur_node].c++;
    return tree[cur_node].c;
}
};

```

## 4 Dynamic Programming Optimization

### 4.1 Convex Hull Trick

```

#define long long long
#define pll pair<long, long>
#define all(c) c.begin(), c.end()
#define fastio ios_base::sync_with_stdio(false);
    cin.tie(0)

struct line{
    long a, b;
    line() {}
    line(long a, long b) : a(a), b(b) {};
    bool operator < (const line &A) const {
        return pll(a,b) < pll(A.a,A.b);
    }
};

bool bad(line A, line B, line C){

```

```

    return (C.b - B.b) * (A.a - B.a) <= (B.b - A.b) *
        (B.a - C.a);
}

void addLine(vector<line> &memo, line cur){
    int k = memo.size();
    while (k >= 2 && bad(memo[k - 2], memo[k - 1],
        cur)){
        memo.pop_back();
        k--;
    }
    memo.push_back(cur);
}

long Fn(line A, long x){
    return A.a * x + A.b;
}

long query(vector<line> &memo, long x){
    int lo = 0, hi = memo.size() - 1;
    while (lo != hi){
        int mi = (lo + hi) / 2;
        if (Fn(memo[mi], x) > Fn(memo[mi + 1], x)){
            lo = mi + 1;
        }
        else hi = mi;
    }
    return Fn(memo[lo], x);
}

const int N = 1e6 + 1;
long dp[N];

int main()
{
    fastio;
    int n, c; cin >> n >> c;
    vector<line> memo;
    for (int i = 1; i <= n; i++){
        long val; cin >> val;
        addLine(memo, {-2 * val, val * val + dp[i -
            1]});
        dp[i] = query(memo, val) + val * val + c;
    }
    cout << dp[n] << '\n';
    return 0;
}

```

### 4.2 Divide and Conquer

```
/**

```

```

* recurrence:
*   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for all
*   j > i;
*
* "comp" computes dp[k][i] for all i in 0(n log n) (k
*   is fixed)
*
* Problems:
*   https://icpc.kattis.com/problems/branch
*   http://codeforces.com/contest/321/problem/E
* */

void comp(int l, int r, int le, int re) {
    if (l > r) return;

    int mid = (l + r) >> 1;

    int best = max(mid + 1, le);
    dp[cur][mid] = dp[cur ^ 1][best] + cost(mid, best -
        1);
    for (int i = best; i <= re; i++) {
        if (dp[cur][mid] > dp[cur ^ 1][i] + cost(mid, i -
            1)) {
            best = i;
            dp[cur][mid] = dp[cur ^ 1][i] + cost(mid, i - 1);
        }
    }

    comp(l, mid - 1, le, best);
    comp(mid + 1, r, best, re);
}

```

## 5 Geometry

### 5.1 Closest Pair Problem

```

struct point {
    double x, y;
    int id;
    point() {}
    point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
    double a = p.x - o.x, b = p.y - o.y;
    return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x,
    vector<point> &y) {

```



```

if (p.size() < 4) {
    double best = 1e100;
    for (int i = 0; i < p.size(); ++i)
        for (int j = i + 1; j < p.size(); ++j)
            best = min(best, dist(p[i], p[j]));
    return best;
}

int ls = (p.size() + 1) >> 1;
double l = (p[ls - 1].x + p[ls].x) * 0.5;
vector<point> xl(ls), xr(p.size() - ls);
unordered_set<int> left;
for (int i = 0; i < ls; ++i) {
    xl[i] = x[i];
    left.insert(x[i].id);
}
for (int i = ls; i < p.size(); ++i) {
    xr[i - ls] = x[i];
}

vector<point> yl, yr;
vector<point> pl, pr;
yl.reserve(ls); yr.reserve(p.size() - ls);
pl.reserve(ls); pr.reserve(p.size() - ls);
for (int i = 0; i < p.size(); ++i) {
    if (left.count(y[i].id))
        yl.push_back(y[i]);
    else
        yr.push_back(y[i]);

    if (left.count(p[i].id))
        pl.push_back(p[i]);
    else
        pr.push_back(p[i]);
}

double dl = cp(pl, xl, yl);
double dr = cp(pr, xr, yr);
double d = min(dl, dr);
vector<point> yp; yp.reserve(p.size());
for (int i = 0; i < p.size(); ++i) {
    if (fabs(y[i].x - l) < d)
        yp.push_back(y[i]);
}
for (int i = 0; i < yp.size(); ++i) {
    for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
        d = min(d, dist(yp[i], yp[j]));
    }
}
return d;
}

```

```

double closest_pair(vector<point> &p) {
    vector<point> x(p.begin(), p.end());
    sort(x.begin(), x.end(), [](const point &a, const
        point &b) {
            return a.x < b.x;
        });
    vector<point> y(p.begin(), p.end());
    sort(y.begin(), y.end(), [](const point &a, const
        point &b) {
            return a.y < b.y;
        });
    return cp(p, x, y);
}

```

## 5.2 Convex Diameter

```

struct point{
    int x, y;
};

struct vec{
    int x, y;
};

vec operator - (const point &A, const point &B){
    return vec{A.x - B.x, A.y - B.y};
}

int cross(vec A, vec B){
    return A.x*B.y - A.y*B.x;
}

int cross(point A, point B, point C){
    int val = A.x*(B.y - C.y) + B.x*(C.y - A.y) +
        C.x*(A.y - B.y);
    if(val == 0)
        return 0; // coline
    if(val < 0)
        return 1; // clockwise
    return -1; //counter clockwise
}

vector<point> findConvexHull(vector<point> points){
    vector<point> convex;
    sort(points.begin(), points.end(), [](const point
        &A, const point &B){
            return (A.x == B.x)? (A.y < B.y): (A.x < B.x);
        });
    vector<point> Up, Down;
    point A = points[0], B = points.back();
    Up.push_back(A);

```

```

Down.push_back(A);

for(int i = 0; i < points.size(); i++){
    if(i == points.size()-1 || cross(A, points[i],
        B) > 0){
        while(Up.size() > 2 &&
            cross(Up[Up.size()-2], Up[Up.size()-1],
                points[i]) <= 0)
            Up.pop_back();
        Up.push_back(points[i]);
    }
    if(i == points.size()-1 || cross(A, points[i],
        B) < 0){
        while(Down.size() > 2 &&
            cross(Down[Down.size()-2],
                Down[Down.size()-1], points[i]) >= 0)
            Down.pop_back();
        Down.push_back(points[i]);
    }
}

for(int i = 0; i < Up.size(); i++)
    convex.push_back(Up[i]);
for(int i = Down.size()-2; i > 0; i--)
    convex.push_back(Down[i]);
return convex;
}

int dist(point A, point B){
    return (A.x - B.x)*(A.x - B.x) + (A.y - B.y)*(A.y -
        B.y);
}

double findConvexDiameter(vector<point> convexHull){
    int n = convexHull.size();

    int is = 0, js = 0;
    for(int i = 1; i < n; i++){
        if(convexHull[i].y > convexHull[is].y)
            is = i;
        if(convexHull[js].y > convexHull[i].y)
            js = i;
    }

    int maxd = dist(convexHull[is], convexHull[js]);
    int i, maxi, j, maxj;
    i = maxi = is;
    j = maxj = js;
    do{
        int ni = (i+1)%n, nj = (j+1)%n;
        if(cross(convexHull[ni] - convexHull[i],
            convexHull[nj] - convexHull[j]) <= 0){
            j = nj;
        }else{

```

```

        i = ni;
    }
    int d = dist(convexHull[i], convexHull[j]);
    if(d > maxd){
        maxd = d;
        maxi = i;
        maxj = j;
    }
}while(i != is || j != js);
return sqrt(maxd);
}

```

### 5.3 Pick Theorem

```

struct point{
    ll x, y;
};

//Pick: S = I + B/2 - 1

ld polygonArea(vector <point> &points){
    int n = (int)points.size();
    ld area = 0.0;
    int j = n-1;
    for(int i = 0; i < n; i++){
        area += (points[j].x + points[i].x) *
            (points[j].y - points[i].y);
        j = i;
    }

    return abs(area/2.0);
}

ll boundary(vector <point> points){
    int n = (int)points.size();
    ll num_bound = 0;
    for(int i = 0; i < n; i++){
        ll dx = (points[i].x - points[(i+1)%n].x);
        ll dy = (points[i].y - points[(i+1)%n].y);
        num_bound += abs(__gcd(dx, dy)) - 1;
    }
    return num_bound;
}

```

### 5.4 Square

```
typedef long double ld;
```

```

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
    return ( x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

struct point{
    ld x, y;
    point(ld a, ld b) : x(a), y(b) {}
    point() {}
};

struct square{
    ld x1, x2, y1, y2,
    a, b, c;
    point edges[4];
    square(ld _a, ld _b, ld _c) {
        a = _a, b = _b, c = _c;
        x1 = a - c * 0.5;
        x2 = a + c * 0.5;
        y1 = b - c * 0.5;
        y2 = b + c * 0.5;
        edges[0] = point(x1, y1);
        edges[1] = point(x2, y1);
        edges[2] = point(x2, y2);
        edges[3] = point(x1, y2);
    }
};

ld min_dist(point &a, point &b) {
    ld x = a.x - b.x,
    y = a.y - b.y;
    return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
    if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) != -1 &&
        cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) != -1)
        return true;
    return false;
}

bool inside(square &s1, square &s2) {
    for (int i = 0; i < 4; ++i)
        if (point_in_box(s2, s1.edges[i]))
            return true;

    return false;
}

bool inside_vert(square &s1, square &s2) {
    if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1, s2.y2) !=
        1) ||

```

```

        (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2, s2.y2) !=
            1))
        return true;
    return false;
}

bool inside_hori(square &s1, square &s2) {
    if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1, s2.x2) !=
        1) ||
        (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2, s2.x2) !=
            1))
        return true;
    return false;
}

ld min_dist(square &s1, square &s2) {
    if (inside(s1, s2) || inside(s2, s1))
        return 0;

    ld ans = 1e100;
    for (int i = 0; i < 4; ++i)
        for (int j = 0; j < 4; ++j)
            ans = min(ans, min_dist(s1.edges[i],
                s2.edges[j]));

    if (inside_hori(s1, s2) || inside_hori(s2, s1)) {
        if (cmp(s1.y1, s2.y2) != -1)
            ans = min(ans, s1.y1 - s2.y2);
        else
            if (cmp(s2.y1, s1.y2) != -1)
                ans = min(ans, s2.y1 - s1.y2);
    }

    if (inside_vert(s1, s2) || inside_vert(s2, s1)) {
        if (cmp(s1.x1, s2.x2) != -1)
            ans = min(ans, s1.x1 - s2.x2);
        else
            if (cmp(s2.x1, s1.x2) != -1)
                ans = min(ans, s2.x1 - s1.x2);
    }

    return ans;
}

```

### 5.5 Triangle

Let a, b, c be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

## 6 Graphs

### 6.1 Bridges

```
struct Graph {
    vector<vector<Edge>> g;
    vector<int> vi, low, d, pi, is_b; // vi = visited
    int bridges_computed;
    int ticks, edges;

    Graph(int n, int m) {
        g.assign(n, vector<Edge>());
        id_b.assign(m, 0);
        vi.resize(n);
        low.resize(n);
        d.resize(n);
        pi.resize(n);
        edges = 0;
        bridges_computed = 0;
    }

    void addEdge(int u, int v) {
        g[u].push_back(Edge(v, edges));
        g[v].push_back(Edge(u, edges));
        edges++;
    }

    void dfs(int u) {
        vi[u] = true;
        d[u] = low[u] = ticks++;
        for (int i = 0; i < g[u].size(); i++) {
            int v = g[u][i].to;
            if (v == pi[u]) continue;
            if (!vi[v]) {
                pi[v] = u;
                dfs(v);
                if (d[u] < low[v]) is_b[g[u][i].id] = true;
                low[u] = min(low[u], low[v]);
            } else {
                low[u] = min(low[u], low[v]);
            }
        }
    }
};
```

```
    }
}

// multiple edges from a to b are not allowed.
// (they could be detected as a bridge).
// if we need to handle this, just count how many
// edges there are from a to b.
void compBridges() {
    fill(pi.begin(), pi.end(), -1);
    fill(vi.begin(), vi.end(), false);
    fill(d.begin(), d.end(), 0);
    fill(low.begin(), low.end(), 0);
    ticks = 0;
    for (int i = 0; i < g.size(); i++)
        if (!vi[i]) dfs(i);
    bridges_computed = 1;
}

map<int, vector<Edge>> bridgesTree() {
    if (!bridges_computed) compBridges();
    int n = g.size();
    Dsu dsu(n);
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (!is_b[e.id]) dsu.Join(i, e.to);
    map<int, vector<Edge>> tree;
    for (int i = 0; i < n; i++)
        for (auto e : g[i])
            if (is_b[e.id])
                tree[dsu.Find(i)].emplace_back(dsu.Find(e.to), e.id);
    return tree;
}
};
```

### 6.2 Dijkstra

```
struct edge {
    int to;
    long long w;
    edge() {}
    edge(int a, long long b) : to(a), w(b) {}
    bool operator<(const edge &e) const {
        return w > e.w;
    }
};

typedef <vector<vector<edge>> graph;
const long long inf = 1000000LL * 10000000LL;
```

```
pair<vector<int>, vector<long long>> dijkstra(graph& g,
    int start) {
    int n = g.size();
    vector<long long> d(n, inf);
    vector<int> p(n, -1);
    d[start] = 0;
    priority_queue<edge> q;
    q.push(edge(start, 0));
    while (!q.empty()) {
        int node = q.top().to;
        long long dist = q.top().w;
        q.pop();
        if (dist > d[node]) continue;
        for (int i = 0; i < g[node].size(); i++) {
            int to = g[node][i].to;
            long long w_extra = g[node][i].w;
            if (dist + w_extra < d[to]) {
                p[to] = node;
                d[to] = dist + w_extra;
                q.push(edge(to, d[to]));
            }
        }
    }
    return {p, d};
}
```

### 6.3 Directed MST

```
struct Edge { int a, b; ll w; };
struct Node { /// lazy skew heap node
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() { prop(); return key; }
};

Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ? b : a->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}

void pop(Node& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
```

```

RollbackUF uf(n);
vector<Node*> heap(n);
for (Edge e : g) heap[e.b] = merge(heap[e.b],
    new Node{e});
ll res = 0;
vi seen(n, -1), path(n), par(n);
seen[r] = r;
vector<Edge> Q(n), in(n, {-1,-1}), comp;
deque<tuple<int, int, vector<Edge>>> cyps;
rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
        if (!heap[u]) return {-1,{};};
        Edge e = heap[u]->top();
        heap[u]->delta -= e.w,
        pop(heap[u]);
        Q[qi] = e, path[qi++] = u,
        seen[u] = s;
        res += e.w, u = uf.find(e.a);
        if (seen[u] == s) { /// found
            cycle, contract
            Node* cyc = 0;
            int end = qi, time =
                uf.time();
            do cyc = merge(cyc, heap[w]
                = path[--qi]);
            while (uf.join(u, w));
            u = uf.find(u), heap[u] =
                cyc, seen[u] = -1;
            cyps.push_front({u, time,
                {&Q[qi], &Q[end]}});
        }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
}

for (auto& [u,t,comp] : cyps) { // restore sol
    (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] =
        e;
    in[uf.find(inEdge.b)] = inEdge;
}
rep(i,0,n) par[i] = in[i].a;
return {res, par};
}

```

## 6.4 Edge Coloring

```

vi edgeColoring(int N, vector<pii> eds) {

```

```

    vi cc(N + 1), ret(sz(eds)), fan(N), free(N),
    loc;
    for (pii e : eds) ++cc[e.first], ++cc[e.second];
    int u, v, ncols = *max_element(all(cc)) + 1;
    vector<vi> adj(N, vi(ncols, -1));
    for (pii e : eds) {
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(ncols, 0);
        int at = u, end = u, d, c = free[u], ind
            = 0, i = 0;
        while (d = free[v], !loc[d] && (v =
            adj[u][d]) != -1)
            loc[d] = ++ind, cc[ind] = d,
            fan[ind] = v;
        cc[loc[d]] = c;
        for (int cd = d; at != -1; cd ^= c ^ d,
            at = adj[at][cd])
            swap(adj[at][cd], adj[end =
                at][cd ^ c ^ d]);
        while (adj[fan[i]][d] != -1) {
            int left = fan[i], right =
                fan[++i], e = cc[i];
            adj[u][e] = left;
            adj[left][e] = u;
            adj[right][e] = -1;
            free[right] = e;
        }
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = free[y] = 0;
                adj[y][z] != -1; z++);
    }
    rep(i,0,sz(eds))
        for (tie(u, v) = eds[i]; adj[u][ret[i]]
            != v; ++ret[i]);
    return ret;
}

```

## 6.5 Eulerian Path

```

struct DirectedEulerPath
{
    int n;
    vector<vector<int>> > g;
    vector<int> path;

    void init(int _n){
        n = _n;

```

```

        g = vector<vector<int>> > (n + 1,
            vector<int> ());
        path.clear();
    }

    void add_edge(int u, int v){
        g[u].push_back(v);
    }

    void dfs(int u)
    {
        while(g[u].size())
        {
            int v = g[u].back();
            g[u].pop_back();
            dfs(v);
        }
        path.push_back(u);
    }

    bool getPath(){
        int ctEdges = 0;
        vector<int> outDeg, inDeg;
        outDeg = inDeg = vector<int> (n + 1, 0);
        for(int i = 1; i <= n; i++)
        {
            ctEdges += g[i].size();
            outDeg[i] += g[i].size();
            for(auto &u:g[i])
                inDeg[u]++;
        }
        int ctMiddle = 0, src = 1;
        for(int i = 1; i <= n; i++)
        {
            if(abs(inDeg[i] - outDeg[i]) > 1)
                return 0;
            if(inDeg[i] == outDeg[i])
                ctMiddle++;
            if(outDeg[i] > inDeg[i])
                src = i;
        }
        if(ctMiddle != n && ctMiddle + 2 != n)
            return 0;
        dfs(src);
        reverse(path.begin(), path.end());
        return (path.size() == ctEdges + 1);
    }
};

```

## 6.6 Floyd - Warshall

---

```

const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
    int n = sz(m);
    rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
    rep(k,0,n) rep(i,0,n) rep(j,0,n)
        if (m[i][k] != inf && m[k][j] != inf) {
            auto newDist = max(m[i][k] +
                               m[k][j], -inf);
            m[i][j] = min(m[i][j], newDist);
        }
    rep(k,0,n) if (m[k][k] < 0) rep(i,0,n)
        rep(j,0,n)
            if (m[i][k] != inf && m[k][j] != inf)
                m[i][j] = -inf;
}

```

---

## 6.7 Ford - Bellman

---

```

const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; } };
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds,
int s) {
    nodes[s].dist = 0;
    sort(all(eds), [](Ed a, Ed b) { return a.s() <
        b.s(); });

    int lim = sz(nodes) / 2 + 2; // /3+100 with
    shuffled vertices
    rep(i,0,lim) for (Ed ed : eds) {
        Node cur = nodes[ed.a], &dest =
            nodes[ed.b];
        if (abs(cur.dist) == inf) continue;
        ll d = cur.dist + ed.w;
        if (d < dest.dist) {
            dest.prev = ed.a;
            dest.dist = (i < lim-1 ? d :
                -inf);
        }
    }
    rep(i,0,lim) for (Ed e : eds) {
        if (nodes[e.a].dist == -inf)
            nodes[e.b].dist = -inf;
    }
}

```

---

## 6.8 Gomory Hu

---

```

#include "PushRelabel.cpp"

typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i,1,N) {
        PushRelabel D(N); // Dinic also works
        for (Edge t : ed) D.addEdge(t[0], t[1],
            t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i,
            par[i])});
        rep(j,i+1,N)
            if (par[j] == par[i] &&
                D.leftOfMinCut(j)) par[j] =
                i;
    }
    return tree;
}

```

---

## 6.9 Karp Min Mean Cycle

---

```

/**
 * Finds the min mean cycle, if you need the max mean
 * cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
 * */

const int MN = 1000;
struct edge{
    int v;
    long long w;
    edge(){} edge(int v, int w) : v(v), w(w) {}
};

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge>> g) {
    int n = g.size();

    g.resize(n + 1); // this is important

    for (int i = 0; i < n; ++i)
        if (!g[i].empty())

```

---

```

        g[n].push_back(edge(i,0));
        ++n;

    for(int i = 0; i < n; ++i)
        fill(d[i], d[i] + (n+1), INT_MAX);

    d[n - 1][0] = 0;

    for (int k = 1; k <= n; ++k) for (int u = 0; u < n;
        ++u) {
        if (d[u][k - 1] == INT_MAX) continue;
        for (int i = g[u].size() - 1; i >= 0; --i)
            d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k -
                1] + g[u][i].w);
    }

    bool flag = true;

    for (int i = 0; i < n && flag; ++i)
        if (d[i][n] != INT_MAX)
            flag = false;

    if (flag) {
        return true; // return true if there is no a cycle.
    }

    double ans = 1e15;

    for (int u = 0; u + 1 < n; ++u) {
        if (d[u][n] == INT_MAX) continue;
        double W = -1e15;

        for (int k = 0; k < n; ++k)
            if (d[u][k] != INT_MAX)
                W = max(W, (double)(d[u][n] - d[u][k]) / (n -
                    k));

        ans = min(ans, W);
    }

    // printf("%.2lf\n", ans);
    cout << fixed << setprecision(2) << ans << endl;

    return false;
}

```

---

## 6.10 Konig's Theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

## 6.11 LCA

---

```
#include "../Data Structures/RMQ.h"

struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)),
        rmq((dfs(C,0,-1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) {
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v),
            ret.push_back(time[v]);
            dfs(C, y, v);
        }
    }

    int lca(int a, int b) {
        if (a == b) return a;
        tie(a, b) = minmax(time[a], time[b]);
        return path[rmq.query(a, b)];
    }
    //dist(a,b){return depth[a] + depth[b] -
    //2*depth[lca(a,b)];}
};
```

---

## 6.12 Math

### Number of Spanning Trees

Create an  $N \times N$  matrix `mat`, and for each edge  $a \rightarrow b \in G$ , do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if  $G$  is undirected). Remove the  $i$ th row and column and take the determinant; this yields the number of directed spanning trees rooted at  $i$  (if  $G$  is undirected, remove any row/column).

### Erdős–Gallai theorem

A simple graph with node degrees  $d_1 \geq \dots \geq d_n$  exists iff  $d_1 + \dots + d_n$  is even and for every  $k = 1 \dots n$ ,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

## 6.13 Minimum Path Cover in DAG

Given a directed acyclic graph  $G = (V, E)$ , we are to find the minimum number of vertex-disjoint paths to cover each vertex in  $V$ .

We can construct a bipartite graph  $G' = (V_{out} \cup V_{in}, E')$  from  $G$ , where :

$$V_{out} = \{v \in V : v \text{ has positive out-degree}\}$$

$$V_{in} = \{v \in V : v \text{ has positive in-degree}\}$$

$$E' = \{(u, v) \in V_{out} \times V_{in} : (u, v) \in E\}$$

Then it can be shown, via König's theorem, that  $G'$  has a matching of size  $m$  if and only if there exists  $n - m$  vertex-disjoint paths that cover each vertex in  $G$ , where  $n$  is the number of vertices in  $G$  and  $m$  is the maximum cardinality bipartite matching in  $G'$ .

Therefore, the problem can be solved by finding the maximum cardinality matching in  $G'$  instead.

**NOTE:** If the paths are not necessarily disjoint, find the transitive closure and solve the problem for disjoint paths.

## 6.14 Planar Graph (Euler)

Euler's formula states that if a finite, connected, planar graph is drawn in the plane without any edge intersections, and  $v$  is the number of vertices,  $e$  is the number of edges and  $f$  is the number of faces (regions bounded by edges, including the outer, infinitely large region), then:

$$f + v = e + 2$$

It can be extended to non connected planar graphs with  $c$  connected components:

$$f + v = e + c + 1$$

## 6.15 Push Relabel

---

```
struct PushRelabel {
    struct Edge {
        int dest, back;
        ll f, c;
    };
    vector<vector<Edge>> g;
    vector<ll> ec;
    vector<Edge*> cur;
    vector<vi> hs; vi H;
    PushRelabel(int n) : g(n), ec(n), cur(n),
        hs(2*n), H(n) {}

    void addEdge(int s, int t, ll cap, ll rcap=0) {
        if (s == t) return;
        g[s].push_back({t, sz(g[t]), 0, cap});
        g[t].push_back({s, sz(g[s])-1, 0, rcap});
    }

    void addFlow(Edge& e, ll f) {
        Edge &back = g[e.dest][e.back];
        if (!ec[e.dest] && f)
            hs[H[e.dest]].push_back(e.dest);
        e.f += f; e.c -= f; ec[e.dest] += f;
        back.f -= f; back.c += f; ec[back.dest] -= f;
    }

    ll calc(int s, int t) {
        int v = sz(g); H[s] = v; ec[t] = 1;
        vi co(2*v); co[0] = v-1;
        rep(i,0,v) cur[i] = g[i].data();
        for (Edge& e : g[s]) addFlow(e, e.c);

        for (int hi = 0;;) {
            while (hs[hi].empty()) if (!hi--)
                return -ec[s];
            int u = hs[hi].back();
            hs[hi].pop_back();
            while (ec[u] > 0) // discharge u
                if (cur[u] == g[u].data()
                    + sz(g[u])) {
                    H[u] = 1e9;
                    for (Edge& e :
                        g[u]) if (e.c
                            && H[u] >
                            H[e.dest]+1)
                        H[u] =
                            H[e.dest]+1,
                            cur[u]
                            = &e;
                }
        }
    }
};
```

```

        if (++co[H[u]],
            !--co[hi] &&
            hi < v)
            rep(i,0,v)
                if (hi
                    < H[i]
                    && H[i]
                    < v)
                    --co[H[i]],
                    H[i]
                    =
                    v
                    +
                    1;

            hi = H[u];
        } else if (cur[u]->c &&
            H[u] ==
            H[cur[u]->dest]+1)
            addFlow(*cur[u],
                min(ec[u],
                    cur[u]->c));
            else ++cur[u];
    }
}
bool leftOfMinCut(int a) { return H[a] >=
    sz(g); }
};

```

## 6.16 SCC Kosaraju

// SCC = Strongly Connected Components

```

struct SCC {
    vector<vector<int>> g, gr;
    vector<bool> used;
    vector<int> order, component;
    int total_components;

    SCC(vector<vector<int>>& adj) {
        g = adj;
        int n = g.size();
        gr.resize(n);
        for (int i = 0; i < n; i++)
            for (auto to : g[i])
                gr[to].push_back(i);

        used.assign(n, false);
        for (int i = 0; i < n; i++)
            if (!used[i])
                GenTime(i);
    }
};

```

```

        used.assign(n, false);
        component.assign(n, -1);
        total_components = 0;
        for (int i = n - 1; i >= 0; i--) {
            int v = order[i];
            if (!used[v]) {
                vector<int> cur_component;
                Dfs(cur_component, v);
                for (auto node : cur_component)
                    component[node] = total_components;
            }
        }

        void GenTime(int node) {
            used[node] = true;
            for (auto to : g[node])
                if (!used[to])
                    GenTime(to);
            order.push_back(node);
        }

        void Dfs(vector<int>& cur, int node) {
            used[node] = true;
            cur.push_back(node);
            if (!used[to])
                Dfs(cur, to);
        }

        vector<vector<int>> CondensedGraph() {
            vector<vector<int>> ans(total_components);
            for (int i = 0; i < int(g.size()); i++) {
                for (int to : g[i]) {
                    int u = component[i], v = component[to];
                    if (u != v)
                        ans[u].push_back(v);
                }
            }
            return ans;
        }
};

```

## 6.17 Tarjan SCC

```

const int N = 20002;
struct tarjan_scc {
    int scc[MN], low[MN], d[MN], stacked[MN];
    int ticks, current_scc;
    deque<int> s; // used as stack
    tarjan_scc() {}
    void init() {

```

```

        memset(scc, -1, sizeof(scc));
        memset(d, -1, sizeof(d));
        memset(stacked, 0, sizeof(stacked));
        s.clear();
        ticks = current_scc = 0;
    }

    void compute(vector<vector<int>> &g, int u) {
        d[u] = low[u] = ticks++;
        s.push_back(u);
        stacked[u] = true;
        for (int i = 0; i < g[u].size(); i++) {
            int v = g[u][i];
            if (d[v] == -1) compute(g, v);
            if (stacked[v]) low[u] = min(low[u], low[v]);
        }
        if (d[u] == low[u]) {
            int v;
            do {
                v = s.back(); s.pop_back();
                stacked[v] = false;
                scc[v] = current_scc;
            } while (u != v);
            current_scc++;
        }
    }
};

```

## 6.18 Topological Sort

```

vi topoSort(const vector<vi>& gr) {
    vi indeg(sz(gr)), ret;
    for (auto& li : gr) for (int x : li) indeg[x]++;
    queue<int> q; // use priority-queue for lexic.
    largest ans.
    rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(i);
    while (!q.empty()) {
        int i = q.front(); // top() for priority
        queue
        ret.push_back(i);
        q.pop();
        for (int x : gr[i])
            if (--indeg[x] == 0) q.push(x);
    }
    return ret;
}

```

## 7 Linear Algebra

### 7.1 Matrix Determinant

---

```
double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) >
            fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k]
                -= v * a[i][k];
        }
    }
    return res;
}
```

---

### 7.2 Matrix Inverse

---

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n,
        vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) {
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]),
            swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -=
                f*tmp[i][k];
        }
    }
}
```

---

```
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    }

    /// forget A at this point, just eliminate tmp
    backward
    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    }

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] =
        tmp[i][j];
    return n;
}
```

---

### 7.3 PolyRoots

---

```
#include "Polynomial.cpp"

vector<double> polyRoots(Poly p, double xmin, double
    xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);
    dr.push_back(xmin-1);
    dr.push_back(xmax+1);
    sort(all(dr));
    rep(i,0,sz(dr)-1) {
        double l = dr[i], h = dr[i+1];
        bool sign = p(l) > 0;
        if (sign ^ (p(h) > 0)) {
            rep(it,0,60) { // while (h - l >
                1e-8)
                double m = (l + h) / 2, f
                    = p(m);
                if ((f <= 0) ^ sign) l = m;
                else h = m;
            }
            ret.push_back((l + h) / 2);
        }
    }
    return ret;
}
```

---

### 7.4 Polynomial

---

```
struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val += x) +=
            a[i];
        return val;
    }
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i]
            = a[i+1]*x0+b, b=c;
        a.pop_back();
    }
};
```

---

## 8 Misc

### 8.1 Dates

---

```
//
// Time - Leap years
//

// A[i] has the accumulated number of days from months
// previous to i
const int A[13] = { 0, 0, 31, 59, 90, 120, 151, 181,
    212, 243, 273, 304, 334 };
// same as A, but for a leap year
const int B[13] = { 0, 0, 31, 60, 91, 121, 152, 182,
    213, 244, 274, 305, 335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y /
    400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 ==
    0 && y % 100 != 0); }
// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4 = 4*365 + 1;
const int p1 = 365;
int date_to_days(int d, int m, int y)
{
```

---



```

    return (y - 1) * 365 + leap_years(y - 1) +
           (is_leap(y) ? B[m] : A[m]) + d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
    bool top100; // are we in the top 100 years of a 400
                // block?
    bool top4;   // are we in the top 4 years of a 100
                // block?
    bool top1;   // are we in the top year of a 4 block?

    y = 1;
    top100 = top4 = top1 = false;

    y += ((days-1) / p400) * 400;
    d = (days-1) % p400 + 1;

    if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
    else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

    if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
    else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

    if (d > p1*3) top1 = true, d -= p1*3, y += 3;
    else y += (d-1) / p1, d = (d-1) % p1 + 1;

    const int *ac = top1 && (!top4 || top100) ? B : A;
    for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
    d -= ac[m];
}

```

## 8.2 Debugging Tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

## 8.3 Interval Container

```

set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
    if (L == R) return is.end();
    auto it = is.lower_bound({L, R}), before = it;
    while (it != is.end() && it->first <= R) {

```

```

        R = max(R, it->second);
        before = it = is.erase(it);
    }
    if (it != is.begin() && (--it)->second >= L) {
        L = min(L, it->first);
        R = max(R, it->second);
        is.erase(it);
    }
    return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
    if (L == R) return;
    auto it = addInterval(is, L, R);
    auto r2 = it->second;
    if (it->first == L) is.erase(it);
    else (int&)it->second = L;
    if (R != r2) is.emplace(R, r2);
}

```

## 8.4 Optimization Tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

### 8.4.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`  
if `(i & 1 << b) D[i] += D[i^(1 << b)]`; computes all sums of subsets.

### 8.4.2 Pragas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.

- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

## 8.5 Ternary Search

```

template<class F>
int ternSearch(int a, int b, F f) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
    return a;
}

```

## 9 Number Theory

### 9.1 Chinese Remainder Theorem

```

/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 */
long long crt(vector<long long> &a, vector<long long>
             &x) {
    long long z = 0;
    long long n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        long long tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}

```

## 9.2 Convolution

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

inline bool is_pow2(LL x) {
    return (x & (x-1)) == 0;
}

inline int ceil_log2(LL x) {
    int ans = 0;
    --x;
    while (x != 0) {
        x >>= 1;
        ans++;
    }
    return ans;
}

/* Returns the convolution of the two given vectors in
   time proportional to n*log(n).
   * The number of roots of unity to use nroots_unity
   must be set so that the product of the first
   * nroots_unity primes of the vector nth_roots_unity is
   greater than the maximum value of the
   * convolution. Never use sizes of vectors bigger than
   2^24, if you need to change the values of
   * the nth roots of unity to appropriate primes for
   those sizes.
   */
vector<LL> convolve(const vector<LL> &a, const
    vector<LL> &b, int nroots_unity = 2) {
    int N = 1 << ceil_log2(a.size() + b.size());
    vector<LL> ans(N, 0), fA(N), fB(N), fC(N);
    LL modulo = 1;
    for (int times = 0; times < nroots_unity; times++) {
        fill(fA.begin(), fA.end(), 0);
        fill(fB.begin(), fB.end(), 0);
        for (int i = 0; i < a.size(); i++) fA[i] = a[i];
        for (int i = 0; i < b.size(); i++) fB[i] = b[i];
        LL prime = nth_roots_unity[times].first;
        LL inv_modulo = mod_inv(modulo % prime, prime);
        LL normalize = mod_inv(N, prime);
        ntfft(fA, 1, nth_roots_unity[times]);
        ntfft(fB, 1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i])
            % prime;
        ntfft(fC, -1, nth_roots_unity[times]);
        for (int i = 0; i < N; i++) {
            LL curr = (fC[i] * normalize) % prime;
            LL k = (curr - (ans[i] % prime) + prime) % prime;
            k = (k * inv_modulo) % prime;
```

```
        ans[i] += modulo * k;
    }
    modulo *= prime;
}
return ans;
}
```

## 9.3 Diophantine Equations

```
long long gcd(long long a, long long b, long long &x,
    long long &y) {
    if (a == 0) {
        x = 0;
        y = 1;
        return b;
    }
    long long x1, y1;
    long long d = gcd(b % a, a, x1, y1);
    x = y1 - (b / a) * x1;
    y = x1;
    return d;
}

bool find_any_solution(long long a, long long b, long
    long c, long long &x0,
    long long &y0, long long &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}

void shift_solution(long long &x, long long &y, long
    long a, long long b,
    long long cnt) {
    x += cnt * b;
    y -= cnt * a;
}

long long find_all_solutions(long long a, long long b,
    long long c,
    long long minx, long long maxx, long long miny,
    long long maxy) {
    long long x, y, g;
    if (!find_any_solution(a, b, c, x, y, g)) return 0;
```

```
a /= g;
b /= g;
```

```
long long sign_a = a > 0 ? +1 : -1;
long long sign_b = b > 0 ? +1 : -1;
```

```
shift_solution(x, y, a, b, (minx - x) / b);
if (x < minx) shift_solution(x, y, a, b, sign_b);
if (x > maxx) return 0;
long long lx1 = x;
```

```
shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx) shift_solution(x, y, a, b, -sign_b);
long long rx1 = x;
```

```
shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny) shift_solution(x, y, a, b, -sign_a);
if (y > maxy) return 0;
long long lx2 = x;
```

```
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy) shift_solution(x, y, a, b, sign_a);
long long rx2 = x;
```

```
if (lx2 > rx2) swap(lx2, rx2);
long long lx = max(lx1, lx2);
long long rx = min(rx1, rx2);
```

```
if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}
```

## 9.4 Discrete Logarithm

```
// Computes x which a ^ x = b mod n.
```

```
long long d_log(long long a, long long b, long long n) {
    long long m = ceil(sqrt(n));
    long long aj = 1;
    map<long long, long long> M;
    for (int i = 0; i < m; ++i) {
        if (!M.count(aj))
            M[aj] = i;
        aj = (aj * a) % n;
    }
```

```
long long coef = mod_pow(a, n - 2, n);
coef = mod_pow(coef, m, n);
// coef = a ^ (-m)
long long gamma = b;
```

```

for (int i = 0; i < m; ++i) {
    if (M.count(gamma)) {
        return i * m + M[gamma];
    } else {
        gamma = (gamma * coef) % n;
    }
}
return -1;
}

```

## 9.5 Ext Euclidean

```

void ext_euclid(long long a, long long b, long long &x,
               long long &y, long long &g) {
    x = 0, y = 1, g = b;
    long long m, n, q, r;
    for (long long u = 1, v = 0; a != 0; g = a, a = r) {
        q = g / a, r = g % a;
        m = x - u * q, n = y - v * q;
        x = u, y = v, u = m, v = n;
    }
}

```

## 9.6 Fast Eratosthenes

```

const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1);
    pr.reserve(int(LIM/log(LIM)*1.1));
    vector<pii> cp;
    for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
        cp.push_back({i, i * i / 2});
        for (int j = i * i; j <= S; j += 2 * i)
            sieve[j] = 1;
    }
    for (int L = 1; L <= R; L += S) {
        array<bool, S> block{};
        for (auto &[p, idx] : cp)
            for (int i=idx; i < S+L; idx = (i+p)) block[i-L] = 1;
        rep(i,0,min(S, R - L))
            if (!block[i]) pr.push_back((L + i) * 2 + 1);
    }
    for (int i : pr) isPrime[i] = 1;
}

```

```

return pr;
}

```

## 9.7 Highest Exponent Factorial

```

int highest_exponent(int p, const int &n){
    int ans = 0;
    int t = p;
    while(t <= n){
        ans += n/t;
        t*=p;
    }
    return ans;
}

```

## 9.8 Miller - Rabin

```

const int rounds = 20;

// checks whether a is a witness that n is not prime, 1 < a < n
bool witness(long long a, long long n) {
    // check as in Miller Rabin Primality Test described
    long long u = n - 1;
    int t = 0;
    while (u % 2 == 0) {
        t++;
        u >>= 1;
    }
    long long next = mod_pow(a, u, n);
    if (next == 1) return false;
    long long last;
    for (int i = 0; i < t; ++i) {
        last = next;
        next = mod_mul(last, last, n);
        if (next == 1) {
            return last != n - 1;
        }
    }
    return next != 1;
}

// Checks if a number is prime with prob 1 - 1 / (2 ^ it)
// D(miller_rabin(99999999999999997LL) == 1);
// D(miller_rabin(99999999999971LL) == 1);
// D(miller_rabin(7907) == 1);

```

```

bool miller_rabin(long long n, int it = rounds) {
    if (n <= 1) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    for (int i = 0; i < it; ++i) {
        long long a = rand() % (n - 1) + 1;
        if (witness(a, n)) {
            return false;
        }
    }
    return true;
}

```

## 9.9 Mod Integer

```

template<class T, T mod>
struct mint_t {
    T val;
    mint_t() : val(0) {}
    mint_t(T v) : val(v % mod) {}

    mint_t operator + (const mint_t& o) const {
        return (val + o.val) % mod;
    }
    mint_t operator - (const mint_t& o) const {
        return (val - o.val) % mod;
    }
    mint_t operator * (const mint_t& o) const {
        return (val * o.val) % mod;
    }
};

typedef mint_t<long long, 998244353> mint;

```

## 9.10 Mod Inv

```

long long mod_inv(long long n, long long m) {
    long long x, y, gcd;
    ext_euclid(n, m, x, y, gcd);
    if (gcd != 1)
        return 0;
    return (x + m) % m;
}

```

## 9.11 Mod Mul

---

```
// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long mod) {
    long long x = 0, y = a % mod;
    while (b > 0) {
        if (b & 1)
            x = (x + y) % mod;
        y = (y * 2) % mod;
        b /= 2;
    }
    return x % mod;
}
```

---

## 9.12 Mod Pow

---

```
// Computes (a ^ exp) % mod.
long long mod_pow(long long a, long long exp, long long mod) {
    long long ans = 1;
    while (exp > 0) {
        if (exp & 1)
            ans = mod_mul(ans, a, mod);
        a = mod_mul(a, a, mod);
        exp >>= 1;
    }
    return ans;
}
```

---

## 9.13 Number Theoretic Transform

---

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime,
   generator)
   * where the prime has an Nth root of unity for N being
   a power of two.
   * The generator is a number g s.t g^(p-1)=1 (mod p)
   * but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
    {1224736769, 330732430}, {1711276033, 927759239}, {167772161, 167489322},
    {469762049, 343261969}, {754974721, 643797295}, {1107296257, 883865065}
};

PLL ext_euclid(LL a, LL b) {
    if (b == 0)
        return make_pair(1, 0);
    pair<LL, LL> rc = ext_euclid(b, a % b);
```

---

```
return make_pair(rc.second, rc.first - (a / b) *
    rc.second);
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
    PLL p = ext_euclid(x, modulo);
    if ( (p.first * x + p.second * modulo) != 1 )
        return -1;
    return (p.first+modulo) % modulo;
}
```

```
//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL
    &root_unity) {
    int n = a.size();
    LL prime = root_unity.first;
    LL basew = mod_pow(root_unity.second, (prime-1) / n,
        prime);
    if (dir < 0) basew = mod_inv(basew, prime);
    for (int m = n; m >= 2; m >>= 1) {
        int mh = m >> 1;
        LL w = 1;
        for (int i = 0; i < mh; i++) {
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                LL x = (a[j] - a[k] + prime) % prime;
                a[j] = (a[j] + a[k]) % prime;
                a[k] = (w * x) % prime;
            }
            w = (w * basew) % prime;
        }
        basew = (basew * basew) % prime;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
}
```

---

## 9.14 Pollard Rho Factorize

---

```
long long pollard_rho(long long n) {
    long long x, y, i = 1, k = 2, d;
    x = y = rand() % n;
    while (1) {
        ++i;
        x = mod_mul(x, x, n);
        x += 2;
```

```
if (x >= n) x -= n;
if (x == y) return 1;
d = __gcd(abs(x - y), n);
if (d != 1) return d;
if (i == k) {
    y = x;
    k *= 2;
}
}
return 1;
}
```

```
// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
    vector<long long> ans;
    if (n == 1)
        return ans;
    if (miller_rabin(n)) {
        ans.push_back(n);
    } else {
        long long d = 1;
        while (d == 1)
            d = pollard_rho(n);
        vector<long long> dd = factorize(d);
        ans = factorize(n / d);
        for (int i = 0; i < dd.size(); ++i)
            ans.push_back(dd[i]);
    }
    return ans;
}
```

---

## 9.15 Primes

---

```
namespace primes {
    const int MP = 100001;
    bool sieve[MP];
    long long primes[MP];
    int num_p;
    void fill_sieve() {
        num_p = 0;
        sieve[0] = sieve[1] = true;
        for (long long i = 2; i < MP; ++i) {
            if (!sieve[i]) {
                primes[num_p++] = i;
                for (long long j = i * i; j < MP; j += i)
                    sieve[j] = true;
            }
        }
    }
}
```

```
// Finds prime numbers between a and b, using basic
// primes up to sqrt(b)
// a must be greater than 1.
vector<long long> seg_sieve(long long a, long long b)
{
    long long ant = a;
    a = max(a, 3LL);
    vector<bool> pmap(b - a + 1);
    long long sqrt_b = sqrt(b);
    for (int i = 0; i < num_p; ++i) {
        long long p = primes[i];
        if (p > sqrt_b) break;
        long long j = (a + p - 1) / p;
        for (long long v = (j == 1) ? p + p : j * p; v <=
            b; v += p) {
            pmap[v - a] = true;
        }
    }
    vector<long long> ans;
    if (ant == 2) ans.push_back(2);
    int start = a % 2 ? 0 : 1;
    for (int i = start, I = b - a + 1; i < I; i += 2)
        if (pmap[i] == false)
            ans.push_back(a + i);
    return ans;
}
```

```
vector<pair<int, int>> factor(int n) {
    vector<pair<int, int>> ans;
    if (n == 0) return ans;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            int expo = 0;
            while ((n % primes[i]) == 0) {
                expo++;
                n /= primes[i];
            }
            ans.emplace_back(primes[i], expo);
        }
    }

    if (n > 1) {
        ans.emplace_back(n, 1);
    }
    return ans;
}
```

## 9.16 Totient Sieve

```
for (int i = 1; i < MN; i++)
```

```
phi[i] = i;

for (int i = 1; i < MN; i++)
    if (!sieve[i]) // is prime
        for (int j = i; j < MN; j += i)
            phi[j] -= phi[j] / i;
```

## 9.17 Totient

```
long long totient(long long n) {
    if (n == 1) return 0;
    long long ans = n;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
        if ((n % primes[i]) == 0) {
            while ((n % primes[i]) == 0) n /= primes[i];
            ans -= ans / primes[i];
        }
    }
    if (n > 1) {
        ans -= ans / n;
    }
    return ans;
}
```

# 10 Probability and Statistics

## 10.1 Continuous Distributions

### 10.1.1 Uniform distribution

If the probability density function is constant between  $a$  and  $b$  and 0 elsewhere it is  $U(a, b)$ ,  $a < b$ .

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

### 10.1.2 Exponential distribution

The time between events in a Poisson process is  $\text{Exp}(\lambda)$ ,  $\lambda > 0$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

### 10.1.3 Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

## 10.2 Discrete Distributions

### 10.2.1 Binomial distribution

The number of successes in  $n$  independent yes/no experiments, each which yields success with probability  $p$  is  $\text{Bin}(n, p)$ ,  $n = 1, 2, \dots$ ,  $0 \leq p \leq 1$ .

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$  is approximately  $\text{Po}(np)$  for small  $p$ .

### 10.2.2 First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability  $p$  is  $\text{Fs}(p)$ ,  $0 \leq p \leq 1$ .

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

### 10.2.3 Poisson distribution

The number of events occurring in a fixed period of time  $t$  if these events occur with a known average rate  $\kappa$  and independently of the time since the last event is  $\text{Po}(\lambda)$ ,  $\lambda = t\kappa$ .

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

### 10.3 Probability Theory

Let  $X$  be a discrete random variable with probability  $p_X(x)$  of assuming the value  $x$ . It will then have an expected value (mean)  $\mu = E(X) = \sum_x x p_X(x)$  and variance  $\sigma^2 = V(X) = E(X^2) - (E(X))^2 = \sum_x (x - E(X))^2 p_X(x)$  where  $\sigma$  is the standard deviation. If  $X$  is instead continuous it will have a probability density function  $f_X(x)$  and the sums above will instead be integrals with  $p_X(x)$  replaced by  $f_X(x)$ .

Expectation is linear:

$$E(aX + bY) = aE(X) + bE(Y)$$

For independent  $X$  and  $Y$ ,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

## 11 Strings

### 11.1 Hashing

```
struct H {
    typedef uint64_t ull;
    ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
    (A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) :
    B); return r; }
    OP(+, "d"(o.x)) OP(*, "mul %1\n", "r"(o.x) :
    "rdx")
    H operator-(H o) { return *this + ~o.x; }
    ull get() const { return x + !~x; }
    bool operator==(H o) const { return get() ==
    o.get(); }
    bool operator<(H o) const { return get() <
    o.get(); }
};

static const H C = (1l)1e11+3; // (order ~ 3e9; random
    also ok)

struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1),
    pw(ha) {
        pw[0] = 1;
        rep(i,0,sz(str))
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    }
};
```

```
    }
};

vector<H> getHashes(string& str, int length) {
    if (sz(str) < length) return {};
    H h = 0, pw = 1;
    rep(i,0,length)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    rep(i,length,sz(str)) {
        ret.push_back(h = h * C + str[i] - pw *
            str[i-length]);
    }
    return ret;
}

H hashString(string& s){H h{}; for(char c:s)
    h=h*C+c;return h;}
```

### 11.2 Incremental Aho Corasick

```
class IncrementalAhoCorasick {
    static const int Alphabets = 26;
    static const int AlphabetBase = 'a';
    struct Node {
        Node *fail;
        Node *next[Alphabets];
        int sum;
        Node() : fail(NULL), next{}, sum(0) {}
    };

    struct String {
        string str;
        int sign;
    };

public:
    //totalLen = sum of (len + 1)
    void init(int totalLen) {
        nodes.resize(totalLen);
        nNodes = 0;
        strings.clear();
        roots.clear();
        sizes.clear();
        que.resize(totalLen);
    }

    void insert(const string &str, int sign) {
        strings.push_back(String{ str, sign });
        roots.push_back(nodes.data() + nNodes);
        sizes.push_back(1);
    }
};
```

```
nNodes += (int)str.size() + 1;
auto check = [&]() { return sizes.size() > 1 &&
    sizes.end()[-1] == sizes.end()[-2]; };
if(!check())
    makePMA(strings.end() - 1, strings.end(),
        roots.back(), que);
while(check()) {
    int m = sizes.back();
    roots.pop_back();
    sizes.pop_back();
    sizes.back() += m;
    if(!check())
        makePMA(strings.end() - m * 2, strings.end(),
            roots.back(), que);
}

int match(const string &str) const {
    int res = 0;
    for(const Node *t : roots)
        res += matchPMA(t, str);
    return res;
}

private:
    static void makePMA(vector<String>::const_iterator
        begin, vector<String>::const_iterator end, Node
        *nodes, vector<Node*> &que) {
        int nNodes = 0;
        Node *root = new(&nodes[nNodes++]) Node();
        for(auto it = begin; it != end; ++it) {
            Node *t = root;
            for(char c : it->str) {
                Node *&n = t->next[c - AlphabetBase];
                if(n == nullptr)
                    n = new(&nodes[nNodes++]) Node();
                t = n;
            }
            t->sum += it->sign;
        }
        int qt = 0;
        for(Node *&n : root->next) {
            if(n != nullptr) {
                n->fail = root;
                que[qt++] = n;
            } else {
                n = root;
            }
        }
        for(int qh = 0; qh != qt; ++qh) {
            Node *t = que[qh];
            int a = 0;
            for(Node *n : t->next) {

```

```

    if(n != nullptr) {
        que[qt++] = n;
        Node *r = t->fail;
        while(r->next[a] == nullptr)
            r = r->fail;
        n->fail = r->next[a];
        n->sum += r->next[a]->sum;
    }
    ++a;
}
}
}

static int matchPMA(const Node *t, const string &str)
{
    int res = 0;
    for(char c : str) {
        int a = c - AlphabetBase;
        while(t->next[a] == nullptr)
            t = t->fail;
        t = t->next[a];
        res += t->sum;
    }
    return res;
}

vector<Node> nodes;
int nNodes;
vector<String> strings;
vector<Node*> roots;
vector<int> sizes;
vector<Node*> que;
};

```

```

int main() {
    int m;
    while(~scanf("%d", &m)) {
        IncrementalAhoCorasic iac;
        iac.init(600000);
        rep(i, m) {
            int ty;
            char s[300001];
            scanf("%d%s", &ty, s);
            if(ty == 1) {
                iac.insert(s, +1);
            } else if(ty == 2) {
                iac.insert(s, -1);
            } else if(ty == 3) {
                int ans = iac.match(s);
                printf("%d\n", ans);
                fflush(stdout);
            } else {

```

```

        abort();
    }
}
return 0;
}

```

### 11.3 KMP

```

vi pi(const string& s) {
    vi p(sz(s));
    rep(i, 1, sz(s)) {
        int g = p[i-1];
        while (g && s[i] != s[g]) g = p[g-1];
        p[i] = g + (s[i] == s[g]);
    }
    return p;
}

vi match(const string& s, const string& pat) {
    vi p = pi(pat + '\0' + s), res;
    rep(i, sz(p)-sz(s), sz(p))
        if (p[i] == sz(pat)) res.push_back(i - 2 * sz(pat));
    return res;
}

```

### 11.4 Minimal String Rotation

```

// Lexicographically minimal string rotation
int lmsr() {
    string s;
    cin >> s;
    int n = s.size();
    s += s;
    vector<int> f(s.size(), -1);
    int k = 0;
    for (int j = 1; j < 2 * n; ++j) {
        int i = f[j - k - 1];
        while (i != -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1])
                k = j - i - 1;
            i = f[i];
        }
        if (i == -1 && s[j] != s[k + i + 1]) {
            if (s[j] < s[k + i + 1]) {
                k = j;
            }

```

```

        f[j - k] = -1;
    } else {
        f[j - k] = i + 1;
    }
}
return k;
}

```

### 11.5 Suffix Array

```

const int MAXN = 200005;
const int MAX_DIGIT = 256;
void countingSort(vector<int>& SA, vector<int>& RA, int k = 0) {
    int n = SA.size();
    vector<int> cnt(max(MAX_DIGIT, n), 0);
    for (int i = 0; i < n; i++)
        if (i + k < n)
            cnt[RA[i + k]]++;
    else
        cnt[0]++;
    for (int i = 1; i < cnt.size(); i++)
        cnt[i] += cnt[i - 1];
    vector<int> tempSA(n);
    for (int i = n - 1; i >= 0; i--)
        if (SA[i] + k < n)
            tempSA[--cnt[RA[SA[i] + k]]] = SA[i];
        else
            tempSA[--cnt[0]] = SA[i];
    SA = tempSA;
}

vector<int> constructSA(string s) {
    int n = s.length();
    vector<int> SA(n);
    vector<int> RA(n);
    vector<int> tempRA(n);
    for (int i = 0; i < n; i++) {
        RA[i] = s[i];
        SA[i] = i;
    }
    for (int step = 1; step < n; step <= 1) {
        countingSort(SA, RA, step);
        countingSort(SA, RA, 0);
        int c = 0;
        tempRA[SA[0]] = c;
        for (int i = 1; i < n; i++) {
            if (RA[SA[i]] == RA[SA[i - 1]] && RA[SA[i] + step] == RA[SA[i - 1] + step])
                tempRA[SA[i]] = tempRA[SA[i - 1]];

```



```

        else
            tempRA[SA[i]] = tempRA[SA[i - 1]] + 1;
    }
    RA = tempRA;
    if (RA[SA[n - 1]] == n - 1) break;
}
return SA;
}

```

```

vector<int> computeLCP(const string& s, const
    vector<int>& SA) {
    int n = SA.size();
    vector<int> LCP(n), PLCP(n), c(n, 0);
    for (int i = 0; i < n; i++)
        c[SA[i]] = i;
    int k = 0;
    for (int j, i = 0; i < n-1; i++) {
        if (c[i] - 1 < 0)
            continue;
        j = SA[c[i] - 1];
        k = max(k - 1, 0);
        while (i+k < n && j+k < n && s[i+k] == s[j+k])
            k++;
        PLCP[i] = k;
    }
    for (int i = 0; i < n; i++)
        LCP[i] = PLCP[SA[i]];
    return LCP;
}

```

## 11.6 Suffix Automation

```

/*
 * Suffix automaton:
 * This implementation was extended to maintain
 * (online) the
 * number of different substrings. This is equivalent
 * to compute
 * the number of paths from the initial state to all
 * the other
 * states.
 *
 * The overall complexity is O(n)
 * can be tested here:
 * https://www.urionlinejudge.com.br/judge/en/problems/view/1530
 */
struct state {
    int len, link;
    long long num_paths;
}

```

```

    map<int, int> next;
};

const int MN = 200011;
state sa[MN << 1];
int sz, last;
long long tot_paths;

void sa_init() {
    sz = 1;
    last = 0;
    sa[0].len = 0;
    sa[0].link = -1;
    sa[0].next.clear();
    sa[0].num_paths = 1;
    tot_paths = 0;
}

void sa_extend(int c) {
    int cur = sz++;
    sa[cur].len = sa[last].len + 1;
    sa[cur].next.clear();
    sa[cur].num_paths = 0;
    int p;
    for (p = last; p != -1 && !sa[p].next.count(c); p =
        sa[p].link) {
        sa[p].next[c] = cur;
        sa[cur].num_paths += sa[p].num_paths;
        tot_paths += sa[p].num_paths;
    }

    if (p == -1) {
        sa[cur].link = 0;
    } else {
        int q = sa[p].next[c];
        if (sa[p].len + 1 == sa[q].len) {
            sa[cur].link = q;
        } else {
            int clone = sz++;
            sa[clone].len = sa[p].len + 1;
            sa[clone].next = sa[q].next;
            sa[clone].num_paths = 0;
            sa[clone].link = sa[q].link;
            for (; p != -1 && sa[p].next[c] == q; p =
                sa[p].link) {
                sa[p].next[c] = clone;
                sa[q].num_paths -= sa[p].num_paths;
                sa[clone].num_paths += sa[p].num_paths;
            }
            sa[q].link = sa[cur].link = clone;
        }
    }
    last = cur;
}

```

```

}

```

## 11.7 Suffix Tree

```

struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~
        2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;

    void ukkadd(int i, int c) { suff:
        if (r[v] <= q) {
            if (t[v][c] == -1) { t[v][c] = m;
                l[m] = i;
                p[m++] = v; v = s[v]; q = r[v];
                goto suff; }
            v = t[v][c]; q = l[v];
        }
        if (q == -1 || c == toi(a[q])) q++; else {
            l[m+1] = i; p[m+1] = m; l[m] = l[v];
            r[m] = q;
            p[m] = p[v]; t[m][c] = m+1;
            t[m][toi(a[q])] = v;
            l[v] = q; p[v] = m;
            t[p[m]][toi(a[l[m]])] = m;
            v = s[p[m]]; q = l[m];
            while (q < r[m]) {
                v = t[v][toi(a[q])];
                q += r[v] - l[v]; }
            if (q == r[m]) s[m] = v; else
                s[m] = m+2;
            q = r[v] - (q - r[m]); m += 2; goto suff;
        }
    }

    SuffixTree(string a) : a(a) {
        fill(r, r+N, sz(a));
        memset(s, 0, sizeof s);
        memset(t, -1, sizeof t);
        fill(t[1], t[1]+ALPHA, 0);
        s[0] = 1; l[0] = l[1] = -1; r[0] = r[1]
            = p[0] = p[1] = 0;
        rep(i, 0, sz(a)) ukkadd(i, toi(a[i]));
    }

    // example: find longest common substring (uses
    ALPHA = 28)
    pii best;
    int lcs(int node, int i1, int i2, int olen) {

```



```

    if (l[node] <= i1 && i1 < r[node])
        return 1;
    if (l[node] <= i2 && i2 < r[node])
        return 2;
    int mask = 0, len = node ? olen +
        (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], i1, i2,
            len);
    if (mask == 3)
        best = max(best, {len, r[node] -
            len});
    return mask;
}
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t +
        (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
}
};

```

## 11.8 Z Algorithm

```

vector<int> compute_z(const string &s){
    int n = s.size();
    vector<int> z(n,0);
    int l,r;
    r = l = 0;
    for(int i = 1; i < n; ++i){
        if(i > r) {
            l = r = i;
            while(r < n and s[r - l] == s[r])r++;
            z[i] = r - l;r--;
        }else{
            int k = i-l;
            if(z[k] < r - i +1) z[i] = z[k];
            else {
                l = i;
                while(r < n and s[r - l] == s[r])r++;
                z[i] = r - l;r--;
            }
        }
    }
}

```

```

    }
    return z;
}

int main(){
    //string line;cin>>line;
    string line = "alfalfa";
    vector<int> z = compute_z(line);

    for(int i = 0; i < z.size(); ++i ){
        if(i)cout<<" ";
        cout<<z[i];
    }
    cout<<endl;

    // must print "0 0 0 4 0 0 1"

    return 0;
}

```