Team notebook

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1 Algorithms

1.1 Mo's Algorithm

```
https://www.spoj.com/problems/FREQ2/
vector <int> MoQueries(int n, vector <query> Q){
   block_size = sqrt(n);
   sort(Q.begin(), Q.end(), [](const query &A,
       const query &B){
       return (A.1/block_size != B.1/block_size)?
           (A.l/block_size < B.l/block_size) :</pre>
           (A.r < B.r);
   });
   vector <int> res;
   res.resize((int)Q.size());
   int L = 1, R = 0;
   for(query q: Q){
       while (L > q.1) add(--L);
       while (R < q.r) add(++R);
       while (L < q.1) del(L++);
       while (R > q.r) del(R--);
       res[q.pos] = calc(1, R-L+1);
   }
   return res;
```

1.2 Mo's Algorithms on Trees

```
Given a tree with N nodes and Q queries. Each
    node has an integer weight.
Each query provides two numbers u and v, ask for
    how many different integers weight of nodes
there are on path from u to v.
Modify DFS:
For each node u, maintain the start and the end
    DFS time. Let's call them ST(u) and EN(u).
=> For each query, a node is considered if its
    occurrence count is one.
Query solving:
Let's query be (u, v). Assume that ST(u) <=
    ST(v). Denotes P as LCA(u, v).
Case 1: P = u
Our query would be in range [ST(u), ST(v)].
Case 2: P != u
Our query would be in range [EN(u), ST(v)] +
    [ST(p), ST(p)]
*/
```

```
void update(int &L, int &R, int qL, int qR){
    while (L > qL) add(--L);
    while (R < qR) add(++R);
    while (L < qL) del(L++);</pre>
    while (R > qR) del(R--);
}
vector <int> MoQueries(int n, vector <query> Q){
    block_size = sqrt((int)nodes.size());
    sort(Q.begin(), Q.end(), [](const query &A,
        const query &B){
       return (ST[A.1]/block_size !=
           ST[B.1]/block size)?
           (ST[A.1]/block_size <
           ST[B.1]/block size) : (ST[A.r] <
           ST[B.r]);
    });
    vector <int> res;
    res.resize((int)Q.size());
    LCA lca;
    lca.initialize(n);
    int L = 1, R = 0;
    for(query q: Q){
       int u = q.1, v = q.r;
       if(ST[u] > ST[v]) swap(u, v); // assume
           that S[u] <= S[v]
       int parent = lca.get(u, v);
       if(parent == u){
           int qL = ST[u], qR = ST[v];
           update(L, R, qL, qR);
       }else{
           int qL = EN[u], qR = ST[v];
           update(L, R, qL, qR);
           if(cnt_val[a[parent]] == 0)
              res[q.pos] += 1;
       }
       res[q.pos] += cur_ans;
    return res;
```

}

1.3 Parallel Binary Search

```
int lo[N], mid[N], hi[N];
vector<int> vec[N];
void clear() //Reset
       memset(bit, 0, sizeof(bit));
}
void apply(int idx) //Apply ith update/query
       if(ql[idx] <= qr[idx])</pre>
               update(ql[idx], qa[idx]),
                   update(qr[idx]+1, -qa[idx]);
       else
       {
               update(1, qa[idx]);
               update(qr[idx]+1, -qa[idx]);
               update(ql[idx], qa[idx]);
       }
}
bool check(int idx) //Check if the condition is
    satisfied
{
       int req=reqd[idx];
       for(auto &it:owns[idx])
               req-=pref(it);
               if(req<0)</pre>
                      break;
       if(req<=0)</pre>
               return 1;
       return 0;
}
void work()
```

```
for(int i=1;i<=q;i++)</pre>
                vec[i].clear();
        for(int i=1;i<=n;i++)</pre>
               if(mid[i]>0)
                        vec[mid[i]].push_back(i);
        clear():
        for(int i=1;i<=q;i++)</pre>
                apply(i);
                for(auto &it:vec[i]) //Add
                    appropriate check conditions
                        if(check(it))
                               hi[it]=i:
                        else
                               lo[it]=i+1:
}
void parallel_binary()
        for(int i=1;i<=n;i++)</pre>
               lo[i]=1, hi[i]=q+1;
        bool changed = 1;
        while(changed)
                changed=0;
                for(int i=1;i<=n;i++)</pre>
                        if(lo[i]<hi[i])</pre>
                                changed=1;
                               mid[i]=(lo[i] +
                                    hi[i])/2:
                       }
                        else
                               mid[i]=-1;
                work();
        }
}
```

2 Combinatorics

2.1 Factorial Approximate

Approximate Factorial:

$$n! = \sqrt{2.\pi \cdot n} \cdot \left(\frac{n}{e}\right)^n \tag{1}$$

2.2 Factorial

						9		
$\overline{n!}$	1 2 6	5 24 1	20 72	0 5040	40320	362880	3628800	
n	11	12	13	14	15	16	17	
n!	4.0e7	7 4.8€	8 6.2e	9 8.7e	10 1.3e	$12 \ 2.1e1$	3 3.6e14	
n	20	25	30	40	50 10	00 - 150) 171	
n!	2e18	2e25	3e32	8e47 3	Be64 9e1	157 6e26	$62 > DBL_MA$	X

2.3 Fast Fourier Transform

```
/**
* Fast Fourier Transform.
* Useful to compute convolutions.
* computes:
* C(f \operatorname{star} g)[n] = \operatorname{sum}_m(f[m] * g[n - m])
* for all n.
* test: icpc live archive, 6886 - Golf Bot
* */
using namespace std;
#include <bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'
const int MN = 262144 << 1;</pre>
int d[MN + 10], d2[MN + 10];
const double PI = acos(-1.0);
struct cpx {
```

```
double real, image;
  cpx(double _real, double _image) {
   real = _real;
   image = _image;
 cpx(){}
};
cpx operator + (const cpx &c1, const cpx &c2) {
 return cpx(c1.real + c2.real, c1.image +
      c2.image);
cpx operator - (const cpx &c1, const cpx &c2) {
 return cpx(c1.real - c2.real, c1.image -
      c2.image):
cpx operator * (const cpx &c1, const cpx &c2) {
 return cpx(c1.real*c2.real - c1.image*c2.image,
      c1.real*c2.image + c1.image*c2.real);
int rev(int id, int len) {
 int ret = 0;
 for (int i = 0; (1 << i) < len; i++) {
   ret <<= 1:
   if (id & (1 << i)) ret |= 1;
 return ret;
cpx A[1 << 20];
void FFT(cpx *a, int len, int DFT) {
 for (int i = 0; i < len; i++)</pre>
   A[rev(i, len)] = a[i];
 for (int s = 1; (1 << s) <= len; s++) {
   int m = (1 << s);
   cpx wm = cpx(cos(DFT * 2 * PI / m), sin(DFT)
       * 2 * PI / m));
   for(int k = 0; k < len; k += m) {</pre>
     cpx w = cpx(1, 0);
     for(int j = 0; j < (m >> 1); j++) {
```

```
cpx t = w * A[k + j + (m >> 1)];
       cpx u = A[k + j];
       A[k + j] = u + t;
       A[k + j + (m >> 1)] = u - t;
       w = w * wm;
  if (DFT == -1) for (int i = 0; i < len; i++)</pre>
      A[i].real /= len, A[i].image /= len;
 for (int i = 0; i < len; i++) a[i] = A[i];</pre>
}
cpx in[1 << 20];
void solve(int n) {
 memset(d, 0, sizeof d);
 int t:
  for (int i = 0; i < n; ++i) {</pre>
   cin >> t;
   d[t] = true;
  int m;
  cin >> m;
 vector<int> q(m);
 for (int i = 0; i < m; ++i)</pre>
   cin >> q[i];
 for (int i = 0; i < MN; ++i) {</pre>
   if (d[i])
     in[i] = cpx(1, 0);
      in[i] = cpx(0, 0);
 FFT(in, MN, 1);
 for (int i = 0; i < MN; ++i) {</pre>
   in[i] = in[i] * in[i];
 FFT(in, MN, -1);
 int ans = 0;
 for (int i = 0; i < q.size(); ++i) {</pre>
```

```
if (in[q[i]].real > 0.5 || d[q[i]]) {
    ans++;
  }
}
cout << ans << endl;
}
int main() {
  ios_base::sync_with_stdio(false);cin.tie(NULL);
  int n;
  while (cin >> n)
    solve(n);
  return 0;
}
```

2.4 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$ Sums of powers:

$$\sum_{i=1}^{n} n^{m} = \frac{1}{m+1} \sum_{k=0}^{m} {m+1 \choose k} B_{k} \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_{m}^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_{m}^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k), \ c(0,0) = 1$$

$$\sum_{k=0}^{n} c(n,k)x^{k} = x(x+1)\dots(x+n-1)$$

c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$
$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^{n}$$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, k+1 j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{i=1}^{k} (-1)^{i} \binom{n+1}{i} (k+1-j)^{n}$$

Bell numbers

Total number of partitions of n distinct elements. B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, ... For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

Labeled unrooted trees

on n vertices: n^{n-2} # on k existing trees of size n_i : $n_1 n_2 \cdots n_k n^{k-2}$ # with degrees d_i : $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1} {2n \choose n} = {2n \choose n} - {2n \choose n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1$$
, $C_{n+1} = \frac{2(2n+1)}{n+2}C_n$, $C_{n+1} = \sum C_i C_{n-i}$

 $C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...$ [noitemsep]sub-diagonal monotone paths in an $n \times n$ grid. strings with n pairs of parenthesis, correctly nested. binary trees with with n+1 leaves (0 or 2 children). ordered trees with n+1 vertices. ways a convex polygon with n+2 sides can be cut into triangles by connecting vertices with straight lines. permutations of [n] with no 3-term increasing subseq.

2.5 Lucas Theorem

For non-negative integers m and n and a prime p, the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{n_i} \pmod{p},$$

where:

$$m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0,$$

and:

$$n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$$

are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \le n$.

2.6 Multinomial

```
/**
 * Description: Computes $\displaystyle
    \binom{k_1 + \dots + k_n}{k_1, k_2, \dots,
    k_n} = \frac{(\sum k_i)!}{k_1!k_2!...k_n!}$.

* Status: Tested on kattis:lexicography
 */

#pragma once

long long multinomial(vector<int>& v) {
    long long c = 1, m = v.empty() ? 1 : v[0];
    for (long long i = 1; i < v.size(); i++) {
        for (long long j = 0; j < v[i]; j++) {
            c = c * ++m / (j + 1);
        }
    }
    return c;
}</pre>
```

2.7 Others

Cycles Let $g_S(n)$ be the number of *n*-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

Derangements Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n =$$

Burnside's lemma Given a group G of symmetries and a set X, the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g (g.x = x).

If f(n) counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = Z_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

2.8 Permutation To Int

2.9 Sigma Function

The Sigma Function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when x = 0 is called the divisor function, that counts the number of positive divisors of n.

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

If n is written as prime factorization:

$$n = \prod_{i=1}^{k} P_i^{e_k}$$

We can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^{k} g(e_k + 1)$$

where g(x) is the sum of the first x positive numbers:

$$g(x) = (x * (x+1))/2$$

3 Data Structures

3.1 Binary Index Tree

```
struct BIT {
    int n;
    int t[2 * N];
    void add(int where, long long what) {
       for (where++; where <= n; where += where &</pre>
            -where) {
           t[where] += what;
       }
    }
    void add(int from, int to, long long what) {
        add(from, what);
        add(to + 1, -what);
    }
    long long query(int where) {
        long long sum = t[0];
        for (where++; where > 0; where -= where &
             -where) {
            sum += t[where]:
        return sum;
    }
};
```

3.2 Disjoint Set Uninon (DSU)

```
void Union(int u, int v){
    int x = parent[u] + parent[v];
    if(parent[u] > parent[v]){
        parent[v] = x;
        parent[u] = v;
    }else{
        parent[u] = x;
        parent[v] = u;
    }
};
```

3.3 Fake Update

```
vector <int> fake bit[MAXN]:
void fake_update(int x, int y, int limit_x){
    for(int i = x; i < limit_x; i += i&(-i))</pre>
       fake_bit[i].pb(y);
}
void fake_get(int x, int y){
    for(int i = x; i >= 1; i -= i&(-i))
       fake_bit[i].pb(y);
}
vector <int> bit[MAXN];
void update(int x, int y, int limit_x, int val){
    for(int i = x; i < limit_x; i += i&(-i)){
       for(int j =
            lower_bound(fake_bit[i].begin(),
            fake_bit[i].end(), y) -
            fake_bit[i].begin(); j <</pre>
            fake_bit[i].size(); j += j&(-j))
           bit[i][j] = max(bit[i][j], val);
   }
}
int get(int x, int y){
```

```
int ans = 0:
   for(int i = x; i >= 1; i -= i&(-i)){
       for(int j =
           lower_bound(fake_bit[i].begin(),
           fake_bit[i].end(), y) -
           fake_bit[i].begin(); j >= 1; j -=
           i&(-i))
           ans = max(ans, bit[i][j]);
   }
   return ans:
}
int main(){
    io
   int n; cin >> n;
   vector <int> Sx, Sy;
   for(int i = 1; i <= n; i++){</pre>
       cin >> a[i].fi >> a[i].se;
       Sx.pb(a[i].fi);
       Sy.pb(a[i].se);
   unique_arr(Sx);
   unique_arr(Sy);
   // unique all value
   for(int i = 1; i <= n; i++){</pre>
       a[i].fi = lower_bound(Sx.begin(),
            Sx.end(), a[i].fi) - Sx.begin();
       a[i].se = lower_bound(Sy.begin(),
            Sy.end(), a[i].se) - Sy.begin();
   }
   // do fake BIT update and get operator
   for(int i = 1; i <= n; i++){</pre>
       fake_get(a[i].fi-1, a[i].se-1);
       fake_update(a[i].fi, a[i].se,
            (int)Sx.size());
   }
   for(int i = 0; i < Sx.size(); i++){</pre>
       fake_bit[i].pb(INT_MIN); // avoid zero
       sort(fake_bit[i].begin(),
            fake_bit[i].end());
       fake_bit[i].resize(unique(fake_bit[i].begin(),
            fake_bit[i].end()) -
```

3.4 Fenwick Tree

```
template <typename T>
class FenwickTree{
 vector <T> fenw:
 int n;
public:
  void initialize(int _n){
   this \rightarrow n = n;
   fenw.resize(n+1);
  void update(int id, T val) {
   while (id \leq n) {
     fenw[id] += val:
     id += id\&(-id);
     }
 }
 T get(int id){
   T ans{};
   while(id >= 1){
     ans += fenw[id];
     id = id\&(-id);
   return ans;
```

};

3.5 Hash Table

```
/*

* Micro hash table, can be used as a set.

* Very efficient vs std::set

*

*/

const int MN = 1001;

struct ht {
  int _s[(MN + 10) >> 5];
  int len;
  void set(int id) {
    len++;
    _s[id >> 5] |= (1LL << (id & 31));
  }

bool is_set(int id) {
  return _s[id >> 5] & (1LL << (id & 31));
  }

};
```

3.6 STL Treap

```
struct Node {
    Node *1 = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(1) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val);
        each(n->r, f); }
}
```

```
pair<Node*, Node*> split(Node* n, int k) {
       if (!n) return {};
       if (cnt(n->1) >= k) { // "n->val >= k" for
           lower bound(k)
              auto pa = split(n->1, k);
              n->1 = pa.second;
              n->recalc();
              return {pa.first, n};
       } else {
               auto pa = split(n->r, k - cnt(n->1)
                  - 1); // and just "k"
              n->r = pa.first;
              n->recalc();
              return {n, pa.second};
       }
}
Node* merge(Node* 1, Node* r) {
       if (!1) return r;
       if (!r) return 1;
       if (1->y > r->y) {
              1->r = merge(1->r, r);
              1->recalc();
              return 1;
       } else {
              r->1 = merge(1, r->1);
              r->recalc():
              return r;
       }
}
Node* ins(Node* t, Node* n, int pos) {
       auto pa = split(t, pos);
       return merge(merge(pa.first, n),
           pa.second);
}
// Example application: move the range [1, r) to
    index k
void move(Node*& t, int 1, int r, int k) {
       Node *a, *b, *c;
       tie(a,b) = split(t, 1); tie(b,c) =
           split(b, r - 1);
       if (k <= 1) t = merge(ins(a, b, k), c);</pre>
```

```
else t = merge(a, ins(c, b, k - r));
}
```

3.7 Segment Tree

```
#include <bits/stdc++.h>
using namespace std;
const int N = 1e5 + 10;
int node[4*N]:
void modify(int seg, int 1, int r, int p, int
    val){
   if(1 == r){
       node[seg] += val;
       return:
   int mid = (1 + r)/2;
   if(p \le mid){
       modify(2*seg + 1, 1, mid, p, val);
       modify(2*seg + 2, mid + 1, r, p, val);
   node[seg] = node[2*seg + 1] + node[2*seg + 2];
int sum(int seg, int 1, int r, int a, int b){
   if(1 > b || r < a) return 0;</pre>
   if(1 >= a && r <= b) return node[seg];</pre>
   int mid = (1 + r)/2:
   return sum(2*seg + 1, 1, mid, a, b) +
        sum(2*seg + 2, mid + 1, r, a, b);
}
```

3.8 Sparse Table

```
template <typename T, typename func =
  function<T(const T, const T)>>
```

```
struct SparseTable {
   func calc;
   int n;
   vector<vector<T>> ans;
   SparseTable() {}
   SparseTable(const vector<T>& a, const func&
        f) : n(a.size()), calc(f) {
       int last = trunc(log2(n)) + 1;
       ans.resize(n);
       for (int i = 0; i < n; i++){</pre>
           ans[i].resize(last);
       }
       for (int i = 0; i < n; i++){</pre>
           ans[i][0] = a[i]:
       }
       for (int j = 1; j < last; j++){</pre>
           for (int i = 0; i <= n - (1 << j);</pre>
               i++){
               ans[i][j] = calc(ans[i][j - 1],
                   ans[i + (1 << (j - 1))][j -
                   1]);
           }
       }
   }
```

3.9 Trie

```
const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

void clear(){
        tree[nodes].c = 0;
```

```
memset(tree[nodes].a, -1, sizeof
        tree[nodes].a);
   nodes++;
 void init(){
   nodes = 0;
   clear();
 int add(const string &s, bool query = 0){
   int cur_node = 0;
   for(int i = 0; i < s.size(); ++i){</pre>
     int id = gid(s[i]);
     if(tree[cur_node].a[id] == -1){
       if(query) return 0;
       tree[cur_node].a[id] = nodes;
       clear();
     }
     cur_node = tree[cur_node].a[id];
   if(!query) tree[cur_node].c++;
   return tree[cur_node].c;
};
```