Team notebook

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Contents

1 Algorithms

1.1 Mo's Algorithm

```
/*
    https://www.spoj.com/problems/FREQ2/
*/
vector <int> MoQueries(int n, vector <query> Q){

    block_size = sqrt(n);
    sort(Q.begin(), Q.end(), [](const query &A, const
         query &B){
        return (A.l/block_size != B.l/block_size)?
             (A.l/block_size < B.l/block_size) : (A.r <
             B.r);
    });
    vector <int> res;
    res.resize((int)Q.size());

    int L = 1, R = 0;
    for(query q: Q){
        while (L > q.l) add(--L);
        while (R < q.r) add(++R);

        while (L < q.l) del(L++);
        while (R > q.r) del(R--);

        res[q.pos] = calc(1, R-L+1);
    }
    return res;
}
```

1.2 Mo's Algorithms on Trees

```
/*
Given a tree with N nodes and Q queries. Each node has
     an integer weight.
Each query provides two numbers u and v, ask for how
     many different integers weight of nodes
there are on path from u to v.

----------
Modify DFS:
----------
For each node u, maintain the start and the end DFS
     time. Let's call them ST(u) and EN(u).
=> For each query, a node is considered if its
     occurrence count is one.

--------------
Query solving:
--------------
Let's query be (u, v). Assume that ST(u) <= ST(v).
     Denotes P as LCA(u, v).

Case 1: P = u
Our query would be in range [ST(u), ST(v)].

Case 2: P != u
Our query would be in range [EN(u), ST(v)] + [ST(p),
     ST(p)]
*/

void update(int &L, int &R, int qL, int qR){
    while (L > qL) add(--L);
    while (R < qR) add(++R);

    while (L < qL) del(L++);
    while (R > qR) del(R--);
}

vector <int> MoQueries(int n, vector <query> Q){
    block_size = sqrt((int)nodes.size());
    sort(Q.begin(), Q.end(), [](const query &A, const
         query &B){
        return (ST[A.l]/block_size !=
             ST[B.l]/block_size)? (ST[A.l]/block_size <
             ST[B.l]/block_size) : (ST[A.r] < ST[B.r]);
    });
    vector <int> res;
    res.resize((int)Q.size());

    LCA lca;
    lca.initialize(n);

    int L = 1, R = 0;
    for(query q: Q){
        int u = q.l, v = q.r;
        if(ST[u] > ST[v]) swap(u, v); // assume that
             S[u] <= S[v]
        int parent = lca.get(u, v);

        if(parent == u){
            int qL = ST[u], qR = ST[v];
            update(L, R, qL, qR);
        }else{
            int qL = EN[u], qR = ST[v];
            update(L, R, qL, qR);
            if(cnt_val[a[parent]] == 0)
                res[q.pos] += 1;
        }

        res[q.pos] += cur_ans;
    }
    return res;
}
```

1.3 Parallel Binary Search

```
int lo[N], mid[N], hi[N];
vector<int> vec[N];

void clear() //Reset
{
        memset(bit, 0, sizeof(bit));
}

void apply(int idx) //Apply ith update/query
{
        if(ql[idx] <= qr[idx])
                update(ql[idx], qa[idx]),
                     update(qr[idx]+1, -qa[idx]);
        else
        {
                update(1, qa[idx]);
                update(qr[idx]+1, -qa[idx]);
                update(ql[idx], qa[idx]);
        }
}

bool check(int idx) //Check if the condition is
     satisfied
{
        int req=reqd[idx];
        for(auto &it:owns[idx])
        {
                req-=pref(it);
                if(req<0)
                        break;
        }
        if(req<=0)
```

```
                return 1;
        return 0;
}

void work()
{
        for(int i=1;i<=q;i++)
                vec[i].clear();
        for(int i=1;i<=n;i++)
                if(mid[i]>0)
                        vec[mid[i]].push_back(i);
        clear();
        for(int i=1;i<=q;i++)
        {
                apply(i);
                for(auto &it:vec[i]) //Add appropriate
                     check conditions
                {
                        if(check(it))
                                hi[it]=i;
                        else
                                lo[it]=i+1;
                }
        }
}

void parallel_binary()
{
        for(int i=1;i<=n;i++)
                lo[i]=1, hi[i]=q+1;
        bool changed = 1;
        while(changed)
        {
                changed=0;
                for(int i=1;i<=n;i++)
                {
                        if(lo[i]<hi[i])
                        {
                                changed=1;
                                mid[i]=(lo[i] + hi[i])/2;
                        }
                        else
                                mid[i]=-1;
                }
                work();
        }
}
```

2 Combinatorics

2.1 Factorial Approximate

Approximate Factorial:

$$n! = \sqrt{2.\pi.n}.(\frac{n}{e})^n \tag{1}$$

2.2 Factorial

n	1 2 3	4	5	6	7	8	9	10	
$n!$	1 2 6	24	120	720	5040	40320	362880	3628800	
n	11	12	13	14	15	16	17		
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14		
n	20	25	30	40	50	100	150	171	
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX	

2.3 Fast Fourier Transform

```
/**
 * Fast Fourier Transform.
 * Useful to compute convolutions.
 * computes:
 *    C(f star g)[n] = sum_m(f[m] * g[n - m])
 * for all n.
 * test: icpc live archive, 6886 - Golf Bot
 * */


using namespace std;
#include <bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'

const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];


const double PI = acos(-1.0);

struct cpx {
  double real, image;
  cpx(double _real, double _image) {
    real = _real;
    image = _image;
  }
  cpx(){}
};

cpx operator + (const cpx &c1, const cpx &c2) {
  return cpx(c1.real + c2.real, c1.image + c2.image);
}

cpx operator - (const cpx &c1, const cpx &c2) {
  return cpx(c1.real - c2.real, c1.image - c2.image);
}

cpx operator * (const cpx &c1, const cpx &c2) {
  return cpx(c1.real*c2.real - c1.image*c2.image,
       c1.real*c2.image + c1.image*c2.real);
}

int rev(int id, int len) {
  int ret = 0;
  for (int i = 0; (1 << i) < len; i++) {
    ret <<= 1;
    if (id & (1 << i)) ret |= 1;
  }
  return ret;
}

cpx A[1 << 20];

void FFT(cpx *a, int len, int DFT) {
  for (int i = 0; i < len; i++)
    A[rev(i, len)] = a[i];
  for (int s = 1; (1 << s) <= len; s++) {
    int m = (1 << s);
    cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT * 2 *
         PI / m));
    for(int k = 0; k < len; k += m) {
      cpx w = cpx(1, 0);
      for(int j = 0; j < (m >> 1); j++) {
        cpx t = w * A[k + j + (m >> 1)];
        cpx u = A[k + j];
        A[k + j] = u + t;
        A[k + j + (m >> 1)] = u - t;
        w = w * wm;
      }
    }
  }
  if (DFT == -1) for (int i = 0; i < len; i++)
       A[i].real /= len, A[i].image /= len;
  for (int i = 0; i < len; i++) a[i] = A[i];
  return;
}

cpx in[1 << 20];

void solve(int n) {
  memset(d, 0, sizeof d);
  int t;
```

```
  for (int i = 0; i < n; ++i) {
    cin >> t;
    d[t] = true;
  }
  int m;
  cin >> m;
  vector<int> q(m);
  for (int i = 0; i < m; ++i)
    cin >> q[i];

  for (int i = 0; i < MN; ++i) {
    if (d[i])
      in[i] = cpx(1, 0);
    else
      in[i] = cpx(0, 0);
  }

  FFT(in, MN, 1);
  for (int i = 0; i < MN; ++i) {
    in[i] = in[i] * in[i];
  }
  FFT(in, MN, -1);

  int ans = 0;
  for (int i = 0; i < q.size(); ++i) {
    if (in[q[i]].real > 0.5 || d[q[i]]) {
      ans++;
    }
  }
  cout << ans << endl;
}

int main() {
  ios_base::sync_with_stdio(false);cin.tie(NULL);
  int n;
  while (cin >> n)
    solve(n);
  return 0;
}
```

2.4 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^\infty f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

[noitemsep]sub-diagonal monotone paths in an $n \times n$ grid. strings with n pairs of parenthesis, correctly nested. binary trees with with $n+1$ leaves (0 or 2 children). ordered trees with $n+1$ vertices. ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines. permutations of $[n]$ with no 3-term increasing subseq.

2.5 Lucas Theorem

For non-negative integers m and n and a prime p, the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^{k}\binom{m_i}{n_i} \pmod p,$$

where :

$$m = m_k p^k + m_{k-1}p^{k-1} + \cdots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1}p^{k-1} + \cdots + n_1 p + n_0$$

are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \leq n$.

2.6 Multinomial

```
/**
 * Description: Computes $\displaystyle \binom{k_1 +
     \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum
     k_i)!}{k_1!k_2!...k_n!}$.
 * Status: Tested on kattis:lexicography
 */
#pragma once

long long multinomial(vector<int>& v) {
        long long c = 1, m = v.empty() ? 1 : v[0];
        for (long long i = 1; i < v.size(); i++) {
```

```
        for (long long j = 0; j < v[i]; j++) {
            c = c * ++m / (j + 1);
        }
    }
        return c;
}
```

2.7 Others

Cycles Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

Derangements Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

Burnside's lemma Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = Z_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

2.8 Permutation To Int

```
/**
 * Description: Permutation -> integer conversion. (Not
     order preserving.)
 * Integer -> permutation can use a lookup table.
 * Time: O(n)
 **/

int permToInt(vector<int>& v) {
        int use = 0, i = 0, r = 0;
        for(int x : v) r = r * ++i +
             __builtin_popcount(use & -(1<<x)),
                use |= 1 << x;                 // (note:
                    minus, not ~!)
        return r;
}
```

2.9 Sigma Function

The Sigma Function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when $x = 0$ is called the divisor function, that counts the number of positive divisors of n.

Now, we are interested in find

$$\sum_{d|n}\sigma_0(d)$$

If n is written as prime factorization:

$$n = \prod_{i=1}^{k} P_i^{e_k}$$

We can demonstrate that:

$$\sum_{d|n}\sigma_0(d) = \prod_{i=1}^{k} g(e_k+1)$$

where $g(x)$ is the sum of the first x positive numbers:

$$g(x) = (x*(x+1))/2$$

3 Data Structures

3.1 Binary Index Tree

```
struct BIT {
    int n;
    int t[2 * N];

    void add(int where, long long what) {
       for (where++; where <= n; where += where &
            -where) {
           t[where] += what;
       }
    }

    void add(int from, int to, long long what) {
        add(from, what);
        add(to + 1, -what);
    }

    long long query(int where) {
        long long sum = t[0];

        for (where++; where > 0; where -= where &
             -where) {
            sum += t[where];
        }

        return sum;
    }
};
```

3.2 Disjoint Set Uninon (DSU)

```
class DSU{
public:
    vector <int> parent;
    void initialize(int n){
        parent.resize(n+1, -1);
    }

    int findSet(int u){
        while(parent[u] > 0)
            u = parent[u];
        return u;
    }

    void Union(int u, int v){
        int x = parent[u] + parent[v];
        if(parent[u] > parent[v]){
            parent[v] = x;
            parent[u] = v;
        }else{
            parent[u] = x;
            parent[v] = u;
        }
    }
};
```

3.3 Fake Update

```
vector <int> fake_bit[MAXN];

void fake_update(int x, int y, int limit_x){
    for(int i = x; i < limit_x; i += i&(-i))
        fake_bit[i].pb(y);
}

void fake_get(int x, int y){
    for(int i = x; i >= 1; i -= i&(-i))
        fake_bit[i].pb(y);
}

vector <int> bit[MAXN];

void update(int x, int y, int limit_x, int val){
    for(int i = x; i < limit_x; i += i&(-i)){
        for(int j = lower_bound(fake_bit[i].begin(),
             fake_bit[i].end(), y) -
             fake_bit[i].begin(); j <
             fake_bit[i].size(); j += j&(-j))
            bit[i][j] = max(bit[i][j], val);
    }
}

int get(int x, int y){
    int ans = 0;
    for(int i = x; i >= 1; i -= i&(-i)){
        for(int j = lower_bound(fake_bit[i].begin(),
             fake_bit[i].end(), y) -
             fake_bit[i].begin(); j >= 1; j -= j&(-j))
            ans = max(ans, bit[i][j]);
    }
    return ans;
}

int main(){
    _io
    int n; cin >> n;
    vector <int> Sx, Sy;
    for(int i = 1; i <= n; i++){
        cin >> a[i].fi >> a[i].se;
        Sx.pb(a[i].fi);
        Sy.pb(a[i].se);
    }
    unique_arr(Sx);
    unique_arr(Sy);
    // unique all value
    for(int i = 1; i <= n; i++){
        a[i].fi = lower_bound(Sx.begin(), Sx.end(),
             a[i].fi) - Sx.begin();
        a[i].se = lower_bound(Sy.begin(), Sy.end(),
             a[i].se) - Sy.begin();
    }

    // do fake BIT update and get operator
    for(int i = 1; i <= n; i++){
        fake_get(a[i].fi-1, a[i].se-1);
        fake_update(a[i].fi, a[i].se, (int)Sx.size());
    }

    for(int i = 0; i < Sx.size(); i++){
        fake_bit[i].pb(INT_MIN); // avoid zero
        sort(fake_bit[i].begin(), fake_bit[i].end());
        fake_bit[i].resize(unique(fake_bit[i].begin(),
             fake_bit[i].end()) - fake_bit[i].begin());
        bit[i].resize((int)fake_bit[i].size(), 0);
    }

    // real update, get operator
    int res = 0;
    for(int i = 1; i <= n; i++){
        int maxCurLen = get(a[i].fi-1, a[i].se-1) + 1;
        res = max(res, maxCurLen);
        update(a[i].fi, a[i].se, (int)Sx.size(),
             maxCurLen);
    }
}
```

3.4 Fenwick Tree

```
template <typename T>
class FenwickTree{
  vector <T> fenw;
  int n;
public:
  void initialize(int _n){
    this->n = _n;
    fenw.resize(n+1);
  }

  void update(int id, T val) {
    while (id <= n) {
      fenw[id] += val;
      id += id&(-id);
      }
  }

  T get(int id){
    T ans{};
    while(id >= 1){
      ans += fenw[id];
      id -= id&(-id);
    }
    return ans;
  }
};
```

3.5 Hash Table

```
/*
 * Micro hash table, can be used as a set.
 *  Very efficient vs std::set
 *
 */

const int MN = 1001;
struct ht {
  int _s[(MN + 10) >> 5];
  int len;
  void set(int id) {
    len++;
    _s[id >> 5] |= (1LL << (id & 31));
  }
  bool is_set(int id) {
    return _s[id >> 5] & (1LL << (id & 31));
  }
};
```

3.6 Range Minimum Query

```
/*
    return min(v[a], v[a + 1], ..., v[b - 1]) in
         constant time
*/

template<class T>
struct RMQ {
        vector<vector<T>> jmp;
        RMQ(const vector<T>& V) : jmp(1, V) {
                for (int pw = 1, k = 1; pw * 2 <= sz(V);
                    pw *= 2, ++k) {
                        jmp.emplace_back(sz(V) - pw * 2 +
                             1);
                        rep(j,0,sz(jmp[k]))
                                jmp[k][j] = min(jmp[k -
                                     1][j], jmp[k - 1][j +
                                     pw]);
                }
        }
```

```
	T query(int a, int b) {
		assert(a < b); // or return inf if a == b
		int dep = 31 - __builtin_clz(b - a);
		return min(jmp[dep][a], jmp[dep][b - (1
		    << dep)]);
	}
};
```

3.7 STL Treap

```
struct Node {
	Node *l = 0, *r = 0;
	int val, y, c = 1;
	Node(int val) : val(val), y(rand()) {}
	void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
	if (n) { each(n->l, f); f(n->val); each(n->r,
	    f); }
}

pair<Node*, Node*> split(Node* n, int k) {
	if (!n) return {};
	if (cnt(n->l) >= k) { // "n->val >= k" for
	    lower_bound(k)
		auto pa = split(n->l, k);
		n->l = pa.second;
		n->recalc();
		return {pa.first, n};
	} else {
		auto pa = split(n->r, k - cnt(n->l) -
		    1); // and just "k"
		n->r = pa.first;
		n->recalc();
		return {n, pa.second};
	}
}

Node* merge(Node* l, Node* r) {
	if (!l) return r;
	if (!r) return l;
	if (l->y > r->y) {
		l->r = merge(l->r, r);
		l->recalc();
		return l;
	} else {
		r->l = merge(l, r->l);
		r->recalc();
		return r;
	}
}

Node* ins(Node* t, Node* n, int pos) {
	auto pa = split(t, pos);
	return merge(merge(pa.first, n), pa.second);
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
	Node *a, *b, *c;
	tie(a,b) = split(t, l); tie(b,c) = split(b, r -
	    l);
	if (k <= l) t = merge(ins(a, b, k), c);
	else t = merge(a, ins(c, b, k - r));
}
```

3.8 Segment Tree

```
#include <bits/stdc++.h>
using namespace std;

const int N = 1e5 + 10;

int node[4*N];

void modify(int seg, int l, int r, int p, int val){
    if(l == r){
        node[seg] += val;
        return;
    }
    int mid = (l + r)/2;
    if(p <= mid){
        modify(2*seg + 1, l, mid, p, val);
    }else{
        modify(2*seg + 2, mid + 1, r, p, val);
    }
    node[seg] = node[2*seg + 1] + node[2*seg + 2];
}

int sum(int seg, int l, int r, int a, int b){
    if(l > b || r < a) return 0;
    if(l >= a && r <= b) return node[seg];
    int mid = (l + r)/2;
    return sum(2*seg + 1, l, mid, a, b) + sum(2*seg +
        2, mid + 1, r, a, b);
}
```

3.9 Sparse Table

```
template <typename T, typename func = function<T(const
    T, const T)>>
struct SparseTable {
    func calc;
    int n;
    vector<vector<T>> ans;

    SparseTable() {}

    SparseTable(const vector<T>& a, const func& f) :
        n(a.size()), calc(f) {
        int last = trunc(log2(n)) + 1;
        ans.resize(n);
        for (int i = 0; i < n; i++){
            ans[i].resize(last);
        }
        for (int i = 0; i < n; i++){
            ans[i][0] = a[i];
        }
        for (int j = 1; j < last; j++){
            for (int i = 0; i <= n - (1 << j); i++){
                ans[i][j] = calc(ans[i][j - 1], ans[i +
                    (1 << (j - 1))][j - 1]);
            }
        }
    }

    T query(int l, int r){
        assert(0 <= l && l <= r && r < n);
        int k = trunc(log2(r - l + 1));
        return calc(ans[l][k], ans[r - (1 << k) +
            1][k]);
    }
};
```

3.10 Trie

```
const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

struct trie{
  struct node{
    int c;
    int a[MN];
  };

  node tree[MS];
```

```
  int nodes;

  void clear(){
    tree[nodes].c = 0;
    memset(tree[nodes].a, -1, sizeof tree[nodes].a);
    nodes++;
  }

  void init(){
    nodes = 0;
    clear();
  }

  int add(const string &s, bool query = 0){
    int cur_node = 0;
    for(int i = 0; i < s.size(); ++i){
      int id = gid(s[i]);
      if(tree[cur_node].a[id] == -1){
        if(query) return 0;
        tree[cur_node].a[id] = nodes;
        clear();
      }
      cur_node = tree[cur_node].a[id];
    }
    if(!query) tree[cur_node].c++;
    return tree[cur_node].c;
  }

};
```

4 Dynamic Programming Optimization

4.1 Convex Hull Trick

```
#define long long long
#define pll pair <long, long>
#define all(c) c.begin(), c.end()
#define fastio ios_base::sync_with_stdio(false);
     cin.tie(0)

struct line{
    long a, b;
    line() {};
    line(long a, long b) : a(a), b(b) {};
    bool operator < (const line &A) const {
            return pll(a,b) < pll(A.a,A.b);
        }
};

bool bad(line A, line B, line C){
    return (C.b - B.b) * (A.a - B.a) <= (B.b - A.b) *
        (B.a - C.a);
}

void addLine(vector<line> &memo, line cur){
    int k = memo.size();
    while (k >= 2 && bad(memo[k - 2], memo[k - 1],
        cur)){
        memo.pop_back();
        k--;
    }
    memo.push_back(cur);
}

long Fn(line A, long x){
    return A.a * x + A.b;
}

long query(vector<line> &memo, long x){
    int lo = 0, hi = memo.size() - 1;
    while (lo != hi){
        int mi = (lo + hi) / 2;
        if (Fn(memo[mi], x) > Fn(memo[mi + 1], x)){
            lo = mi + 1;
        }
        else hi = mi;
    }
    return Fn(memo[lo], x);
}

const int N = 1e6 + 1;
long dp[N];

int main()
{
    fastio;
    int n, c; cin >> n >> c;
    vector<line> memo;
    for (int i = 1; i <= n; i++){
        long val; cin >> val;
        addLine(memo, {-2 * val, val * val + dp[i -
            1]});
        dp[i] = query(memo, val) + val * val + c;
    }
    cout << dp[n] << '\n';
    return 0;
}
```

4.2 Divide and Conquer

```
/**
 * recurrence:
 *   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for all
     j > i;
 *
 * "comp" computes dp[k][i] for all i in O(n log n) (k
     is fixed)
 *
 * Problems:
 *  https://icpc.kattis.com/problems/branch
 *  http://codeforces.com/contest/321/problem/E
 * */

void comp(int l, int r, int le, int re) {
  if (l > r) return;

  int mid = (l + r) >> 1;

  int best = max(mid + 1, le);
  dp[cur][mid] = dp[cur ^ 1][best] + cost(mid, best -
      1);
  for (int i = best; i <= re; i++) {
    if (dp[cur][mid] > dp[cur ^ 1][i] + cost(mid, i -
        1)) {
      best = i;
      dp[cur][mid] = dp[cur ^ 1][i] + cost(mid, i - 1);
    }
  }

  comp(l, mid - 1, le, best);
  comp(mid + 1, r, best, re);
}
```

5 Geometry

5.1 Closest Pair Problem

```
struct point {
  double x, y;
  int id;
  point() {}
  point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
  double a = p.x - o.x, b = p.y - o.y;
  return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x,
    vector<point> &y) {
```

```
  if (p.size() < 4) {
    double best = 1e100;
    for (int i = 0; i < p.size(); ++i)
      for (int j = i + 1; j < p.size(); ++j)
        best = min(best, dist(p[i], p[j]));
    return best;
  }

  int ls = (p.size() + 1) >> 1;
  double l = (p[ls - 1].x + p[ls].x) * 0.5;
  vector<point> xl(ls), xr(p.size() - ls);
  unordered_set<int> left;
  for (int i = 0; i < ls; ++i) {
    xl[i] = x[i];
    left.insert(x[i].id);
  }
  for (int i = ls; i < p.size(); ++i) {
    xr[i - ls] = x[i];
  }

  vector<point> yl, yr;
  vector<point> pl, pr;
  yl.reserve(ls); yr.reserve(p.size() - ls);
  pl.reserve(ls); pr.reserve(p.size() - ls);
  for (int i = 0; i < p.size(); ++i) {
    if (left.count(y[i].id))
      yl.push_back(y[i]);
    else
      yr.push_back(y[i]);

    if (left.count(p[i].id))
      pl.push_back(p[i]);
    else
      pr.push_back(p[i]);
  }

  double dl = cp(pl, xl, yl);
  double dr = cp(pr, xr, yr);
  double d = min(dl, dr);
  vector<point> yp; yp.reserve(p.size());
  for (int i = 0; i < p.size(); ++i) {
    if (fabs(y[i].x - l) < d)
      yp.push_back(y[i]);
  }
  for (int i = 0; i < yp.size(); ++i) {
    for (int j = i + 1; j < yp.size() && j < i + 7;
        ++j) {
      d = min(d, dist(yp[i], yp[j]));
    }
  }
  return d;
}
```

```
double closest_pair(vector<point> &p) {
  vector<point> x(p.begin(), p.end());
  sort(x.begin(), x.end(), [](const point &a, const
      point &b) {
    return a.x < b.x;
  });
  vector<point> y(p.begin(), p.end());
  sort(y.begin(), y.end(), [](const point &a, const
      point &b) {
    return a.y < b.y;
  });
  return cp(p, x, y);
}
```

5.2 Convex Diameter

```
struct point{
    int x, y;
};

struct vec{
    int x, y;
};

vec operator - (const point &A, const point &B){
    return vec{A.x - B.x, A.y - B.y};
}

int cross(vec A, vec B){
    return A.x*B.y - A.y*B.x;
}

int cross(point A, point B, point C){
    int val = A.x*(B.y - C.y) + B.x*(C.y - A.y) +
        C.x*(A.y - B.y);
    if(val == 0)
        return 0; // coline
    if(val < 0)
        return 1; // clockwise
    return -1; //counter clockwise
}

vector <point> findConvexHull(vector <point> points){
    vector <point> convex;
    sort(points.begin(), points.end(), [](const point
        &A, const point &B){
        return (A.x == B.x)? (A.y < B.y): (A.x < B.x);
    });
    vector <point> Up, Down;
    point A = points[0], B = points.back();
    Up.push_back(A);
    Down.push_back(A);

    for(int i = 0; i < points.size(); i++){
        if(i == points.size()-1 || cross(A, points[i],
            B) > 0){
            while(Up.size() > 2 &&
                cross(Up[Up.size()-2], Up[Up.size()-1],
                points[i]) <= 0)
                Up.pop_back();
            Up.push_back(points[i]);
        }
        if(i == points.size()-1 || cross(A, points[i],
            B) < 0){
            while(Down.size() > 2 &&
                cross(Down[Down.size()-2],
                Down[Down.size()-1], points[i]) >= 0)
                Down.pop_back();
            Down.push_back(points[i]);
        }
    }
    for(int i = 0; i < Up.size(); i++)
        convex.push_back(Up[i]);
    for(int i = Down.size()-2; i > 0; i--)
        convex.push_back(Down[i]);
    return convex;
}

int dist(point A, point B){
    return (A.x - B.x)*(A.x - B.x) + (A.y - B.y)*(A.y -
        B.y);
}

double findConvexDiameter(vector <point> convexHull){
    int n = convexHull.size();

    int is = 0, js = 0;
    for(int i = 1; i < n; i++){
        if(convexHull[i].y > convexHull[is].y)
            is = i;
        if(convexHull[js].y > convexHull[i].y)
            js = i;
    }

    int maxd = dist(convexHull[is], convexHull[js]);
    int i, maxi, j, maxj;
    i = maxi = is;
    j = maxj = js;
    do{
        int ni = (i+1)%n, nj = (j+1)%n;
        if(cross(convexHull[ni] - convexHull[i],
            convexHull[nj] - convexHull[j]) <= 0){
            j = nj;
        }else{
```

```
        i = ni;
      }
      int d = dist(convexHull[i], convexHull[j]);
      if(d > maxd){
        maxd = d;
        maxi = i;
        maxj = j;
      }
   }while(i != is || j != js);
   return sqrt(maxd);
}
```

5.3 Pick Theorem

```
struct point{
   ll x, y;
};

//Pick: S = I + B/2 - 1

ld polygonArea(vector <point> &points){
   int n = (int)points.size();
   ld area = 0.0;
   int j = n-1;
   for(int i = 0; i < n; i++){
      area += (points[j].x + points[i].x) *
          (points[j].y - points[i].y);
      j = i;
   }

   return abs(area/2.0);
}

ll boundary(vector <point> points){
   int n = (int)points.size();
   ll num_bound = 0;
   for(int i = 0; i < n; i++){
      ll dx = (points[i].x - points[(i+1)%n].x);
      ll dy = (points[i].y - points[(i+1)%n].y);
      num_bound += abs(__gcd(dx, dy)) - 1;
   }
   return num_bound;
}
```

5.4 Square

```
typedef long double ld;

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
   return ( x <= y + tol) ? (x + tol < y) ? -1 : 0 : 1;
}

struct point{
  ld x, y;
  point(ld a, ld b) : x(a), y(b) {}
  point() {}
};

struct square{
  ld x1, x2, y1, y2,
     a, b, c;
  point edges[4];
  square(ld _a, ld _b, ld _c) {
    a = _a, b = _b, c = _c;
    x1 = a - c * 0.5;
    x2 = a + c * 0.5;
    y1 = b - c * 0.5;
    y2 = b + c * 0.5;
    edges[0] = point(x1, y1);
    edges[1] = point(x2, y1);
    edges[2] = point(x2, y2);
    edges[3] = point(x1, y2);
  }
};

ld min_dist(point &a, point &b) {
  ld x = a.x - b.x,
     y = a.y - b.y;
  return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
  if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) != -1 &&
      cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) != -1)
    return true;
  return false;
}

bool inside(square &s1, square &s2) {
  for (int i = 0; i < 4; ++i)
    if (point_in_box(s2, s1.edges[i]))
      return true;

  return false;
}

bool inside_vert(square &s1, square &s2) {
  if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1, s2.y2) !=
       1) ||
      (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2, s2.y2) !=
         1))
    return true;
 return false;
}

bool inside_hori(square &s1, square &s2) {
  if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1, s2.x2) !=
       1) ||
      (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2, s2.x2) !=
         1))
    return true;
 return false;
}

ld min_dist(square &s1, square &s2) {
  if (inside(s1, s2) || inside(s2, s1))
    return 0;

  ld ans = 1e100;
  for (int i = 0; i < 4; ++i)
    for (int j = 0; j < 4; ++j)
      ans = min(ans, min_dist(s1.edges[i],
          s2.edges[j]));


  if (inside_hori(s1, s2) || inside_hori(s2, s1)) {
    if (cmp(s1.y1, s2.y2) != -1)
      ans = min(ans, s1.y1 - s2.y2);
    else
    if (cmp(s2.y1, s1.y2) != -1)
      ans = min(ans, s2.y1 - s1.y2);
  }

  if (inside_vert(s1, s2) || inside_vert(s2, s1)) {
    if (cmp(s1.x1, s2.x2) != -1)
      ans = min(ans, s1.x1 - s2.x2);
    else
    if (cmp(s2.x1, s1.x2) != -1)
      ans = min(ans, s2.x1 - s1.x2);
  }

  return ans;
}
```

5.5 Triangle

Let a, b, c be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

6 Graphs

6.1 Bridges

```
struct Graph {
    vector<vector<Edge>> g;
    vector<int> vi, low, d, pi, is_b; // vi = visited
    int bridges_computed;
    int ticks, edges;

    Graph(int n, int m) {
        g.assign(n, vector<Edge>());
        id_b.assign(m, 0);
        vi.resize(n);
        low.resize(n);
        d.resize(n);
        pi.resize(n);
        edges = 0;
        bridges_computed = 0;
    }

    void addEge(int u, int v) {
        g[u].push_back(Edge(v, edges));
        g[v].push_back(Edge(u, edges));
        edges++;
    }

    void dfs(int u) {
        vi[u] = true;
        d[u] = low[u] = ticks++;
        for (int i = 0; i < g[u].size(); i++) {
            int v = g[u][i].to;
            if (v == pi[u]) continue;
            if (!vi[v]) {
                pi[v] = u;
                dfs(v);
                if(d[u] < low[v]) is_b[g[u][i].id] =
                    true;
                low[u] = min(low[u], low[v]);
            } else {
                low[u] = min(low[u], low[v]);
            }
        }
    }

    // multiple edges from a to b are not alloweerd.
    // (they could be detected as a bridge).
    // if we need to handle this, just count how many
         edges there are from a to b.
    void compBridges() {
        fill(pi.begin(), pi.end(), -1);
        fill(vi.begin(), vi.end(), false);
        fill(d.begin(), d.end(), 0);
        fill(low.begin(), low.end(), 0);
        ticks = 0;
        for (int i = 0; i < g.size(); i++)
            if (!vi[i]) dfs(i);
        bridges_computed = 1;
    }

    map<int, vector<Edge>> bridgesTree() {
        if (!bridges_computed) compBridges();
        int n = g.size();
        Dsu dsu(n);
        for (int i = 0; i < n; i++)
            for (auto e : g[i])
                if (!is_b[e.id]) dsu.Join(i, e.to);
        map<int. vector<Edge>> tree;
        for (int i = 0; i < n; i++)
            for (auto e : g[i])
                if (is_b[e.id])
                    tree[dsu.Find(i)].emplace_back(dsu.Find(e.to),
                         e.id);
        return tree;
    }
};
```

6.2 Dijkstra

```
struct edge {
    int to;
    long long w;
    edge() {}
    edge(int a, long long b) : to(a), w(b) {}
    bool operator<(const edge &e) const {
        return w > e.w;
    }
};

typedef <vector<vector<edge>> graph;
const long long inf = 1000000LL * 10000000LL;

pair<vector<int>, vector<long long>> dijkstra(graph& g,
     int start) {
    int n = g.size();
    vector<long long> d(n, inf);
    vector<int> p(n, -1);
    d[start] = 0;
    priority_queue<edge> q;
    q.push(edge(start, 0));
    while (!q.empty()) {
        int node = q.top().to;
        long long dist = q.top().w;
        q.pop();
        if (dist > d[node]) continue;
        for (int i = 0; i < g[node].size(); i++) {
            int to = g[node][i].to;
            long long w_extra = g[node][i].w;
            if (dist + w_extra < d[to]) {
                p[to] = node;
                d[to] = dist + w_extra;
                q.push(edge(to, d[to]));
            }
        }
    }
    return {p, d};
}
```

6.3 Directed MST

```
struct Edge { int a, b; ll w; };
struct Node { /// lazy skew heap node
        Edge key;
        Node *l, *r;
        ll delta;
        void prop() {
                key.w += delta;
                if (l) l->delta += delta;
                if (r) r->delta += delta;
                delta = 0;
        }
        Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
        if (!a || !b) return a ?: b;
        a->prop(), b->prop();
        if (a->key.w > b->key.w) swap(a, b);
        swap(a->l, (a->r = merge(b, a->r)));
        return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
```

```
	RollbackUF uf(n);
	vector<Node*> heap(n);
	for (Edge e : g) heap[e.b] = merge(heap[e.b],
	     new Node{e});
	ll res = 0;
	vi seen(n, -1), path(n), par(n);
	seen[r] = r;
	vector<Edge> Q(n), in(n, {-1,-1}), comp;
	deque<tuple<int, int, vector<Edge>>> cycs;
	rep(s,0,n) {
		int u = s, qi = 0, w;
		while (seen[u] < 0) {
			if (!heap[u]) return {-1,{}};
			Edge e = heap[u]->top();
			heap[u]->delta -= e.w,
			    pop(heap[u]);
			Q[qi] = e, path[qi++] = u,
			    seen[u] = s;
			res += e.w, u = uf.find(e.a);
			if (seen[u] == s) { /// found
			    cycle, contract
				Node* cyc = 0;
				int end = qi, time =
				    uf.time();
				do cyc = merge(cyc, heap[w
				    = path[--qi]]);
				while (uf.join(u, w));
				u = uf.find(u), heap[u] =
				    cyc, seen[u] = -1;
				cycs.push_front({u, time,
				    {&Q[qi], &Q[end]}});
			}
		}
		rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
	}

	for (auto& [u,t,comp] : cycs) { // restore sol
	     (optional)
		uf.rollback(t);
		Edge inEdge = in[u];
		for (auto& e : comp) in[uf.find(e.b)] =
		     e;
		in[uf.find(inEdge.b)] = inEdge;
	}
	rep(i,0,n) par[i] = in[i].a;
	return {res, par};
}
```

6.4 Edge Coloring

```
vi edgeColoring(int N, vector<pii> eds) {
	vi cc(N + 1), ret(sz(eds)), fan(N), free(N),
	    loc;
	for (pii e : eds) ++cc[e.first], ++cc[e.second];
	int u, v, ncols = *max_element(all(cc)) + 1;
	vector<vi> adj(N, vi(ncols, -1));
	for (pii e : eds) {
		tie(u, v) = e;
		fan[0] = v;
		loc.assign(ncols, 0);
		int at = u, end = u, d, c = free[u], ind
		    = 0, i = 0;
		while (d = free[v], !loc[d] && (v =
		    adj[u][d]) != -1)
			loc[d] = ++ind, cc[ind] = d,
			    fan[ind] = v;
		cc[loc[d]] = c;
		for (int cd = d; at != -1; cd ^= c ^ d,
		    at = adj[at][cd])
			swap(adj[at][cd], adj[end =
			    at][cd ^ c ^ d]);
		while (adj[fan[i]][d] != -1) {
			int left = fan[i], right =
			    fan[++i], e = cc[i];
			adj[u][e] = left;
			adj[left][e] = u;
			adj[right][e] = -1;
			free[right] = e;
		}
		adj[u][d] = fan[i];
		adj[fan[i]][d] = u;
		for (int y : {fan[0], u, end})
			for (int& z = free[y] = 0;
			    adj[y][z] != -1; z++);
	}
	rep(i,0,sz(eds))
		for (tie(u, v) = eds[i]; adj[u][ret[i]]
		    != v;) ++ret[i];
	return ret;
}
```

6.5 Eulerian Path

```
struct DirectedEulerPath
{
	int n;
	vector<vector<int> > g;
	vector<int> path;

	void init(int _n){
		n = _n;
		g = vector<vector<int> > (n + 1,
		    vector<int> ());
		path.clear();
	}

	void add_edge(int u, int v){
		g[u].push_back(v);
	}

	void dfs(int u)
	{
		while(g[u].size())
		{
			int v = g[u].back();
			g[u].pop_back();
			dfs(v);
		}
		path.push_back(u);
	}

	bool getPath(){
		int ctEdges = 0;
		vector<int> outDeg, inDeg;
		outDeg = inDeg = vector<int> (n + 1, 0);
		for(int i = 1; i <= n; i++)
		{
			ctEdges += g[i].size();
			outDeg[i] += g[i].size();
			for(auto &u:g[i])
				inDeg[u]++;
		}
		int ctMiddle = 0, src = 1;
		for(int i = 1; i <= n; i++)
		{
			if(abs(inDeg[i] - outDeg[i]) > 1)
				return 0;
			if(inDeg[i] == outDeg[i])
				ctMiddle++;
			if(outDeg[i] > inDeg[i])
				src = i;
		}
		if(ctMiddle != n && ctMiddle + 2 != n)
			return 0;
		dfs(src);
		reverse(path.begin(), path.end());
		return (path.size() == ctEdges + 1);
	}
};
```

6.6 Floyd - Warshall

```
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
	int n = sz(m);
	rep(i,0,n) m[i][i] = min(m[i][i], 0LL);
	rep(k,0,n) rep(i,0,n) rep(j,0,n)
		if (m[i][k] != inf && m[k][j] != inf) {
			auto newDist = max(m[i][k] +
			    m[k][j], -inf);
			m[i][j] = min(m[i][j], newDist);
		}
	rep(k,0,n) if (m[k][k] < 0) rep(i,0,n)
	    rep(j,0,n)
		if (m[i][k] != inf && m[k][j] != inf)
		    m[i][j] = -inf;
}
```

6.7 Ford - Bellman

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a;
    }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds,
    int s) {
	nodes[s].dist = 0;
	sort(all(eds), [](Ed a, Ed b) { return a.s() <
	    b.s(); });

	int lim = sz(nodes) / 2 + 2; // /3+100 with
	    shuffled vertices
	rep(i,0,lim) for (Ed ed : eds) {
		Node cur = nodes[ed.a], &dest =
		    nodes[ed.b];
		if (abs(cur.dist) == inf) continue;
		ll d = cur.dist + ed.w;
		if (d < dest.dist) {
			dest.prev = ed.a;
			dest.dist = (i < lim-1 ? d :
			    -inf);
		}
	}
	rep(i,0,lim) for (Ed e : eds) {
		if (nodes[e.a].dist == -inf)
			nodes[e.b].dist = -inf;
	}
}
```

6.8 Gomory Hu

```
#include "PushRelabel.cpp"

typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
	vector<Edge> tree;
	vi par(N);
	rep(i,1,N) {
		PushRelabel D(N); // Dinic also works
		for (Edge t : ed) D.addEdge(t[0], t[1],
		    t[2], t[2]);
		tree.push_back({i, par[i], D.calc(i,
		    par[i])});
		rep(j,i+1,N)
			if (par[j] == par[i] &&
			    D.leftOfMinCut(j)) par[j] =
			    i;
	}
	return tree;
}
```

6.9 Karp Min Mean Cycle

```
/**
 * Finds the min mean cycle, if you need the max mean
    cycle
 * just add all the edges with negative cost and print
 * ans * -1
 *
 * test: uva, 11090 - Going in Cycle!!
 * */

const int MN = 1000;
struct edge{
  int v;
  long long w;
  edge(){} edge(int v, int w) : v(v), w(w) {}
};

long long d[MN][MN];
// This is a copy of g because increments the size
// pass as reference if this does not matter.
int karp(vector<vector<edge> > g) {
  int n = g.size();

  g.resize(n + 1); // this is important

  for (int i = 0; i < n; ++i)
    if (!g[i].empty())
      g[n].push_back(edge(i,0));
  ++n;

  for(int i = 0;i<n;++i)
    fill(d[i],d[i]+(n+1),INT_MAX);

  d[n - 1][0] = 0;

  for (int k = 1; k <= n; ++k) for (int u = 0; u < n;
      ++u) {
    if (d[u][k - 1] == INT_MAX) continue;
    for (int i = g[u].size() - 1; i >= 0; --i)
      d[g[u][i].v][k] = min(d[g[u][i].v][k], d[u][k -
          1] + g[u][i].w);
  }

  bool flag = true;

  for (int i = 0; i < n && flag; ++i)
    if (d[i][n] != INT_MAX)
      flag = false;

  if (flag) {
    return true; // return true if there is no a cycle.
  }

  double ans = 1e15;

  for (int u = 0; u + 1 < n; ++u) {
    if (d[u][n] == INT_MAX) continue;
    double W = -1e15;

    for (int k = 0; k < n; ++k)
      if (d[u][k] != INT_MAX)
        W = max(W, (double)(d[u][n] - d[u][k]) / (n -
            k));

    ans = min(ans, W);
  }

  // printf("%.2lf\n", ans);
  cout << fixed << setprecision(2) << ans << endl;

  return false;
}
```

6.10 Konig's Theorem

In any bipartite graph, the number of edges in a maximum matching equals the number of vertices in a minimum vertex cover

6.11 LCA

```
#include "../Data Structures/RMQ.h"

struct LCA {
	int T = 0;
	vi time, path, ret;
	RMQ<int> rmq;

	LCA(vector<vi>& C) : time(sz(C)),
	    rmq((dfs(C,0,-1), ret)) {}
	void dfs(vector<vi>& C, int v, int par) {
		time[v] = T++;
		for (int y : C[v]) if (y != par) {
			path.push_back(v),
			    ret.push_back(time[v]);
			dfs(C, y, v);
		}
	}

	int lca(int a, int b) {
		if (a == b) return a;
		tie(a, b) = minmax(time[a], time[b]);
		return path[rmq.query(a, b)];
	}
	//dist(a,b){return depth[a] + depth[b] -
	    2*depth[lca(a,b)];}
};
```

6.12 Math

Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--`, `mat[b][b]++` (and `mat[b][a]--`, `mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

6.13 Push Relabel

```
struct PushRelabel {
	struct Edge {
		int dest, back;
		ll f, c;
	};
	vector<vector<Edge>> g;
	vector<ll> ec;
	vector<Edge*> cur;
	vector<vi> hs; vi H;
	PushRelabel(int n) : g(n), ec(n), cur(n),
	    hs(2*n), H(n) {}

	void addEdge(int s, int t, ll cap, ll rcap=0) {
		if (s == t) return;
		g[s].push_back({t, sz(g[t]), 0, cap});
		g[t].push_back({s, sz(g[s])-1, 0, rcap});
	}

	void addFlow(Edge& e, ll f) {
		Edge &back = g[e.dest][e.back];
		if (!ec[e.dest] && f)
		    hs[H[e.dest]].push_back(e.dest);
		e.f += f; e.c -= f; ec[e.dest] += f;
		back.f -= f; back.c += f; ec[back.dest]
		    -= f;
	}
	ll calc(int s, int t) {
		int v = sz(g); H[s] = v; ec[t] = 1;
		vi co(2*v); co[0] = v-1;
		rep(i,0,v) cur[i] = g[i].data();
		for (Edge& e : g[s]) addFlow(e, e.c);

		for (int hi = 0;;) {
			while (hs[hi].empty()) if (!hi--)
			    return -ec[s];
			int u = hs[hi].back();
			    hs[hi].pop_back();
			while (ec[u] > 0) // discharge u
				if (cur[u] == g[u].data()
				    + sz(g[u])) {
					H[u] = 1e9;
					for (Edge& e :
					    g[u]) if (e.c
					    && H[u] >
					    H[e.dest]+1)
						H[u] =
						    H[e.dest]+1,
						    cur[u]
						    = &e;
					if (++co[H[u]],
					    !--co[hi] &&
					    hi < v)
						rep(i,0,v)
						    if (hi
						    < H[i]
						    && H[i]
						    < v)
						     --co[H[i]],
						        H[i]
						        =
						        v
						        +
						        1;
					hi = H[u];
				} else if (cur[u]->c &&
				    H[u] ==
				    H[cur[u]->dest]+1)
					addFlow(*cur[u],
					    min(ec[u],
					    cur[u]->c));
				else ++cur[u];
		}
	}
	bool leftOfMinCut(int a) { return H[a] >=
	    sz(g); }
};
```

6.14 SCC Kosaraju

```
// SCC = Strongly Connected Components

struct SCC {
    vector<vector<int>> g, gr;
    vector<bool> used;
    vector<int> order, component;
    int total_components;

    SCC(vector<vector<int>>& adj) {
        g = adj;
        int n = g.size();
        gr.resize(n);
        for (int i = 0; i < n; i++)
            for (auto to : g[i])
                gr[to].push_back(i);

        used.assign(n, false);
        for (int i = 0; i < n; i++)
        if (!used[i])
            GenTime(i);

        used.assign(n, false);
        component.assign(n, -1);
        total_components = 0;
```

```
        for (int i = n - 1; i >= 0; i--) {
            int v = order[i];
            if (!used[v]) {
                vector<int> cur_component;
                Dfs(cur_component, v);
                for (auto node : cur_component)
                    component[node] = total_components;
            }
        }
    }

    void GenTime(int node) {
        used[node] = true;
        for (auto to : g[node])
            if (!used[to])
                GenTime(to);
        order.push_back(node);
    }

    void Dfs(vector<int>& cur, int node) {
        used[node] = true;
        cur.push_back(node);
        if (!used[to])
            Dfs(cur, to);
    }

    vector<vector<int>> CondensedGraph() {
        vector<vector<int>> ans(total_components);
        for (int i = 0; i < int(g.size()); i++) {
            for (int to : g[i]) {
                int u = component[i], v = component[to];
                if (u != v)
                    ans[u].push_back(v);
            }
        }
        return ans;
    }
};
```

6.15 Topological Sort

```
vi topoSort(const vector<vi>& gr) {
        vi indeg(sz(gr)), ret;
        for (auto& li : gr) for (int x : li) indeg[x]++;
        queue<int> q; // use priority_queue for lexic.
            largest ans.
        rep(i,0,sz(gr)) if (indeg[i] == 0) q.push(i);
        while (!q.empty()) {
                int i = q.front(); // top() for priority
                    queue
                ret.push_back(i);
                q.pop();
                for (int x : gr[i])
                        if (--indeg[x] == 0) q.push(x);
        }
        return ret;
}
```

7 Linear Algebra

7.1 Matrix Determinant

```
double det(vector<vector<double>>& a) {
        int n = sz(a); double res = 1;
        rep(i,0,n) {
                int b = i;
                rep(j,i+1,n) if (fabs(a[j][i]) >
                    fabs(a[b][i])) b = j;
                if (i != b) swap(a[i], a[b]), res *= -1;
                res *= a[i][i];
                if (res == 0) return 0;
                rep(j,i+1,n) {
                        double v = a[j][i] / a[i][i];
                        if (v != 0) rep(k,i+1,n) a[j][k]
                            -= v * a[i][k];
                }
        }
        return res;
}
```

7.2 Matrix Inverse

```
int matInv(vector<vector<double>>& A) {
        int n = sz(A); vi col(n);
        vector<vector<double>> tmp(n,
            vector<double>(n));
        rep(i,0,n) tmp[i][i] = 1, col[i] = i;

        rep(i,0,n) {
                int r = i, c = i;
                rep(j,i,n) rep(k,i,n)
                        if (fabs(A[j][k]) > fabs(A[r][c]))
                                r = j, c = k;
                if (fabs(A[r][c]) < 1e-12) return i;
                A[i].swap(A[r]); tmp[i].swap(tmp[r]);
                rep(j,0,n)
                        swap(A[j][i], A[j][c]),
                            swap(tmp[j][i], tmp[j][c]);
                swap(col[i], col[c]);
                double v = A[i][i];
                rep(j,i+1,n) {
                        double f = A[j][i] / v;
                        A[j][i] = 0;
                        rep(k,i+1,n) A[j][k] -= f*A[i][k];
                        rep(k,0,n) tmp[j][k] -=
                            f*tmp[i][k];
                }
                rep(j,i+1,n) A[i][j] /= v;
                rep(j,0,n) tmp[i][j] /= v;
                A[i][i] = 1;
        }

        /// forget A at this point, just eliminate tmp
            backward
        for (int i = n-1; i > 0; --i) rep(j,0,i) {
                double v = A[j][i];
                rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
        }

        rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] =
            tmp[i][j];
        return n;
}
```

7.3 PolyRoots

```
#include "Polynomial.cpp"

vector<double> polyRoots(Poly p, double xmin, double
    xmax) {
        if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
        vector<double> ret;
        Poly der = p;
        der.diff();
        auto dr = polyRoots(der, xmin, xmax);
        dr.push_back(xmin-1);
        dr.push_back(xmax+1);
        sort(all(dr));
        rep(i,0,sz(dr)-1) {
                double l = dr[i], h = dr[i+1];
                bool sign = p(l) > 0;
                if (sign ^ (p(h) > 0)) {
                        rep(it,0,60) { // while (h - l >
                            1e-8)
                                double m = (l + h) / 2, f
                                    = p(m);
                                if ((f <= 0) ^ sign) l = m;
                                else h = m;
                        }
```

```
                ret.push_back((l + h) / 2);
            }
        }
        return ret;
}
```

7.4 Polynomial

```
struct Poly {
        vector<double> a;
        double operator()(double x) const {
                double val = 0;
                for (int i = sz(a); i--;) (val *= x) +=
                    a[i];
                return val;
        }
        void diff() {
                rep(i,1,sz(a)) a[i-1] = i*a[i];
                a.pop_back();
        }
        void divroot(double x0) {
                double b = a.back(), c; a.back() = 0;
                for(int i=sz(a)-1; i--;) c = a[i], a[i]
                    = a[i+1]*x0+b, b=c;
                a.pop_back();
        }
};
```

8 Misc

8.1 Dates

```
//
// Time - Leap years
//

// A[i] has the accumulated number of days from months
     previous to i
const int A[13] = { 0, 0, 31, 59, 90, 120, 151, 181,
     212, 243, 273, 304, 334 };
// same as A, but for a leap year
const int B[13] = { 0, 0, 31, 60, 91, 121, 152, 182,
     213, 244, 274, 305, 335 };
// returns number of leap years up to, and including, y
int leap_years(int y) { return y / 4 - y / 100 + y /
     400; }
bool is_leap(int y) { return y % 400 == 0 || (y % 4 ==
     0 && y % 100 != 0); }
// number of days in blocks of years
const int p400 = 400*365 + leap_years(400);
const int p100 = 100*365 + leap_years(100);
const int p4  = 4*365 + 1;
const int p1  = 365;
int date_to_days(int d, int m, int y)
{
  return (y - 1) * 365 + leap_years(y - 1) +
       (is_leap(y) ? B[m] : A[m]) + d;
}
void days_to_date(int days, int &d, int &m, int &y)
{
  bool top100; // are we in the top 100 years of a 400
       block?
  bool top4;   // are we in the top 4 years of a 100
       block?
  bool top1;   // are we in the top year of a 4 block?

  y = 1;
  top100 = top4 = top1 = false;

  y += ((days-1) / p400) * 400;
  d = (days-1) % p400 + 1;

  if (d > p100*3) top100 = true, d -= 3*p100, y += 300;
  else y += ((d-1) / p100) * 100, d = (d-1) % p100 + 1;

  if (d > p4*24) top4 = true, d -= 24*p4, y += 24*4;
  else y += ((d-1) / p4) * 4, d = (d-1) % p4 + 1;

  if (d > p1*3) top1 = true, d -= p1*3, y += 3;
  else y += (d-1) / p1, d = (d-1) % p1 + 1;

  const int *ac = top1 && (!top4 || top100) ? B : A;
  for (m = 1; m < 12; ++m) if (d <= ac[m + 1]) break;
  d -= ac[m];
}
```

8.2 Interval Container

```
set<pii>::iterator addInterval(set<pii>& is, int L, int
     R) {
        if (L == R) return is.end();
        auto it = is.lower_bound({L, R}), before = it;
        while (it != is.end() && it->first <= R) {
                R = max(R, it->second);
                before = it = is.erase(it);
        }
        if (it != is.begin() && (--it)->second >= L) {
                L = min(L, it->first);
                R = max(R, it->second);
                is.erase(it);
        }
        return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
        if (L == R) return;
        auto it = addInterval(is, L, R);
        auto r2 = it->second;
        if (it->first == L) is.erase(it);
        else (int&)it->second = L;
        if (R != r2) is.emplace(R, r2);
}
```

8.3 Optimization Tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

8.3.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))` `if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

8.3.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

8.4 Ternary Search

```
template<class F>
int ternSearch(int a, int b, F f) {
	assert(a <= b);
	while (b - a >= 5) {
		int mid = (a + b) / 2;
		if (f(mid) < f(mid+1)) a = mid; // (A)
		else b = mid+1;
	}
	rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
	return a;
}
```

9 Number Theory

9.1 Chinese Remainder Theorem

```
/**
 * Chinese remainder theorem.
 * Find z such that z % x[i] = a[i] for all i.
 * */
long long crt(vector<long long> &a, vector<long long>
    &x) {
  long long z = 0;
  long long n = 1;
  for (int i = 0; i < x.size(); ++i)
    n *= x[i];

  for (int i = 0; i < a.size(); ++i) {
    long long tmp = (a[i] * (n / x[i])) % n;
    tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
    z = (z + tmp) % n;
  }

  return (z + n) % n;
}
```

9.2 Convolution

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

inline bool is_pow2(LL x) {
  return (x & (x-1)) == 0;
}

inline int ceil_log2(LL x) {
  int ans = 0;
  --x;
  while (x != 0) {
    x >>= 1;
    ans++;
  }
  return ans;
}

/* Returns the convolution of the two given vectors in
     time proportional to n*log(n).
 * The number of roots of unity to use nroots_unity
     must be set so that the product of the first
 * nroots_unity primes of the vector nth_roots_unity is
     greater than the maximum value of the
 * convolution. Never use sizes of vectors bigger than
     2^24, if you need to change the values of
 * the nth roots of unity to appropriate primes for
     those sizes.
 */
vector<LL> convolve(const vector<LL> &a, const
     vector<LL> &b, int nroots_unity = 2) {
  int N = 1 << ceil_log2(a.size() + b.size());
  vector<LL> ans(N,0), fA(N), fB(N), fC(N);
  LL modulo = 1;
  for (int times = 0; times < nroots_unity; times++) {
    fill(fA.begin(), fA.end(), 0);
    fill(fB.begin(), fB.end(), 0);
    for (int i = 0; i < a.size(); i++) fA[i] = a[i];
    for (int i = 0; i < b.size(); i++) fB[i] = b[i];
    LL prime = nth_roots_unity[times].first;
    LL inv_modulo = mod_inv(modulo % prime, prime);
    LL normalize = mod_inv(N, prime);
    ntfft(fA, 1, nth_roots_unity[times]);
    ntfft(fB, 1, nth_roots_unity[times]);
    for (int i = 0; i < N; i++) fC[i] = (fA[i] * fB[i])
         % prime;
    ntfft(fC, -1, nth_roots_unity[times]);
    for (int i = 0; i < N; i++) {
      LL curr = (fC[i] * normalize) % prime;
      LL k = (curr - (ans[i] % prime) + prime) % prime;
      k = (k * inv_modulo) % prime;
      ans[i] += modulo * k;
    }
    modulo *= prime;
  }
  return ans;
}
```

9.3 Diophantine Equations

```
long long gcd(long long a, long long b, long long &x,
     long long &y) {
  if (a == 0) {
    x = 0;
    y = 1;
    return b;
  }
  long long x1, y1;
  long long d = gcd(b % a, a, x1, y1);
  x = y1 - (b / a) * x1;
  y = x1;
  return d;
}

bool find_any_solution(long long a, long long b, long
     long c, long long &x0,
    long long &y0, long long &g) {
  g = gcd(abs(a), abs(b), x0, y0);
  if (c % g) {
    return false;
  }

  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}

void shift_solution(long long &x, long long &y, long
     long a, long long b,
    long long cnt) {
  x += cnt * b;
  y -= cnt * a;
}

long long find_all_solutions(long long a, long long b,
     long long c,
    long long minx, long long maxx, long long miny,
    long long maxy) {
  long long x, y, g;
  if (!find_any_solution(a, b, c, x, y, g)) return 0;
  a /= g;
  b /= g;

  long long sign_a = a > 0 ? +1 : -1;
  long long sign_b = b > 0 ? +1 : -1;

  shift_solution(x, y, a, b, (minx - x) / b);
  if (x < minx) shift_solution(x, y, a, b, sign_b);
```

```
  if (x > maxx) return 0;
  long long lx1 = x;

  shift_solution(x, y, a, b, (maxx - x) / b);
  if (x > maxx) shift_solution(x, y, a, b, -sign_b);
  long long rx1 = x;

  shift_solution(x, y, a, b, -(miny - y) / a);
  if (y < miny) shift_solution(x, y, a, b, -sign_a);
  if (y > maxy) return 0;
  long long lx2 = x;

  shift_solution(x, y, a, b, -(maxy - y) / a);
  if (y > maxy) shift_solution(x, y, a, b, sign_a);
  long long rx2 = x;

  if (lx2 > rx2) swap(lx2, rx2);
  long long lx = max(lx1, lx2);
  long long rx = min(rx1, rx2);

  if (lx > rx) return 0;
  return (rx - lx) / abs(b) + 1;
}
```

9.4 Discrete Logarithm

```
// Computes x which a ^ x = b mod n.

long long d_log(long long a, long long b, long long n) {
  long long m = ceil(sqrt(n));
  long long aj = 1;
  map<long long, long long> M;
  for (int i = 0; i < m; ++i) {
    if (!M.count(aj))
      M[aj] = i;
    aj = (aj * a) % n;
  }

  long long coef = mod_pow(a, n - 2, n);
  coef = mod_pow(coef, m, n);
  // coef = a ^ (-m)
  long long gamma = b;
  for (int i = 0; i < m; ++i) {
    if (M.count(gamma)) {
      return i * m + M[gamma];
    } else {
      gamma = (gamma * coef) % n;
    }
  }
  return -1;
}
```

9.5 Ext Euclidean

```
void ext_euclid(long long a, long long b, long long &x,
     long long &y, long long &g) {
  x = 0, y = 1, g = b;
  long long m, n, q, r;
  for (long long u = 1, v = 0; a != 0; g = a, a = r) {
    q = g / a, r = g % a;
    m = x - u * q, n = y - v * q;
    x = u, y = v, u = m, v = n;
  }
}
```

9.6 Fast Eratosthenes

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
        const int S = (int)round(sqrt(LIM)), R = LIM /
            2;
        vi pr = {2}, sieve(S+1);
            pr.reserve(int(LIM/log(LIM)*1.1));
        vector<pii> cp;
        for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
                cp.push_back({i, i * i / 2});
                for (int j = i * i; j <= S; j += 2 * i)
                    sieve[j] = 1;
        }
        for (int L = 1; L <= R; L += S) {
                array<bool, S> block{};
                for (auto &[p, idx] : cp)
                        for (int i=idx; i < S+L; idx =
                            (i+=p)) block[i-L] = 1;
                rep(i,0,min(S, R - L))
                        if (!block[i]) pr.push_back((L +
                            i) * 2 + 1);
        }
        for (int i : pr) isPrime[i] = 1;
        return pr;
}
```

9.7 Highest Exponent Factorial

```
int highest_exponent(int p, const int &n){
  int ans = 0;
  int t = p;
  while(t <= n){
    ans += n/t;
    t*=p;
  }
  return ans;
}
```

9.8 Miller - Rabin

```
const int rounds = 20;

// checks whether a is a witness that n is not prime, 1
     < a < n
bool witness(long long a, long long n) {
  // check as in Miller Rabin Primality Test described
  long long u = n - 1;
  int t = 0;
  while (u % 2 == 0) {
    t++;
    u >>= 1;
  }
  long long next = mod_pow(a, u, n);
  if (next == 1) return false;
  long long last;
  for (int i = 0; i < t; ++i) {
    last = next;
    next = mod_mul(last, last, n);
    if (next == 1) {
      return last != n - 1;
    }
  }
  return next != 1;
}


// Checks if a number is prime with prob 1 - 1 / (2 ^
     it)
//  D(miller_rabin(999999999999999997LL) == 1);
//  D(miller_rabin(9999999999971LL) == 1);
//  D(miller_rabin(7907) == 1);
bool miller_rabin(long long n, int it = rounds) {
  if (n <= 1) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  for (int i = 0; i < it; ++i) {
    long long a = rand() % (n - 1) + 1;
    if (witness(a, n)) {
```

```
      return false;
    }
  }
  return true;
}
```

9.9 Mod Integer

```
template<class T, T mod>
struct mint_t {
  T val;
  mint_t() : val(0) {}
  mint_t(T v) : val(v % mod) {}

  mint_t operator + (const mint_t& o) const {
    return (val + o.val) % mod;
  }
  mint_t operator - (const mint_t& o) const {
    return (val - o.val) % mod;
  }
  mint_t operator * (const mint_t& o) const {
    return (val * o.val) % mod;
  }
};

typedef mint_t<long long, 998244353> mint;
```

9.10 Mod Inv

```
long long mod_inv(long long n, long long m) {
  long long x, y, gcd;
  ext_euclid(n, m, x, y, gcd);
  if (gcd != 1)
    return 0;
  return (x + m) % m;
}
```

9.11 Mod Mul

```
// Computes (a * b) % mod
long long mod_mul(long long a, long long b, long long
     mod) {
  long long x = 0, y = a % mod;
  while (b > 0) {
    if (b & 1)
      x = (x + y) % mod;
    y = (y * 2) % mod;
    b /= 2;
  }
  return x % mod;
}
```

9.12 Mod Pow

```
// Computes ( a ^ exp ) % mod.
long long mod_pow(long long a, long long exp, long long
     mod) {
  long long ans = 1;
  while (exp > 0) {
    if (exp & 1)
      ans = mod_mul(ans, a, mod);
    a = mod_mul(a, a, mod);
    exp >>= 1;
  }
  return ans;
}
```

9.13 Number Theoretic Transform

```
typedef long long int LL;
typedef pair<LL, LL> PLL;

/* The following vector of pairs contains pairs (prime,
     generator)
 * where the prime has an Nth root of unity for N being
      a power of two.
 * The generator is a number g s.t g^(p-1)=1 (mod p)
 * but is different from 1 for all smaller powers */
vector<PLL> nth_roots_unity {
  {1224736769,330732430},{1711276033,927759239},{167772161,167489322},
   {469762049,343261969},{754974721,643797295},{1107296257,883865065}};

PLL ext_euclid(LL a, LL b) {
  if (b == 0)
    return make_pair(1,0);
  pair<LL,LL> rc = ext_euclid(b, a % b);
  return make_pair(rc.second, rc.first - (a / b) *
       rc.second);
}

//returns -1 if there is no unique modular inverse
LL mod_inv(LL x, LL modulo) {
  PLL p = ext_euclid(x, modulo);
  if ( (p.first * x + p.second * modulo) != 1 )
    return -1;
  return (p.first+modulo) % modulo;
}


//Number theory fft. The size of a must be a power of 2
void ntfft(vector<LL> &a, int dir, const PLL
     &root_unity) {
  int n = a.size();
  LL prime = root_unity.first;
  LL basew = mod_pow(root_unity.second, (prime-1) / n,
       prime);
  if (dir < 0) basew = mod_inv(basew, prime);
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    LL w = 1;
    for (int i = 0; i < mh; i++) {
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        LL x = (a[j] - a[k] + prime) % prime;
        a[j] = (a[j] + a[k]) % prime;
        a[k] = (w * x) % prime;
      }
      w = (w * basew) % prime;
    }
    basew = (basew * basew) % prime;
  }
  int i = 0;
  for (int j = 1; j < n - 1; j++) {
    for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  }
}
```

9.14 Pollard Rho Factorize

```
long long pollard_rho(long long n) {
  long long x, y, i = 1, k = 2, d;
  x = y = rand() % n;
  while (1) {
    ++i;
    x = mod_mul(x, x, n);
    x += 2;
    if (x >= n) x -= n;
    if (x == y) return 1;
    d = __gcd(abs(x - y), n);
    if (d != 1) return d;
    if (i == k) {
      y = x;
      k *= 2;
```

```cpp
    }
  }
  return 1;
}


// Returns a list with the prime divisors of n
vector<long long> factorize(long long n) {
  vector<long long> ans;
  if (n == 1)
    return ans;
  if (miller_rabin(n)) {
    ans.push_back(n);
  } else {
    long long d = 1;
    while (d == 1)
      d = pollard_rho(n);
    vector<long long> dd = factorize(d);
    ans = factorize(n / d);
    for (int i = 0; i < dd.size(); ++i)
      ans.push_back(dd[i]);
  }
  return ans;
}
```

9.15 Primes

```cpp
namespace primes {
  const int MP = 100001;
  bool sieve[MP];
  long long primes[MP];
  int num_p;
  void fill_sieve() {
    num_p = 0;
    sieve[0] = sieve[1] = true;
    for (long long i = 2; i < MP; ++i) {
      if (!sieve[i]) {
        primes[num_p++] = i;
        for (long long j = i * i; j < MP; j += i)
          sieve[j] = true;
      }
    }
  }

  // Finds prime numbers between a and b, using basic
       primes up to sqrt(b)
  // a must be greater than 1.
  vector<long long> seg_sieve(long long a, long long b)
       {
    long long ant = a;
    a = max(a, 3LL);
    vector<bool> pmap(b - a + 1);
    long long sqrt_b = sqrt(b);
    for (int i = 0; i < num_p; ++i) {
      long long p = primes[i];
      if (p > sqrt_b) break;
      long long j = (a + p - 1) / p;
      for (long long v = (j == 1) ? p + p : j * p; v <=
           b; v += p) {
        pmap[v - a] = true;
      }
    }
    vector<long long> ans;
    if (ant == 2) ans.push_back(2);
    int start = a % 2 ? 0 : 1;
    for (int i = start, I = b - a + 1; i < I; i += 2)
      if (pmap[i] == false)
        ans.push_back(a + i);
    return ans;
  }

  vector<pair<int, int>> factor(int n) {
    vector<pair<int, int>> ans;
    if (n == 0) return ans;
    for (int i = 0; primes[i] * primes[i] <= n; ++i) {
      if ((n % primes[i]) == 0) {
        int expo = 0;
        while ((n % primes[i]) == 0) {
          expo++;
          n /= primes[i];
        }
        ans.emplace_back(primes[i], expo);
      }
    }

    if (n > 1) {
      ans.emplace_back(n, 1);
    }
    return ans;
  }
}
```

9.16 Totient Sieve

```cpp
  for (int i = 1; i < MN; i++)
    phi[i] = i;

  for (int i = 1; i < MN; i++)
    if (!sieve[i]) // is prime
      for (int j = i; j < MN; j += i)
        phi[j] -= phi[j] / i;
```

9.17 Totient

```cpp
long long totient(long long n) {
  if (n == 1) return 0;
  long long ans = n;
  for (int i = 0; primes[i] * primes[i] <= n; ++i) {
    if ((n % primes[i]) == 0) {
      while ((n % primes[i]) == 0) n /= primes[i];
      ans -= ans / primes[i];
    }
  }
  if (n > 1) {
    ans -= ans / n;
  }
  return ans;
}
```

10 Probability and Statistics

10.1 Continuous Distributions

10.1.1 Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a<b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

10.1.2 Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

10.1.3 Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu,\sigma^2)$, $\sigma>0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

10.2 Discrete Distributions

10.2.1 Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

10.2.2 First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

10.2.3 Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!},\ k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

10.3 Probability Theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = E(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = E(X^2) - (E(X))^2 = \sum_x (x - E(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$E(aX + bY) = aE(X) + bE(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

11 Strings

11.1 Hashing

```
struct H {
	typedef uint64_t ull;
	ull x; H(ull x=0) : x(x) {}
#define OP(O,A,B) H operator O(H o) { ull r = x; asm \
	(A "addq %%rdx, %0\n adcq $0,%0" : "+a"(r) :
	    B); return r; }
	OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) :
	    "rdx")
	H operator-(H o) { return *this + ~o.x; }
	ull get() const { return x + !~x; }
	bool operator==(H o) const { return get() ==
	    o.get(); }
	bool operator<(H o) const { return get() <
	    o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random
    also ok)

struct HashInterval {
	vector<H> ha, pw;
	HashInterval(string& str) : ha(sz(str)+1),
	    pw(ha) {
		pw[0] = 1;
		rep(i,0,sz(str))
			ha[i+1] = ha[i] * C + str[i],
			pw[i+1] = pw[i] * C;
	}
	H hashInterval(int a, int b) { // hash [a, b)
		return ha[b] - ha[a] * pw[b - a];
	}
};

vector<H> getHashes(string& str, int length) {
	if (sz(str) < length) return {};
	H h = 0, pw = 1;
	rep(i,0,length)
		h = h * C + str[i], pw = pw * C;
	vector<H> ret = {h};
	rep(i,length,sz(str)) {
		ret.push_back(h = h * C + str[i] - pw *
		    str[i-length]);
	}
	return ret;
}

H hashString(string& s){H h{}; for(char c:s)
    h=h*C+c;return h;}
```

11.2 Incremental Aho Corasick

```
class IncrementalAhoCorasic {
  static const int Alphabets = 26;
  static const int AlphabetBase = 'a';
  struct Node {
    Node *fail;
    Node *next[Alphabets];
    int sum;
    Node() : fail(NULL), next{}, sum(0) { }
  };

  struct String {
    string str;
    int sign;
  };

public:
  //totalLen = sum of (len + 1)
  void init(int totalLen) {
    nodes.resize(totalLen);
    nNodes = 0;
    strings.clear();
    roots.clear();
    sizes.clear();
    que.resize(totalLen);
  }

  void insert(const string &str, int sign) {
    strings.push_back(String{ str, sign });
    roots.push_back(nodes.data() + nNodes);
    sizes.push_back(1);
```

```
    nNodes += (int)str.size() + 1;
    auto check = [&]() { return sizes.size() > 1 &&
        sizes.end()[-1] == sizes.end()[-2]; };
    if(!check())
      makePMA(strings.end() - 1, strings.end(),
          roots.back(), que);
    while(check()) {
      int m = sizes.back();
      roots.pop_back();
      sizes.pop_back();
      sizes.back() += m;
      if(!check())
        makePMA(strings.end() - m * 2, strings.end(),
            roots.back(), que);
    }
  }

  int match(const string &str) const {
    int res = 0;
    for(const Node *t : roots)
      res += matchPMA(t, str);
    return res;
  }

private:
  static void makePMA(vector<String>::const_iterator
      begin, vector<String>::const_iterator end, Node
      *nodes, vector<Node*> &que) {
    int nNodes = 0;
    Node *root = new(&nodes[nNodes ++]) Node();
    for(auto it = begin; it != end; ++ it) {
      Node *t = root;
      for(char c : it->str) {
        Node *&n = t->next[c - AlphabetBase];
        if(n == nullptr)
          n = new(&nodes[nNodes ++]) Node();
        t = n;
      }
      t->sum += it->sign;
    }
    int qt = 0;
    for(Node *&n : root->next) {
      if(n != nullptr) {
        n->fail = root;
        que[qt ++] = n;
      } else {
        n = root;
      }
    }
    for(int qh = 0; qh != qt; ++ qh) {
      Node *t = que[qh];
      int a = 0;
      for(Node *n : t->next) {
        if(n != nullptr) {
          que[qt ++] = n;
          Node *r = t->fail;
          while(r->next[a] == nullptr)
            r = r->fail;
          n->fail = r->next[a];
          n->sum += r->next[a]->sum;
        }
        ++ a;
      }
    }
  }

  static int matchPMA(const Node *t, const string &str)
      {
    int res = 0;
    for(char c : str) {
      int a = c - AlphabetBase;
      while(t->next[a] == nullptr)
        t = t->fail;
      t = t->next[a];
      res += t->sum;
    }
    return res;
  }


  vector<Node> nodes;
  int nNodes;
  vector<String> strings;
  vector<Node*> roots;
  vector<int> sizes;
  vector<Node*> que;
};

int main() {
  int m;
  while(~scanf("%d", &m)) {
    IncrementalAhoCorasic iac;
    iac.init(600000);
    rep(i, m) {
      int ty;
      char s[300001];
      scanf("%d%s", &ty, s);
      if(ty == 1) {
        iac.insert(s, +1);
      } else if(ty == 2) {
        iac.insert(s, -1);
      } else if(ty == 3) {
        int ans = iac.match(s);
        printf("%d\n", ans);
        fflush(stdout);
      } else {
        abort();
      }
    }
  }
  return 0;
}
```

11.3 KMP

```
vi pi(const string& s) {
        vi p(sz(s));
        rep(i,1,sz(s)) {
                int g = p[i-1];
                while (g && s[i] != s[g]) g = p[g-1];
                p[i] = g + (s[i] == s[g]);
        }
        return p;
}

vi match(const string& s, const string& pat) {
        vi p = pi(pat + '\0' + s), res;
        rep(i,sz(p)-sz(s),sz(p))
                if (p[i] == sz(pat)) res.push_back(i - 2
                    * sz(pat));
        return res;
}
```

11.4 Minimal String Rotation

```
// Lexicographically minimal string rotation
int lmsr() {
  string s;
  cin >> s;
  int n = s.size();
  s += s;
  vector<int> f(s.size(), -1);
  int k = 0;
  for (int j = 1; j < 2 * n; ++j) {
    int i = f[j - k - 1];
    while (i != -1 && s[j] != s[k + i + 1]) {
      if (s[j] < s[k + i + 1])
        k = j - i - 1;
      i = f[i];
    }
    if (i == -1 && s[j] != s[k + i + 1]) {
      if (s[j] < s[k + i + 1]) {
        k = j;
      }
```

```
      f[j - k] = -1;
    } else {
      f[j - k] = i + 1;
    }
  }
  return k;
}
```

11.5 Suffix Array

```
const int MAXN = 200005;

const int MAX_DIGIT = 256;
void countingSort(vector<int>& SA, vector<int>& RA, int
     k = 0) {
    int n = SA.size();
    vector<int> cnt(max(MAX_DIGIT, n), 0);
    for (int i = 0; i < n; i++)
        if (i + k < n)
            cnt[RA[i + k]]++;
        else
            cnt[0]++;
    for (int i = 1; i < cnt.size(); i++)
        cnt[i] += cnt[i - 1];
    vector<int> tempSA(n);
    for (int i = n - 1; i >= 0; i--)
        if (SA[i] + k < n)
            tempSA[--cnt[RA[SA[i] + k]]] = SA[i];
        else
            tempSA[--cnt[0]] = SA[i];
    SA = tempSA;
}

vector <int> constructSA(string s) {
    int n = s.length();
    vector <int> SA(n);
    vector <int> RA(n);
    vector <int> tempRA(n);
    for (int i = 0; i < n; i++) {
        RA[i] = s[i];
        SA[i] = i;
    }
    for (int step = 1; step < n; step <<= 1) {
        countingSort(SA, RA, step);
        countingSort(SA, RA, 0);
        int c = 0;
        tempRA[SA[0]] = c;
        for (int i = 1; i < n; i++) {
            if (RA[SA[i]] == RA[SA[i - 1]] && RA[SA[i] +
                 step] == RA[SA[i - 1] + step])
                    tempRA[SA[i]] = tempRA[SA[i - 1]];
            else
                tempRA[SA[i]] = tempRA[SA[i - 1]] + 1;
        }
        RA = tempRA;
        if (RA[SA[n - 1]] == n - 1) break;
    }
    return SA;
}

vector<int> computeLCP(const string& s, const
     vector<int>& SA) {
    int n = SA.size();
    vector<int> LCP(n), PLCP(n), c(n, 0);
    for (int i = 0; i < n; i++)
        c[SA[i]] = i;
    int k = 0;
    for (int j, i = 0; i < n-1; i++) {
        if(c[i] - 1 < 0)
            continue;
        j = SA[c[i] - 1];
        k = max(k - 1, 0);
        while (i+k < n && j+k < n && s[i + k] == s[j +
             k])
            k++;
        PLCP[i] = k;
    }
    for (int i = 0; i < n; i++)
        LCP[i] = PLCP[SA[i]];
    return LCP;
}
```

11.6 Suffix Automation

```
/*
 * Suffix automaton:
 * This implementation was extended to maintain
     (online) the
 * number of different substrings. This is equivalent
     to compute
 * the number of paths from the initial state to all
     the other
 * states.
 *
 * The overall complexity is O(n)
 * can be tested here:
     https://www.urionlinejudge.com.br/judge/en/problems/view/1530
 * */

struct state {
  int len, link;
  long long num_paths;
  map<int, int> next;
};

const int MN = 200011;
state sa[MN << 1];
int sz, last;
long long tot_paths;

void sa_init() {
  sz = 1;
  last = 0;
  sa[0].len = 0;
  sa[0].link = -1;
  sa[0].next.clear();
  sa[0].num_paths = 1;
  tot_paths = 0;
}

void sa_extend(int c) {
  int cur = sz++;
  sa[cur].len = sa[last].len + 1;
  sa[cur].next.clear();
  sa[cur].num_paths = 0;
  int p;
  for (p = last; p != -1 && !sa[p].next.count(c); p =
       sa[p].link) {
    sa[p].next[c] = cur;
    sa[cur].num_paths += sa[p].num_paths;
    tot_paths += sa[p].num_paths;
  }

  if (p == -1) {
    sa[cur].link = 0;
  } else {
    int q = sa[p].next[c];
    if (sa[p].len + 1 == sa[q].len) {
      sa[cur].link = q;
    } else {
      int clone = sz++;
      sa[clone].len = sa[p].len + 1;
      sa[clone].next = sa[q].next;
      sa[clone].num_paths = 0;
      sa[clone].link = sa[q].link;
      for (; p!= -1 && sa[p].next[c] == q; p =
           sa[p].link) {
        sa[p].next[c] = clone;
        sa[q].num_paths -= sa[p].num_paths;
        sa[clone].num_paths += sa[p].num_paths;
      }
      sa[q].link = sa[cur].link = clone;
    }
  }
  last = cur;
```

```
}
```

11.7 Suffix Tree

```
struct SuffixTree {
	enum { N = 200010, ALPHA = 26 }; // N ~
		2*maxlen+10
	int toi(char c) { return c - 'a'; }
	string a; // v = cur node, q = cur position
	int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;

	void ukkadd(int i, int c) { suff:
		if (r[v]<=q) {
			if (t[v][c]==-1) { t[v][c]=m;
				l[m]=i;
				  p[m++]=v; v=s[v]; q=r[v];
					goto suff; }
			v=t[v][c]; q=l[v];
		}
		if (q==-1 || c==toi(a[q])) q++; else {
			l[m+1]=i; p[m+1]=m; l[m]=l[v];
				r[m]=q;
			p[m]=p[v]; t[m][c]=m+1;
				t[m][toi(a[q])]=v;
			l[v]=q; p[v]=m;
				t[p[m]][toi(a[l[m]])]=m;
			v=s[p[m]]; q=l[m];
			while (q<r[m]) {
				v=t[v][toi(a[q])];
				q+=r[v]-l[v]; }
			if (q==r[m]) s[m]=v; else
				s[m]=m+2;
			q=r[v]-(q-r[m]); m+=2; goto suff;
		}
	}

	SuffixTree(string a) : a(a) {
		fill(r,r+N,sz(a));
		memset(s, 0, sizeof s);
		memset(t, -1, sizeof t);
		fill(t[1],t[1]+ALPHA,0);
		s[0] = 1; l[0] = l[1] = -1; r[0] = r[1]
			= p[0] = p[1] = 0;
		rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
	}

	// example: find longest common substring (uses
		ALPHA = 28)
	pii best;
	int lcs(int node, int i1, int i2, int olen) {
		if (l[node] <= i1 && i1 < r[node])
			return 1;
		if (l[node] <= i2 && i2 < r[node])
			return 2;
		int mask = 0, len = node ? olen +
			(r[node] - l[node]) : 0;
		rep(c,0,ALPHA) if (t[node][c] != -1)
			mask |= lcs(t[node][c], i1, i2,
				len);
		if (mask == 3)
			best = max(best, {len, r[node] -
				len});
		return mask;
	}
	static pii LCS(string s, string t) {
		SuffixTree st(s + (char)('z' + 1) + t +
			(char)('z' + 2));
		st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
		return st.best;
	}
};
```

11.8 Z Algorithm

```
vector<int> compute_z(const string &s){
  int n = s.size();
  vector<int> z(n,0);
  int l,r;
  r = l = 0;
  for(int i = 1; i < n; ++i){
    if(i > r) {
      l = r = i;
      while(r < n and s[r - l] == s[r])r++;
      z[i] = r - l;r--;
    }else{
      int k = i-l;
      if(z[k] < r - i +1) z[i] = z[k];
      else {
        l = i;
        while(r < n and s[r - l] == s[r])r++;
        z[i] = r - l;r--;
      }
    }
  }
  return z;
}

int main(){

  //string line;cin>>line;
  string line = "alfalfa";
  vector<int> z = compute_z(line);

  for(int i = 0; i < z.size(); ++i ){
    if(i)cout<<" ";
    cout<<z[i];
  }
  cout<<endl;

  // must print "0 0 0 4 0 0 1"

  return 0;
}
```