

Team notebook

HCMUS-PenguinSpammers

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Contents

1 Algorithms	1
1.1 Mo's Algorithm	1
1.2 Mo's Algorithms on Trees	1
1.3 Parallel Binary Search	2
2 Combinatorics	2
2.1 Factorial Approximate	2
2.2 Factorial	2
2.3 Fast Fourier Transform	2
2.4 General purpose numbers	3
2.5 Lucas Theorem	4
2.6 Multinomial	4
2.7 Others	4
2.8 Permutation To Int	5
2.9 Sigma Function	5
3 Data Structures	5
3.1 Binary Index Tree	5
3.2 Disjoint Set Union (DSU)	5
3.3 Fake Update	5
3.4 Fenwick Tree	6
3.5 Hash Table	6
3.6 Range Minimum Query	7
3.7 STL Treap	7
3.8 Segment Tree	7
3.9 Sparse Table	8
3.10 Trie	8

4 Dynamic Programming Optimization	8
4.1 Convex Hull Trick	8
4.2 Divide and Conquer	9
5 Geometry	9
5.1 Closest Pair Problem	9
5.2 Convex Diameter	10
5.3 Pick Theorem	11
5.4 Square	11
5.5 Triangle	12
6 Graphs	12
6.1 SCC Kosaraju	12

1 Algorithms

1.1 Mo's Algorithm

```
/*
https://www.spoj.com/problems/FREQ2/
*/
vector<int> MoQueries(int n, vector<query> Q){
    block_size = sqrt(n);
    sort(Q.begin(), Q.end(), [](const query &A,
        const query &B){
        return (A.l/block_size != B.l/block_size)?
            (A.l/block_size < B.l/block_size) :
            (A.r < B.r);
    });
```

```
});
vector<int> res;
res.resize((int)Q.size());

int L = 1, R = 0;
for(query q: Q){
    while (L > q.l) add(--L);
    while (R < q.r) add(++R);

    while (L < q.l) del(L++);
    while (R > q.r) del(R--);

    res[q.pos] = calc(1, R-L+1);
}
return res;
}
```

1.2 Mo's Algorithms on Trees

```
/*
Given a tree with N nodes and Q queries. Each
node has an integer weight.
Each query provides two numbers u and v, ask for
how many different integers weight of nodes
there are on path from u to v.
-----
Modify DFS:
-----
*/
```

For each node u , maintain the start and the end DFS time. Let's call them $ST(u)$ and $EN(u)$.
 \Rightarrow For each query, a node is considered if its occurrence count is one.

Query solving:

Let's query be (u, v) . Assume that $ST(u) \leq ST(v)$. Denotes P as $LCA(u, v)$.

Case 1: $P = u$

Our query would be in range $[ST(u), ST(v)]$.

Case 2: $P \neq u$

Our query would be in range $[EN(u), ST(v)] + [ST(p), ST(p)]$

*/

```
void update(int &L, int &R, int qL, int qR){
    while (L > qL) add(--L);
    while (R < qR) add(++R);

    while (L < qL) del(L++);
    while (R > qR) del(R--);
}
```

```
vector<int> MoQueries(int n, vector<query> Q){
    block_size = sqrt((int)nodes.size());
    sort(Q.begin(), Q.end(), [](const query &A,
        const query &B){
        return (ST[A.l]/block_size !=
            ST[B.l]/block_size)?
            (ST[A.l]/block_size <
                ST[B.l]/block_size) : (ST[A.r] <
                ST[B.r]);
    });
    vector<int> res;
    res.resize((int)Q.size());

    LCA lca;
    lca.initialize(n);

    int L = 1, R = 0;
```

```
for(query q: Q){
    int u = q.l, v = q.r;
    if(ST[u] > ST[v]) swap(u, v); // assume
        that S[u] <= S[v]
    int parent = lca.get(u, v);

    if(parent == u){
        int qL = ST[u], qR = ST[v];
        update(L, R, qL, qR);
    }else{
        int qL = EN[u], qR = ST[v];
        update(L, R, qL, qR);
        if(cnt_val[a[parent]] == 0)
            res[q.pos] += 1;
    }

    res[q.pos] += cur_ans;
}
return res;
}
```

1.3 Parallel Binary Search

```
int lo[N], mid[N], hi[N];
vector<int> vec[N];

void clear() //Reset
{
    memset(bit, 0, sizeof(bit));
}

void apply(int idx) //Apply ith update/query
{
    if(ql[idx] <= qr[idx])
        update(ql[idx], qa[idx]),
        update(qr[idx]+1, -qa[idx]);

    else
    {
        update(1, qa[idx]);
        update(qr[idx]+1, -qa[idx]);
        update(ql[idx], qa[idx]);
    }
}
```

```
}

bool check(int idx) //Check if the condition is
    satisfied
{
    int req=reqd[idx];
    for(auto &it:owns[idx])
    {
        req-=pref(it);
        if(req<0)
            break;
    }
    if(req<=0)
        return 1;
    return 0;
}

void work()
{
    for(int i=1;i<=q;i++)
        vec[i].clear();
    for(int i=1;i<=n;i++)
        if(mid[i]>0)
            vec[mid[i]].push_back(i);

    clear();
    for(int i=1;i<=q;i++)
    {
        apply(i);
        for(auto &it:vec[i]) //Add
            appropriate check conditions
            {
                if(check(it))
                    hi[it]=i;
                else
                    lo[it]=i+1;
            }
    }
}

void parallel_binary()
{
    for(int i=1;i<=n;i++)
        lo[i]=1, hi[i]=q+1;
    bool changed = 1;
```

```

while(changed)
{
    changed=0;
    for(int i=1;i<=n;i++)
    {
        if(lo[i]<hi[i])
        {
            changed=1;
            mid[i]=(lo[i] +
                    hi[i])/2;
        }
        else
            mid[i]=-1;
    }
    work();
}

```

2 Combinatorics

2.1 Factorial Approximate

Approximate Factorial:

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \quad (1)$$

2.2 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

2.3 Fast Fourier Transform

```

/**
 * Fast Fourier Transform.

```

```

 * Useful to compute convolutions.
 * computes:
 *   C(f star g)[n] = sum_m(f[m] * g[n - m])
 * for all n.
 * test: icpc live archive, 6886 - Golf Bot
 * */

```

```

using namespace std;
#include <bits/stdc++.h>
#define D(x) cout << #x " = " << (x) << endl
#define endl '\n'

```

```

const int MN = 262144 << 1;
int d[MN + 10], d2[MN + 10];

```

```

const double PI = acos(-1.0);

```

```

struct cpx {
    double real, image;
    cpx(double _real, double _image) {
        real = _real;
        image = _image;
    }
    cpx(){}
};

```

```

cpx operator + (const cpx &c1, const cpx &c2) {
    return cpx(c1.real + c2.real, c1.image +
               c2.image);
}

```

```

cpx operator - (const cpx &c1, const cpx &c2) {
    return cpx(c1.real - c2.real, c1.image -
               c2.image);
}

```

```

cpx operator * (const cpx &c1, const cpx &c2) {
    return cpx(c1.real*c2.real - c1.image*c2.image,
               c1.real*c2.image + c1.image*c2.real);
}

```

```

int rev(int id, int len) {

```

```

int ret = 0;
for (int i = 0; (1 << i) < len; i++) {
    ret <= 1;
    if (id & (1 << i)) ret |= 1;
}
return ret;
}

```

```

cpx A[1 << 20];

```

```

void FFT(cpx *a, int len, int DFT) {
    for (int i = 0; i < len; i++)
        A[rev(i, len)] = a[i];
    for (int s = 1; (1 << s) <= len; s++) {
        int m = (1 << s);
        cpx wm = cpx(cos( DFT * 2 * PI / m), sin(DFT
            * 2 * PI / m));
        for(int k = 0; k < len; k += m) {
            cpx w = cpx(1, 0);
            for(int j = 0; j < (m >> 1); j++) {
                cpx t = w * A[k + j + (m >> 1)];
                cpx u = A[k + j];
                A[k + j] = u + t;
                A[k + j + (m >> 1)] = u - t;
                w = w * wm;
            }
        }
        if (DFT == -1) for (int i = 0; i < len; i++)
            A[i].real /= len, A[i].image /= len;
        for (int i = 0; i < len; i++) a[i] = A[i];
        return;
    }
}

```

```

cpx in[1 << 20];

```

```

void solve(int n) {
    memset(d, 0, sizeof d);
    int t;
    for (int i = 0; i < n; ++i) {
        cin >> t;
        d[t] = true;
    }
    int m;

```

```

cin >> m;
vector<int> q(m);
for (int i = 0; i < m; ++i)
    cin >> q[i];

for (int i = 0; i < MN; ++i) {
    if (d[i])
        in[i] = cpx(1, 0);
    else
        in[i] = cpx(0, 0);
}

FFT(in, MN, 1);
for (int i = 0; i < MN; ++i) {
    in[i] = in[i] * in[i];
}
FFT(in, MN, -1);

int ans = 0;
for (int i = 0; i < q.size(); ++i) {
    if (in[q[i]].real > 0.5 || d[q[i]]) {
        ans++;
    }
}
cout << ans << endl;
}

int main() {
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int n;
    while (cin >> n)
        solve(n);
    return 0;
}

```

2.4 General purpose numbers

Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
 $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ \approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), \quad c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k) x^k = x(x+1) \dots (x+n-1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j :s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j :s s.t. $\pi(j) \geq j$, k j :s s.t. $\pi(j) > j$.

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

Bell numbers

Total number of partitions of n distinct elements.

$B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$ For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

Labeled unrooted trees

on n vertices: n^{n-2}

on k existing trees of size n_i : $n_1 n_2 \dots n_k n^{k-2}$

with degrees d_i : $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

[noitemsep]sub-diagonal monotone paths in an $n \times n$ grid. strings with n pairs of parenthesis, correctly nested. binary trees with $n+1$ leaves (0 or 2 children). ordered trees with $n+1$ vertices. ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines. permutations of $[n]$ with no 3-term increasing subseq.

2.5 Lucas Theorem

For non-negative integers m and n and a prime p , the following congruence relation holds: :

$$\binom{m}{n} \equiv \prod_{i=0}^k \binom{m_i}{n_i} \pmod{p},$$

where :

$$m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0,$$

and :

$$n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0$$

are the base p expansions of m and n respectively. This uses the convention that $\binom{m}{n} = 0$ if $m \leq n$.

2.6 Multinomial

```
/**
 * Description: Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$ .
 * Status: Tested on kattis:lexicography
 */
#pragma once

long long multinomial(vector<int>& v) {
    long long c = 1, m = v.empty() ? 1 : v[0];
    for (long long i = 1; i < v.size(); i++) {
        for (long long j = 0; j < v[i]; j++) {
            c = c * ++m / (j + 1);
        }
    }
    return c;
}
```

2.7 Others

Cycles Let $g_S(n)$ be the number of n -permutations whose cycle lengths all belong to the set S . Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp \left(\sum_{n \in S} \frac{x^n}{n} \right)$$

Derangements Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$$

Burnside's lemma Given a group G of symmetries and a set X , the number of elements of X up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts “configurations” (of some sort) of length n , we can ignore rotational symmetry using $G = Z_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

2.8 Permutation To Int

```
/**
 * Description: Permutation -> integer
 *              conversion. (Not order preserving.)
 * Integer -> permutation can use a lookup table.
 * Time: O(n)
 */

int permToInt(vector<int>& v) {
    int use = 0, i = 0, r = 0;
    for(int x : v) r = r * ++i +
        __builtin_popcount(use & ~(1<<x)),
        use |= 1 << x; //
        (note: minus, not ~!)
    return r;
}
```

2.9 Sigma Function

The Sigma Function is defined as:

$$\sigma_x(n) = \sum_{d|n} d^x$$

when $x = 0$ is called the divisor function, that counts the number of positive divisors of n .

Now, we are interested in find

$$\sum_{d|n} \sigma_0(d)$$

If n is written as prime factorization:

$$n = \prod_{i=1}^k P_i^{e_k}$$

We can demonstrate that:

$$\sum_{d|n} \sigma_0(d) = \prod_{i=1}^k g(e_k + 1)$$

where $g(x)$ is the sum of the first x positive numbers:

$$g(x) = (x * (x + 1)) / 2$$

3 Data Structures

3.1 Binary Index Tree

```
struct BIT {
    int n;
    int t[2 * N];

    void add(int where, long long what) {
        for (where++; where <= n; where += where &
            -where) {
            t[where] += what;
        }
    }

    void add(int from, int to, long long what) {
        add(from, what);
        add(to + 1, -what);
    }

    long long query(int where) {
        long long sum = t[0];

        for (where++; where > 0; where -= where &
            -where) {
            sum += t[where];
        }
    }
}
```

```

        return sum;
    }
};

```

3.2 Disjoint Set Union (DSU)

```

class DSU{
public:
    vector<int> parent;
    void initialize(int n){
        parent.resize(n+1, -1);
    }

    int findSet(int u){
        while(parent[u] > 0)
            u = parent[u];
        return u;
    }

    void Union(int u, int v){
        int x = parent[u] + parent[v];
        if(parent[u] > parent[v]){
            parent[v] = x;
            parent[u] = v;
        }else{
            parent[u] = x;
            parent[v] = u;
        }
    }
};

```

3.3 Fake Update

```

vector<int> fake_bit[MAXN];

void fake_update(int x, int y, int limit_x){
    for(int i = x; i < limit_x; i += i&(-i))
        fake_bit[i].pb(y);
}

```

```

void fake_get(int x, int y){
    for(int i = x; i >= 1; i -= i&(-i))
        fake_bit[i].pb(y);
}

vector<int> bit[MAXN];

void update(int x, int y, int limit_x, int val){
    for(int i = x; i < limit_x; i += i&(-i)){
        for(int j =
            lower_bound(fake_bit[i].begin(),
                fake_bit[i].end(), y) -
                fake_bit[i].begin(); j <
                fake_bit[i].size(); j += j&(-j))
            bit[i][j] = max(bit[i][j], val);
        }
    }

    int get(int x, int y){
        int ans = 0;
        for(int i = x; i >= 1; i -= i&(-i)){
            for(int j =
                lower_bound(fake_bit[i].begin(),
                    fake_bit[i].end(), y) -
                    fake_bit[i].begin(); j >= 1; j -=
                    j&(-j))
                ans = max(ans, bit[i][j]);
            }
        return ans;
    }

    int main(){
        _io
        int n; cin >> n;
        vector<int> Sx, Sy;
        for(int i = 1; i <= n; i++){
            cin >> a[i].fi >> a[i].se;
            Sx.pb(a[i].fi);
            Sy.pb(a[i].se);
        }
        unique_arr(Sx);
        unique_arr(Sy);
        // unique all value
    }
}

```

```

for(int i = 1; i <= n; i++){
    a[i].fi = lower_bound(Sx.begin(),
        Sx.end(), a[i].fi) - Sx.begin();
    a[i].se = lower_bound(Sy.begin(),
        Sy.end(), a[i].se) - Sy.begin();
}

// do fake BIT update and get operator
for(int i = 1; i <= n; i++){
    fake_get(a[i].fi-1, a[i].se-1);
    fake_update(a[i].fi, a[i].se,
        (int)Sx.size());
}

for(int i = 0; i < Sx.size(); i++){
    fake_bit[i].pb(INT_MIN); // avoid zero
    sort(fake_bit[i].begin(),
        fake_bit[i].end());
    fake_bit[i].resize(unique(fake_bit[i].begin(),
        fake_bit[i].end()) -
        fake_bit[i].begin());
    bit[i].resize((int)fake_bit[i].size(), 0);
}

// real update, get operator
int res = 0;
for(int i = 1; i <= n; i++){
    int maxCurLen = get(a[i].fi-1, a[i].se-1)
        + 1;
    res = max(res, maxCurLen);
    update(a[i].fi, a[i].se, (int)Sx.size(),
        maxCurLen);
}
}

```

3.4 Fenwick Tree

```

template<typename T>
class FenwickTree{
    vector<T> fenw;
    int n;
public:

```

```

void initialize(int _n){
    this->n = _n;
    fenw.resize(n+1);
}

void update(int id, T val) {
    while (id <= n) {
        fenw[id] += val;
        id += id&(-id);
    }
}

T get(int id){
    T ans{};
    while(id >= 1){
        ans += fenw[id];
        id -= id&(-id);
    }
    return ans;
}
};

```

3.5 Hash Table

```

/*
 * Micro hash table, can be used as a set.
 * Very efficient vs std::set
 */

const int MN = 1001;
struct ht {
    int _s[(MN + 10) >> 5];
    int len;
    void set(int id) {
        len++;
        _s[id >> 5] |= (1LL << (id & 31));
    }
    bool is_set(int id) {
        return _s[id >> 5] & (1LL << (id & 31));
    }
};

```

3.6 Range Minimum Query

```

/*
    return min(v[a], v[a + 1], ..., v[b - 1]) in
        constant time
*/

template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <=
            sz(V); pw *= 2, ++k) {
            jmp.emplace_back(sz(V) - pw
                * 2 + 1);
            rep(j, 0, sz(jmp[k]))
                jmp[k][j] =
                    min(jmp[k -
                        1][j], jmp[k -
                            1][j + pw]);
        }
    }
    T query(int a, int b) {
        assert(a < b); // or return inf if
            a == b
        int dep = 31 - __builtin_clz(b - a);
        return min(jmp[dep][a], jmp[dep][b
            - (1 << dep)]);
    }
};

```

3.7 STL Treap

```

struct Node {
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

```

```

};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val);
        each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {};
    if (cnt(n->l) >= k) { // "n->val >= k" for
        lower_bound(k)
        auto pa = split(n->l, k);
        n->l = pa.second;
        n->recalc();
        return {pa.first, n};
    } else {
        auto pa = split(n->r, k - cnt(n->l)
            - 1); // and just "k"
        n->r = pa.first;
        n->recalc();
        return {n, pa.second};
    }
}

Node* merge(Node* l, Node* r) {
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        l->recalc();
        return l;
    } else {
        r->l = merge(l, r->l);
        r->recalc();
        return r;
    }
}

Node* ins(Node* t, Node* n, int pos) {
    auto pa = split(t, pos);
};

```

```

        return merge(merge(pa.first, n),
                      pa.second);
    }

    // Example application: move the range [l, r] to
    // index k
    void move(Node& t, int l, int r, int k) {
        Node *a, *b, *c;
        tie(a,b) = split(t, l); tie(b,c) =
            split(b, r - l);
        if (k <= l) t = merge(merge(a, b, k), c);
        else t = merge(a, merge(c, b, k - r));
    }

```

3.8 Segment Tree

```

#include <bits/stdc++.h>
using namespace std;

const int N = 1e5 + 10;

int node[4*N];

void modify(int seg, int l, int r, int p, int
val){
    if(l == r){
        node[seg] += val;
        return;
    }
    int mid = (l + r)/2;
    if(p <= mid){
        modify(2*seg + 1, l, mid, p, val);
    }else{
        modify(2*seg + 2, mid + 1, r, p, val);
    }
    node[seg] = node[2*seg + 1] + node[2*seg + 2];
}

int sum(int seg, int l, int r, int a, int b){
    if(l > b || r < a) return 0;
    if(l >= a && r <= b) return node[seg];
    int mid = (l + r)/2;

```

```

        return sum(2*seg + 1, l, mid, a, b) +
            sum(2*seg + 2, mid + 1, r, a, b);
    }

```

3.9 Sparse Table

```

template <typename T, typename func =
    function<T(const T, const T)>>
struct SparseTable {
    func calc;
    int n;
    vector<vector<T>> ans;

    SparseTable() {}

    SparseTable(const vector<T>& a, const func&
f) : n(a.size()), calc(f) {
        int last = trunc(log2(n)) + 1;
        ans.resize(n);
        for (int i = 0; i < n; i++){
            ans[i].resize(last);
        }
        for (int i = 0; i < n; i++){
            ans[i][0] = a[i];
        }
        for (int j = 1; j < last; j++){
            for (int i = 0; i <= n - (1 << j);
                i++){
                ans[i][j] = calc(ans[i][j - 1],
                    ans[i + (1 << (j - 1))][j -
                    1]);
            }
        }
    }

    T query(int l, int r){
        assert(0 <= l && l <= r && r < n);
        int k = trunc(log2(r - l + 1));
        return calc(ans[l][k], ans[r - (1 << k) +
            1][k]);
    }
}

```

```
};
```

3.10 Trie

```

const int MN = 26; // size of alphabet
const int MS = 100010; // Number of states.

```

```

struct trie{
    struct node{
        int c;
        int a[MN];
    };

    node tree[MS];
    int nodes;

    void clear(){
        tree[nodes].c = 0;
        memset(tree[nodes].a, -1, sizeof
            tree[nodes].a);
        nodes++;
    }

    void init(){
        nodes = 0;
        clear();
    }

    int add(const string &s, bool query = 0){
        int cur_node = 0;
        for(int i = 0; i < s.size(); ++i){
            int id = gid(s[i]);
            if(tree[cur_node].a[id] == -1){
                if(query) return 0;
                tree[cur_node].a[id] = nodes;
                clear();
            }
            cur_node = tree[cur_node].a[id];
        }
        if(!query) tree[cur_node].c++;
        return tree[cur_node].c;
    }
}

```



```
};
```

4 Dynamic Programming Optimization

4.1 Convex Hull Trick

```
#define long long long
#define pll pair<long, long>
#define all(c) c.begin(), c.end()
#define fastio ios_base::sync_with_stdio(false);
cin.tie(0)

struct line{
    long a, b;
    line() {};
    line(long a, long b) : a(a), b(b) {};
    bool operator < (const line &A) const {
        return pll(a,b) < pll(A.a,A.b);
    }
};

bool bad(line A, line B, line C){
    return (C.b - B.b) * (A.a - B.a) <= (B.b -
        A.b) * (B.a - C.a);
}

void addLine(vector<line> &memo, line cur){
    int k = memo.size();
    while (k >= 2 && bad(memo[k - 2], memo[k -
        1], cur)){
        memo.pop_back();
        k--;
    }
    memo.push_back(cur);
}

long Fn(line A, long x){
    return A.a * x + A.b;
```

```
}

long query(vector<line> &memo, long x){
    int lo = 0, hi = memo.size() - 1;
    while (lo != hi){
        int mi = (lo + hi) / 2;
        if (Fn(memo[mi], x) > Fn(memo[mi + 1], x)){
            lo = mi + 1;
        }
        else hi = mi;
    }
    return Fn(memo[lo], x);
}

const int N = 1e6 + 1;
long dp[N];

int main()
{
    fastio;
    int n, c; cin >> n >> c;
    vector<line> memo;
    for (int i = 1; i <= n; i++){
        long val; cin >> val;
        addLine(memo, {-2 * val, val * val + dp[i
            - 1]});
        dp[i] = query(memo, val) + val * val + c;
    }
    cout << dp[n] << '\n';
    return 0;
}
```

4.2 Divide and Conquer

```
/**
 * recurrence:
 *   dp[k][i] = min dp[k-1][j] + c[i][j - 1], for
 *   all j > i;
 *
 * "comp" computes dp[k][i] for all i in O(n log
 *   n) (k is fixed)
 *
 */
```

```
* Problems:
* https://icpc.kattis.com/problems/branch
* http://codeforces.com/contest/321/problem/E
* */
```

```
void comp(int l, int r, int le, int re) {
    if (l > r) return;

    int mid = (l + r) >> 1;

    int best = max(mid + 1, le);
    dp[cur][mid] = dp[cur ^ 1][best] + cost(mid,
        best - 1);
    for (int i = best; i <= re; i++) {
        if (dp[cur][mid] > dp[cur ^ 1][i] + cost(mid,
            i - 1)) {
            best = i;
            dp[cur][mid] = dp[cur ^ 1][i] + cost(mid, i
                - 1);
        }
    }

    comp(l, mid - 1, le, best);
    comp(mid + 1, r, best, re);
}
```

5 Geometry

5.1 Closest Pair Problem

```
struct point {
    double x, y;
    int id;
    point() {}
    point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
    double a = p.x - o.x, b = p.y - o.y;
    return sqrt(a * a + b * b);
}
```

```

double cp(vector<point> &p, vector<point> &x,
vector<point> &y) {
    if (p.size() < 4) {
        double best = 1e100;
        for (int i = 0; i < p.size(); ++i)
            for (int j = i + 1; j < p.size(); ++j)
                best = min(best, dist(p[i], p[j]));
        return best;
    }

    int ls = (p.size() + 1) >> 1;
    double l = (p[ls - 1].x + p[ls].x) * 0.5;
    vector<point> xl(ls), xr(p.size() - ls);
    unordered_set<int> left;
    for (int i = 0; i < ls; ++i) {
        xl[i] = x[i];
        left.insert(x[i].id);
    }
    for (int i = ls; i < p.size(); ++i) {
        xr[i - ls] = x[i];
    }

    vector<point> yl, yr;
    vector<point> pl, pr;
    yl.reserve(ls); yr.reserve(p.size() - ls);
    pl.reserve(ls); pr.reserve(p.size() - ls);
    for (int i = 0; i < p.size(); ++i) {
        if (left.count(y[i].id))
            yl.push_back(y[i]);
        else
            yr.push_back(y[i]);

        if (left.count(p[i].id))
            pl.push_back(p[i]);
        else
            pr.push_back(p[i]);
    }

    double dl = cp(pl, xl, yl);
    double dr = cp(pr, xr, yr);
    double d = min(dl, dr);
    vector<point> yp; yp.reserve(p.size());
    for (int i = 0; i < p.size(); ++i) {

```

```

        if (fabs(y[i].x - l) < d)
            yp.push_back(y[i]);
    }
    for (int i = 0; i < yp.size(); ++i) {
        for (int j = i + 1; j < yp.size() && j < i +
            7; ++j) {
            d = min(d, dist(yp[i], yp[j]));
        }
    }
    return d;
}

double closest_pair(vector<point> &p) {
    vector<point> x(p.begin(), p.end());
    sort(x.begin(), x.end(), [](const point &a,
        const point &b) {
        return a.x < b.x;
    });
    vector<point> y(p.begin(), p.end());
    sort(y.begin(), y.end(), [](const point &a,
        const point &b) {
        return a.y < b.y;
    });
    return cp(p, x, y);
}

```

5.2 Convex Diameter

```

struct point{
    int x, y;
};

struct vec{
    int x, y;
};

vec operator - (const point &A, const point &B){
    return vec{A.x - B.x, A.y - B.y};
}

int cross(vec A, vec B){
    return A.x*B.y - A.y*B.x;
}

```

```

}

int cross(point A, point B, point C){
    int val = A.x*(B.y - C.y) + B.x*(C.y - A.y) +
        C.x*(A.y - B.y);
    if(val == 0)
        return 0; // coline
    if(val < 0)
        return 1; // clockwise
    return -1; //counter clockwise
}

vector<point> findConvexHull(vector<point>
points){
    vector<point> convex;
    sort(points.begin(), points.end(), [](const
        point &A, const point &B){
        return (A.x == B.x)? (A.y < B.y): (A.x <
            B.x);
    });
    vector<point> Up, Down;
    point A = points[0], B = points.back();
    Up.push_back(A);
    Down.push_back(A);

    for(int i = 0; i < points.size(); i++){
        if(i == points.size()-1 || cross(A,
            points[i], B) > 0){
            while(Up.size() > 2 &&
                cross(Up[Up.size()-2],
                    Up[Up.size()-1], points[i]) <= 0)
                Up.pop_back();
            Up.push_back(points[i]);
        }
        if(i == points.size()-1 || cross(A,
            points[i], B) < 0){
            while(Down.size() > 2 &&
                cross(Down[Down.size()-2],
                    Down[Down.size()-1], points[i]) >=
                    0)
                Down.pop_back();
            Down.push_back(points[i]);
        }
    }
}

```

```

for(int i = 0; i < Up.size(); i++)
    convex.push_back(Up[i]);
for(int i = Down.size()-2; i > 0; i--)
    convex.push_back(Down[i]);
return convex;
}

int dist(point A, point B){
    return (A.x - B.x)*(A.x - B.x) + (A.y -
        B.y)*(A.y - B.y);
}

double findConvexDiameter(vector <point>
    convexHull){
    int n = convexHull.size();

    int is = 0, js = 0;
    for(int i = 1; i < n; i++){
        if(convexHull[i].y > convexHull[is].y)
            is = i;
        if(convexHull[js].y > convexHull[i].y)
            js = i;
    }

    int maxd = dist(convexHull[is],
        convexHull[js]);
    int i, maxi, j, maxj;
    i = maxi = is;
    j = maxj = js;
    do{
        int ni = (i+1)%n, nj = (j+1)%n;
        if(cross(convexHull[ni] - convexHull[i],
            convexHull[nj] - convexHull[j]) <= 0){
            j = nj;
        }else{
            i = ni;
        }
        int d = dist(convexHull[i], convexHull[j]);
        if(d > maxd){
            maxd = d;
            maxi = i;
            maxj = j;
        }
    }while(i != is || j != js);
}

```

```

return sqrt(maxd);
}

```

5.3 Pick Theorem

```

struct point{
    ll x, y;
};

//Pick:  $S = I + B/2 - 1$ 

ld polygonArea(vector <point> &points){
    int n = (int)points.size();
    ld area = 0.0;
    int j = n-1;
    for(int i = 0; i < n; i++){
        area += (points[j].x + points[i].x) *
            (points[j].y - points[i].y);
        j = i;
    }

    return abs(area/2.0);
}

ll boundary(vector <point> points){
    int n = (int)points.size();
    ll num_bound = 0;
    for(int i = 0; i < n; i++){
        ll dx = (points[i].x - points[(i+1)%n].x);
        ll dy = (points[i].y - points[(i+1)%n].y);
        num_bound += abs(__gcd(dx, dy)) - 1;
    }
    return num_bound;
}

```

5.4 Square

```

typedef long double ld;

```

```

const ld eps = 1e-12;
int cmp(ld x, ld y = 0, ld tol = eps) {
    return (x <= y + tol) ? (x + tol < y) ? -1 :
        0 : 1;
}

struct point{
    ld x, y;
    point(ld a, ld b) : x(a), y(b) {}
    point() {}
};

struct square{
    ld x1, x2, y1, y2,
        a, b, c;
    point edges[4];
    square(ld _a, ld _b, ld _c) {
        a = _a, b = _b, c = _c;
        x1 = a - c * 0.5;
        x2 = a + c * 0.5;
        y1 = b - c * 0.5;
        y2 = b + c * 0.5;
        edges[0] = point(x1, y1);
        edges[1] = point(x2, y1);
        edges[2] = point(x2, y2);
        edges[3] = point(x1, y2);
    }
};

ld min_dist(point &a, point &b) {
    ld x = a.x - b.x,
        y = a.y - b.y;
    return sqrt(x * x + y * y);
}

bool point_in_box(square s1, point p) {
    if (cmp(s1.x1, p.x) != 1 && cmp(s1.x2, p.x) !=
        -1 &&
        cmp(s1.y1, p.y) != 1 && cmp(s1.y2, p.y) !=
        -1)
        return true;
    return false;
}

```

```

bool inside(square &s1, square &s2) {
    for (int i = 0; i < 4; ++i)
        if (point_in_box(s2, s1.edges[i]))
            return true;

    return false;
}

bool inside_vert(square &s1, square &s2) {
    if ((cmp(s1.y1, s2.y1) != -1 && cmp(s1.y1,
        s2.y2) != 1) ||
        (cmp(s1.y2, s2.y1) != -1 && cmp(s1.y2,
        s2.y2) != 1))
        return true;
    return false;
}

bool inside_hori(square &s1, square &s2) {
    if ((cmp(s1.x1, s2.x1) != -1 && cmp(s1.x1,
        s2.x2) != 1) ||
        (cmp(s1.x2, s2.x1) != -1 && cmp(s1.x2,
        s2.x2) != 1))
        return true;
    return false;
}

ld min_dist(square &s1, square &s2) {
    if (inside(s1, s2) || inside(s2, s1))
        return 0;

    ld ans = 1e100;
    for (int i = 0; i < 4; ++i)
        for (int j = 0; j < 4; ++j)
            ans = min(ans, min_dist(s1.edges[i],
                s2.edges[j]));

    if (inside_hori(s1, s2) || inside_hori(s2, s1))
        {
            if (cmp(s1.y1, s2.y2) != -1)
                ans = min(ans, s1.y1 - s2.y2);
            else
                if (cmp(s2.y1, s1.y2) != -1)
                    ans = min(ans, s2.y1 - s1.y2);
        }
}

```

```

}

if (inside_vert(s1, s2) || inside_vert(s2, s1))
    {
        if (cmp(s1.x1, s2.x2) != -1)
            ans = min(ans, s1.x1 - s2.x2);
        else
            if (cmp(s2.x1, s1.x2) != -1)
                ans = min(ans, s2.x1 - s1.x2);
    }

return ans;
}

```

5.5 Triangle

Let a, b, c be length of the three sides of a triangle.

$$p = (a + b + c) * 0.5$$

The inradius is defined by:

$$iR = \sqrt{\frac{(p-a)(p-b)(p-c)}{p}}$$

The radius of its circumcircle is given by the formula:

$$cR = \frac{abc}{\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}}$$

6 Graphs

6.1 SCC Kosaraju

// SCC = Strongly Connected Components

```

struct SCC {
    vector<vector<int>> g, gr;
    vector<bool> used;

```

```

    vector<int> order, component;
    int total_components;

    SCC(vector<vector<int>>& adj) {
        g = adj;
        int n = g.size();
        gr.resize(n);
        for (int i = 0; i < n; i++)
            for (auto to : g[i])
                gr[to].push_back(i);

        used.assign(n, false);
        for (int i = 0; i < n; i++)
            if (!used[i])
                GenTime(i);

        used.assign(n, false);
        component.assign(n, -1);
        total_components = 0;
        for (int i = n - 1; i >= 0; i--) {
            int v = order[i];
            if (!used[v]) {
                vector<int> cur_component;
                Dfs(cur_component, v);
                for (auto node : cur_component)
                    component[node] =
                        total_components;
            }
        }

        void GenTime(int node) {
            used[node] = true;
            for (auto to : g[node])
                if (!used[to])
                    GenTime(to);
            order.push_back(node);
        }

        void Dfs(vector<int>& cur, int node) {
            used[node] = true;
            cur.push_back(node);
            if (!used[to])
                Dfs(cur, to);
        }
    }
}

```

```
}  
  
vector<vector<int>>> CondensedGraph() {  
    vector<vector<int>>> ans(total_components);  
    for (int i = 0; i < int(g.size()); i++) {  
        for (int to : g[i]) {  
  
            int u = component[i], v =  
                component[to];  
            if (u != v)  
                ans[u].push_back(v);  
        }  
    }  
  
    return ans;  
};
```
