Trade option's time value

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October 2024

Abstract: This research briefly presents an understanding on how to trade option's time value by getting exposed to the well known option's greeks Gamma, Theta and Vega. First, there will be some reminders on option's profit and loss and the identification of the two main factors driving an option's time value PnL. Then, some common option at-the-money strategies will be introduced to understand how to trade gamma-theta/vega based on the trader's desired exposure. Finally, we will present some backtestings with an algorithmic trader trading gamma-theta/vega using Deribit Bitcoin future options. The algo trader uses "in-house" machine learning techniques in order to forecast the driving factors of the option's time value profit and loss and applies an automatic delta-hedging algorithm to hedge the exposure against the underlying's spot price.

***Keywords*:** Trading, Options, Quantitative Finance, Risk Management

1 What do we mean by trading option's time value

This section is meant to bring back some basics knowledge about an option's profit and loss explanation using the well known greeks. The main objective is to briefly deep dive into it in order to understand why Gamma/Vega exposure is essential and which factors are driving the profit and loss on some short time period. Let's say we have a vanilla european option at time t with price V_t, underlying S_t and implied volatility σ_t. For simplification, we assume interest rate are equal to zero and that it is constant through time. Trading this option at time t will give you exposure on diverse factors translated into mathematical derivatives, called the "greeks":

- delta (Δ_t) : measures the rate of change of the theoretical option value with respect to changes in the underlying asset's price
$$\Delta_t = \frac{\partial V_t}{\partial S_t} = \phi N(\phi d_t+)$$
where ϕ is equal to 1 if the option is a call and -1 if it is a put.

- vega (υ_t) : measures sensitivity to volatility. Vega is the derivative of the option value with respect to the volatility of the underlying asset.
$$\upsilon_t = \frac{\partial V_t}{\partial \sigma_t} = S_t N'(d_t+)\sqrt{T-t}$$

- gamma (Γ_t) : measures the rate of change in the delta with respect to changes in the underlying price.
$$\Gamma_t = \frac{\partial^2 V_t}{\partial \sigma_t^2} = \frac{N'(d_t+)}{S_t\sigma_t\sqrt{T-t}}$$

- theta (θ_t) : measures the sensitivity of the value of the derivative to the passage of time.
$$\theta_t = \frac{\partial V_t}{\partial t} = \frac{-S_t\sigma_t N'(d_t+)}{2\sqrt{T-t}} = \frac{1}{2}S_t^2\sigma_t^2\Gamma_t$$
when r and q (interest rate and convenience yield) are both set to zero.

There exists other derivative in higher order but we will not look at them for now. With these 4 sensitivities, we can approximate the change in option's price (therefore the profit and loss π_V) over a small time interval dt as:

$$\pi_V \simeq \Delta_t dS_t + \frac{1}{2}\Gamma_t (dS_t)^2 + \upsilon_t d\sigma_t + \theta_t dt$$

what we mean by "trading the option's time value" is actually to trade a portfolio without any directional exposure on the underlying, we call such a portfolio the "delta neutral portfolio". More specifically, This portfolio can be structured as an option with a fraction Δ_t of the option's underlying S_t. The value of this portfolio is:

$$P_t = V_t - \Delta_t S_t$$

Therefore, the profit and loss of the delta neutral portfolio over a small time interval dt is:

$$\pi_P = \pi_V - \Delta_t dS_t$$

which leads to:

$$\pi_P = \frac{1}{2}\Gamma_t (dS_t)^2 + \upsilon_t d\sigma_t + \theta_t dt$$

by using the following equality:

$$\frac{1}{2}\Gamma_t (dS_t)^2 = \frac{1}{2}\Gamma_t S_t^2 \frac{1}{dt}(\frac{dS_t}{S_t})^2 dt$$

we can express the realized variance as:

$$\sigma_{RE}^2 = \frac{1}{dt}(\frac{dS_t}{S_t})^2$$

As we assumed that interest rates are equal to zero, we can express θ_t as a function of Γ_t which will give the following profit and loss:

$$\pi_P = \frac{1}{2}\Gamma_t S^2 dt(\sigma_{RE} + \sigma_t)(\sigma_{RE} - \sigma_t) + \upsilon_t d\sigma_t$$

with some tricks in the algebra, we finally have:

$$\pi_P = \Gamma_t S^2 dt(\frac{1}{2}(\sigma_{RE} - \sigma_t)^2 + \sigma_t(\sigma_{RE} - \sigma_t)) + \upsilon_t(\sigma_{t+dt} - \sigma_t) \tag{1}$$

The equation (1) shows that the delta neutral portfolio is based on the exposures given by Γ_t and υ_t. It is also driven by two unknown factors:

- $\delta_{RE} = (\sigma_{RE} - \sigma_t)$ which represents the error in pricing by the market at time t.
- $\delta_{IV} = (\sigma_{t+dt} - \sigma_t)$ which could represents the repricing done by the market over the time interval dt assuming that the option is stuck to the same moneyness at $t + dt$. It also could represents the change in implied volatility because of the movement of the option's moneyness (and therefore the movement of the sport price).

2 Option strategies to get exposure

It will be discussed here how to get exposure based on forecasted value of δ_{RE} and δ_{IV}, respectively represented by δ^M_{RE} and δ^M_{IV}. Those two values are supposedly forecasted from two independent models. The forecasted delta neutral portfolio's PnL can then be calculated by:

$$\pi^M_P = \Gamma_t S^2 dt(\frac{1}{2}(\delta^M_{RE})^2 + \sigma_t \delta^M_{RE}) + \upsilon_t \delta^M_{IV}$$

It is important here to understand the behavior of Γ_t and υ_t as the moneyness (strikes) and the time to expiry change. Both greeks will find their highest values when the option is stuck at-the-money as both mathematical formula are just a factorisation of the Gaussian probability distribution function. However, they have an inverse relationship with respect to time to expiry. The gamma (vega) will decrease (increase) as the time to expiry increases. It is important to understand this in order to trade the right delta neutral option's strategy and get the desired exposure. Following, we list the following delta neutral option's strategies for each exposure:

- long vega / long gamma : long calendar spread straddle
- long vega / neutral gamma : long long-expiry straddle
- long vega / short gamma : long calendar spread
- neutral vega / long gamma : long short-expiry straddle
- neutral vega / short gamma : short short-expiry straddle
- short vega / long gamma : short calendar spread
- short vega / neutral gamma : short long-expiry straddle
- short vega / short gamma : short calendar spread straddle

When traded ATM, those strategies will give you the highest exposure possible. However the choices in expiries will depend on the difference of δ^M_{RE} and δ^M_{IV} in order to maximize the forecasted PnL π^M_P. Finally, a trader will choose to be long (short) gamma when $\delta^M_{RE} > 0$ (< 0) and neutral if δ^M_{RE} is equal to zero. The trader will choose to be long (short) vega when $\delta^M_{IV} > 0$ (< 0) and neutral when δ^M_{IV} is equal to zero. It is to be noted that the incorporation of higher order greeks into the forecasted PnL such as the vanna and the volga could help making a better choice in term of expiries. It would also influenced the choice of exposure on the repricing of market's volatility. Nonetheless, using the vanna would result in the need of a directional bet (and thus a model) on the underlying's spot.

3 Backtest of a simple strategy with cryptocurency's options

This section will show the results of a small algorithmic volatility trader executing (in simulation) Bitcoin Future options available on the Deribit exchange. The trader will apply two simple models to forecast δ_{RE}^{M} and δ_{IV}^{M}. Every hour, the trader collects the ATM option chain and trade the best strategies in term of forecasted PnL. As the option book is building, the trader delta hedged is position hourly and close forecasted loosing option positions (only when the spread to close is under a certain threshold). This mechanism allows the trader to position the book sensitivies with respect to its view of future market's volatility movements while staying neutral to the movements of the underlying spot price.

3.1 Algo trader's model

We briefly introduce to the models used by the algorithmic trader to take decisions on which option's strategies to trade. It is to be noted that the forecasted PnL of each options are mapped to their return on investment, which includes initial margin, premiums, trading fees and delta hedging fees.

3.1.1 Directional Gamma: forecast σ_{RE}

The gamma/theta PnL is driven by a single random variable σ_{RE}. The model developed for the algorithmic trader is a pseudo "in-house" machine learning model using 3 types of data, all related to the perpetual future on Bitcoin, to forecast the next hour realised volatility σ_{RE}. The data types are:

- the historical realised volatility computed at different horizon
- the historical volume
- the historical open interests

Those data represent some common features that should drive the price's volatility of the bitcoin. The perpetual future contract represents the most liquid instrument traded on Deribit to get exposure to the bitcoin spot price. We will not get further into what the model is exactly as it is not the point of the research. However, the model outputs a forecasted σ_{RE} which is used to estimate the future Gamma-theta PnL of any options traded on the market.

3.1.2 Directional Vega: forecast the SSVI volatility surface

When the trader's book is building among different options, there as many random variables σ_{t+dt} as there are option's positions. Moreover, as stated before, the pure vega bet requires also a bet on the underlying spot's price as it will moves the option's moneyness and therefore moves the option's implied volatility (even though the volatility surface did not show movements). The only moment when a vega bet does not require a bet on the underlying's spot price is when the volatility surface is symmetric around the ATM term structure, which is rarely obeserved on the market. Thus, a good vega bet requires a robust model capturing the full volatility surface's dynamic plus a model forecasting the underlying spot's price. That being said, we will forget about forecasting the underlying's spot price here and focus on forecasting the Surface Stochastic Volatiity Inspired (SSVI) which has been introduced in [1]. The main idea of forecasting a parametric volatility surface is to tackle the high dimensional problem presented when trying to position into a directional Vega. The SSVI is build from three paramaters (γ, η and ρ) and a vector of ATM total variance ω_t where $\omega_t = t(\sigma_t^{ATM})^2$. The SSVI Surface is calibrated each hour from market's quotes and therefore, just like the directional gamma, the algorithmic's trader use an "in house" machine learning model using historical values of the SSVI surface's parameters and historical values of ATM total variance at fixed maturity points. The forecasted SSVI, being the output of the model, is then used to estimate the forecasted Vega PnL of any options traded on the market.

3.2 Results

The backtest is done with a virtual deposit of USD 100'000. On deribit exchange, the option's premiums and the margins are settle in bitcoin so there is an initial spot trade to obtain bitcoin and the delta exposure is hedged with a trade on the Perpetual Future. First, in the annexes, one can find the initial and maintenance margins for each backtest. They are all in a very acceptable range which means that the trader never gets margin called while trading its strategies.

3.2.1 February 2nd, 2023 to March 2nd, 2023

It can be seen in figure 1 that the algo trader is loosing a little bit more than 3'000USD with a very volatile PnL by trading 2'753 times on 348 different options. Most of the loss is made in trading fees (around 2'500USD) which are very expensive for a retail trader on the Deribit exchange (0.03% of the notional, capped at 12.5% of the option's premium). However, the delta exposure, displayed in figure 5, shows that the delta hedging is effective altough very expensive in terms of fees. Based on the figure 7, it can safely be said that the trader is loosing money because of wrong gamma bets which makes the PnL fluctuating downwards.

Figure 1: Total USD profit and loss and its decomposition from February 2nd, 2023 5pm to March 2nd, 2023 3pm (693 hours of trading)

3.2.2 March 7th, 2023 to April 4th, 2023

It can be seen in figure 2 that the algo trader is making around 10'000USD with a very consistent PnL by trading 2'734 times on 424 different options. The delta exposure, displayed in figure 4, shows that the delta hedging is also effective here. During this backtest, one can see in figures 8 and 11, that the trader is almost consistently long gamma and mostly short vega. The loosing PnL on the delta hedge is offset and enhanced by the gains on the option's book which creates a nice and not too volatile book's profit and loss.

Figure 2: Total USD profit and loss and its decomposition from March 7th, 2023 6pm to April 4th, 2023 7pm (671 hours of trading)

3.2.3 April 9th, 2023 to May 6th, 2023

Again here, it can be seen in figure 2 that the algo trader is making around 5'000USD with a volatile PnL by trading 2'526 times on 393 different options. The delta exposure, displayed in figure 4, shows that the delta hedging is also effective. During this backtest, one can see in figures 9 and 12, that the trader is almost consistently long gamma and navigate between short and long vega. The loosing PnL on the delta hedge is also offset and enhanced by the gains on the option's book which creates volatile book's profit and loss, less consistent than during the month of march.

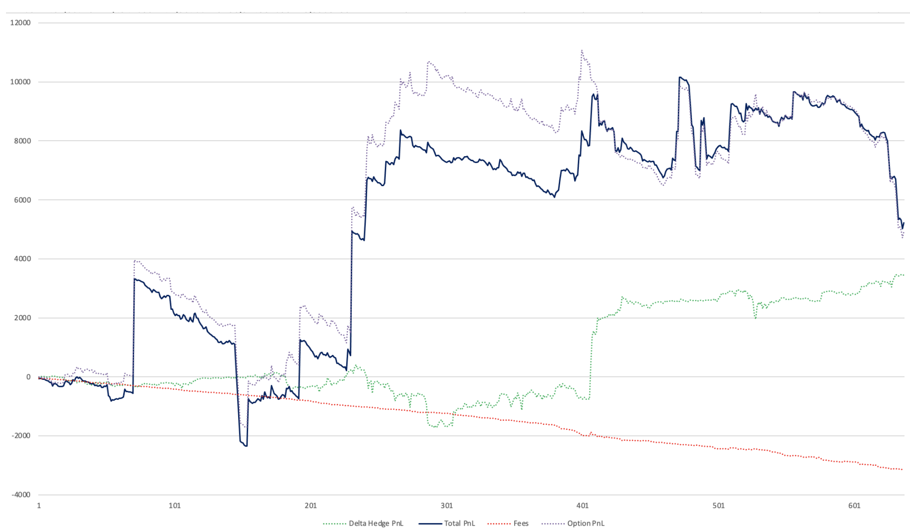

Figure 3: Total USD profit and loss and its decomposition from April 9th, 2023 10pm to May 6th 2023 10am (636 hours of trading)

References

[1] Jim Gatheral and Antoine Jacquier. *Arbitrage-free SVI volatility surfaces*. 2013. arXiv: 1204.0646 [q-fin.PR]. URL: https://arxiv.org/abs/1204.0646.

4 Annexes

4.1 Delta exposure

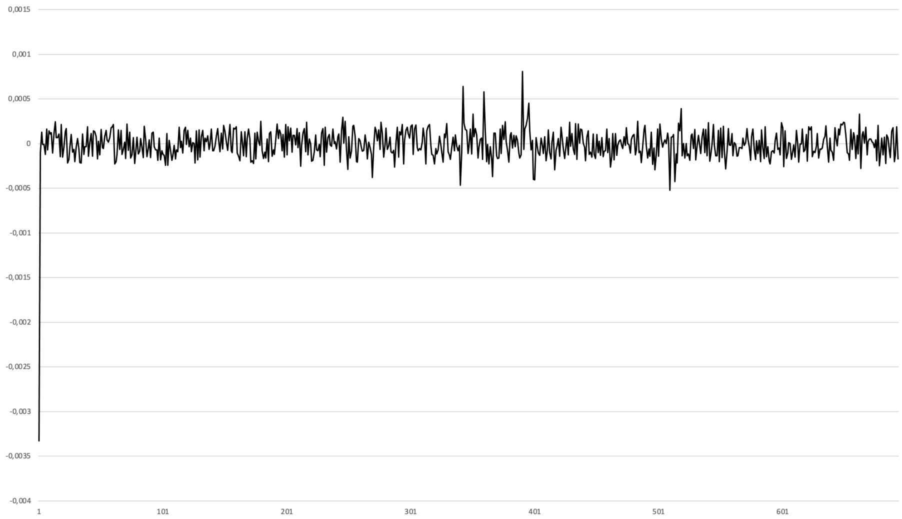

Figure 4: Book's delta exposure (in bitcoin) February 2nd, 2023 5pm to March 2nd, 2023 3pm (693 hours of trading)

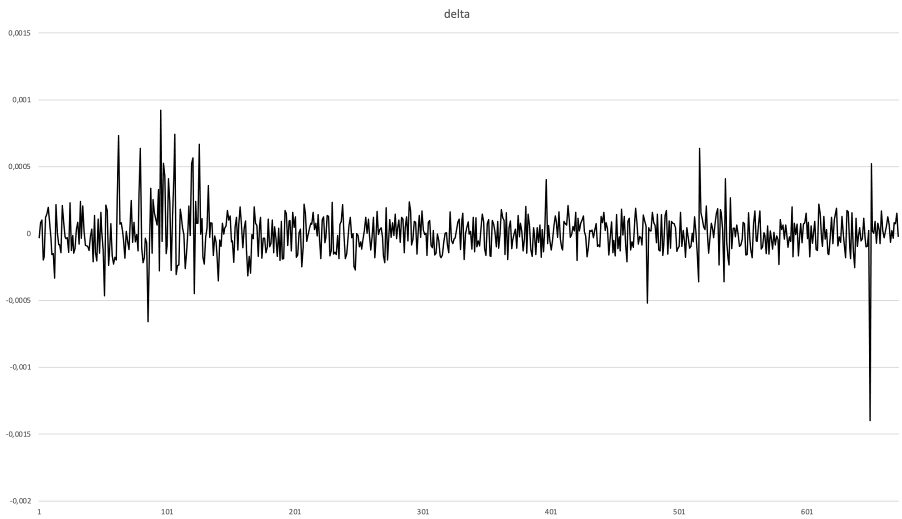

Figure 5: Book's delta exposure (in bitcoin) from March 7th, 2023 6pm to April 4th 2023 7pm (671 hours of trading)

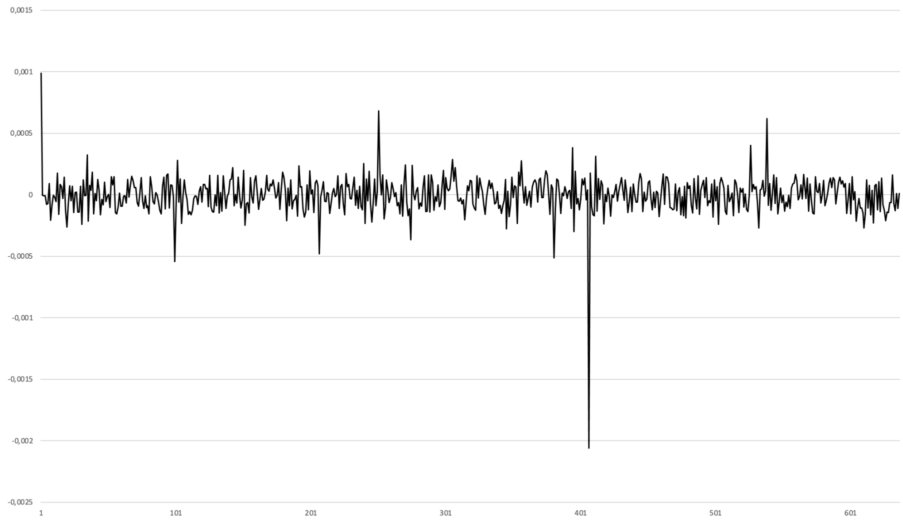

Figure 6: Book's delta exposure (in bitcoin) from April 9th, 2023 10pm to May 6th 2023 10am (636 hours of trading)

4.2 Gamma exposure

Figure 7: Book's gamma exposure from February 2nd, 2023 5pm to March 2nd, 2023 3pm (693 hours of trading)

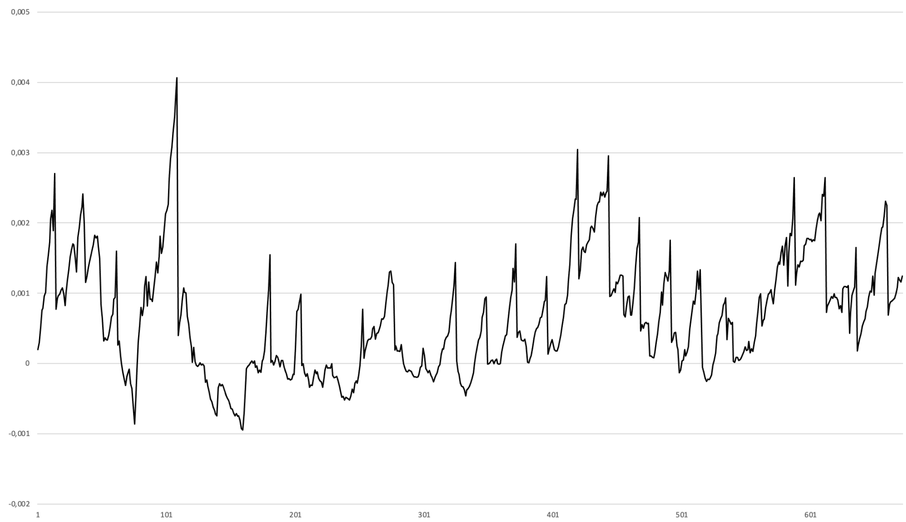

Figure 8: Book's gamma exposure from March 7th, 2023 6pm to April 4th 2023 7pm (671 hours of trading)

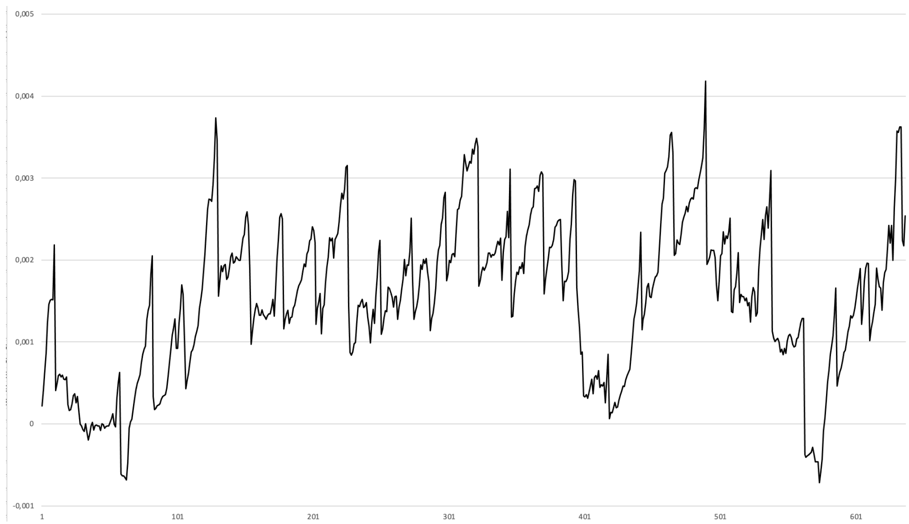

Figure 9: Book's gamma exposure from April 9th, 2023 10pm to May 6th 2023 10am (636 hours of trading)

4.3 Vega exposure

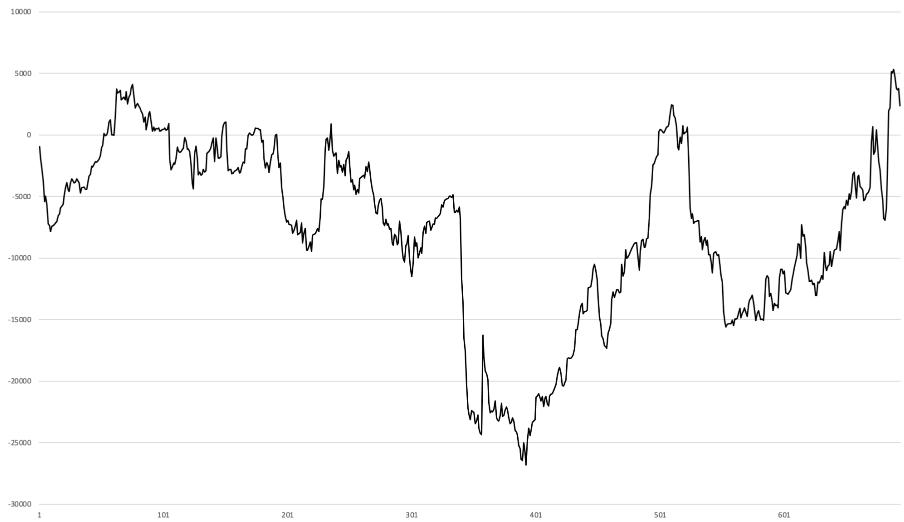

Figure 10: Book's vega exposure (in USD) from February 2nd, 2023 5pm to March 2nd, 2023 3pm (693 hours of trading)

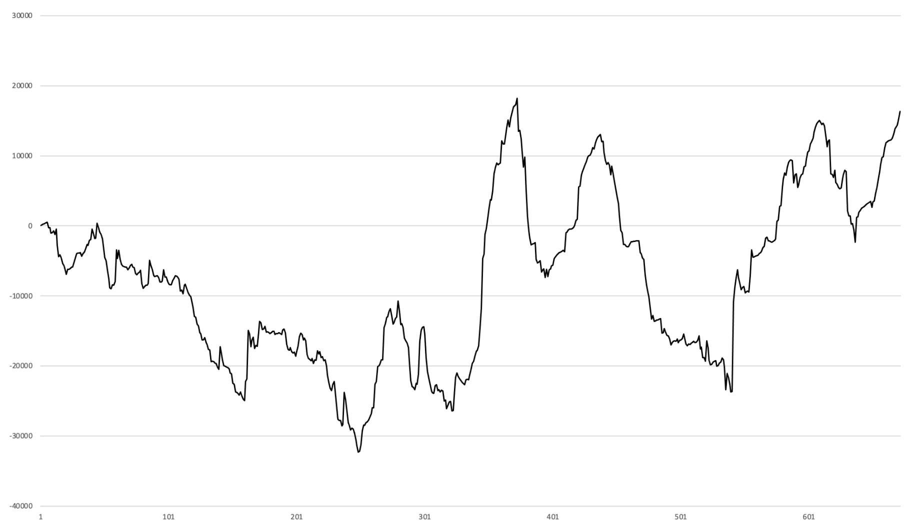

Figure 11: Book's vega exposure (in USD) from March 7th, 2023 6pm to April 4th 2023 7pm (671 hours of trading)

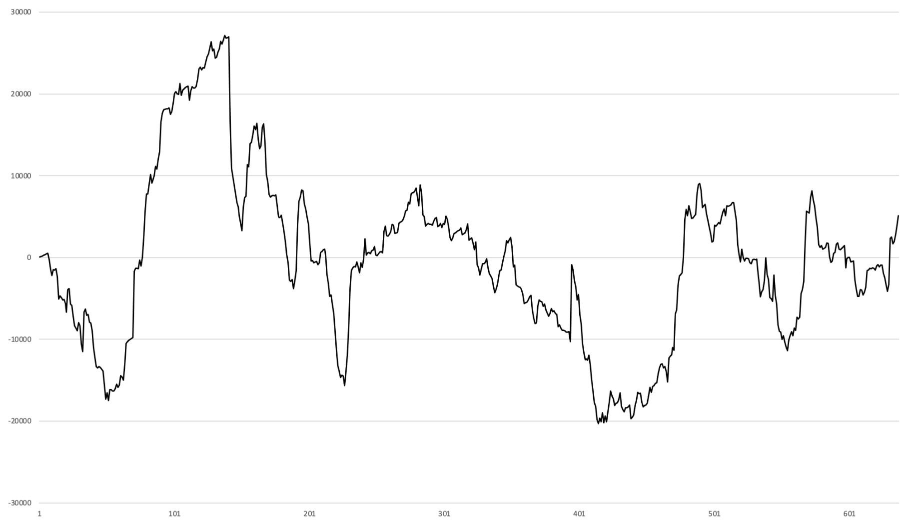

Figure 12: Book's vega exposure (in USD) from April 9th, 2023 10pm to May 6th 2023 10am (636 hours of trading)

4.4 Initial and maintenance margins

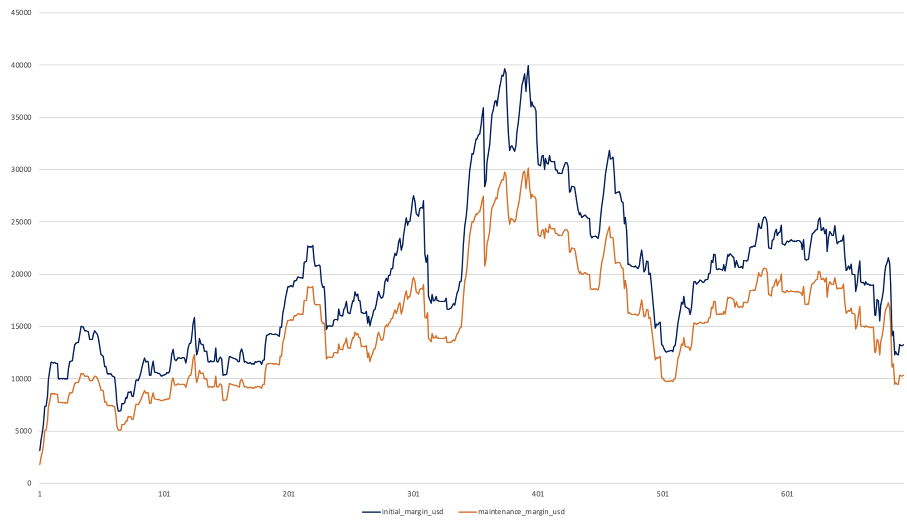

Figure 13: Book's margins from February 2nd, 2023 5pm to March 2nd, 2023 3pm (693 hours of trading)

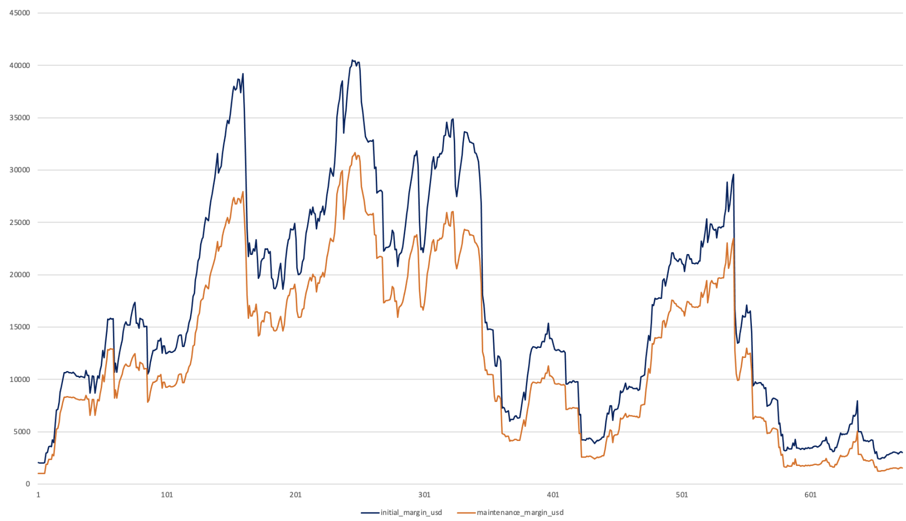

Figure 14: Book's margins from March 7th, 2023 6pm to April 4th 2023 7pm (671 hours of trading)

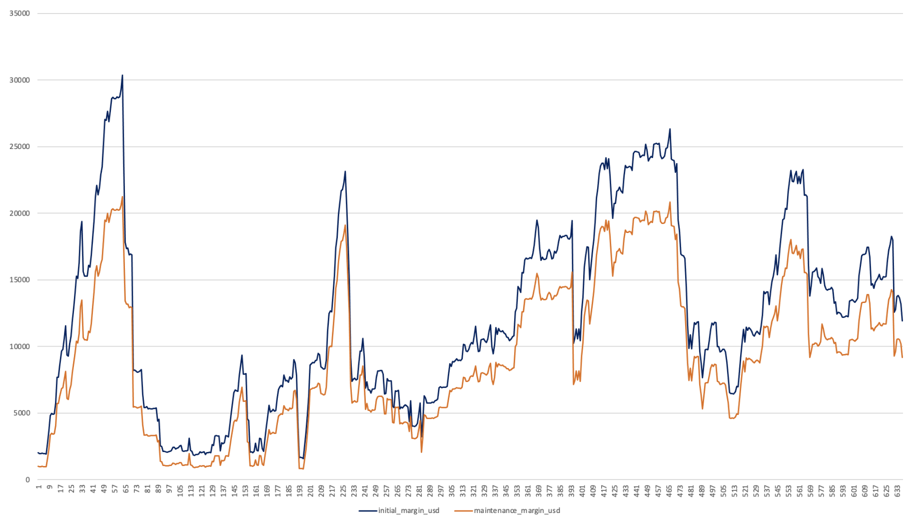

Figure 15: Book's margins rom April 9th, 2023 10pm to May 6th 2023 10am (636 hours of trading)