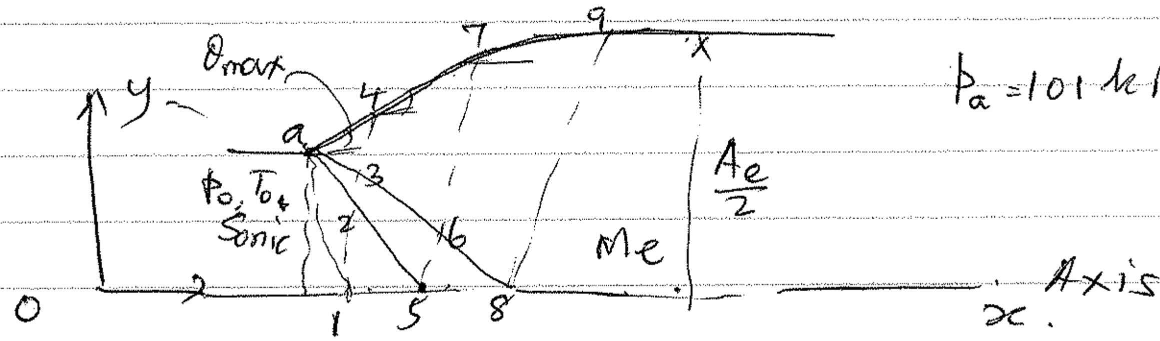
4/10/09

MAE 3062:01 Nozzle ~~Design~~ Term-Project

To design a "short" nozzle - i.e. centered expansion from Throat to exit

$p_o = 1700\ kPa$
$T_o = 3000\ K$

$p_a = 101\ kPa$



Half - nozzle.

For an ideal supersonic expansion

$$\frac{p_o}{p_e} = \frac{1700}{101} = 16.82$$

From ~~Table A.1~~ Equations 3.30

$$\frac{p_o}{p_e} = \left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\gamma/\gamma-1}$$

$$\therefore M_e^2 = \left[\left(\frac{p_o}{p_e}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\frac{2}{\gamma-1} \qquad \text{For } \gamma = 1.22$$

$$M_e^2 = \left[(16.82)^{\frac{0.22}{1.22}} - 1\right]\frac{2}{0.22} = 5.93$$

$$\therefore M_e = 2.44$$

$$\frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \quad \text{from } (3.28)$$

$$\therefore T_e = \frac{T_0}{\left(1 + \frac{\gamma-1}{2} M_e^2\right)} = \frac{3000}{\left[1 + \frac{1.22-1}{2}(2.436)^2\right]}$$

$$T_e = 1815.2\,K$$

$$\therefore a_e = \sqrt{\gamma R T}$$

$$R = 519.6 \frac{J}{kg.K}, \quad \gamma = 1.22$$

$$\therefore a_e = 1072.7 \text{ m/s.}$$

$$\therefore V_e = M_e . a_e = 2617.4 \text{ m/s}$$

$$\therefore \frac{\frac{1}{2} V_e^2}{h_0} = \text{Percentage of } \text{~~K.E~~ total enthalpy} \text{ ~~energy~~ conversion to K.E.}$$

$$h_0 = C_p T_0 = \left(\frac{\gamma R}{\gamma - 1}\right) T_0 = \left[\frac{1.22 \times 519.6}{(1.22-1)}\right] 3000$$

$$= 8{,}644{,}254.5 \frac{J}{kg}$$

$$\therefore \frac{0.5 V_e^2}{h_0} = \frac{3{,}425{,}391.4}{8{,}644{,}254.5} \approx 0.4$$

So, only 40% of the initial enthalpy is converted

Obviously this is not the best conversion efficiency

So, think about how you can improve the conversion ratio, at the expense of accepting some shock waves at high-pressures.

$$\dot{m} = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \quad \text{— (5.21)}$$

theory $$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \quad \text{— (5.20)}$$

Prandtl-Meyer angle can be calculated from

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right) - \tan^{-1}\left(\sqrt{M^2-1}\right) \quad (4.44)$$

Find $\nu(M_e)$ for $M_e = 2.44$

Create a Table of

$\frac{A_e}{A^*}$ & $\nu(M_e)$ for

$M_e =$ 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0, 2.44, ~~3.0, 4.0, 5.0, 6.0 etc.~~

K_+ characteristic $= \theta - \nu(M)$

K_- characteristics $= \theta + \nu(M)$

Choose ~~the~~ characteristics $= n_c$

grid points $N = \frac{n_c(n_c+3)}{2}$.