AutoComplete and Language Model

- Process text corpus to N-gram language model
- Out of vocabulary words
- Smoothing for previously unseen N-grams
- Language model evaluation

Sentence auto-complete

N-Grams: Overview

- What you'll be able to do!
- Create language model (LM) from text corpus to
  - Estimate probability of word sequences
  - Estimate probability of a word following a sequence of words
- Apply this concept to autocomplete a sentence with most likely suggestions

- Other Applications
  - Speech recognition, Spelling correction, Augmentative communication

N-grams and Probabilities

- N-gram
  - What are N-grams?
  - N-grams and conditional probability from corpus

An N-gram is a sequence of N words

Corpus: I am happy because I am learning

Unigrams: { I , am , happy , because , learning }

Bigrams: { I am , am happy , happy because ... }   ✗ I happy

Trigrams: { I am happy , am happy because, ... }

Unigram probability

FPT Education
FPT UNIVERSITY

- Sequence notation

Corpus: This is great … teacher drinks tea.

$w_1 \ w_2 \ w_3$ … $w_{498} \ w_{499} \ w_{500}$

$$m = 500$$

$$w_1^m = w_1 \ w_2 \ \dots \ w_m$$

$$w_1^3 = w_1 \ w_2 \ w_3$$

$$w_{m-2}^m = w_{m-2} \ w_{m-1} \ w_m$$

- Unigram probability

Corpus: I am happy because I am learning

Size of corpus m = 7

$$P(I) = \frac{2}{7} \qquad P(happy) = \frac{1}{7} \qquad P(w) = \frac{C(w)}{m}$$

Bigram probability

Corpus: I am happy because I am learning

$$P(am|I) = \frac{C(I\ am)}{C(I)} = \frac{2}{2} = 1 \qquad P(happy|I) = \frac{C(I\ happy)}{C(I)} = \frac{0}{2} = 0$$

I happy

$$P(learning|am) = \frac{C(am\ learning)}{C(am)} = \frac{1}{2}$$

Probability of a bigram:

$$P(y|x) = \frac{C(x\ y)}{\sum_w C(x\ w)} = \frac{C(x\ y)}{C(x)}$$

Trigram Probability

Corpus: I am happy because I am learning

$$P(happy|I\ am) = \frac{C(I\ am\ happy)}{C(I\ am)} = \frac{1}{2}$$

Probability of a trigram: $$P(w_3|w_1^2) = \frac{C(w_1^2\ w_3)}{C(w_1^2)}$$ $$C(w_1^2\ w_3) = C(w_1\ w_2\ w_3) = C(w_1^3)$$

N-gram probability

Probability of N-gram: $$P(w_N|w_1^{N-1}) = \frac{C(w_1^{N-1}\ w_N)}{C(w_1^{N-1})}$$

$$C(w_1^{N-1}\ w_N) = C(w_1^N)$$

Sequence Probabilities

FPT Education
FPT UNIVERSITY

- Probability of a sequence
  - Given a sentence, what is its probability?

    $P(the\ teacher\ drinks\ tea) =$?

  - Conditional probability and chain rule reminder

$$P(B|A) = \frac{P(A,B)}{P(A)} \implies P(A,B) = P(A)P(B|A)$$

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

$$P(the\ teacher\ drinks\ tea) = P(the)P(teacher|the)P(drinks|the\ teacher) \\ P(tea|the\ teacher\ drinks)$$

Sentence not in corpus

FPT Education
FPT UNIVERSITY

- Problem: Corpus almost never contains the exact sentence we're interested in or even its longer subsequences!

Input: the teacher drinks tea

$$P(the\ teacher\ drinks\ tea) = P(the)P(teacher|the)P(drinks|the\ teacher)\boxed{P(tea|the\ teacher\ drinks)}$$

$$\boxed{P(tea|the\ teacher\ drinks)} = \frac{C(the\ teacher\ drinks\ tea)}{C(the\ teacher\ drinks)}$$

Both likely 0

Approximation of sequence probability

FPT Education
FPT UNIVERSITY

the teacher drinks tea

$$P(tea|the\ teacher\ drinks) \approx P(tea|drinks)$$

$$P(teacher|the)$$
$$P(drinks|teacher)$$
$$P(tea|drinks)$$

$$P(the\ teacher\ drinks\ tea) =$$

$$P(the)P(teacher|the)P(drinks|the\ teacher)\boxed{P(tea|the\ teacher\ drinks)}$$

$$\Downarrow$$

$$P(the)P(teacher|the)P(drinks|teacher)\boxed{P(tea|drinks)}$$

Markov assumption: only last N words matter

Bigram $P(w_n|w_1^{n-1}) \approx P(w_n|w_{n-1})$

N-gram $P(w_n|w_1^{n-1}) \approx P(w_n|w_{n-N+1}^{n-1})$

Entire sentence modeled with bigram

$$P(w_1^n) \approx \prod_{i=1}^{n} P(w_i|w_{i-1})$$

$$P(w_1^n) \approx \boxed{P(w_1)}P(w_2|w_1)...P(w_n|w_{n-1})$$

Starting and Ending Sentences

FPT Education
FPT UNIVERSITY

- Start of sentence token <s>

the teacher drinks tea

$$P(the\ teacher\ drinks\ tea) \approx \boxed{P(the)}P(teacher|the)P(drinks|teacher)P(tea|drinks)$$

<s> the teacher drinks tea

$$P(<s>\ the\ teacher\ drinks\ tea) \approx \boxed{P(the|<s>)}P(teacher|the)P(drinks|teacher)P(tea|drinks)$$

- Start of sentence token <s> for N-grams
  - Trigram:

$$P(the\ teacher\ drinks\ tea) \approx \boxed{P(the)P(teacher|the)}P(drinks|the\ teacher)P(tea|teacher\ drinks)$$

the teacher drinks tea => <s> <s> the teacher drinks tea

$$P(w_1^n) \approx \boxed{P(w_1|<s>\ <s>)P(w_2|<s>\ w_1)}...P(w_n|w_{n-2}\ w_{n-1})$$

  - N-gram model: add N-1 start tokens <s>

End of sentence token </s> - motivation

$$P(y|x) = \frac{C(x\ y)}{\boxed{\sum_w C(x\ w)}} = \frac{C(x\ y)}{\boxed{C(x)}}$$

Corpus:

<s> Lyn drinks chocolate

<s> John drinks

$$\sum_w C(drinks\ w) = 1$$

$$C(drinks) = 2$$

Corpus

<s> yes no

<s> yes yes

<s> no no

Sentences of length 2:

<s> yes yes

<s> yes no

<s> no no

<s> no yes

$$P(<s>\ \text{yes yes}) =$$

$$P(\text{yes} \mid <s>) \times P(\text{yes} \mid \text{yes}) =$$

$$\frac{C(<s>\ \text{yes})}{\sum_w C(<s>\ w)} \times \frac{C(\text{yes yes})}{\sum_w C(\text{yes}\ w)} =$$

$$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

End of sentence token </s> - motivation

FPT Education
FPT UNIVERSITY

Corpus

<s> yes no

<s> yes yes

<s> no no

Sentences of length 2:

<s> yes yes

<s> yes no

<s> no no

<s> no yes

$$P(<s> \text{ yes yes}) = \frac{1}{3}$$

$$P(<s> \text{ yes no}) = \frac{1}{3}$$

$$P(<s> \text{ no no}) = \frac{1}{3}$$

$$P(<s> \text{ no yes}) = 0$$

$$\sum_{2 \text{ word}} P(\cdots) = 1$$

End of sentence token </s> - motivation

FPT Education
FPT UNIVERSITY

Corpus
<s> yes no
<s> yes yes
<s> no no

Sentences of length 3:

<s> yes yes yes
<s> yes yes no
...
<s> no no no

$$P(< s > \text{ yes yes yes}) = \cdots$$
$$P(< s > \text{ yes yes no}) = \ldots$$
$$\cdots = \ldots$$
$$P(< s > \text{ no no no}) = \ldots$$

$$\sum_{3 \text{ word}} P(\cdots) = 1$$

Corpus
<s> yes no
<s> yes yes
<s> no no

$$\sum_{2 \text{ word}} P(\cdots) + \sum_{3 \text{ word}} P(\cdots) + \ldots = 1$$

End of sentence token </s> - solution

- Bigram

  <s> the teacher drinks tea => <s> the teacher drinks tea </s>

$$P(the|<s>)P(teacher|the)P(drinks|teacher)P(tea|drinks)\boxed{P(</s>|tea)}$$

Corpus:
<s> Lyn drinks chocolate </s>
<s> John drinks </s>

$$\sum_{w} C(drinks\ w) = 2$$

$$C(drinks) = 2$$

- End of sentence token </s> for N-grams
  - N-gram => just one </s>
  - E.g. Trigram:

    the teacher drinks tea => <s> <s> the teacher drinks tea </s>

Example - bigram

Corpus

<s> Lyn drinks chocolate </s>

<s> John drinks tea </s>

<s> Lyn eats chocolate </s>

$$P(sentence) = \boxed{\frac{2}{3}} * \boxed{\frac{1}{2}} * \boxed{\frac{1}{2}} * \boxed{\frac{2}{2}} = \frac{1}{6}$$

$$P(John|<s>) = \frac{1}{3}$$

$$P(</s>|tea) = \frac{1}{1}$$

$$P(chocolate|eats) = \frac{1}{2}$$

$$P(Lyn|<s>) = ? = \frac{2}{3}$$

The N-gram Language Model

- Count matrix
- Probability matrix
- Language model
- Log probability to avoid underflow
- Generative language model

Count matrix

$$P(w_n|w_{n-N+1}^{n-1}) = \frac{\boxed{C(w_{n-N+1}^{n-1}, w_n)}}{C(w_{n-N+1}^{n-1})}$$

FPT Education
FPT UNIVERSITY

- Rows: unique corpus (N-1)-grams
- Columns: unique corpus words

Corpus: <s>I study I learn</s>

- Bigram

count matrix

"study I" bigram

	<s>	</s>	I	study	learn
<s>	0	0	1	0	0
</s>	0	0	0	0	0
I	0	0	0	1	1
study	0	0	1	0	0
learn	0	1	0	0	0

Probability matrix

FPT Education
FPT UNIVERSITY

$$P(w_n|w_{n-N+1}^{n-1}) = \frac{C(w_{n-N+1}^{n-1}, w_n)}{\boxed{C(w_{n-N+1}^{n-1})}}$$

- Divide each cell by its row sum

$$sum(row) = \sum_{w \in V} C(w_{n-N+1}^{n-1}, w) = C(w_{n-N+1}^{n-1})$$

Corpus: <s>I study I learn</s>

Count matrix (bigram)

	<s>	</s>	I	study	learn	sum
<s>	0	0	1	0	0	1
</s>	0	0	0	0	0	0
I	0	0	0	1	1	2
study	0	0	1	0	0	1
learn	0	1	0	0	0	1

Probability matrix

	<s>	</s>	I	study	learn
<s>	0	0	1	0	0
</s>	0	0	0	0	0
I	0	0	0	0.5	0.5
study	0	0	1	0	0
learn	0	1	0	0	0

Language model

- probability matrix => language model
  - Sentence probability
  - Next word prediction

	<s>	</s>	I	study	learn
<s>	0	0	1	0	0
</s>	0	0	0	0	0
I	0	0	0	0.5	0.5
study	0	0	1	0	0
learn	0	1	0	0	0

Sentence probability:

<s> I learn </s>

$$P(sentence) =$$
$$P(I|<s>)P(learn|I)P(</s>|learn) =$$
$$1 \times 0.5 \times 1 =$$
$$0.5$$

Log probability

$$P(w_1^n) \approx \prod_{i=1}^{n} P(w_i|w_{i-1})$$

- All probabilities in calculation <=1 and multiplying them brings risk of underflow
- Logarithm properties reminder $$log(a * b) = log(a) + log(b)$$
- Use log of the probabilities in Probability matrix and calculations

$$log(P(w_1^n)) \approx \sum_{i=1}^{n} log(P(w_i|w_{i-1}))$$

- Converts back from log

$$P(w_1^n) = exp(log(P(w_1^n))$$

Generative Language model

FPT Education
FPT UNIVERSITY

Corpus:

<s> Lyn drinks chocolate </s>

<s> John drinks tea </s>

<s> Lyn eats chocolate </s>

1. (<s>, Lyn) or (<s>, John)?
2. (Lyn,eats) or (Lyn,drinks) ?
3. (drinks,tea) or (drinks,chocolate)?
4. (tea,</s>) - always

- Algorithm:
  1. Choose sentence start
  2. Choose next bigram starting with previous word
  3. Continue until </s> is picked

Language Model Evaluation

FPT Education
FPT UNIVERSITY

- Train/Validation/Test split
- Perplexity

- Split corpus to Train/Validation/Test

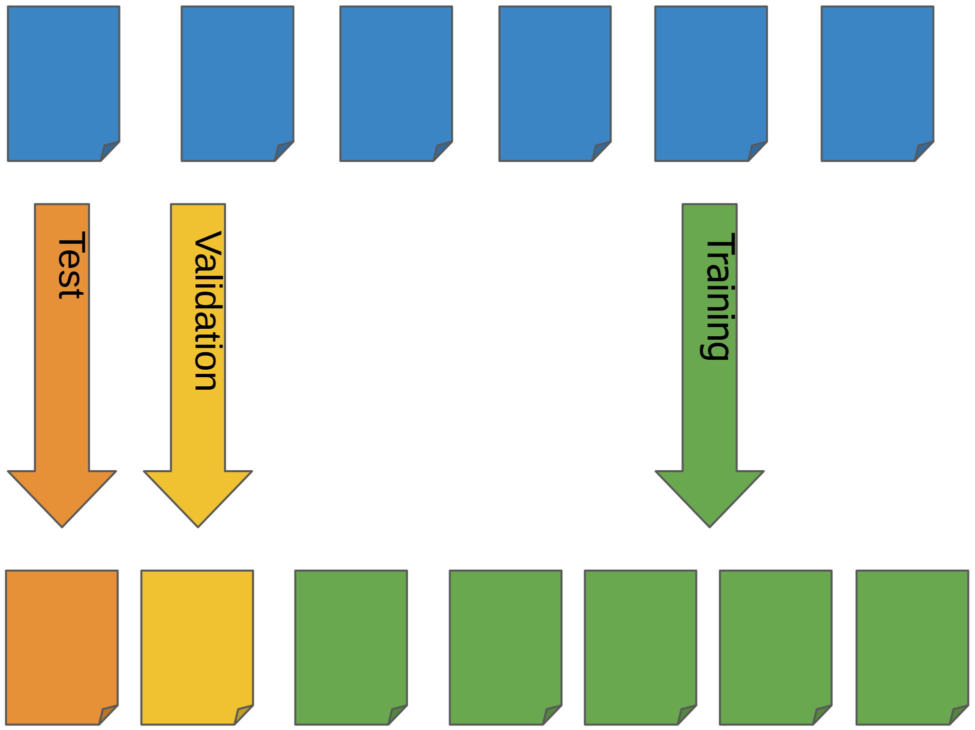
- Evaluate on Training dataset

- For smaller corpora
  - 80% Train
  - 10% Validation
  - 10% Test

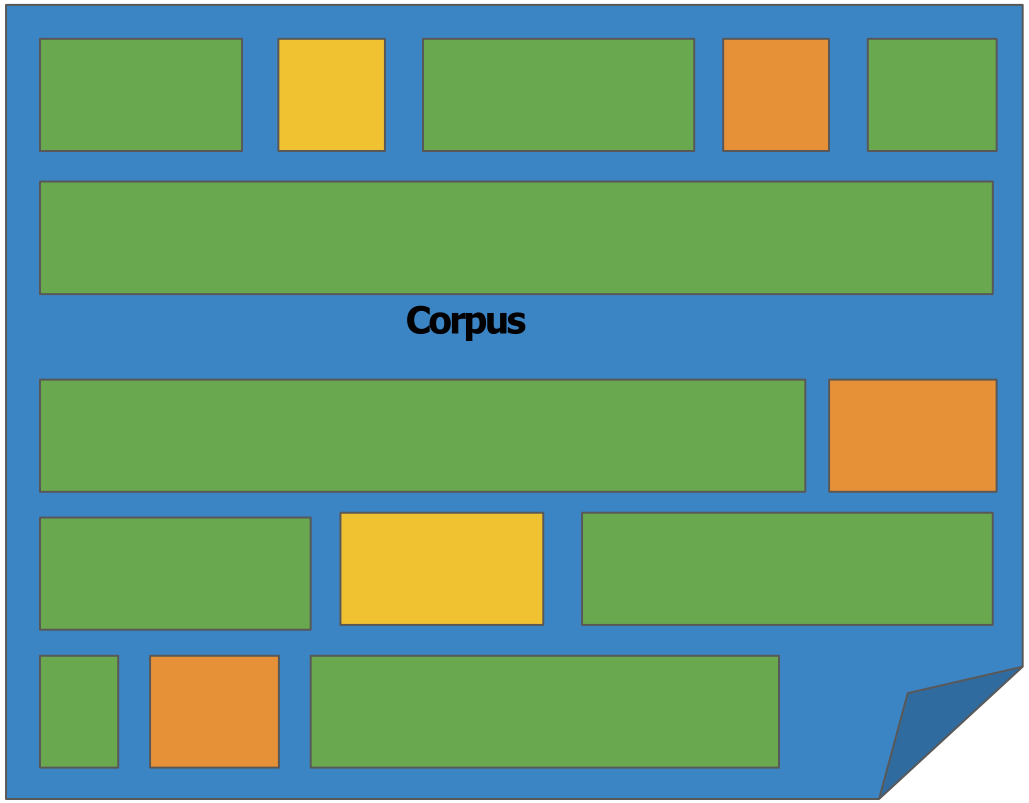
- For large corpora (typical for text)
  - 98% Train
  - 1% Validation
  - 1% Test

Test data - split method

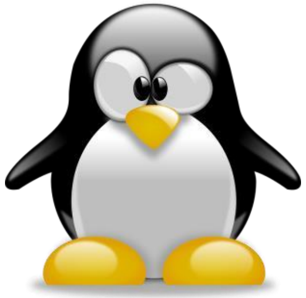
- Continuous text



- Random short sequences



Perplexity



FPT Education
FPT UNIVERSITY

$$PP(W) = P(s_1, s_2, ..., s_m)^{-\frac{1}{m}}$$

s_i

W → test set containing **m** sentences **s** * i-th sentence in the test set, each ending with </s>
m → number of all words in entire test set W including </s> but not including <s>

E.g. m=100

$$P(W) = 0.9 => PP(W) = 0.9^{-\frac{1}{100}} = 1.00105416$$

$$P(W) = 10^{-250} => PP(W) = (10^{-250})^{-\frac{1}{100}} \approx 316$$

- Smaller perplexity = better model
- Character level models PP < word-based models PP

Perplexity for bigram models

FPT Education
FPT UNIVERSITY

$$PP(W) = \sqrt[m]{\prod_{i=1}^{m} \prod_{j=1}^{|s_i|} \frac{1}{P(w_j^{(i)}|w_{j-1}^{(i)})}}$$

$w_j^{(i)}$ * j-th word in i-th sentence

concatenate all sentences in W

$$PP(W) = \sqrt[m]{\prod_{i=1}^{m} \frac{1}{P(w_i|w_{i-1})}}$$

w_i * i-th word in test set

Log perplexity

$$PP(W) = \sqrt[m]{\prod_{i=1}^{m} \frac{1}{P(w_i|w_{i-1})}}$$

$$\Downarrow$$

$$logPP(W) = -\frac{1}{m} \sum_{i=1}^{m} log_2(P(w_i|w_{i-1}))$$

Examples

Training 38 million words, test 1.5 million words, WSJ corpus
Perplexity Unigram: 962 Bigram: 170 Trigram: 109

Unigram
Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
Bigram
Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
Trigram
They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

[Figure from *Speech and Language Processing* by Dan Jurafsky et. al]

Out of Vocabulary Words

- Closed vs. Open vocabularies
- Unknown word = Out of vocabulary word (OOV)
- special tag <UNK> in corpus and in input

- Using <UNK> in corpus
  - Create vocabulary V
  - Replace any word in corpus and not in V by <UNK>
  - Count the probabilities with <UNK> as with any other word

Example

Corpus

<s> Lyn drinks chocolate </s>

<s> John drinks tea </s>

<s> Lyn eats chocolate </s>

Corpus

<s> Lyn drinks chocolate </s>

<s> <UNK> drinks <UNK> </s>

<s> Lyn <UNK> chocolate </s>

Min frequency f=2

Vocabulary

Lyn, drinks, chocolate

Input query

<s>Adam drinks chocolate</s>

<s><UNK> drinks chocolate</s>

How to create vocabulary V

- Criteria:
  - Min word frequency f
  - Max |V|, include words by frequency
- Use <UNK> sparingly
- Perplexity - only compare LMs with the same V

Smoothing Outline

- Missing N-grams in corpus
  - Problem: N-grams made of known words still might be missing in the training corpus "John" , "eats" in corpus ~~"John eats"~~
  - Their counts cannot be used for probability estimation

$$P(w_n|w_{n-N+1}^{n-1}) = \frac{C(w_{n-N+1}^{n-1}, w_n)}{C(w_{n-N+1}^{n-1})}$$

- Can be 0

Smoothing

- Advanced methods:
  Kneser-Ney smoothing
  Good-Turing smoothing

- Add-one smoothing (Laplacian smoothing)

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}, w_n) + 1}{\sum_{\boxed{w \in V}} (C(w_{n-1}, w) + \boxed{1})} = \frac{C(w_{n-1}, w_n) + 1}{C(w_{n-1}) + \boxed{V}}$$

- Add-k smoothing

$$P(w_n|w_{n-1}) = \frac{C(w_{n-1}, w_n) + k}{\sum_{\boxed{w \in V}} (C(w_{n-1}, w) + \boxed{k})} = \frac{C(w_{n-1}, w_n) + k}{C(w_{n-1}) + \boxed{k * V}}$$

Backoff

- If N-gram missing => use (N-1)-gram, …
  - Probability discounting e.g. Katz backoff
  - "Stupid" backoff

Corpus

<s> Lyn drinks chocolate </s>

<s> John drinks tea </s>

<s> Lyn eats chocolate </s>

$$P(chocolate|John\ drinks) = ?$$

$$\Downarrow$$

$$0.4 \times P(chocolate|drinks)$$

Interpolation

$$\hat{P}(chocolate|John\ drinks) = 0.7 \times P(chocolate|John\ drinks)$$
$$+0.2 \times P(chocolate|drinks) + 0.1 \times P(chocolate)$$

$$\hat{P}(w_n|w_{n-2}\ w_{n-1}) = \lambda_1 \times P(w_n|w_{n-2}\ w_{n-1})$$
$$+\lambda_2 \times P(w_n|w_{n-1}) + \lambda_3 \times P(w_n)$$

$$\sum_i \lambda_i = 1$$

Summary

FPT Education
FPT UNIVERSITY

- N-Grams and probabilities
- Approximate sentence probability from N-Grams
- Build language model from corpus
- Fix missing information
  - Out of vocabulary words with <UNK>
  - Missing N-Gram in corpus with smoothing, backoff and interpolation
- Evaluate language model with perplexity