Vector Space Models

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- Outline
  - Vector space models
  - Advantages
  - Applications

- Why learn vector space models?

Where are you heading?

Where are you from?

Different meaning

What is your age?

How old are you?

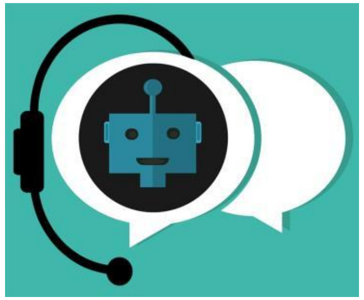
Same Meaning

Vector space models applications

- You eat *cereal* from a *bowl*
- You *buy* something and someone else *sells* it

Information Extraction

Machine Translation



Chatbots

- With vectors based models, you will be able to capture this and many other types of relationships among different sets of words.
- Vector space models are used in information extraction to answer questions, in the style of who, what, where, how and etcetera

Fundamental concept

"You shall know a word by the company it keeps"

Firth, 1957

(Firth, J. R. 1957:11)

- Represent words and documents as vectors
- Representation that captures relative meaning

Word by Word and Word by Doc.

- Co-occurrence ⟶ Vector representation
- Relationships between words/documents

- Word by Word Design

Number of times they *occur together within a certain distance k*

I like simple data

I prefer simple raw data

k=2

	simple	raw	like	I
data	2	1	1	0

n

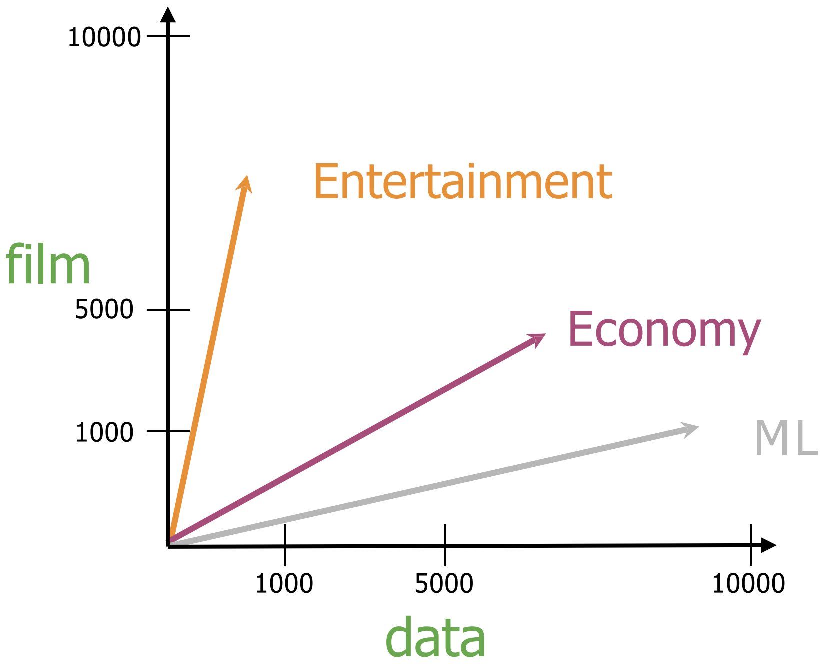
Word by Document Design

Number of times a word *occurs within a certain category*

Corpus

	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

Vector Space

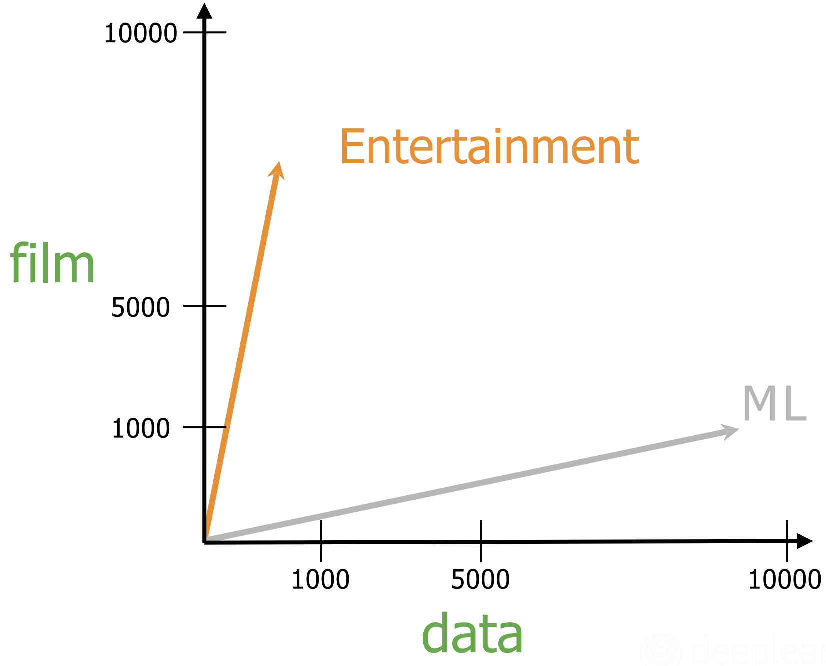


	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Measures of "similarity:"
Angle
Distance

- W/W and W/D, *counts* of occurrence
- Vector Spaces ⟶ Similarity between words/documents

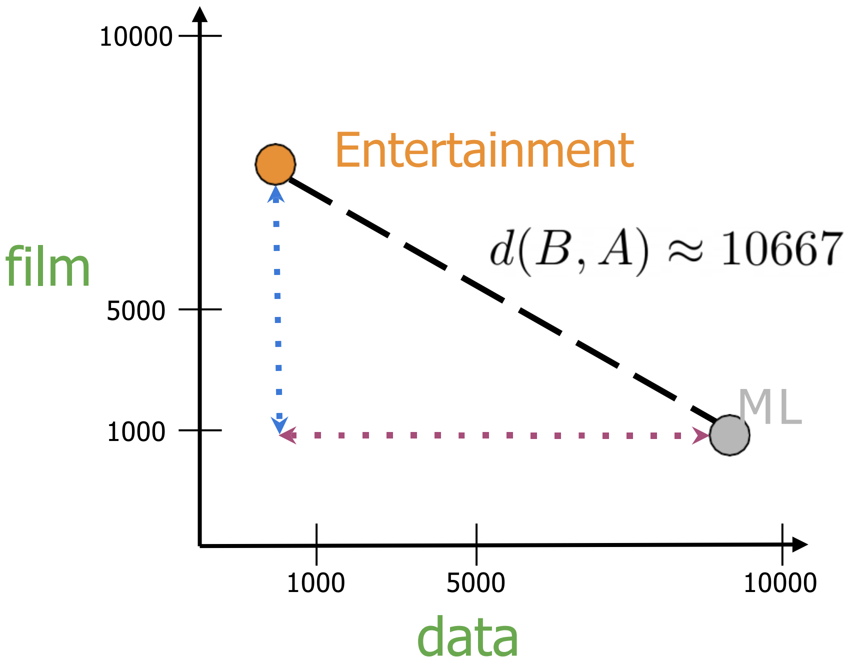
Euclidean distance



Corpus **A**: (500,7000)

Corpus **B**: (9320,1000)

Euclidean distance



Corpus **A**: (500,7000)

Corpus **B**: (9320,1000)

$$d(B, A) = \sqrt{(B_1 - A_1)^2 + (B_2 - A_2)^2}$$

$$c^2 = a^2 + b^2$$

$$d(B, A) = \sqrt{(-8820)^2 + (6000)^2}$$

Euclidean distance for n-dimensional vectors

	data	boba ($\vec{w}$)	ice-cream ($\vec{v}$)
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1-0)^2 + (6-4)^2 + (8-6)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

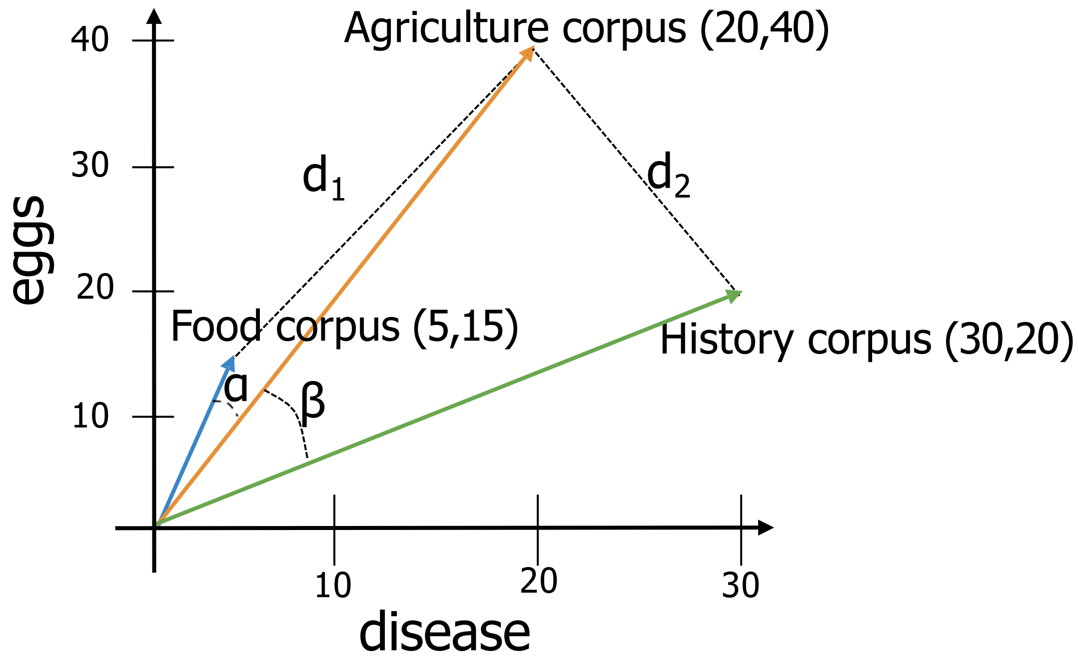
$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^{n} (v_i - w_i)^2} \longrightarrow \text{Norm of } (\vec{v} - \vec{w})$$

Euclidean distance in Python

```python
# Create numpy vectors v and w
v = np.array([1, 6, 8])
w = np.array([0, 4, 6])
# Calculate the Euclidean distance d
d = np.linalg.norm(v-w)
# Print the result
print("The Euclidean distance between v and w is: ", d)
```

```
The Euclidean distance between v and w is: 3
```

Euclidean distance vs Cosine similarity



Euclidean distance: $d_2 < d_1$

Angles comparison: $\beta > \alpha$

The cosine of the angle between the vectors

Cosine Similarity

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- How to get the cosine of the angle between two vectors
- Relation of this metric to similarity

- Previous definitions

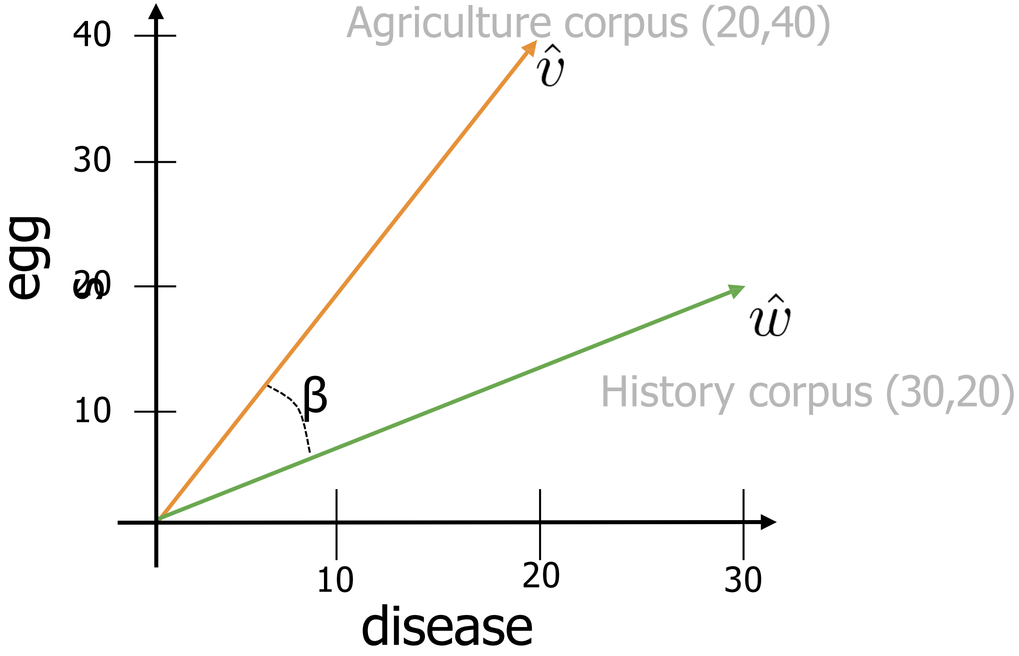
Vector norm

$$\|\vec{v}\| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

Dot product

$$\vec{v}.\vec{w} = \sum_{i=1}^{n} v_i.w_i$$

Cosine Similarity



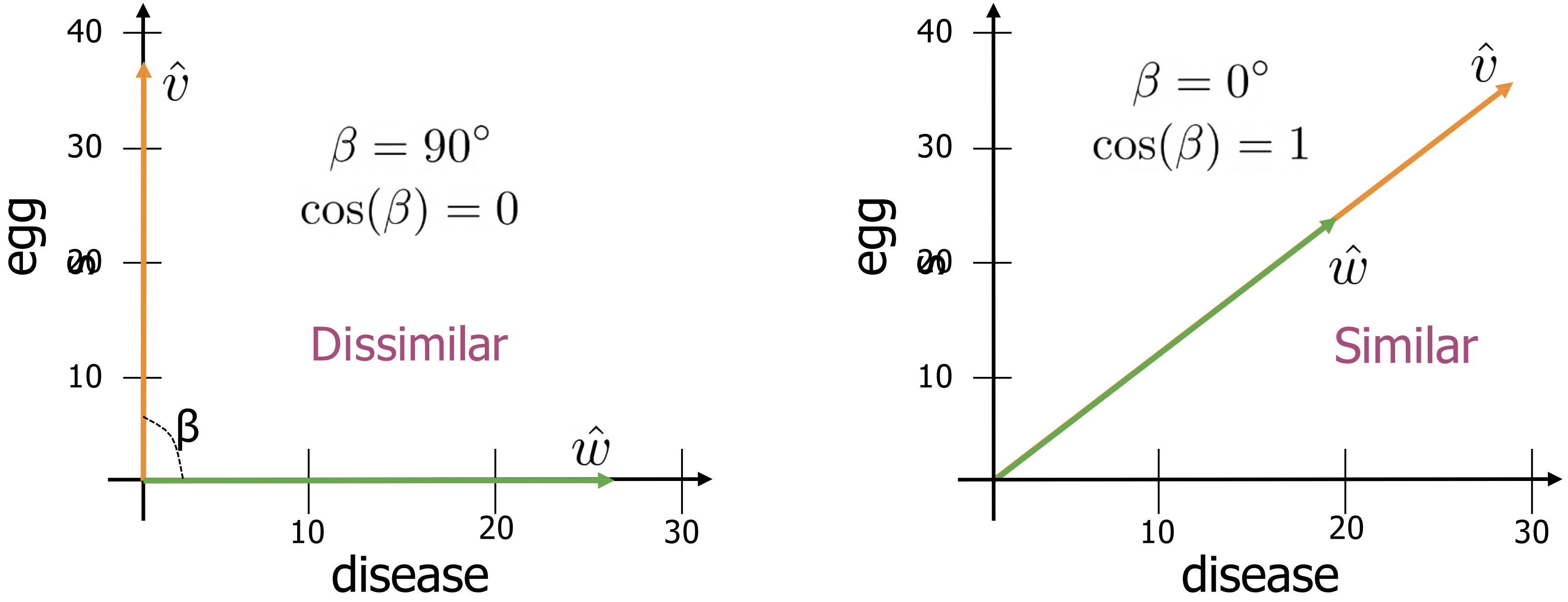
$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}}$$

$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}}$$

$$= 0.87$$

Cosine Similarity



Manipulating Words in Vector Spaces

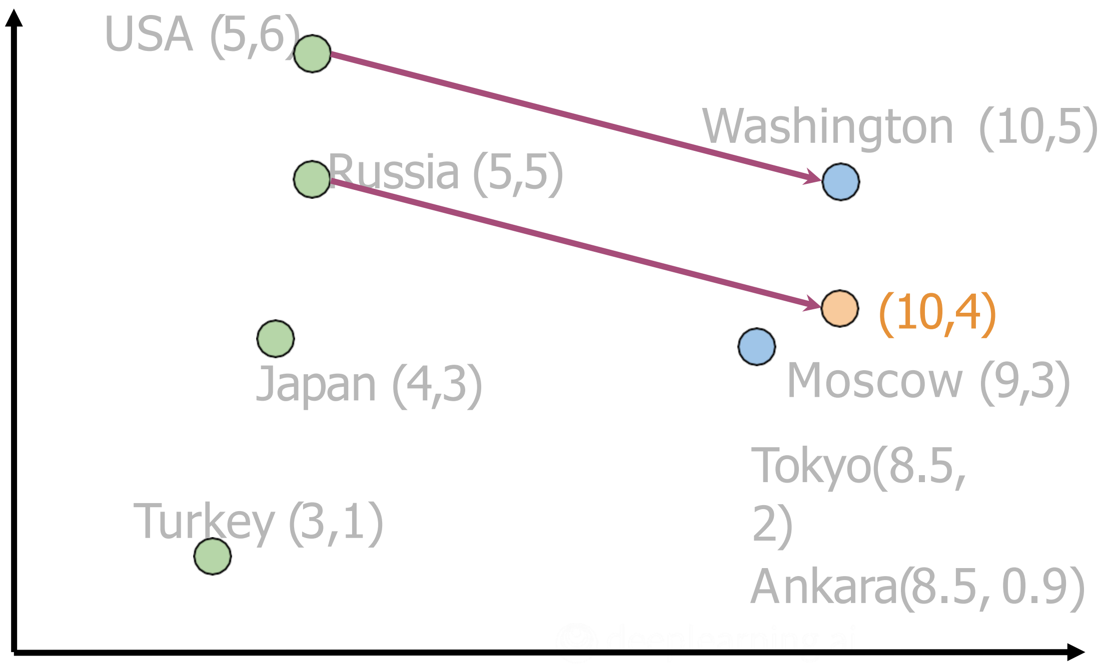
USA

Washington DC

Russia

?

Manipulating word vectors



Washington - USA = $\begin{bmatrix} 5 & -1 \end{bmatrix}$

Russia + $\begin{bmatrix} 5 & -1 \end{bmatrix} = \begin{bmatrix} 10 & 4 \end{bmatrix}$

Moscow

[Mikolov et al, 2013, Distributed Representations of Words and Phrases and their Compositionality]

Visualization and PCA

- Visualization of word vectors

$$\overbrace{\hphantom{0.20 \quad \ldots \quad 0.10}}^{d>2}$$

oil	0.20	…	0.10
gas	2.10	…	3.40
city	9.30	…	52.1
town	6.20	…	34.3

oil & gas

town & city

How can you visualize if your representation captures these relationships?

Visualization of word vectors

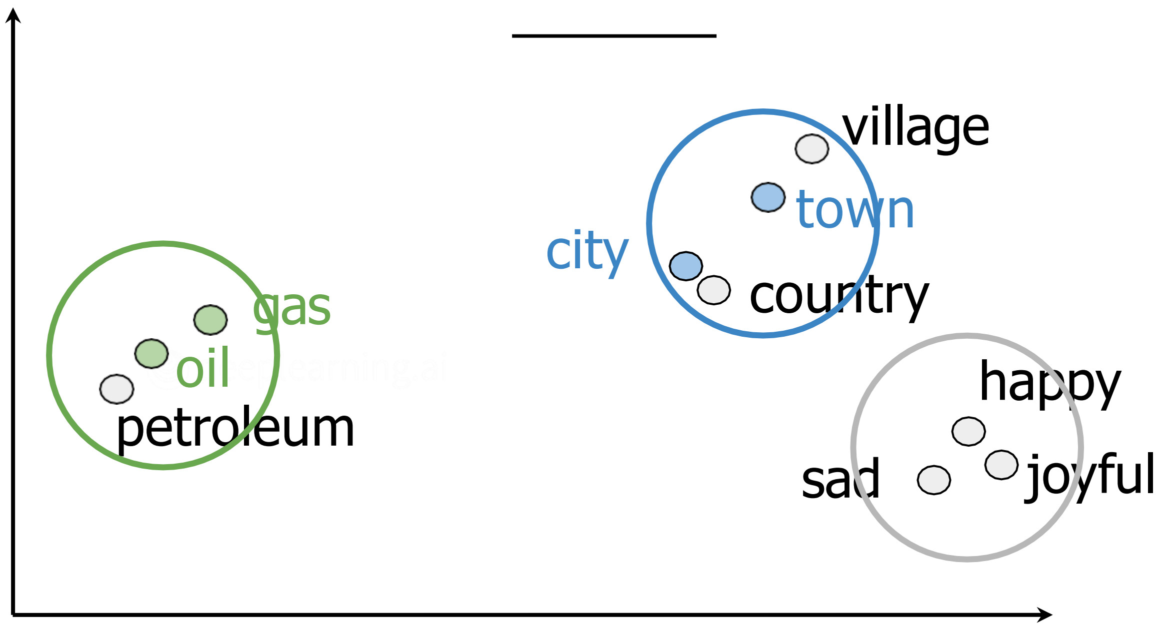
$\overbrace{\qquad\qquad\qquad}^{d > 2}$

oil	0.20	...	0.10
gas	2.10	...	3.40
city	9.30	...	52.1
town	6.20	...	34.3

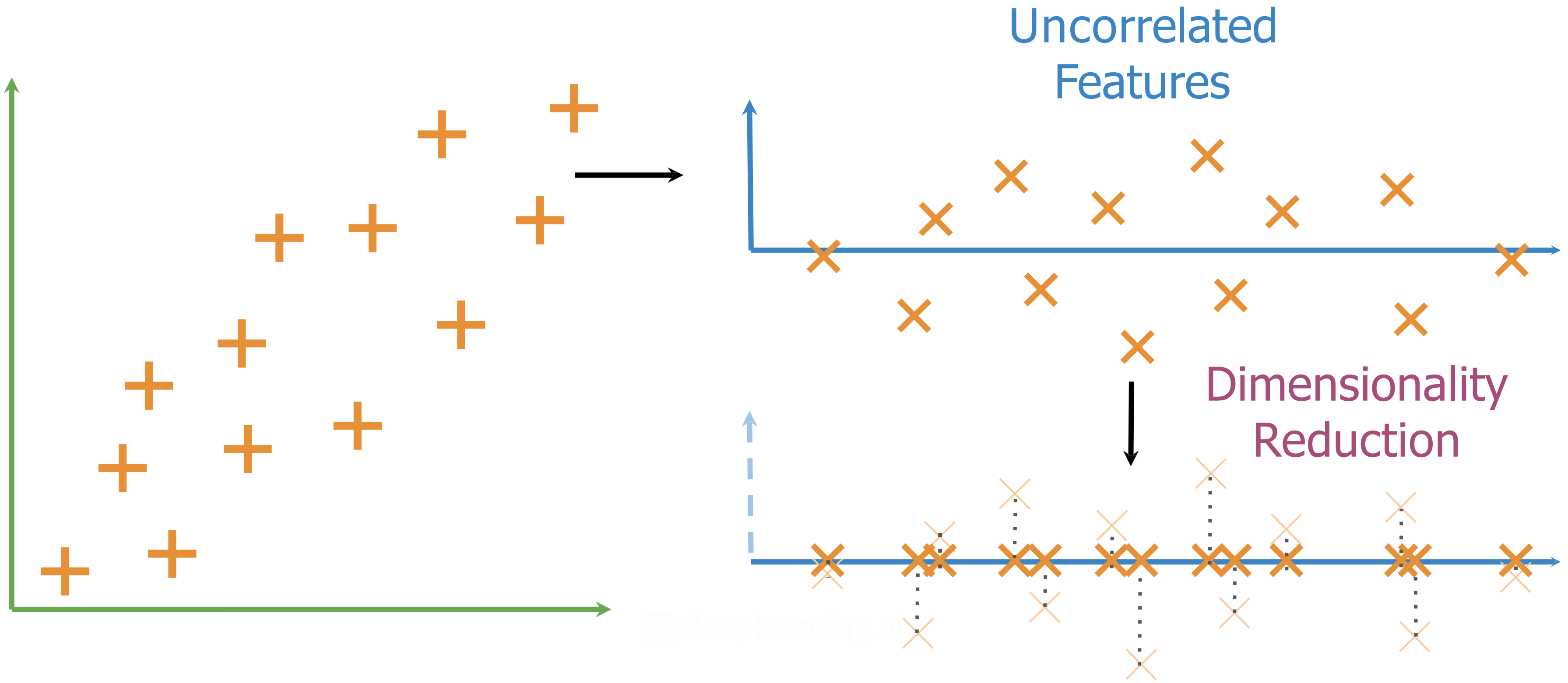
$\xrightarrow{\text{PCA}}$

$\overbrace{\qquad\qquad}^{d = 2}$

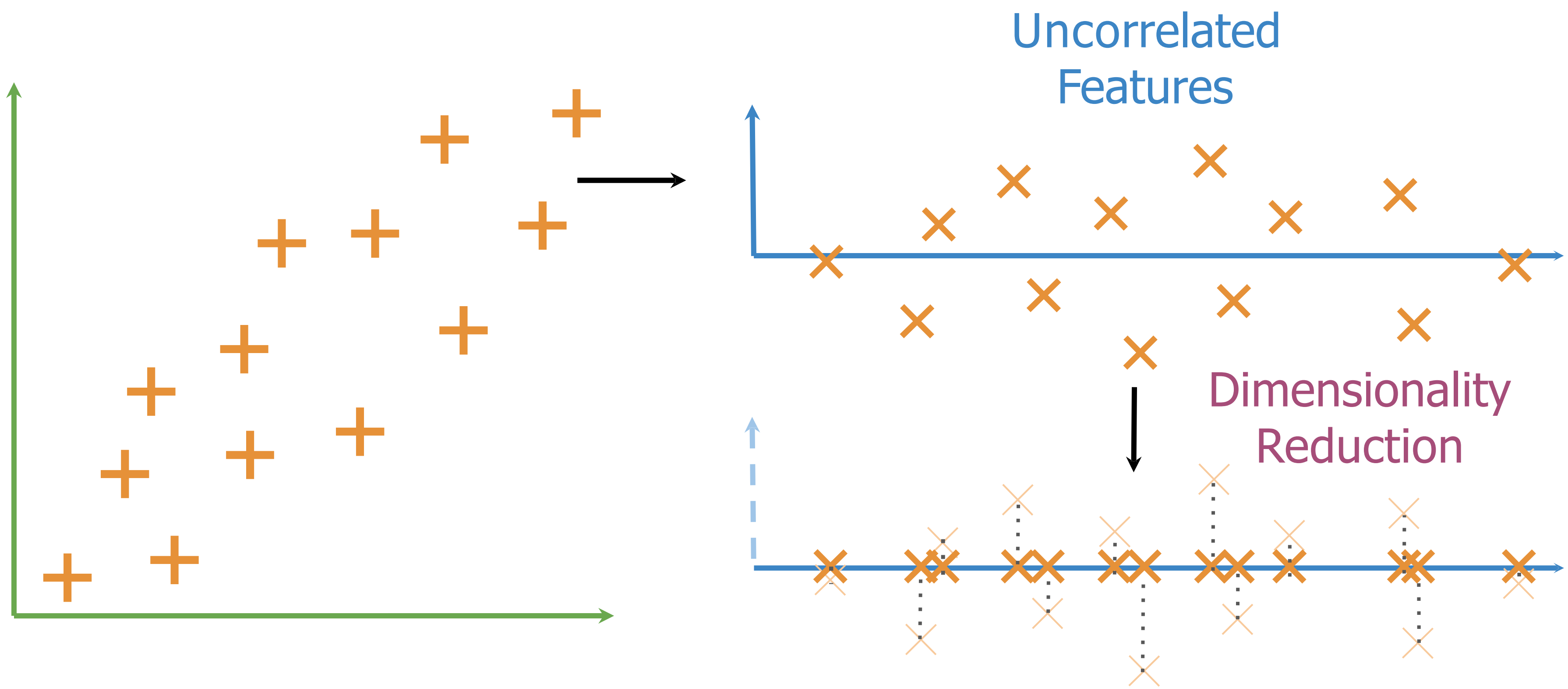
oil	2.30	21.2
gas	1.56	19.3
city	13.4	34.1
town	15.6	29.8



Principal Component Analysis

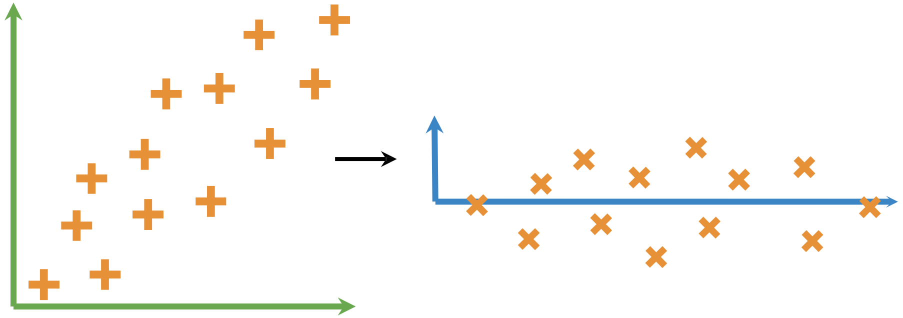


PCA Algorithm



PCA algorithm

- Eigenvector: Uncorrelated features for your data
- Eigenvalue: the amount of information retained by each feature



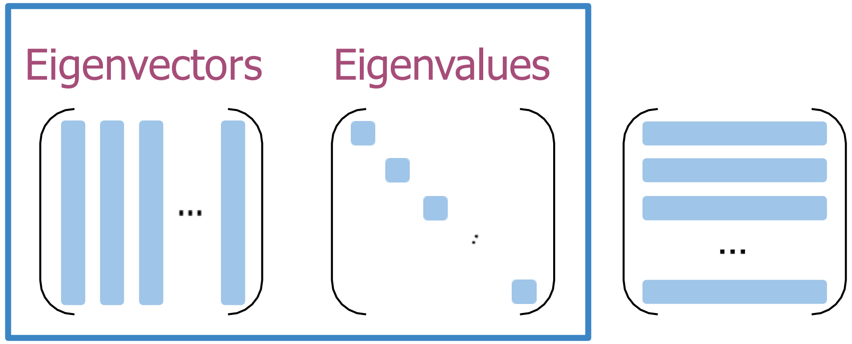
Mean Normalize Data $x_i = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}}$

↓

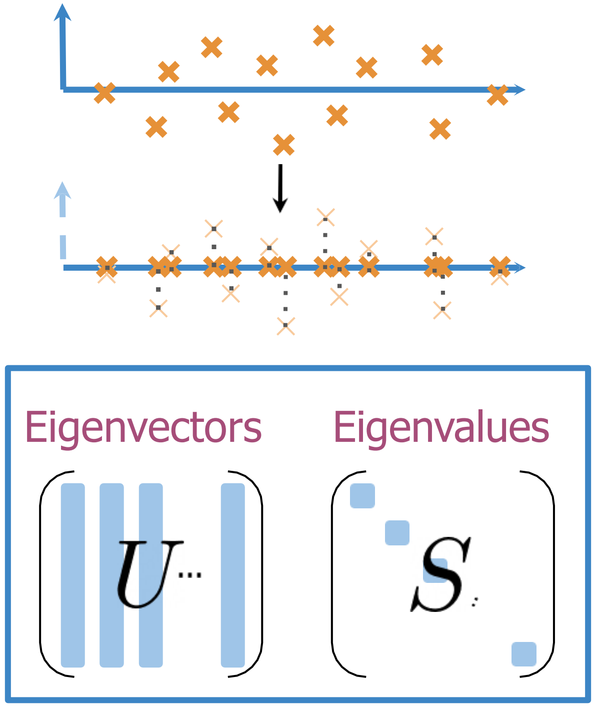
Get Covariance Matrix Σ

↓

Perform SVD $\mathrm{SVD}(\Sigma)$



PCA algorithm



Dot Product to Project Data $X' = XU[:, 0:2]$

Percentage of Retained Variance $\frac{\sum_{i=0}^{1} S_{ii}}{\sum_{j=0}^{d} S_{jj}}$

- Eigenvectors give the direction of uncorrelated features
- Eigenvalues are the variance of the new features
- Dot product gives the projection on uncorrelated features