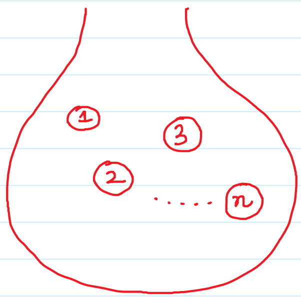
[Combinatorics]

Bag with $\underbrace{n \text{ things}}_{}$ in it
(n different things)



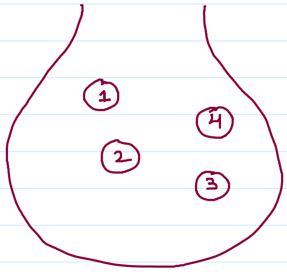
We select k of there things.

	We don't Replace	We Replace
Order matters	$\frac{n!}{(n-k)!}$	n^k
Order doesn't matter.	$\frac{n!}{(n-k)!\,k!} = {}^{n}C_{k}$	$\frac{(n-1+k)!}{(k)!\,(n-1)!} = {}^{n-1+k}C_{k}$

Consider case 1

$\Rightarrow$ we don't replace & order matters.

Suppose we have 4 things $(n=4)$ in the bag & we want to choose 3 things i.e. $(K=3)$



$n=4$, $k=3$

2 1 3
3 1 2
1 2 3
⋮

∵ order matters
∴ all will be considered different choice.

$$n \times (n-1) \ldots\ldots\ldots \times (n-(k-1)) = \boxed{\frac{n!}{(n-k)!}}$$

why not till $(n-k)$?

<u>Note</u>:- we have to select k things among n things

∴ 1st choice has → n possibilities

2nd choice has → $(n-1)$ possibilities ....

k^{th} choice has → $(n-(k-1))$ possibilities

Because total we have to select k things

∴ this corresponds to $(k-1)$ selections.

Consider <u>Case 2</u>

⇒ we don't replace, Order doesn't matter.

∵ order doesn't matter ∴ k balls that i have picked

can be arranged in $k!$ ways.

Suppose :- $k=3$

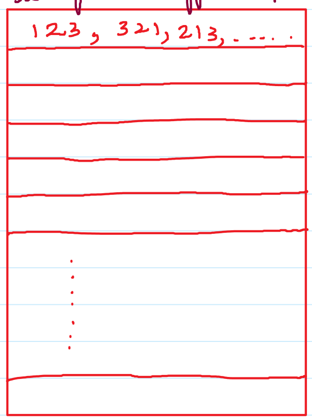
2 1 3

3 possibilities for 1st place × 2 possibilities for 2nd place × 1 possibility for 3rd place $= 3 \times 2 \times 1 = \boxed{3!}$

Set of all different possible ways possible.

cases → $\boxed{123,\ 321,\ 213,\ \ldots}$

Set of all different possible ways possible.



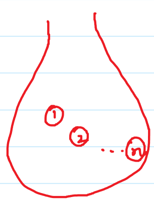
Equivalence classes

Every Equivalent class consist of $k!$ items

⊛ There exist a total of $\frac{n!}{(n-k)!\,k!}$ Equivalent classes.

$$\therefore \quad \frac{n!}{(n-k)!\,k!} = \binom{n}{k} = {}^{n}C_{k}$$

Case ③ Order matters & we Replace.



$$\underbrace{n \times n \times \cdots \times n}_{k \text{ times}} = n^k$$

Consider case ③ } (a ⊛ little interesting to understand)

$\Rightarrow$ order doesn't matter & we replace.

(n things pick k things)

We want to know how many ways are of doing this.

Original Question.

⊛ There are n balls & we want to select k balls with replacing.

Problem conversion :- Suppose There are n baskets each with respective number

Problem conversion :- Suppose There are n baskets each with respective number written on it.
There also exist k (unnumbered) tennis balls lying on the floor.

Now for each ball we pick from the bag before replacing it in the bag we put one tennis ball lying on the ground in the numbered basket same as the no. on the ball picked from the basket.
Now we replace the ball.

Baskets: 1 2 3 4 . . . . – – . . . . n

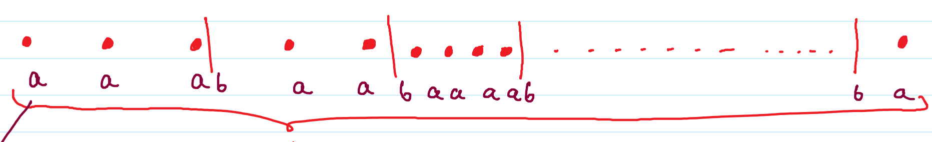
Now we need to answer the problem how many different ways are there to put k balls in n baskets.

Use case of this problem in "MEDECINE" (in pathology)

let Baskets = cells
tennis ball = viruses

How many different ways are there of putting viruses in cells.
So that we can know what is the probability of a cell having a certain no of viruses within it is.

⊛ k tennis balls & we have to put it in n baskets.



k tennis balls

Suppose we have got $\underbrace{(n-1)}$ barriers

there $n-1$ barriers will split the k balls there $n-1$ barriers will split the k balls into n chambers.

∴ The problem now converts to how many ways are there of putting $(n-1)$ barries amongst k balls.

k balls $\quad n-1$ barriers

∴ we want to know how many different ways exist of writing this string with exactly k a's & $n-1$ b's.

Notes: a's are not swappable amongst themselves ∵ it does not lead to a new string.

Similarly b's are not swappable

$$= \boxed{\frac{(k+n-1)!}{(n-1)!\ k!}}$$

Example:- $\overset{1}{a}\overset{2}{a}\overset{3}{a}\overset{4}{b}\overset{5}{a}\overset{6}{a}\overset{7}{b}$

2 b's

5 a's

∴ total 7 things ∴ 7! total possible ways of placing if each of them was a seperate thing

how many equivalent things that are counted as distinct in 7! are there.

$5! \times 2!$ → things in each equivalent classes

∴ a total of $\frac{7!}{5! \times 2!}$ Equivalent classes.

total 7! strings in the set

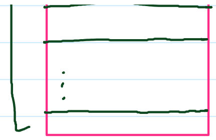
→ $5! \times 2!$ is each Equivalent classes

$\left\{ \frac{7!}{5!\,2!} \right.$

no. of equivalent

$5!\,2!$

no. of equivalent classes



$\Downarrow$

Total no. of distinct strings of k, a's & $n-1$, b's