[Properties of Probability Space]

$(\Omega, \mathcal{F}, P)$ = Probability Space.

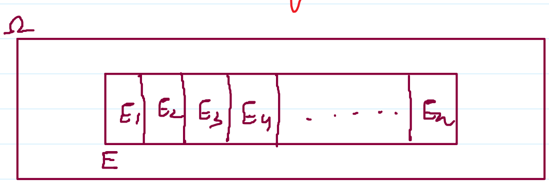
Axioms:-

$P : \mathcal{F} \longrightarrow [0,1]$

1. $P(\phi) = 0 \qquad P(\Omega) = 1$
2. $E = \bigsqcup_{n=1}^{\infty} E_n$

$$P(E) = \sum_{n=1}^{\infty} P(E_n)$$

Countable additivity.



$\mathcal{F}$ — σ algebra of subsets of Ω

i) $\phi, \Omega \in \mathcal{F}$

ii) $E \in \mathcal{F} \quad E^c \in \mathcal{F}$

(if E is an event then E^c is also an event.)
i.e.
if E is an event then the Event: that E does not occur is also an event.

iii) $E_n \in \mathcal{F}$

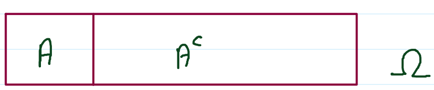
$\bigcup_{n=1}^{\infty} E_n \in \mathcal{F}$

if you attach a countable no of events together then you get an event.

Property 1 :-

$$P(A^c) = 1 - P(A)$$

$$A \cup A^c = \Omega$$



$$P(A \cup A^c) = P(\Omega) = 1$$

$$\underbrace{P(A) + P(A^c)}_{\text{from Axiom (2)}}$$

$\therefore \quad P(A) + P(A^c) = 1$

$$\boxed{P(A^c) = 1 - P(A)}$$

Property 2 :- Monotonicity.

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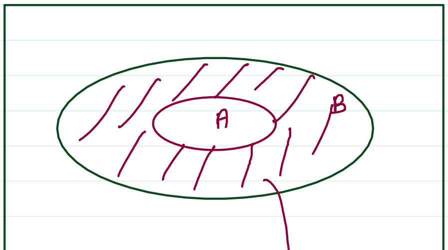
Event: I live in India

If $A, B \in \mathcal{F}$ $\quad B \supseteq A \implies P(B) \geq P(A)$

Event: I live on Earth

$$B = A \cup (B \cap A^c)$$



$B \cap A^c$

Note: A & $B \cap A^c$ are disjoint sets.

$$\therefore \; P(B) = P(A \cup (B \cap A^c))$$

$$= P(A) + P(B \cap A^c) \quad \text{using Axiom 2}$$

Now

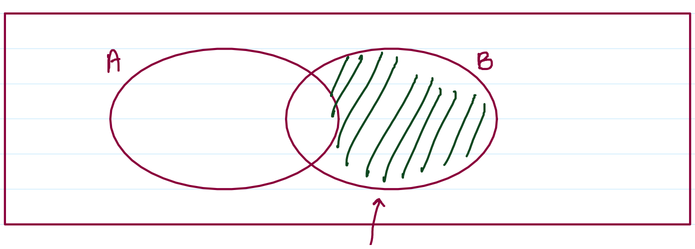
$P(B \cap A^c) \geq 0 \quad \because P : \mathcal{F} : [0,1]$

$$\underbrace{P(A) + P(B \cap A^c)}_{\boxed{P(B) \geq P(A)}} \geq P(A)$$

Property 3

3 holds when A & B are not disjoint otherwise if disjoint use Axiom 2.

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



$$P(A \cup B) = P(A \cup (B \cap A^c))$$

disjoint sets ∴ using Axiom (2)

$$P(A \cup B) = P(A \cup (B \cap A^c))$$

$$= P(A) + P(B \cap A^c)$$

$P(B)$

$$B = (A \cap B) \cup (B \cap A^c)$$

$$P(B) = P((A \cap B) \cup (B \cap A^c))$$

$$P(B) = P(A \cap B) + P(B \cap A^c)$$

$$P(B \cap A^c) = P(B) - P(A \cap B)$$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

if A is an Event
& B is an event
then countable union
is also an event
(from (ii) of σ algebra of subsets of Ω)

∴ we want to know