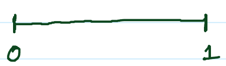
# [Probability Space]

⊛ You start off with a set called sample space $\Omega$, this is the set of all possible outcomes of an experiment.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

Sample space of tossing a dice.

A point in the sample space does not need to have a probability
i.e. Each outcome does not need to have a probability.




$$\Omega = [0, 1]$$

Sample space corresponding to the event of picking a random number from the interval.

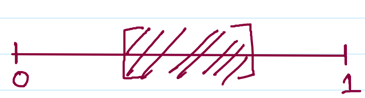
What is the prob of selecting random no. from $[0,1] = \frac{1}{2}$?

Ans $= 0$ (because there are so many points b/w $[0,1]$)

⊛ In a finite sample space we can ascribe probability to each outcome.

but Note Each of the elements of the sample space does not need to have a probability associated with it.

$\Rightarrow$ instead we can consider the probabity associated with an interval of sample space.




This subset of sample space ($\Omega$) would be an event.

$\Rightarrow$ Then you need a set of events denoted by

$$\mathcal{F} \subseteq P(\Omega)$$

Eg

$\Omega = \{H, T\}$ $\qquad$ $P(\Omega) = \{\phi, \{H\}, \{T\}, \{H,T\}\}$

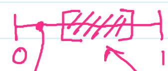
We can ascribe a probability to each event.

∴ $\Omega$ → We cannot always ascribe a probability value to each point of Sample space

but

$\mathcal{F}$ → we can always ascribe a probability to each event & $\boxed{\mathcal{F} \subseteq P(\Omega)}$

∴ In case




$\Omega = [0,1]$

We cannot ascribe probability to this point but we can ascribe a probability value to this interval which denotes a subset of $\Omega$ i.e an event.

## Final ingredient :-

You need a function P called probability.

Such that :-

$$\boxed{P : \mathcal{F} \to [0,1]}$$

The function P (Probability) ascribes a probability value to every event.

$$\boxed{\begin{array}{c} \text{Let, } E \in \mathcal{F} \quad \Rightarrow \quad E \subseteq \Omega \\ P(E) \in [0,1] \end{array}}$$

## Some principles

① $P(\Omega) = 1$ Note: $\Omega \in \mathcal{F}$

$\Omega$ is always an element of set of events.

$P(\phi) = 0$

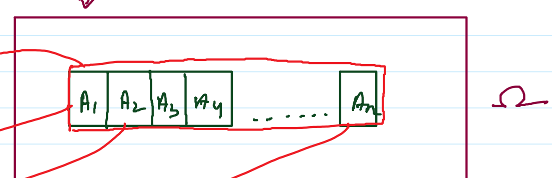
$\phi \in \mathcal{F}$

Empty set is always an Event.

(Event that you don't get any outcomes)

② If $\underbrace{A_n \in \mathcal{F} \quad , \quad A_n \subseteq \Omega}$

$P(\bigsqcup_{n=1}^{n} A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$




$P(A_1)$

$P(A)_2$

$P(A_n)$

$\underbrace{A_i \cap A_j = \phi \qquad i \neq j}$

all are pairwise disjoint.

(pairwise disjoint → $\bigsqcup$ union different from $\cup$)

$\bigsqcup_{n=1}^{\infty} A_n \in \mathcal{F}$ → from Measure theory (don't know how)

but it seems logical.

∵ if $A_n \in \mathcal{F}$ then $\bigsqcup$ (disjoint Union) will lead to a new event which belongs to

∵ if $A_n \in \mathcal{F}$ then $\bigsqcup$ (disjoint Union) will lead to a new event which belongs to Sample space.

Now,

$$\boxed{P\left(\bigsqcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} P(A_n)}$$

→ this property is called countable additivity

$(\Omega, \mathcal{F}, P)$ = Probability Space

Sample space

Set of Events

Probability function.