MEDIAN-RATIONAL HYBRID FILTERS

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ABSTRACT

In this paper we introduce a new class of nonlinear filters called Median-Rational Hybrid Filters (**MRHF**) based on Rational Functions (**RF**). The filter output is the result of a rational operation taking into account three sub-functions, such as two FIR or median sub-filters and one center weighted median filter (CWMF). The proposed MRHF filters have the inherent property that on smooth areas they provide good noise attenuation whereas on changing areas the noise attenuation is traded for good response to the change. The performance of the proposed filter is compared against widely known nonlinear filters such as: Morphological Signal Adaptive Median Filters, Stack Filters, Rank-Order Morphological Filters and simple Rational Filters.

It is shown that a significant subjective improvement in the restored image quality as well as a consistent reduction in the objectively measured mean absolute error and mean square error is obtained.

Keywords: Rational Functions, Rational Filters, Median Filters, Rational Hybrid Filters.

1. INTRODUCTION

Rational Filters are one of the recent and major classes of nonlinear filters. As the name indicates, a rational filter consists of a ratio of two polynomials. It is well known, in fact, that a rational function has several properties (i.e. it is a universal approximator and a good extrapolator, can be trained using a linear algorithm, and requires lower degree terms than Volterra expansions) which can make it very effective in many signal processing tasks.

Rational function filters were used by Leung and Haykin [6] based on the work of Walsh [12] for signal detection and estimation and were later applied by Ramponi in [7] ,[8] for image filtering and enhancement. It was later extended to deal with multidimentional data, in [4], for color image interpolation.

Specifically, in the image restoration problem, the rational filter developed in [7], performs well for relatively high SNR Gaussian contaminated environments. In order to derive a rational filter to deal with various kinds of noise such as Gaussian noise, impulsive noise and mixed Gaussian-impulsive noise, we propose a new class of nonlinear rational type hybrid filters for signal and image processing called Median-Rational Hybrid (**MRH**) Filters. The MRH Filter is based on three sub-operators in which the central sub-operator is a center weighted median CWM filter acting on a plus-shaped mask.

An outline of the paper is as follows. Section 2 describes the Median Rational Hybrid Filters in one and two dimensional cases and points some its important properties. Section 3 includes experimental results; while, section 4 concludes the paper.

2. MEDIAN-RATIONAL HYBRID FILTERS

2.1. 1-D Case

Let us consider the input vector $\mathbf{X}(\mathbf{n}) = [x(n-N), x(n-N+1), \ldots, x(n-1), x(n), x(n+1), \ldots, x(n+N)]$, which contains $(2N+1)$ observation samples at each location $\mathbf{n}$. The output variable $y(n)$ is the result of a rational function using three input sub-functions which form an input vector $\Phi = [\Phi_1, \Phi_2, \Phi_3]^T$, where we fixed the "*central*" sub-function Φ_2 as a center weighted median filter. The proposed MRHF output $y(n)$ is given by:

$$y(n) = \Phi_2(n) + \frac{\sum_{i=1}^{3} \alpha_i \Phi_i(n)}{h + k(\Phi_1(n) - \Phi_3(n))^2} \qquad (1)$$

where, $\alpha = [\alpha_1, \alpha_2, \alpha_3]^T$ characterizes the constant vec-

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tor coefficients of the input sub-functions and satisfies the condition: $\sum_{i=1}^{3} \alpha_i = 0$. In our study, $\alpha = [1, -2, 1]^T$. h and k are some positive constants. The parameter k is used to control the amount of the non-linear effect.

The sub-filters Φ_1 and Φ_3 are chosen so that an acceptable compromise between noise reduction and edge preservation is obtained. It is easy to observe that this Median-Rational Hybrid Filter differs from a linear low-pass filter mainly in the weighting, which is applied to Φ_1 and Φ_3. Indeed, such terms are divided by a factor proportional to the output of an edge-sensing term characterized by $(\Phi_1(n) - \Phi_3(n))^2$. The weight of the median-operation output term is accordingly modified, in order to keep the gain constant. The block scheme in Fig.1 shows an example of the proposed one dimensional MRH Filter structure, where Φ_1 and Φ_3 are considered as two FIR sub-filters. The behavior of this proposed structure for different positive values of parameter k is as follows.

1: $k \simeq 0$, we obtain a linear relation between The three sub-filters cited above:

$$y(n) = \Phi_2(n) + \frac{\sum_{i=1}^{3} \alpha_i \Phi_i(n)}{h}. \quad (2)$$

2: $k \to \infty$, the output of the filter is identical to the central sub-filter output and the rational function has no effect:

$$y(n) = \Phi_2(n). \quad (3)$$

3: For intermediate values of k, the $(\Phi_1(n) - \Phi_3(n))^2$ term perceives the presence of a detail and accordingly reduces the smoothing effect of the operator.

In this way, the MRH Filter acts as a linear operator between three sub-operators the coefficients of which are modulated by the edge-sensitive component.

Deterministic properties: A root signal is a signal that is not affected by further filtering. Thus, we analyze here the MRHF response to various signals such as:

- step edge;
- ramp edge;
- sine wave.

Nearly perfect edge and ramp responses are obtained, as can be seen for Figs.2(a) and (b). The sinusoidal response, on the other hand, is constrained by the median response, i.e., the peaks of the sine wave are

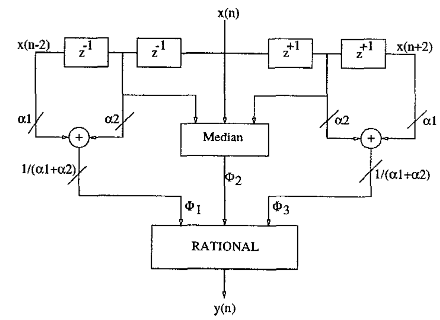
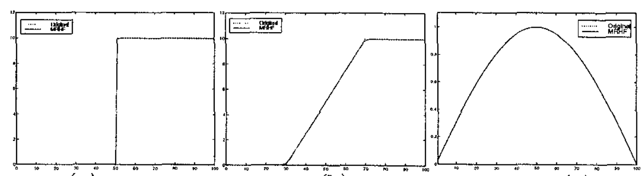


Figure 1: Example of non-recursive 1-D MRH Filter structure

slightly attenuated Fig.2(c). Thus; it proves the good edge response in different orientation of the MRH Filter.



(a) *(b)* *(c)*

Figure 2. MRH Filter responses *(a)*: Step edge response; *(b)*: Ramp edge response; *(c)*: sine wave response.

In the sequel, the extensions to two dimensional MRHF will be described.

2.2. The 2-D MRH Filters

Consider the real-valued 2-D sequence $\{x(\mathbf{n})\}$, and a 3x3 window mask centered around $x(\mathbf{n})$ given by Fig. 3, in order to filter the sample $x(\mathbf{n})$ at the central position. In the following we propose three classes of MRH filters.

MRHF1: Structure of MRHF with FIR sub-filters given by Fig.4.

MRHF2: Structure of MRHF with unidirectional median sub-filters given by Fig.5.

MRHF3: Structure of MRHF with bidirectional median sub-filters given by Fig.6.

The two FIR sub-filters are given by:

$$\Phi_1(\mathbf{n}) = \sum_{i=1}^{4} h_0(i) x_i(\mathbf{n}) \quad (4)$$

$$\Phi_3(\mathbf{n}) = \sum_{i=5}^{8} h_1(i)x_i(\mathbf{n}) \quad (5)$$

Where the filter coefficients are chosen to be:

$$h_0 = [\frac{1}{2(1+\sqrt{2})}, \frac{1}{2+\sqrt{2}}, \frac{1}{2(1+\sqrt{2})}, \frac{1}{2+\sqrt{2}}]^T$$

$$h_1 = [\frac{1}{2+\sqrt{2}}, \frac{1}{2(1+\sqrt{2})}, \frac{1}{2+\sqrt{2}}, \frac{1}{2(1+\sqrt{2})}]^T.$$

$\Phi_2(\mathbf{n})$ is the output of a plus-shaped center weighted median filter with the following filter weights

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

for the above three MRHF structures. It was shown that each sub-filter will preserve signal details within its sub-windows [9]. Hence, the MRHF are very promising detail preserving filtering structures.

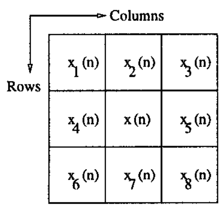


Figure 3. Elements of 3x3 sliding window centered around the pixel $x(n)$ for non-recursive implementation.

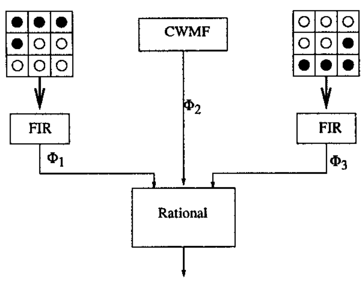


Figure 4. Structure of MRHF using two FIR sub-filters.

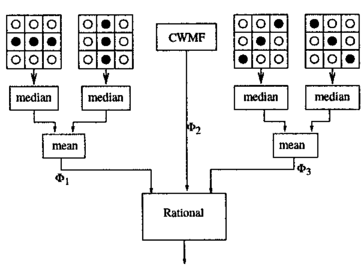


Figure 5. Structure of MRHF using two unidirectional median sub-filters.

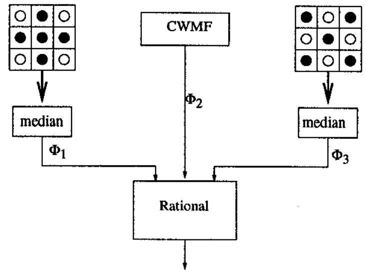


Figure 6. Structure of MRHF using two bidirectional median sub-filters.

3. EXPERIMENTAL RESULTS

The new MRH Filter has been tested using different images mainly from the TUT image database [2] to assess its performance. Six corrupted test images have been derived from the Bridge image in [2]. The perturbed image results by adding to the original image i.i.d. noise having the following contaminated Gaussian probability distribution,

$$\nu \sim (1-\lambda)\mathcal{N}(0,\sigma_n) + \lambda\mathcal{N}(0,\frac{\sigma_n}{\lambda}) \quad (6)$$

with mean $E(\nu)$, in this case 0, and variance

$$\sigma_\nu^2 = \sigma_n^2(1-\lambda+\frac{1}{\lambda}) \quad (7)$$

Three values of λ have been chosen:

1: $\lambda = 0.1, \rightarrow$ very impulsive noise.

2: $\lambda = 0.2, \rightarrow$ mixed Gaussian-impulsive noise.

3: $\lambda = 1, \rightarrow$ purely Gaussian noise.

Two sets of images have been formed according to their SNR values: first set with 3dB of SNR and second set with 9dB of SNR. As an example, Fig.7*(a)* is the clean image Bridge, Fig.7*(b)* is the noisy image with $\lambda = 0.2$, $SNR = 9dB$ and Fig.7*(c)* and *(d)* show the corresponding filtered images by the rational filter and the new MRH Filter respectively. The parameters h, k and the number of passes p are determined empirically according to the experimental minimization of the mean-square error (MSE) criterion of the processed image with respect to the original (uncorrupted) one.

The noisy images are also filtered with several nonlinear filters in order to compare the performance. Four techniques have been selected from the work developed by the European basic research project ESPRIT 7130 NAT Consortium [1], that are well known for their ability of smoothing noise while preserving image details,

i.e., the Morphological Signal Adaptive Median Filters (MSAMF) [11], the Stack Filters (Stack) [10], [13], the Rank-Order Morphological Filters (ROMF)[3], [5] and the simple Rational Filters [7].
For objective measures, we used the following two criteria for a 3x3 filter mask:

1: $MAE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}|\mathbf{y_{i,j}} - \mathbf{d_{i,j}}|$.

2: $MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(\mathbf{y_{i,j}} - \mathbf{d_{i,j}})^2$.

where M, N are the image dimensions, $\mathbf{y_{i,j}}$ is the filter output at pixel (i,j) and, $\mathbf{d_{i,j}}$ is the value of the pixel (i,j) of the desired image.
The best MAE and MSE values obtained from each technique are shown in tables 1, 2, 3 and 4. We conclude that the new MRF Filter attains better performance for the different types of noise compared to the other nonlinear operators.

4. CONCLUSIONS

In this paper, we introduced a new class of nonlinear rational type hybrid filters for signal and image processing. The median-Rational Hybrid (MRH) Filter is a rational function of three sub-filters in which the central one is a CWM Filter. Experimental results demonstrated that the MRH Filter can effectively remove various kinds of additive i.i.d. noise such as Gaussian noise, impulsive noise and mixed Gaussian-impulsive noise.
Several nonlinear filters have also been tested and compared to our new MRH Filters. In most cases, the MRH Filter outperforms all the other filters for both objective criteria used.

Acknowledgment: This work has been supported by the European ESPRIT Project LTR 20229-Noblesse.

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(a)

(b)

(c)

(d)

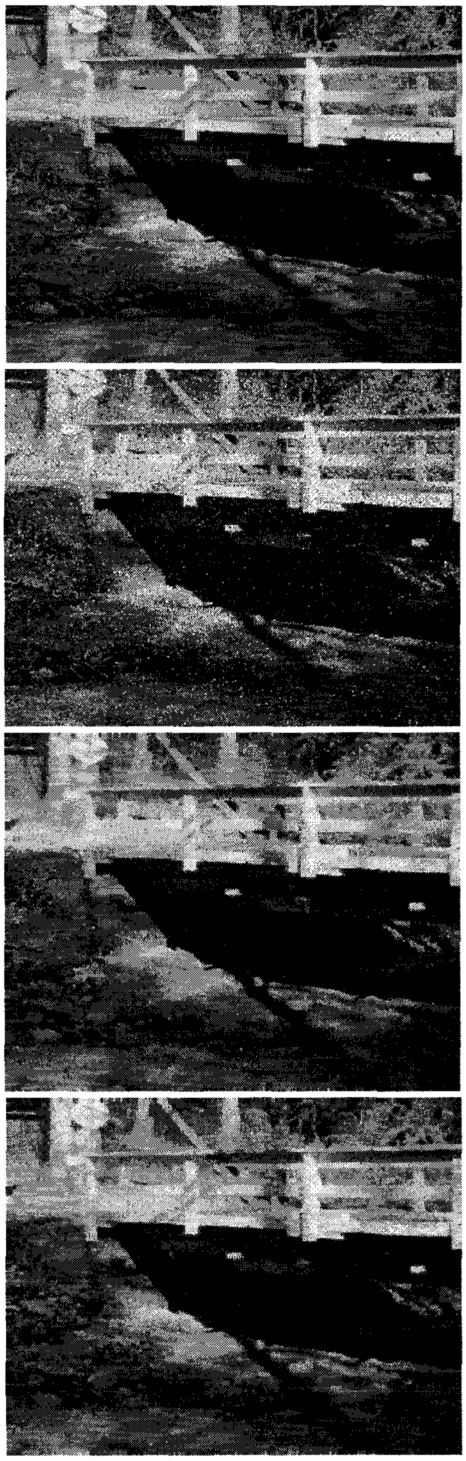

Figure 7. *(a)*: Part of the original bridge image; *(b)*: Corrupted image ($\lambda = 0.2$, $SNR = 9dB$); *(c)*: Filtered image by rational filter; *(d)*: Filtered image by Median-Rational Hybrid Filter (MRHF3).

Images	First set: SNR=3dB		
#	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 1$
Original	23.445	25.172	31.002
MSAMF	10.984	12.031	14.815
Stack	11.408	12.047	16.242
ROMF	11.400	11.400	13.540
Rational	11.640	12.302	13.10
MRHF1	10.956	11.400	12.980
MRHF2	10.580	11.050	13.65
MRHF3	10.465	10.920	13.38

Table 1: Quantitative MAE measures

Images	Second set: SNR=9dB		
#	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 1$
Original	9.390	11.743	15.467
MSAMF	7.225	8.241	10.193
Stack	6.166	7.660	10.245
ROMF	7.500	8.130	10.100
Rational	7.385	9.078	9.160
MRHF1	6.146	7.596	9.397
MRHF2	6.117	7.500	9.52
MRHF3	6.010	7.376	9.427

Table 2: Quantitative MAE measures

Images	First set: SNR=3dB		
#	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 1$
Original	1502.22	1506.98	1498.34
MSAMF	226.83	285.57	390.51
Stack	227.05	255.37	429.27
ROMF	240.46	240.46	320.54
Rational	249.22	268.28	290.00
MRHF1	221.46	234.88	281.80
MRHF2	201.50	218.00	308.00
MRHF3	196.92	213.81	300.10

Table 3: Quantitative MSE measures

Images	Second set: SNR=9dB		
#	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 1$
Original	377.46	374.25	375.73
MSAMF	116.47	147.01	187.31
Stack	85.80	117.44	178.07
ROMF	134.44	136.16	178.05
Rational	117.99	125.94	142.00
MRHF1	86.00	116.58	141.35
MRHF2	86.10	112.46	152.97
MRHF3	84.14	108.87	151.81

Table 4: Quantitative MSE measures