UOA — C2

Linear Algebra for ML.

Agenda

1. Linear Algebra
2. Scalars & Vectors, Operations
3. Matrices — Determinant, Inverse
4. Linear Model
5. Error in Linear Model

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Linear Algebra:

Algebra — play with letters, numbers, symbols.

$$\square - 4 = 6$$

$$x - 4 = 6$$

$$\boxed{x = 10}$$

Linear Algebra: branch of mathematics that is concerned with mathematical structures closed under the operations of addition & scalar multiplication.
It includes linear eqns, Matrices, Vectors, Linear transformations.

Linear Eqns'

It is an eqn. for a straight line.

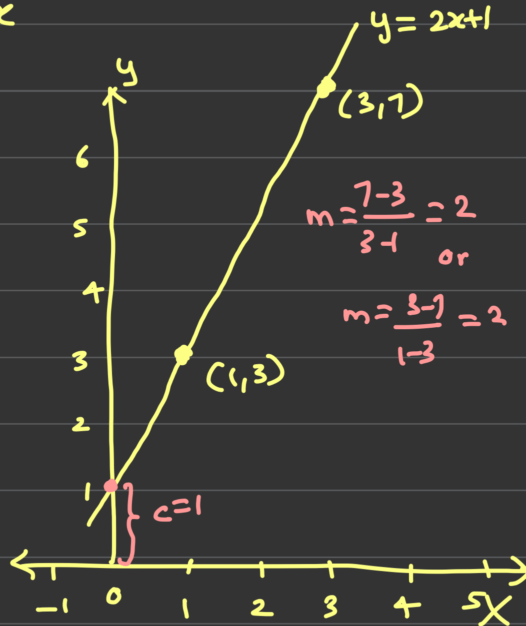
eg: $y = 2x + 1$

$5x = 6 + 3y$

$\frac{y}{2} = 3 - x$

eg: $y = 2x + 1$

Assume x	$y = 2x + 1$
1	3
3	7
-5	-9
⋮	⋮



Eqn. of line:

$y = mx + c$ → Slope intercept form

whr $m = \text{Slope} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

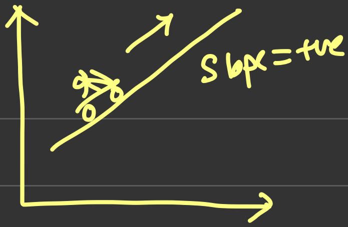
$c = y\text{-Intercept}$

$\left.\begin{array}{l} m = 2 \\ c = 1 \end{array}\right\} \therefore$ Eqn. of line is $\boxed{y = 2x + 1}$

What does $m = 2$ signify?

→ It signifies that!

For every 1 unit you move along X-axis, you climb 2 units along Y-axis.



Scalars & Vectors:

Scalars

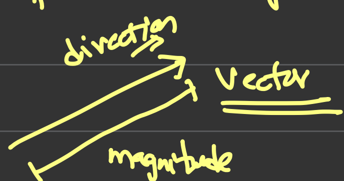
only magnitude & no direction

eg: Volume
- Temperature
- Time
- Speed
- Length
- Area
- Mass
- Energy
- Power
- Resistance

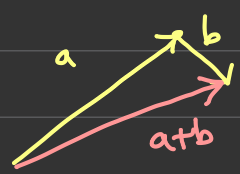
Vectors

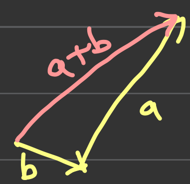
has both magnitude & direction

eg: Velocity
- Force
- Acceleration
- Momentum
- Drag
- Lift
- Pressure
- Gravity
- Magnetic field
- Temperature Change

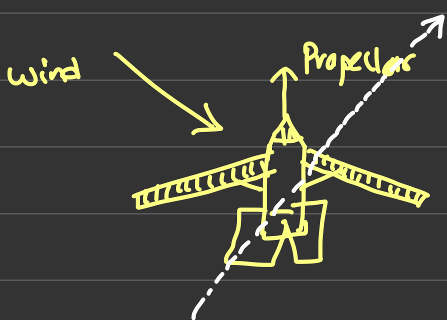


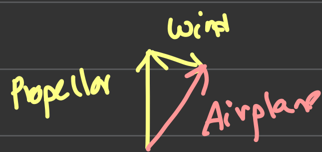
Add two Vectors





eg: A plane is flying along, point North, but there is wind coming from NW.

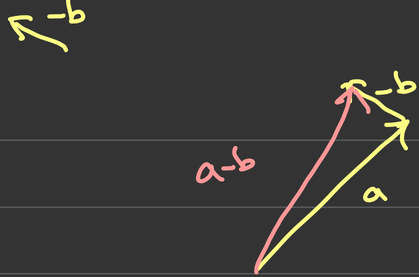




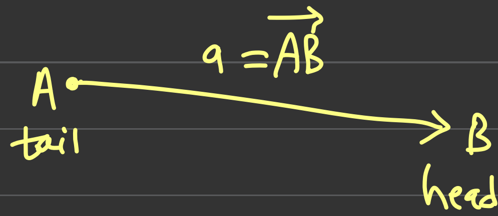
Subtract: ① Reverse direction of vector that you want to subtract

② Add them as usual

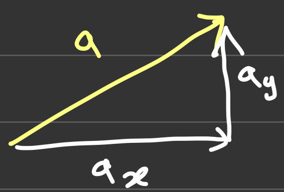
$(a-b)$



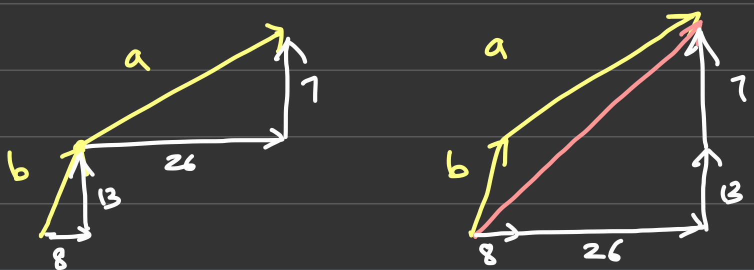
Notation:



Every vector can be broken into x & y parts:



Adding vectors:



13+7=20

8+26 =34

Vector (8,13) & (26,7) add up to (34,20).

eg: If $a = (8, 13)$ & $b = (26, 7)$

$c = ?$

$c = a + b$

$= (8 + 26, 13 + 7)$

$= (34, 20).$

eg! Subtract $k = (4, 5)$ from $v = (12, 2)$

Soln: $a = v + (-k)$

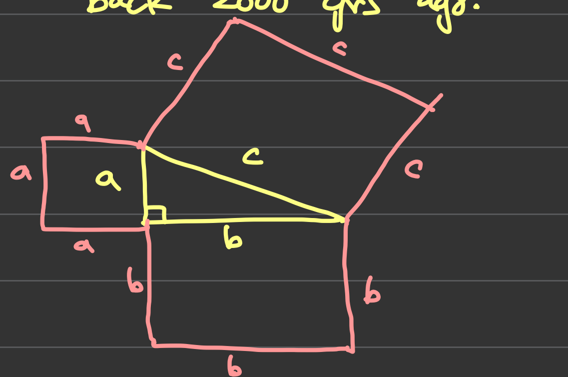
$= (12, 2)$

$+ (-4, -5)$

$= (8, -3)$

Magnitude of Vector! back 2000 yrs ago.

Pythagoras!



$c^2 = a^2 + b^2$

$$\boxed{c = \sqrt{a^2 + b^2}}$$

$a^2 + b^2 = c^2$

whr c = longest side & a & b are the other two sides.

$\vec{AB}$ is a vector

Magnitude : $|\vec{AB}|$ or $||\vec{AB}||$

$$= \sqrt{x^2+y^2}$$

eg: What is magnitude of vector $b = (6,8)$.

Soln: $|b| = \sqrt{6^2+8^2} = \sqrt{36+64}$

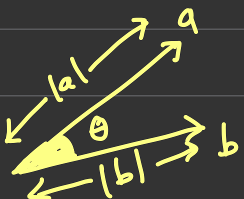
$= \sqrt{100} = 10.$

Note: A vector with magnitude 1 is called as a Unit Vector.

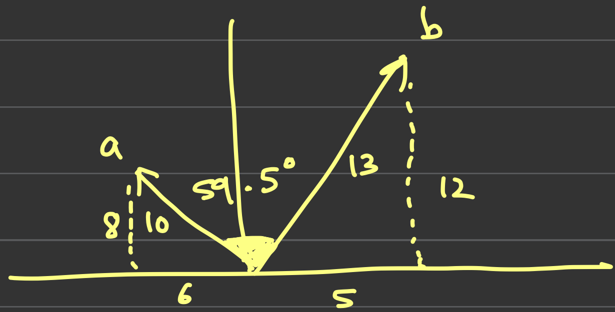
Dot Product: (easy)

Given a & b,

Dot Product is $a \cdot b = |a| \times |b| \times \cos\theta$



eg: Calculate dot product of vectors a & b.



Soln:

way 1

$$a \cdot b = |a| \times |b| \times \cos\theta$$
$$= 10 \times 13 \times \cos(59.5^\circ)$$
$$= 130 \times 0.50$$
$$= 66$$

way 2

$$a \cdot b = a_x \cdot b_x + a_y \cdot b_y$$
$$= -6 \cdot 5 + 8 \cdot 12$$
$$= -30 + 96$$
$$= 66$$

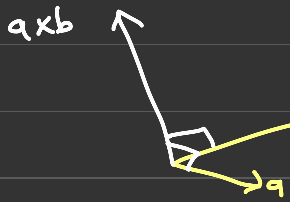
Right Angles! When two vectors are at right angles to each other, then dot product is zero.

$a \cdot b = |a| \times |b| \times \cos 90$

$= 0 \quad (\because \cos 90 = 0)$

Cross Product :

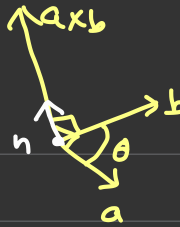
The cross product of two vectors is another vector that is at right angles to both.



It all happens in 3D.

Magnitude (length) of cross product equals the area of parallelogram with vectors a & b for sides!

Calculations:



$$a \times b = |a| \cdot |b| \cdot \sin(\theta) \cdot n$$

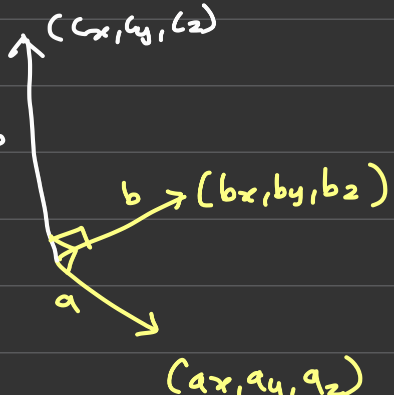
whr $|a|$ – magnitude (length) of a

$|b|$ – —"— b

θ – angle b/w a & b

n – unit vector at right angles to both a & b

Way 2:



$a = (a_x, a_y, a_z)$

$b = (b_x, b_y, b_z)$

$c = (c_x, c_y, c_z)$

$c_x = a_y \cdot b_z - a_z \cdot b_y$

$c_y = a_z b_x - a_x \cdot b_z$

$c_z = a_x b_y - a_y b_x$

eg: Cross product of

$a = (2, 3, 4)$

$b = (5, 6, 7)$

Soln: Solve yourself.

ans $= (-3, 6, -3)$.

Dot product	Cross Product
- o/p is Scalar	- o/p is vector
- use it when you want to get magnitude of the resultant vector	- use it when you want to get both magnitude & direction of the resultant vector.

Matrix:

Addition: / Subtraction: $A_{m\times n} \pm B_{m\times n} = C_{m\times n}$

Determinant of Matrix: It is a special no

Condn: Square Matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = ad - bc$$

$$A = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}$$

$$|A| = 3\cdot 6 - 8\cdot 4$$
$$= 18 - 32 \quad = -14$$

3x3 Matrix:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = +a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

Inverse of Matrix:

$$8 \underset{\text{Reciprocal}}{\overset{\text{Reciprocal}}{\rightleftarrows}} \frac{1}{8} \Rightarrow 8 \times \frac{1}{8} = 1$$

$$A \underset{\text{Inverse}}{\overset{\text{Inverse}}{\rightleftarrows}} A^{-1} \Rightarrow A \times A^{-1} = I$$

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4 \cdot 6 - 7 \cdot 2} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

$A \cdot A^{-1} = I$ (Lets check)

$$A \cdot A^{-1} = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times 0.6 + 7\times -0.2 & 4\times -0.7 + 7\times 0.4 \\ 2\times 0.6 + 6\times -0.2 & 2\times -0.7 + 6\times 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 2.4 - 1.4 & -2.8 + 2.8 \\ 1.2 - 1.2 & -1.4 + 2.4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A^{-1} \cdot A = I$ (Check yourself).

Why do we need Inverse?

Soln: Bcz with matrices, we dont divide.

eg: How do I share 10 mangoes with 2 ppl?

Soln: ① $\frac{10}{2} = 5$ (Divide)

② $10 \times \frac{1}{2}$ i.e. 10×0.5

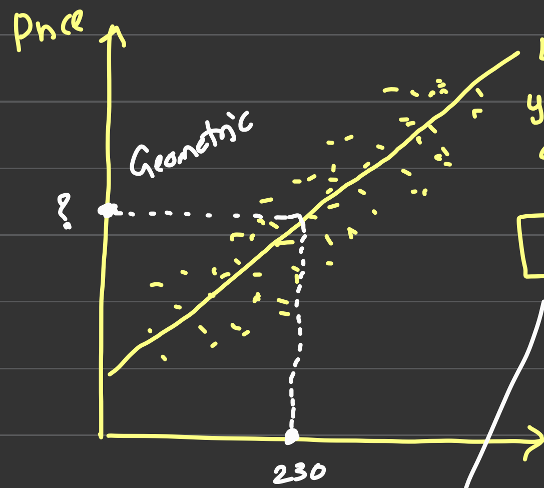
$= 5$ (Reciprocal, not divide).

$\therefore$ They get 5 mangoes each.

Machine Learning Model

Predict price of house from area. → Linear Regression

Area	Price
⋮	⋮



LOBF

$y = mx + c$

assume $m = 3$

$c = 2$

$\boxed{y = 3x + 2}$

Area

Algebraic

If $x = 230$, $y = ?$

$y = 3(230) + 2$