Algorithms and Data Structures

Chapter 18: B-trees
(based on book "Introduction to Algorithms" of Cormen et al.)

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Outline

1. Definition of a B-tree
2. Use-cases
3. Height of a B-tree
4. Operations on B-trees

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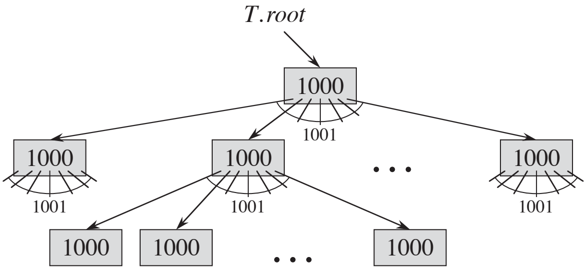

1 node,
1000 keys

1001 nodes,
1,001,000 keys

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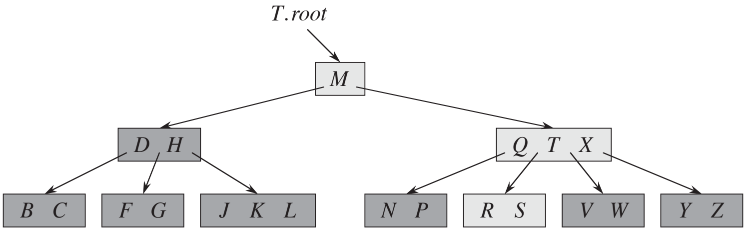

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- Why isn't a minimum degree of $t = 1$ allowed? Trivial.

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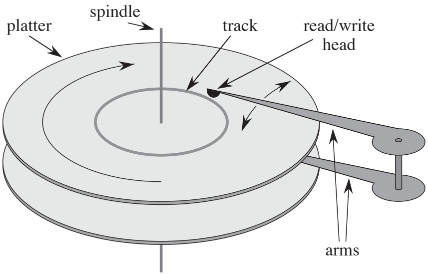
  - E.g. SSD's and HDD's.

Secondary memory is slow – Example: HDD



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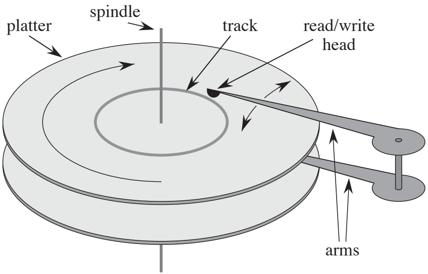

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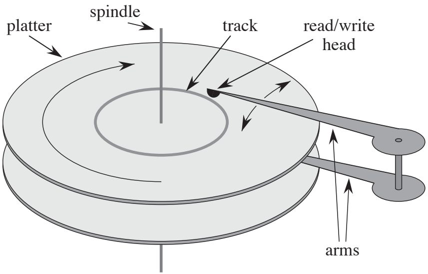

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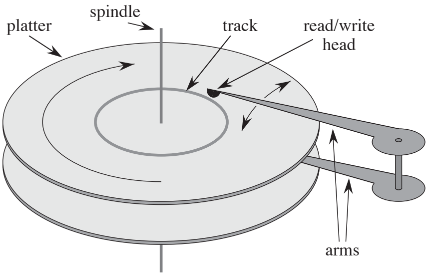

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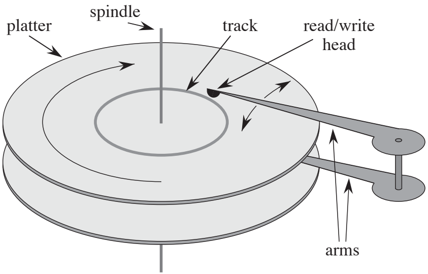

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  $\rightarrow$ latency!

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  - So we want the height to be as small as possible.

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Theorem

If $n \geq 1$, then for any n-key B-tree T of height h and minimum degree $t \geq 2$

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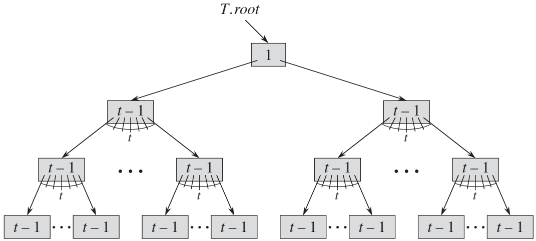

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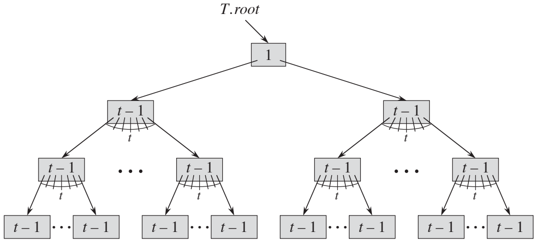

depth	0	1	2	3	...	h
number of nodes	1	2	$2t$	$2t^2$	...	2^{h-1}

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Thus

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t^h \leq \frac{n+1}{2} \quad \Longleftrightarrow \quad \boxed{h \leq \log_t\left(\frac{n+1}{2}\right)}
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- B-trees are limited in depth but contain an enourmous amount of keys.
  $\rightarrow$ Few memory operations, so minimal memory latency.

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  - At each internal node x, the search makes an $(x.n + 1)$-way branching decision.

Searching a B-tree: pseudocode

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B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
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- Recall that $k_1 \leq x.key_1 \leq k_2 \leq x.key_2 \leq \ldots \leq x.key_{x.n} \leq k_{x.n+1}$ where k_i is any key of child $x.c_i$

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- For every node x, $x.n < 2t$, so the while loop takes $O(1)$ time for each encountered node.

Searching a B-tree: runtime

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- For every node x, $x.n < 2t$, so the while loop takes $O(1)$ time for each encountered node.
- The total runtime is $O(th) = O(t \log_t n)$.

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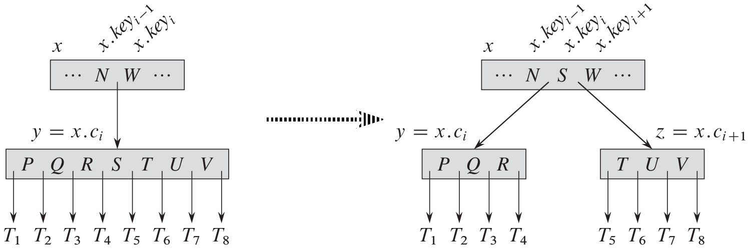
- We cannot simply *create* a new leaf node and insert it, because the resulting tree would fail to be a valid B-tree.
- So, we need to insert the key into an *existing* leaf node.
- What if a node y is *full*, i.e. what if $y.n = 2t - 1$?

Splitting a (full) node y with $y.n = 2t - 1$



$\textsc{B-Tree-Split-Child}(x, i)$

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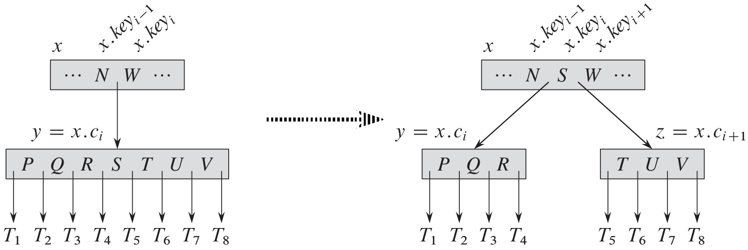

B-Tree-Split-Child(x, i)

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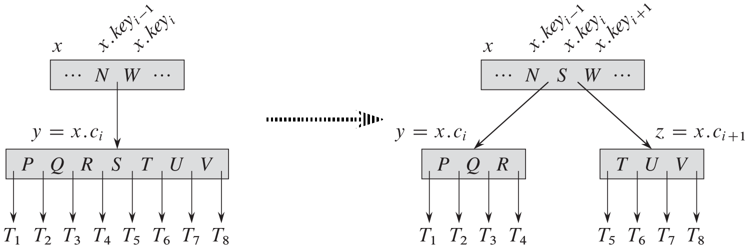

B-TREE-SPLIT-CHILD(x, i)

- x is a non-full node and $y = x.c_i$ is a full child of x.
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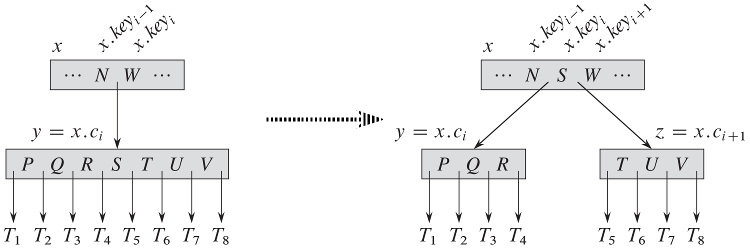

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- x is a non-full node and $y = x.c_i$ is a full child of x.
- Split y about its median key S and move S up into y's parent node x.
- Every key $y.key_i$ that is greater than the median S, is placed in a new node z, which is a new child of x.

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- Calls $\text{B-T}{\small\text{REE}}\text{-I}{\small\text{NSERT}}\text{-N}{\small\text{ONFULL}}(x, k)$ with either $x = r$ or $x = s$

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- Again $O(h) = O(\log_t n)$ disk accesses.

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- Again $O(h) = O(\log_t n)$ disk accesses.
- Again $O(th) = O(t \log_t n)$ time required.

Questions?