Algorithms and Data Structures

Chapter 18: B-trees
(based on book “Introduction to Algorithms” of Cormen et al.)

Vincent Van Schependom
KU Leuven Campus Kulak Kortrijk
Academic year 2024–2025

Outline

1. Definition of a B-tree
2. Use-cases
3. Height of a B-tree
4. Operations on B-trees

Outline

Definition of a B-tree

- Rooted tree with root $T.root$

Definition of a B-tree

- Rooted tree with root $T.root$
- Self-balancing, just like red-black trees.

Definition of a B-tree

- Rooted tree with root $T.root$
- Self-balancing, just like red-black trees.
- All leaves have the same depth, i.e. the tree's height h.

Definition of a B-tree

- Rooted tree with root $T.root$
- Self-balancing, just like red-black trees.
- All leaves have the same depth, i.e. the tree's height h.
- Every node x has $\underline{x.n \text{ keys}}$, stored in monotonically increasing order:

$$x.key_1 \leq x.key_2 \leq \ldots \leq x.key_{x.n}$$

Definition of a B-tree

- Rooted tree with root $T.root$
- Self-balancing, just like red-black trees.
- All leaves have the same depth, i.e. the tree's height h.
- Every node x has $x.n$ keys, stored in monotonically increasing order:

$$x.key_1 \leq x.key_2 \leq \ldots \leq x.key_{x.n}$$

- Each internal node contains $x.n + 1$ pointers to its children:

$$x.c_1, x.c_2, \ldots, x.c_{x.n+1}$$

Definition of a B-tree

- Rooted tree with root $T.root$
- Self-balancing, just like red-black trees.
- All leaves have the same depth, i.e. the tree's height h.
- Every node x has $x.n$ keys, stored in monotonically increasing order:

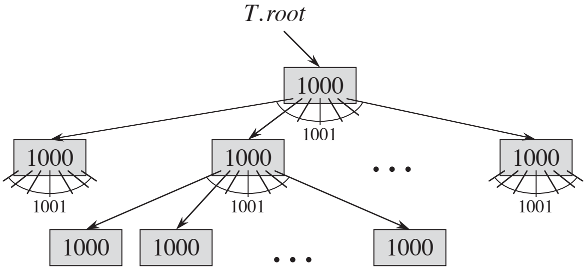
$$x.key_1 \leq x.key_2 \leq \ldots \leq x.key_{x.n}$$

- Each internal node contains $x.n + 1$ pointers to its children:

$$x.c_1, x.c_2, \ldots, x.c_{x.n+1}$$

- Main concept $\rightarrow$ remember well

Example of a B-tree



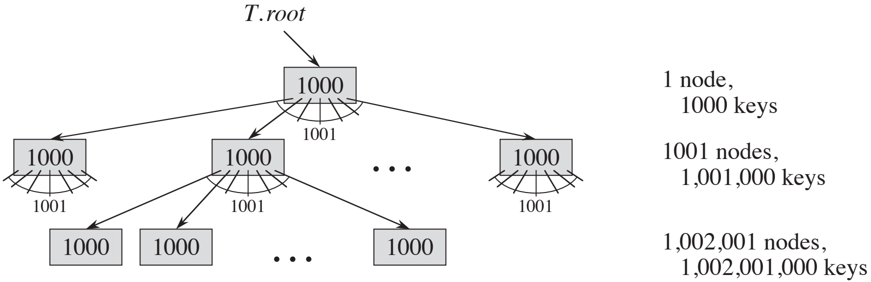
1 node,
1000 keys

1001 nodes,
1,001,000 keys

1,002,001 nodes,
1,002,001,000 keys

- Every node x has $x.n = 1000$ keys $x.key_i$.

Example of a B-tree



- Every node x has $x.n = 1000$ keys $x.key_i$.
- Each internal node x has $x.n + 1 = 1001$ pointers to its children $x.c_i$.

Key ordering

- The keys $x.key_i$ separate the ranges of keys stored in each subtree.

Key ordering

- The keys $x.key_i$ separate the ranges of keys stored in each subtree.
  - Let k_i be any key stored in the subtree with root $x.c_i$

Key ordering

- The keys $x.key_i$ separate the ranges of keys stored in each subtree.
  - Let k_i be any key stored in the subtree with root $x.c_i$
    - k_i is one of the keys of child $x.c_i$ of node x

Key ordering

- The keys $x.key_i$ separate the ranges of keys stored in each subtree.
  - Let k_i be any key stored in the subtree with root $x.c_i$
    - k_i is one of the keys of child $x.c_i$ of node x
    - k_i is one of the keys of a descendant of child $x.c_i$ of node x

Key ordering

- The keys $x.key_i$ separate the ranges of keys stored in each subtree.
  - Let k_i be any key stored in the subtree with root $x.c_i$
    - k_i is one of the keys of child $x.c_i$ of node x
    - k_i is one of the keys of a descendant of child $x.c_i$ of node x

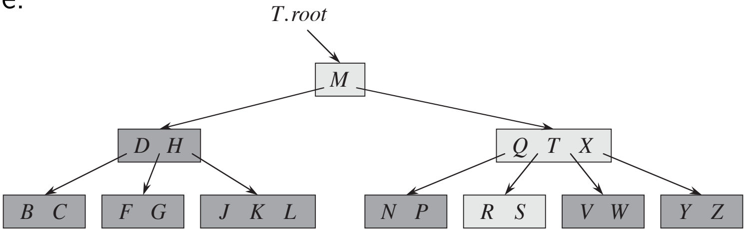
$$k_1 \leq x.key_1 \leq k_2 \leq x.key_2 \leq \ldots \leq x.key_{x.n} \leq k_{x.n+1}$$

Key ordering

- The keys $x.key_i$ separate the ranges of keys stored in each subtree.
  - Let k_i be any key stored in the subtree with root $x.c_i$
    - k_i is one of the keys of child $x.c_i$ of node x
    - k_i is one of the keys of a descendant of child $x.c_i$ of node x

$$k_1 \leq x.key_1 \leq k_2 \leq x.key_2 \leq \ldots \leq x.key_{x.n} \leq k_{x.n+1}$$

- Example:



Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have <u>at least $t - 1$ keys</u>.

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have <u>at least $t - 1$ keys</u>.
  - So, how many children does an internal node need at least?

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t - 1$ keys.
  - So, how many children does an internal node need at least? t children.

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t - 1$ keys.
  - So, how many children does an internal node need at least? t children.
  - If the tree is nonempty, the root must have at least one key.

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t - 1$ keys.
  - So, how many children does an internal node need at least? t children.
  - If the tree is nonempty, the root must have at least one key.
- Upper bound:

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t-1$ keys.
  - So, how many children does an internal node need at least? t children.
  - If the tree is nonempty, the root must have at least one key.
- Upper bound:
  - Every node may contain at most $2t-1$ keys.

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t-1$ keys.
  - So, how many children does an internal node need at least? t children.
  - If the tree is nonempty, the root must have at least one key.
- Upper bound:
  - Every node may contain at most $2t-1$ keys.
  - So, how many children can an internal node have at most?

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t-1$ keys.
  - So, how many children does an internal node need at least? t children.
  - If the tree is nonempty, the root must have at least one key.
- Upper bound:
  - Every node may contain at most $2t-1$ keys.
  - So, how many children can an internal node have at most? $2t$ children.

Minimum degree

- Nodes have lower and upper bounds on the number of keys they can contain.
- Expressed in terms of a fixed integer $t \geq 2$, called the *minimum degree*
- Lower bound:
  - Every node other than the root must have at least $t - 1$ keys.
  - So, how many children does an internal node need at least? t children.
  - If the tree is nonempty, the root must have at least one key.
- Upper bound:
  - Every node may contain at most $2t - 1$ keys.
  - So, how many children can an internal node have at most? $2t$ children.
  - We say that a node is *full* if it contains exactly $2t - 1$ keys.

Outline

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

- Primary memory:

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

- Primary memory:
  - Fast.

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

- Primary memory:
  - Fast.
  - Expensive and limited in capacity.

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

- Primary memory:
  - Fast.
  - Expensive and limited in capacity.
- Secondary memory:

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

- Primary memory:
  - Fast.
  - Expensive and limited in capacity.
- Secondary memory:
  - Cheaper and much larger.

Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

- Primary memory:
  - Fast.
  - Expensive and limited in capacity.
- Secondary memory:
  - Cheaper and much larger.
  - We need such capacity for storing large amounts of data, like in databases.

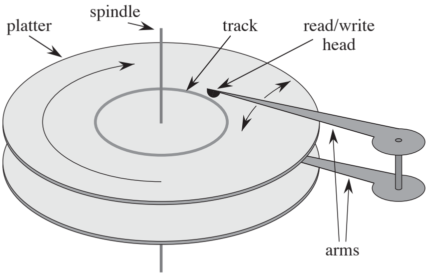
Quick SOCS recap

- Computer systems use a hierarchy of memory technologies.

$$\text{Registers} < \text{Caches} < \text{Main Memory} < \text{Secondary Memory}$$

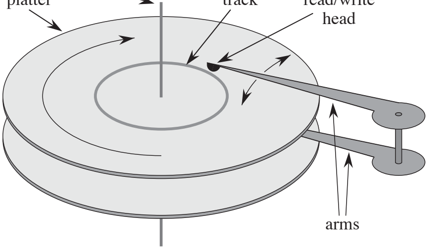
- Primary memory:
  - Fast.
  - Expensive and limited in capacity.
- Secondary memory:
  - Cheaper and much larger.
  - We need such capacity for storing large amounts of data, like in databases.
  - E.g. SSD's and HDD's.

Secondary memory is slow – Example: HDD



- Because of the moving mechanical parts, HDD's are slow.
- SSD's are not mechanical, but still significantly slower dan RAM.

Secondary memory is slow – Example: HDD



- Because of the moving mechanical parts, HDD's are slow.
- SSD's are not mechanical, but still significantly slower dan RAM.

  → latency!

Combating latency

- Access not just one item, but several at a time.

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.
  - B-trees are a very common datastructure for handling this.

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.
  - B-trees are a very common datastructure for handling this.
- Each node is usually as large as a whole disk block.

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.
  - B-trees are a very common datastructure for handling this.
- Each node is usually as large as a whole disk block.
- Reading/writing in secondary memory $\leftrightarrow$ reading/writing in a B-tree

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.
  - B-trees are a very common datastructure for handling this.
- Each node is usually as large as a whole disk block.
- Reading/writing in secondary memory $\leftrightarrow$ reading/writing in a B-tree
  - The number of blocks read or written provides a good approximation of the total time spent accessing the disk drive.

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.
  - B-trees are a very common datastructure for handling this.
- Each node is usually as large as a whole disk block.
- Reading/writing in secondary memory $\leftrightarrow$ reading/writing in a B-tree
  - The number of blocks read or written provides a good approximation of the total time spent accessing the disk drive.
  - Only downward paths.

Combating latency

- Access not just one item, but several at a time.
  - Information is divided into a number of equal-sized *blocks*.
  - B-trees are a very common datastructure for handling this.
- Each node is usually as large as a whole disk block.
- Reading/writing in secondary memory $\leftrightarrow$ reading/writing in a B-tree
  - The number of blocks read or written provides a good approximation of the total time spent accessing the disk drive.
  - Only downward paths.
  - So we want the height to be as small as possible.

Outline

Height of a B-tree

Theorem

If $n \geq 1$, then for any n-key B-tree T of height h and minimum degree $t \geq 2$

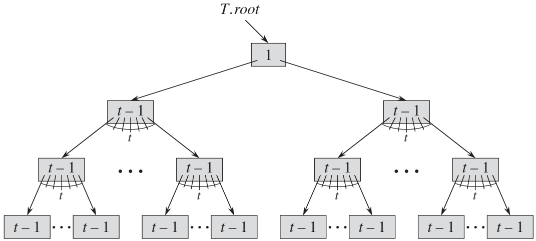
$$h \leq \log_t \left(\frac{n+1}{2}\right)$$

Height of a B-tree

Theorem

If $n \geq 1$, then for any n-key B-tree T of height h and minimum degree $t \geq 2$

$$h \leq \log_t \left(\frac{n+1}{2}\right)$$

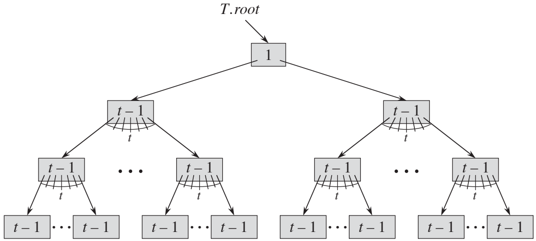


Height of a B-tree

Theorem

If $n \geq 1$, then for any n-key B-tree T of height h and minimum degree $t \geq 2$

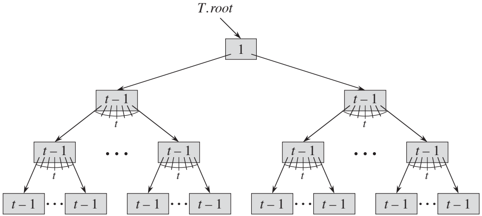
$$h \leq \log_t \left(\frac{n+1}{2} \right)$$



depth	0	1	2	3	...	h
number of nodes	1	2	$2t$	$2t^2$	...	2^{h-1}

Height of a B-tree

Proof.

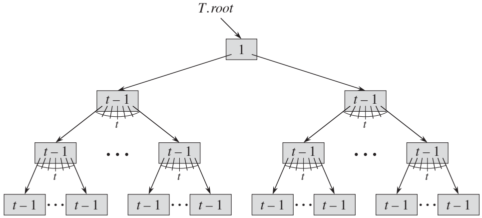


depth	#nodes
0	1
1	2
2	$2t$
3	$2t^2$
$\vdots$	$\vdots$
h	2^{h-1}

Height of a B-tree

Proof. The number of keys n satisfies

$$n \geq \overbrace{\underbrace{1}_{\text{root}} + \underbrace{(t-1)}_{\text{other nodes}}}^{\text{keys per node}} \overbrace{\sum_{i=1}^{h} 2t^{h-1}}^{\#\text{nodes}}$$



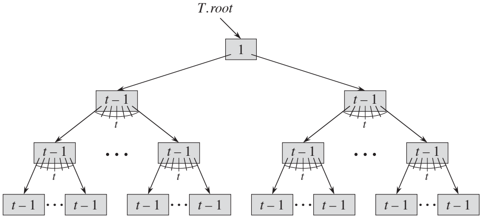
depth	#nodes
0	1
1	2
2	$2t$
3	$2t^2$
$\vdots$	$\vdots$
h	2^{h-1}

Height of a B-tree

Proof. The number of keys n satisfies

$$n \geq \overbrace{\underbrace{1}_{\text{root}} + \underbrace{(t-1)}_{\text{other nodes}}}^{\text{keys per node}} \overbrace{\sum_{i=1}^{h} 2t^{h-1}}^{\#\text{nodes}}$$

$$= 1 + 2(t-1)\left(\frac{t^h - 1}{t-1}\right)$$

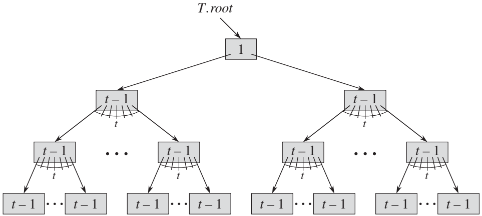


depth	#nodes
0	1
1	2
2	$2t$
3	$2t^2$
$\vdots$	$\vdots$
h	2^{h-1}

Height of a B-tree

Proof. The number of keys n satisfies

$$
\begin{aligned}
n &\geq \overbrace{\underbrace{1}_{\text{root}} + \underbrace{(t-1)}_{\text{other nodes}}}^{\text{keys per node}} \overbrace{\sum_{i=1}^{h} 2t^{h-1}}^{\#\text{nodes}} \\
&= 1 + 2(t-1)\left(\frac{t^h - 1}{t - 1}\right) \\
&= 2t^h - 1
\end{aligned}
$$



depth	#nodes
0	1
1	2
2	$2t$
3	$2t^2$
$\vdots$	$\vdots$
h	2^{h-1}

Height of a B-tree

Proof. The number of keys n satisfies

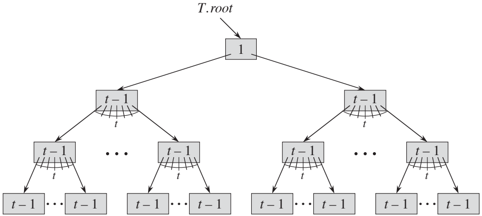
$$n \geq \underbrace{1}_{\text{root}} + \overbrace{\underbrace{(t-1)}_{\text{other nodes}}}^{\text{keys per node}} \overbrace{\sum_{i=1}^{h} 2t^{h-1}}^{\#\text{nodes}}$$

$$= 1 + 2(t-1)\left(\frac{t^h - 1}{t - 1}\right)$$

$$= 2t^h - 1$$

Thus

$$t^h \leq \frac{n+1}{2}$$



depth	#nodes
0	1
1	2
2	$2t$
3	$2t^2$
$\vdots$	$\vdots$
h	2^{h-1}

KU LEUVEN kulak

Height of a B-tree

Proof. The number of keys n satisfies

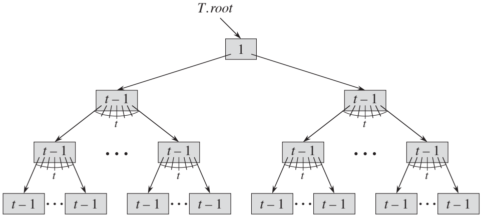
$$n \geq \overbrace{\underbrace{1}_{\text{root}} + \underbrace{(t-1)}_{\text{other nodes}}}^{\text{keys per node}} \overbrace{\sum_{i=1}^{h} 2t^{h-1}}^{\#\text{nodes}}$$

$$= 1 + 2(t-1)\left(\frac{t^h - 1}{t - 1}\right)$$

$$= 2t^h - 1$$

Thus

$$t^h \leq \frac{n+1}{2} \iff \boxed{h \leq \log_t\left(\frac{n+1}{2}\right)} \qquad \square$$



depth	#nodes
0	1
1	2
2	$2t$
3	$2t^2$
$\vdots$	$\vdots$
h	2^{h-1}

Comparison with red-black trees

- Height of the tree grows as $O(\lg n)$ in both cases.

Comparison with red-black trees

- Height of the tree grows as $O(\lg n)$ in both cases.
  - Constant t is left out in asymptotic notation.

Comparison with red-black trees

- Height of the tree grows as $O(\lg n)$ in both cases.
  - Constant t is left out in asymptotic notation.
  - The base logarithm for B-trees is much larger: t vs 2.

Comparison with red-black trees

- Height of the tree grows as $O(\lg n)$ in both cases.
  - Constant t is left out in asymptotic notation.
  - The base logarithm for B-trees is much larger: t vs 2.
- Factor of $\approx \lg t$ saved over red-black trees.

Comparison with red-black trees

- Height of the tree grows as $O(\lg n)$ in both cases.
  - Constant t is left out in asymptotic notation.
  - The base logarithm for B-trees is much larger: t vs 2.
- Factor of $\approx \lg t$ saved over red-black trees.
- B-trees are <u>limited in depth</u> but contain an <u>enourmous amount of keys</u>.

Comparison with red-black trees

- Height of the tree grows as $O(\lg n)$ in both cases.
  - Constant t is left out in asymptotic notation.
  - The base logarithm for B-trees is much larger: t vs 2.
- Factor of $\approx \lg t$ saved over red-black trees.
- B-trees are limited in depth but contain an enourmous amount of keys.

$\rightarrow$ Few memory operations, so minimal memory latency.

Outline

1. Definition of a B-tree
2. Use-cases
3. Height of a B-tree
4. Operations on B-trees

Searching a B-tree

- B-Tree-Search is much like searching a binary tree.

Searching a B-tree

- B-Tree-Search is much like searching a binary tree.
- However, *multiway* (as opposed to binary) branching decisions are made.

Searching a B-tree

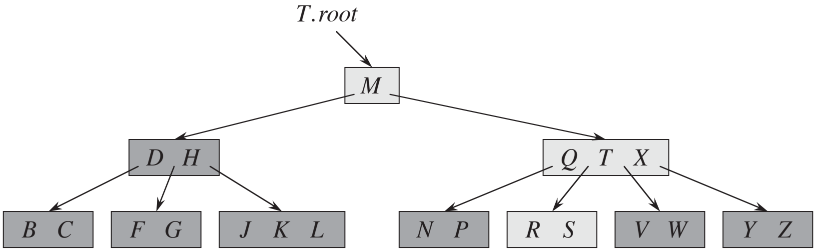
- B-Tree-Search is much like searching a binary tree.
- However, *multiway* (as opposed to binary) branching decisions are made.
  - The amount of *ways* is equal to the number of children.

Searching a B-tree

- B-TREE-SEARCH is much like searching a binary tree.
- However, *multiway* (as opposed to binary) branching decisions are made.
  - The amount of *ways* is equal to the number of children.
  - At each internal node x, the search makes an $(x.n + 1)$-way branching decision.

Searching a B-tree

- B-TREE-SEARCH is much like searching a binary tree.
- However, *multiway* (as opposed to binary) branching decisions are made.
  - The amount of *ways* is equal to the number of children.
  - At each internal node x, the search makes an $(x.n + 1)$-way branching decision.
- Example: searching for the letter S.



Searching a B-tree: pseudocode

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

Searching a B-tree: pseudocode

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- x is a pointer to the root node of a subtree

Searching a B-tree: pseudocode

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- x is a pointer to the root node of a subtree
- k is the key to be searched

Searching a B-tree: pseudocode

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- x is a pointer to the root node of a subtree
- k is the key to be searched
- If k is in the B-tree, B-TREE-SEARCH returns the node-index pair (y, i) such that $y.key_i = k$. Otherwise, the procedure returns NIL.

Searching a B-tree: pseudocode

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- x is a pointer to the root node of a subtree
- k is the key to be searched
- If k is in the B-tree, B-TREE-SEARCH returns the node-index pair (y, i) such that $y.key_i = k$. Otherwise, the procedure returns NIL.

- Recall that $k_1 \leq x.key_1 \leq k_2 \leq x.key_2 \leq \ldots \leq x.key_{x.n} \leq k_{x.n+1}$ where k_i is any key of child $x.c_i$

Searching a B-tree: memory accesses

- The nodes encountered during the recursion form a simple path downward from the root to the tree.

Searching a B-tree: memory accesses

- The nodes encountered during the recursion form a simple path downward from the root to the tree.
- Thus, B-TREE-SEARCH executes $O(h) = O(\log_t n)$ memory accesses.

Searching a B-tree: memory accesses

- The nodes encountered during the recursion form a simple path downward from the root to the tree.
- Thus, B-TREE-SEARCH executes $O(h) = O(\log_t n)$ memory accesses.
  - Recall that $h \leq \log_t \left(\frac{n+1}{2}\right)$.

Searching a B-tree: memory accesses

- The nodes encountered during the recursion form a simple path downward from the root to the tree.
- Thus, $\textsc{B-Tree-Search}$ executes $O(h) = O(\log_t n)$ memory accesses.
  - Recall that $h \leq \log_t \left(\frac{n+1}{2}\right)$.
  - Constants are left out in O-notation.

Searching a B-tree: runtime

```
B-TREE-SEARCH(x, k)
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- For every node x, $x.n < 2t$, so the while loop takes $O(t)$ time for each encountered node.

Searching a B-tree: runtime

B-TREE-SEARCH(x, k)

```
i = 1
while i ≤ x.n and k > x.key_i
    i = i + 1
if i ≤ x.n and k == x.key_i
    return (x, i)
elseif x.leaf
    return NIL
else DISK-READ(x.c_i)
    return B-TREE-SEARCH(x.c_i, k)
```

- For every node x, $x.n < 2t$, so the while loop takes $O(t)$ time for each encountered node.
- The total runtime is $O(th) = O(t \log_t n)$.

Inserting a key

- Significantly more complicated than inserting a key into a binary tree.

Inserting a key

- Significantly more complicated than inserting a key into a binary tree.
- We cannot simply *create* a new leaf node and insert it, because the resulting tree would fail to be a valid B-tree.

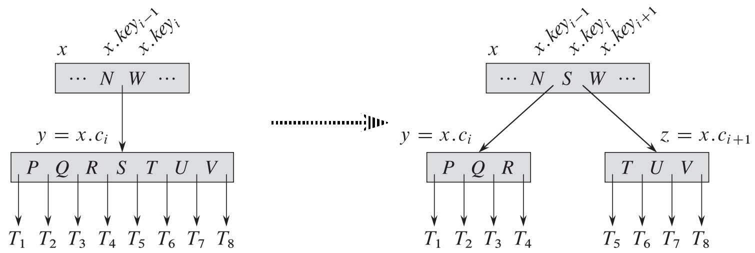
Inserting a key

- Significantly more complicated than inserting a key into a binary tree.
- We cannot simply *create* a new leaf node and insert it, because the resulting tree would fail to be a valid B-tree.
- So, we need to insert the key into an *existing* leaf node.

Inserting a key

- Significantly more complicated than inserting a key into a binary tree.
- We cannot simply *create* a new leaf node and insert it, because the resulting tree would fail to be a valid B-tree.
- So, we need to insert the key into an *existing* leaf node.
- What if a node y is *full*, i.e. what if $y.n = 2t - 1$?

Splitting a full node y with $y.n = 2t - 1$



B-TREE-SPLIT-CHILD(x, i)

- x is a non-full node and $y = x.c_i$ is a full child of x.
- Split y about its median key S and move S up into y's parent node x.
- Every key $y.key_i$ that is greater than the median S, is placed in a new node z, which is a new child of x.

KU LEUVEN kulak

Inserting a key (continued)

Complexity of $\textsc{B-Tree-Insert}(T, k)$?

Inserting a key (continued)

Complexity of $\textsc{B-Tree-Insert}(T, k)$?

- Again $O(h) = O(\log_t n)$ disk accesses.

Inserting a key (continued)

Complexity of $\textsc{B-Tree-Insert}(T, k)$?

- Again $O(h) = O(\log_t n)$ disk accesses.
- Again $O(th) = O(t \log_t n)$ time required.

Questions?