Asymptotic Mean Ergodicity of Average Consensus Estimators

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Introduction

Swarm robotics is the study of multi-agent systems. Groups of agents may be used to solve problems which are either difficult or impossible for a single agent. Each agent usually performs relatively simple tasks and communicates with local neighboring agents to produce emergent global behavior of the swarm.

Average consensus is the problem of having each agent compute the average of a variable over the network.

Applications

Average consensus is a key building block in many distributed algorithms such as the following:

- Formation control
- Distributed Kalman filtering
- Distributed sensor fusion

Objective

Our objective is to design an average consensus estimator with the following properties:

- **Simple** - The algorithm should be simple enough to be implemented on each agent.
- **Scalable** - The complexity of the algorithm should not scale with the number of agents.
- **Robust** - The algorithm should be robust to initialization errors, the addition and removal of agents from the network, as well as dropped packets.
- **Accurate** - The steady-state error should be zero.
- **Fast Convergence** - The algorithm should converge quickly.

Ergodicity

The asymptotic mean ergodicity (AME) property is import for estimators to be robust to changes in the network.

Ergodicity: time-average = statistical average

AME: $\lim_{t\to\infty}$ time mean = statistical mean

If the estimator outputs are AME, then each agent can put its output through a local low-pass filter to calculate the global statistical average.

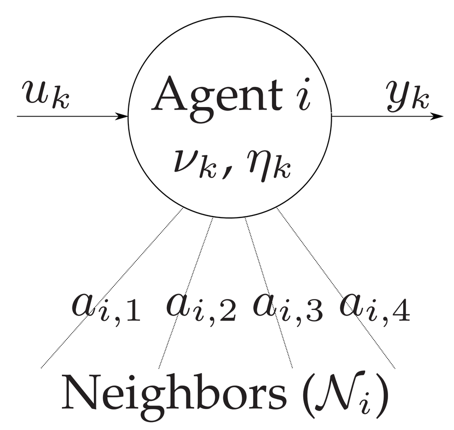
Question

Does there exist an average consensus estimator whose output is asymptotically mean ergodic?

Problem Setup

Consider a group of n agents whose communication topology is modeled as a weighted directed graph G. The adjacency matrix of G is defined as $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ where $a_{ij} > 0$ if agent i can receive information from agent j and zero otherwise. The neighbors of agent i, denoted $\mathcal{N}_i$, is the set of agents from which agent i can receive information.

i	Index of agent
k	Iteration
u_k^i	Input
y_k^i	Output
ν_k^i, η_k^i	States
γ, k_p, k_I	Parameters
$\mathcal{N}_i$	Neighbors
a_{ij}	Graph weights



The communication graphs are assumed to be i.i.d. and connected and balanced on average, but they need not be connected or balanced at any individual time step as illustrated below.

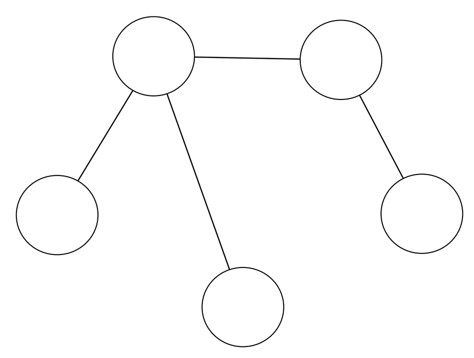


Figure 1: Graph at k

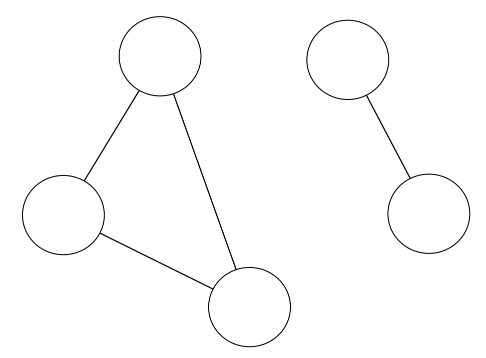


Figure 2: Graph at $k+1$

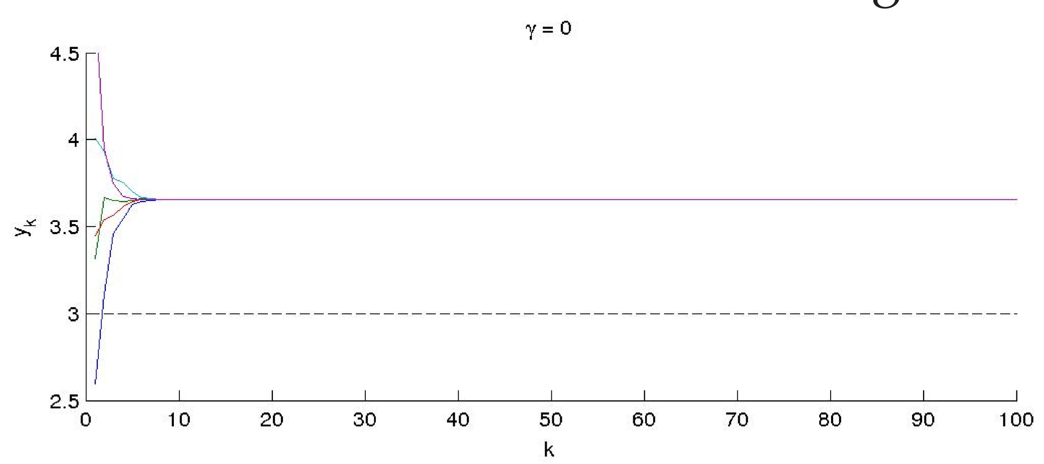
P Estimator

The P estimator is implemented using

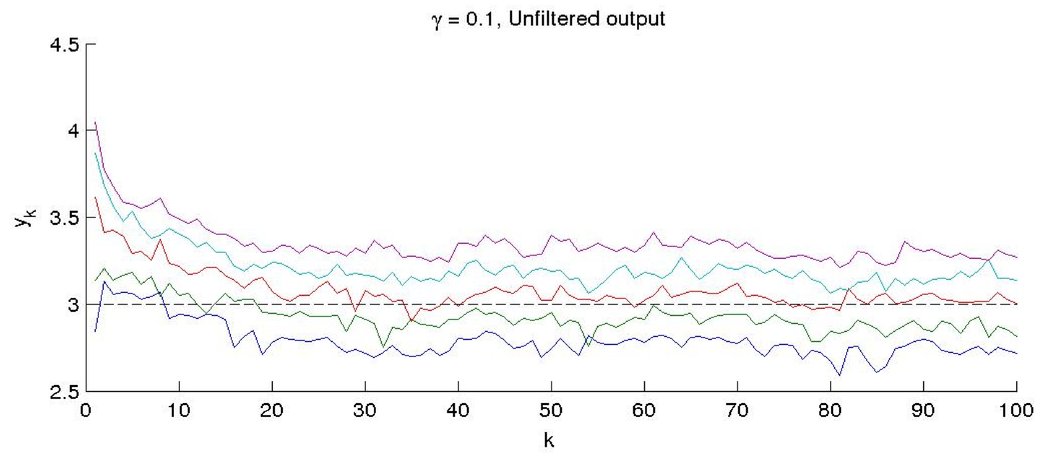
$$\nu_{k+1}^i = (1-\gamma)\nu_k^i - k_p \sum_{j\in\mathcal{N}_i} a_{ij}\left[y_k^i - y_k^j\right] \quad (1)$$

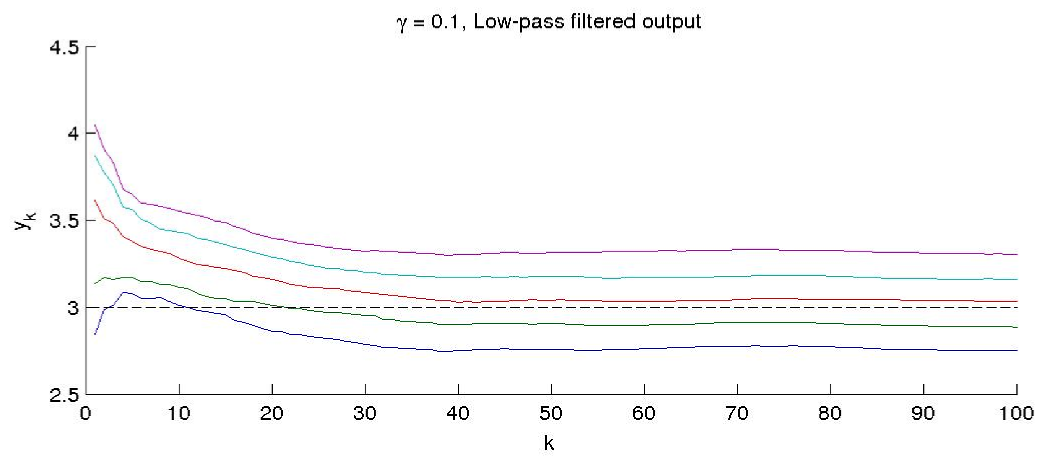
$$y_k^i = \nu_k^i + u_k^i. \quad (2)$$

Simulations over a 5-node network are shown below. Each plot shows the output of each agent and the dashed line shows the correct average.



The system reaches consensus when $\gamma = 0$, but the consensus value is not the correct average.





For $\gamma > 0$ the system is AME, but the statistical mean does not converge to the correct average.

PI Estimator

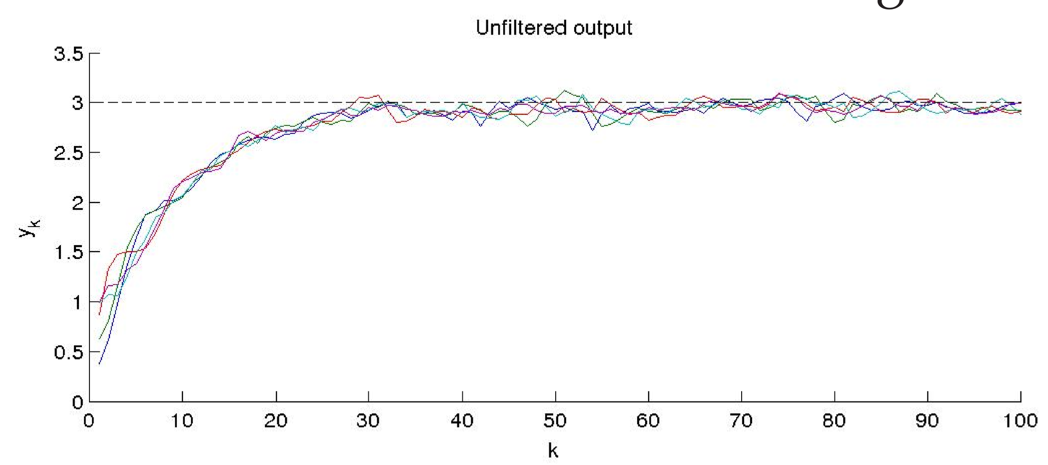
The PI estimator on agent i is implemented using the equations

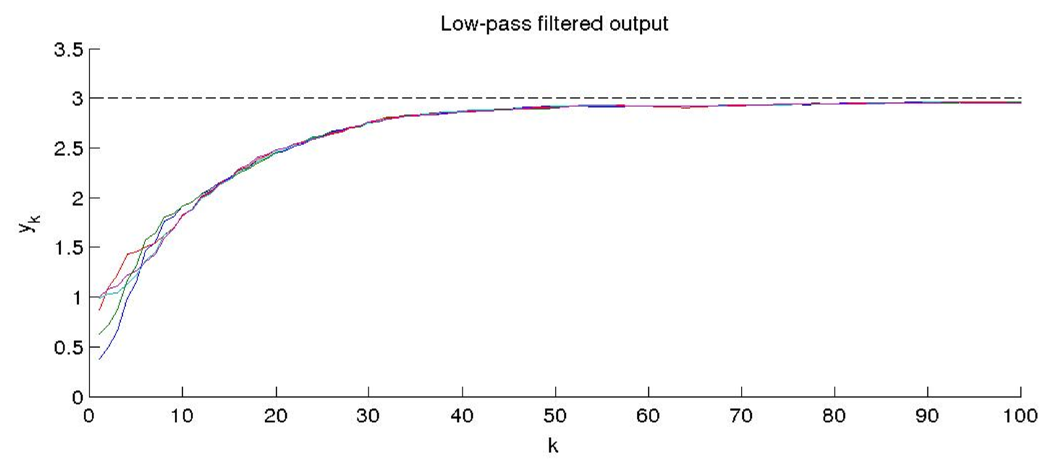
$$\nu_{k+1}^i = (1-\gamma)\nu_k^i + \gamma u_k^i - k_p \sum_{j\in\mathcal{N}_i} a_{ij}\left[\nu_k^i - \nu_k^j\right] - k_I \sum_{j\in\mathcal{N}_i} a_{ij}\left[\eta_k^i - \eta_k^j\right] \quad (3)$$

$$\eta_{k+1}^i = \eta_k^i + k_I \sum_{j\in\mathcal{N}_i} a_{ij}\left[\nu_k^i - \nu_k^j\right] \quad (4)$$

$$y_k^i = \nu_k^i. \quad (5)$$

Simulations over a 5-node network are shown below. Each plot shows the output of each agent and the dashed line shows the correct average.





The PI estimator has the AME property and the expected output converges to the correct average, so the filtered output converges with zero steady-state error. Therefore each agent achieves average consensus over time varying graphs.

Main Contribution

The main contribution from this work is the characterization of the asymptotic mean ergodicity (AME) property for average consensus estimators on random graphs. Results for the P and PI estimators are shown below.

Estimator	Simple	Scalable	Robust	Accurate	AME
P, $\gamma = 0$	Yes	Yes	No	Yes	No*
P, $\gamma \neq 0$	Yes	Yes	Yes	No	Yes
PI	Yes	Yes	Yes	Yes	Yes

General Result: The proven result is more general than the result for the P and PI estimators. We have characterized a general class of algorithms with the AME property for which the P and PI estimators are specific examples. These estimators can have an arbitrary number of state variables.

Future Work

We have now characterized an entire class of estimators which are simple, scalable, robust, accurate, and asymptotically mean ergodic. Future work will involve the optimization of the convergence rate over the entire class of estimators. This will produce average consensus estimators which meet all of the stated objectives.

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