

Tutorial-4

Q1. $T(n) = 3T(n/2) + n^2$

$$T(n) = aT(n/b) + f(n)$$

$$a=3 \quad b=2 \quad f(n)=n^2$$

$$c = \log_b a = \log_2 3 \approx 1.58$$

$$n^c = n^{1.58}$$

$$f(n) = n^2$$

$$f(n) > n^c$$

$$T(n) = \Theta(n^2)$$

Q2. $T(n) = 4T(n/2) + n^2$

$$c = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) = n^2$$

$$T(n) = n^c \log n$$

$$T(n) = \Theta(n^2 \log n)$$

Q3. $T(n) = T(n/2) + 2^n$

$$c = \log_2 1 = 0$$

$$n^c = 1$$

$$f(n) = 2^n$$

$$T(n) = \Theta(2^n)$$

Q4. $T(n) = 2^n T(n/2) + n^n$

$$a = 2^n$$

\Rightarrow Master's theorem not applicable as 'a' is not a constant.

Q5. $T(n) = 16T(n/4) + n$

$$c = \log_4 16 = 2$$

$$n^c = n^2 > f(n)$$

$$T(n) = \Theta(n^2)$$

Q6. $T(n) = 2T(n/2) + n \log n$

$$c = \log_2 2 = 1$$

$$n^c = n$$

$$f(n) = n \log n > n^c$$

$$T(n) = \Theta(n \log n)$$

Q7. $T(n) = 2T(n/2) + n/\log n$

$$c = \log_2 2 = 1$$

$$f(n) = n/\log n < n^c$$

$$T(n) = \Theta(n)$$

Q8. $T(n) = 2T(n/4) + n^{0.51}$

$$c = \log_4 2 = 0.5$$

$$n^c = n^{0.5}$$

$$f(n) = n^{0.51}$$

$$T(n) = \Theta(n^{0.51})$$

Q9. $T(n) = 0.5T(n/2) + 1/n$

$$c = \log_2 0.5 = -1$$

$$n^c = 1/n$$

$$f(n) = 1/n$$

$$T(n) = \Theta\left(\frac{1}{n} \log n\right)$$

Q10. $T(n) = 16T(n/4) + n!$

$$c = \log_4 16 = 2$$

$$n^c = n^2$$

$$f(n) = n!$$

$$T(n) = \Theta(n!)$$

Q11. $T(n) = 4T(n/2) + \log n$

$$c = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) = \log n$$

$$T(n) = \Theta(n^2)$$

Q12. $T(n) = \sqrt{n} T(n/2) + \log n$

→ Master's theorem not applicable as 'a' is not constant.

Q13. $T(n) = 3T(n/2) + n$

$$c = \log_2 3 = 1.5$$

$$n^c = n^{1.5}$$

$$f(n) = n$$

$$T(n) = \Theta(n^{\log_2 3})$$

Q14. $T(n) = 3T(n/3) + \sqrt{n}$

$$c = \log_3 3 = 1$$

$$f(n) = \sqrt{n}$$

$$n^c = n$$

$$T(n) = \Theta(n)$$

Q15. $T(n) = 4T(n/2) + cn$

$$c = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) = cn$$

$$T(n) = \Theta(n^2)$$

Q16. $T(n) = 3T(n/4) + n \log n$

$$c = \log_4 3 \approx 0.8$$

$$n^c = n^{0.8}$$

$$f(n) = n \log n$$

$$T(n) = \Theta(n \log n)$$

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

$$c = \log_2 1 = 0$$

$$n^c = 1$$

$$T(n) = \Theta(n(2 - \cos n))$$

Q17. $T(n) = 3T(n/3) + n/2$

$$c = \log_3 3 = 1$$

$$n^c = n$$

$$f(n) = n/2$$

$$T(n) = \Theta(n)$$

Q18. $T(n) = 6T(n/3) + n^2 \log n$

$$c = \log_3 6 = 1.6$$

$$n^c = n^{1.6}$$

$$f(n) = n^2 \log n$$

$$T(n) = \Theta(n^2 \log n)$$

Q19. $T(n) = 4T(n/2) + n/\log n$

$$c = \log_2 4 = 2$$

$$n^c = n^2$$

$$T(n) = \Theta(n^2)$$

Q20. $T(n) = 64T(n/8) - n^2 \log n$

\Rightarrow Master's theorem is not applicable as $f(n)$ is -ve.

Q21. $T(n) = 7T(n/3) + n^2$

$$c = \log_3 7 = 1.77$$

$$n^c = n^{1.77}$$

$$T(n) = \Theta(n^2)$$