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Tutorial - 6

Q1. What do you mean by minimum spanning tree? What are the applications of MST?

A1. A minimum spanning tree (MST) is a subset of the edges of a connected, edge-weighted undirected graph that connects all the vertices together, without any cycles & with minimum possible total edge weight.

Applications of MST:

i) Design of networks including computer networks, water supply networks
ii) Cluster analysis - clustering points in the plane
iii) Constructing trees for broadcasting in computer networks.

Q2. Please analyse the time & space complexity of Prim, Kruskal, Dijkstra & Bellman ford algorithm.

A2. Kruskal's Algorithm
Time complexity - $O(|E| \log |E|)$
Space complexity = $O(|V|)$.

Prim's Algorithm
Time complexity = $O(|E| \log |V|)$
Space complexity = $O(|V|)$.

Dijkstra Algorithm
Time complexity = $O((|V|+|E|) \log V)$
Space complexity = $O(|V|+|E|)$.

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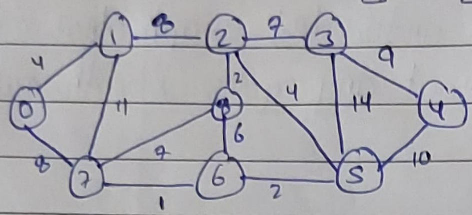
Bellman Ford Algorithm

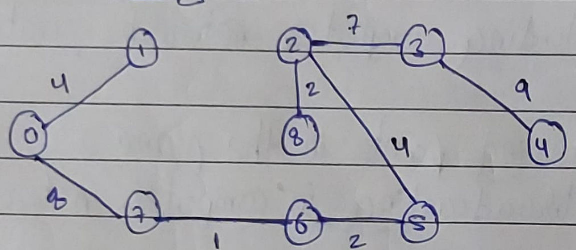
Time complexity = O(VE)

Space complexity = O(V).

Q3. Apply Kruskal & Prim's algo on graph to compute MST & its weight?

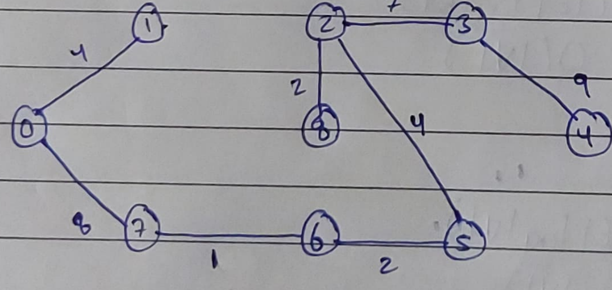


Kruskal's Algorithm



Weight = 4+8+1+2+4+2+7+9 = 37.

Prim's Algorithm



Weight = 37.

Q4. Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in the modified graph in following cases?

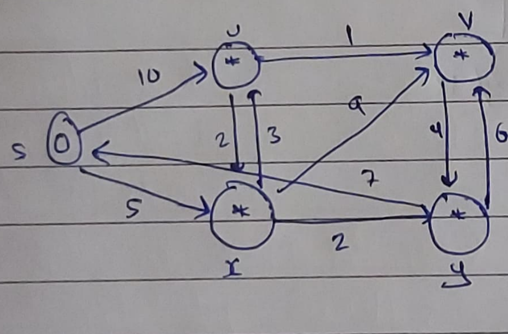
i). If weight of every edge is increased by 10 units.

Ans. The shortest path may change because there may be different no. of edges in different paths from s to t.

ii). If weight of every edge is multiplied by 10 units.

Ans. The shortest path doesn't change because weights of all paths from s to t get multiplied by same amount.

Q5. Apply Dijkstra & Bellman algorithm on graph to compute shortest path to all nodes from node S.



Dijkstra's Algorithm

Source	Destination			
s	u	v	x	y
	∞	∞	∞	∞
	10	∞	5	∞
s, x	8	14	5	7
s, x, y	8	13	5	7
s, x, y, u	8	9	5	7

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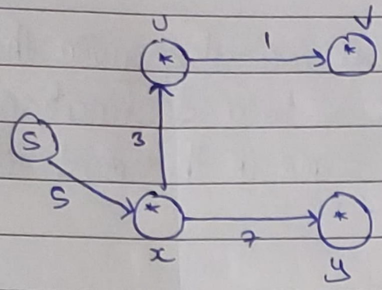
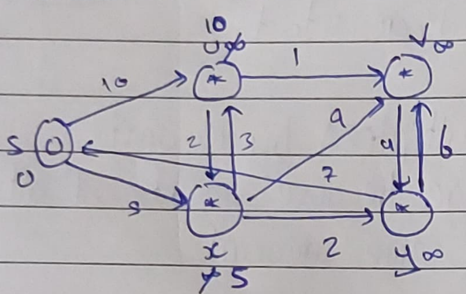
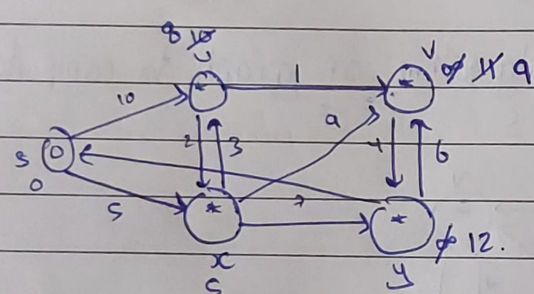


Bellman Ford Algorithm





$s \rightarrow u \Rightarrow 8$

$s \rightarrow x \Rightarrow 5$

$s \rightarrow v \Rightarrow 9$

$s \rightarrow y \Rightarrow 12.$

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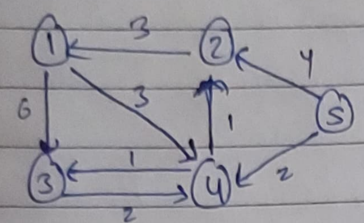
Q6. Apply all pair shortest path algorithm - Floyd Warshall on graph & also analyse time & space complexity of algorithm.



$$D^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$D^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$D^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$D^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 4 & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

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$$D^4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & 6 & 3 & 0 & 2 & \infty \\ 4 & 4 & 1 & 1 & 0 & \infty \\ 5 & 6 & 3 & 3 & 2 & 0 \end{array}$$

$$D^5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & 6 & 3 & 0 & 2 & \infty \\ 4 & 4 & 1 & 1 & 0 & \infty \\ 5 & 6 & 3 & 3 & 2 & 0 \end{array}$$

Time Complexity = $O(n^3)$

Space Complexity = $O(n^2)$.