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Tutorial-2

Q1. What is the time complexity of below code and how?

```
void fun (int n) {
    int j=1, i=0;
    while (i<n) {
        i=i+j;
        j++; }}
```

A1. Values after execution of while loop

Ist time	$i = 1$
2nd time	$i = 3 = 1+2$
3rd time	$i = 6 = 1+2+3$
4th time	$i = 10 = 1+2+3+4$

let for ith time $i = (1+2+3+\dots i) < n$

$$= \frac{i(i+1)}{2} < n$$

$$= i^2 < n$$

$$\boxed{i = sqrt(n)}$$

$\therefore T = O(sqrt\ n)$.

Q2. Write recurrence relation for the recursive function that prints fibonacci series. Solve the recurrence relation to get complexity of the program. What will be the space complexity of this program & why.

A2. $fib(n) = fib(n-1) + fib(n-2)$

```
int fib (int n)
{
    if (n<=1)        — O(1)
```

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return n;

return fib(n-1) + fib(n-2); — $T(n-1) + T(n-2)$

}

$$T(n) = T(n-1) + T(n-2) + 1$$

fib(n)

fib(n-1) fib(n-2)

$\cancel{F(n) = 2F(n-2)+1}$ $\cancel{[\text{let } T(n-1) \approx T(n-2)]}$

$\cancel{T(n-2) = 2*(2F(n-2-2)+)+1}$

$\cancel{= 2*(2F(n}$

fib(n) — 1

fib(n-1) fib(n-2) — 2

fib(n-2) fib(n-3) fib(n-3) fib(n-4) — 4

$1+2+4+8+\ldots$

$a=1 \quad r=2$

$$\Rightarrow \frac{a(r^{\text{terms}}-1)}{r-1}$$

$$= 2^{\text{terms}} - 1$$

$$= 2^{n+1} - 1$$

$$\Rightarrow 2^{n+1}$$

$$\boxed{T = O(2^n)}$$

There is one entry in stack at every function call and it remains inside stack till it returns the value. Maximum entry at any instance $<= n$.

$\boxed{\therefore \text{Space Complexity} = O(n)}$

Q3. Write programs which have complexity - $n \log n$, n^3, $\log(\log n)$.

- n logn

```
for (i=1; i<=n; i*=2)
{
    for (j=1; j<=n; j++)
        sum = sum+i;
}
```

i	j
1	n
2	n
4	n
:	:
logn	n

$T = O(n \log n)$

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- $\underline{n^3}$

```
for (i=0; i<n; i++)
{
    for (j=0; j<n; j++)
    {
        for(k=0; k<n; k++)
            sum+=k;
    }
}
```

- $\underline{\log \log n}$

```
for (j=1; j<n; j*=2)
{
    for (k=j; k>=1; k/=2)
        sum+=j;
}
```

Q4. Solve the following recurrence relation $T(n) = T(n/4) + T(n/2) + cn^2$

A4. $T(n) = 2T(n/2) + cn^2$ $\qquad T(n/2) \geq T(n/4)$

Using masters method

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1 \quad b \geq 1 \quad c = \log_b a$

Comparing n^c & $f(n)$ we get

$$c = \log_2 2 = 1$$

$$f(n) > n^c$$

$$T(n) = \theta(f(n))$$

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$\boxed{T = \theta(n^2)}$

Q5 What is the time complexity of following function fun()?

```
int fun (int n) {
    for (int i=1; i<=n; i++) {
        for (int j=1; j<n; j+=i) {
            // Some O(1) task
}}}
```

As.

for	i=1	j=1,2,3,4,.... n	(run for n times)
for	i=2	j=1,3,5....	(run for n/2 times)
for	i=3	j=1,4,7....	(run for n/3 times)

$T(n) = n + n/2 + n/3 + n/4 + \dots$

$= n(1 + 1/2 + 1/3 + 1/4 + \dots)$

$= n \int_1^n \frac{1}{x}$

$= n \log x \,/_1^n$

$= n \log n$

$\boxed{T(n) = O(n \log n)}$

Q6 What should be the time complexity of

```
for (int i=2; i<=n; i=pow(i,k))
{
        //some O(1) expressions
}
```

where k is a constant.

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A6. I^{st} iteration $i = 2$

II^{nd} iteration $i = 2^k$

III^{rd} iteration $i = 2^{k^2}$

⋮

n^{th} iteration $i = 2^{k^i}$

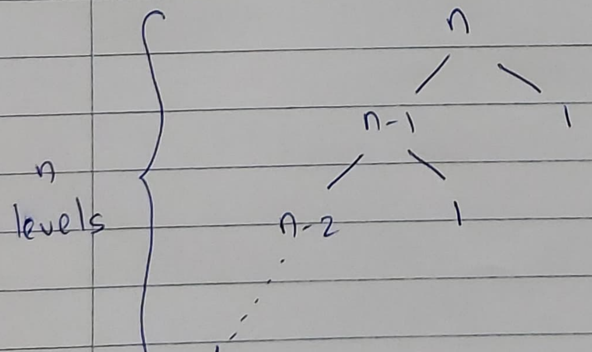
$$n = 2^{k^i}$$
$$\log n = \log 2^{k^i}$$
$$\log n = k^i \log 2$$
$$\log \log n = i \log k$$
$$i = \log_k \log n$$

$$\boxed{T = \log_k \log n}$$

Q7. Write a recurrence relation when quick sort repeatedly divides the array into two parts of 99% & 1%. Derive the time complexity in this case. Show the recursion tree while deriving time complexity & find the difference in heights of both the extreme parts. What do you understand by this analysis?

A7.

$$T(n) = T(n-1) + O(1)$$



$T(n) = [T(n-1) + T(n-2) + \dots + T(1) + O(1)] \times \underset{\substack{\uparrow \\ \text{for merging}}}{n}$

$T(n) = n \times n$

$\boxed{\therefore T(n) = O(n^2)}$

Lowest height = 2
Highest height = n

Difference b/w highest & lowest heights = $n-2$ $\quad n > 1$

Analysis - The given algorithm provides linear result in the form of sorted array.

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Q8. Arrange the following in increasing order of rate of growth.

a) n, n!, logn, loglogn, root(n), log(n!), nlogn, log^2(n), 2^n, 2^(2^n), 4^n, n^2, 100

b) 2(2^n), 4n, 2n, 1, logn, log(log(n)), $\sqrt{\log n}$, log2n, 2logn, n, log(n!), n!, n2, n logn.

c) 8^(2n), $\log_2 n$, $n\log_6 n$, $n\log_2 n$, log(n!), n!, $\log_8(n)$, 96, 8n2, 7n3, 5n.

A8. a) $100 < \log(\log n) < \log n < \log^2 n < root(n) < n < n\log n < n^2 < 2^n < 4^n < 2^{2^n} < \log(n!) < n!$

b) $1 < \log(\log(n)) < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < 2n < 4n < n\log n < n^2 < \log(n!) < n! < 2(2^n)$.

c) $96 < \log_8 n < \log_2 n < 5n < n\log_6 n < n\log_2 n < n! < \log n! < 8^{2n}$