6/6/23

Banking.

Cos θ = Dot product
Scalar

Sin θ = Cross product
Vector

WORK

Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of force.

*** Necessary conditions for work done ***

① A force acts on the body.

② The point of application of the force moves in the direction of force.

Eg:- An engine pulls a train

A man goes up a hill

*** MEASUREMENT OF WORK DONE BY A CONSTANT FORCE ***

A

$$\boxed{W = FS}$$ (When force and displacement are in same direction)

B

$$W = F \cos\theta \, S$$

$$W = FS \cos\theta$$

$$W = F \cdot S$$

(Scalar)

(When force & displacement are inclined at an angle θ)

[redacted]

* Types of Work Done *

① Positive Work Done

If a force acting on a body has a component in the direction of displacement then the work done by the force is +ve.
(when θ is acute)

E.g :- When a body falls freely under gravity the work done by the gravity is positive

② Negative Work Done

If a force acting on a body has a compenent in the opposite direction of displacement. The work done is -ve.
(when θ is obtuse)

Eg :- ① When a body slides against a rough horizontal surface, the work done by the force of friction is -ve as its opposes the motion.

② When a body is lifted upwards, work done by gravity is -ve.

Zero Work Done

- Work done by force is zero if $f=0$, $S=0$ or $\theta = 90°$

Eg :- For a body moving in a circular path, the centripetal force and displacement are perpendicular to each other hence work done is 0.

* Work done in terms of Rectangular components *

$$W = F \cdot S$$

$$= (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})(S_x\hat{i} + S_y\hat{j} + S_z\hat{k})$$

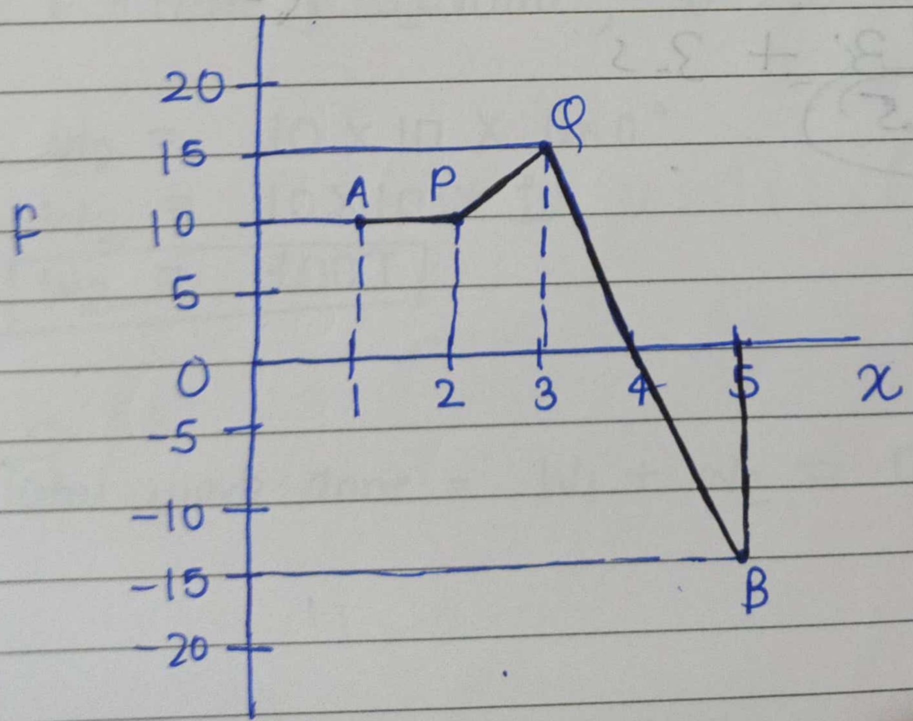
$$= F_xS_x + F_yS_y + F_zS_z$$

* Work Done by a Variable Force *

$$W = \int_{x_i}^{x_j} F\,dx$$

W = Area under Force Displacement Curve

(Q) A body moves from point A to B under the action of force varying as shown in ~~force~~ figure. Find the work done.



$$W = W_{12} + W_{23} + W_{34} + W_{45}$$

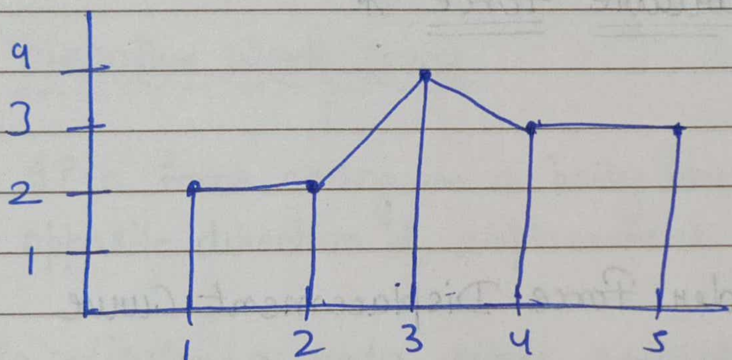
$$W = (10 \times 1) + \frac{1}{2}(10+15) \times 1 + \frac{1}{2} \times 1 \times 15 + \frac{1}{2} \times 1 \times (-15)$$

$$W = 10 + \frac{25}{2} + \cancel{\frac{1}{2} \times 15 \times 1} - \cancel{\frac{1}{2} \times 1 \times 15}$$

$$W = 22.5\ J$$

Pg - 6.7

(3)



7

$$(1 \times 2) + (1 \times 3) + \left(\frac{1}{2} \times (2+4)\right) + \left(\frac{1}{2} \times (3+4)\right)$$

$$2 + 3 + 3 + 3.5$$

$$11.5$$

(Q) A particle moves along the x axis from $x = 0$ to $x = 5\,m$ under the influence of a force given by $F = 7 - 2x + 3x^2$. Find work done in the process.

$$W = \int_0^5 F\,dx$$

$$W = \int_0^5 (7x - 2x + 3x^2)\,dx$$

$$W = \left[7x - \frac{\cancel{2}x^2}{\cancel{2}} + \frac{\cancel{3}x^3}{\cancel{3}}\right]_0^5$$

$$W = \left[7x - x^2 + x^3\right]_0^5$$

$$W = [35 - 25 + 125] - 0 - 0 + 0]$$ (upper limit − lower limit)

$$W = 135\,J$$

(Q) force $= 15 + 0.50x$ $\qquad x = 0$ to $x = 2$

$$\frac{x+1}{2}$$

$$W = \int_0^2 (15 + 0.50x^1)\,dx$$

$$W = \left[15x + \frac{0.50x^2}{2}\right]_0^2$$

$$W = \left[15x + \frac{\overset{25}{\cancel{50}}x^2}{100 \times \cancel{2}}\right]_0^2$$

$$W = 15x + 0.25x^2$$

$$30 + 1 - 0 - 0$$

$$= 31\,J$$

(8) $F = a + bx$

$x = 0$ to d.

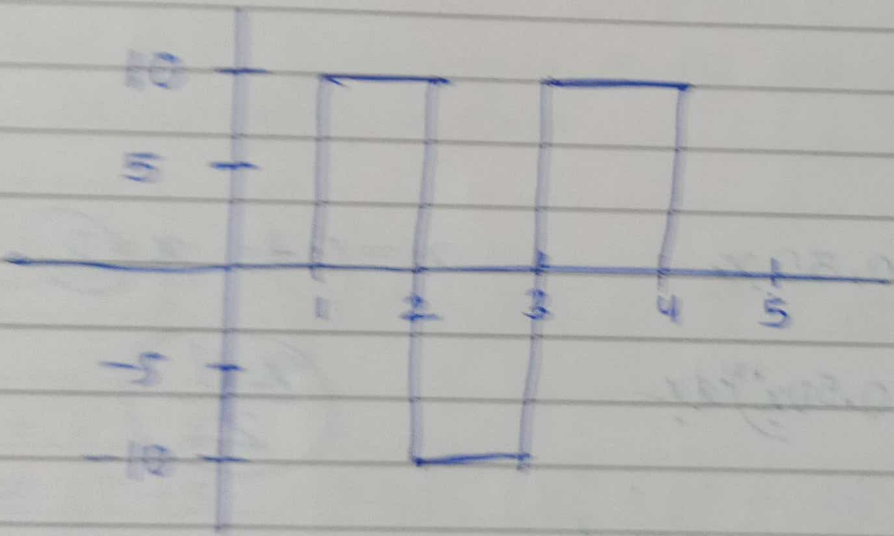
$W = \int_0^d (a + bx)\, dx$

$W = \int_0^d ax + \frac{bx^2}{2}$

$W = \left[ax + \frac{bx^2}{2} \right]_0^d$

$W = ad + \frac{bd^2}{2}$ J

(9)



$(10 \times 1) + (-10 \times 1) + (10 \times 1)$

10

(Q) The relation b/w displacement (x) and time (t) for a body of mass 2 kg moving under the action of force is given by $x = \frac{t^3}{3}$. Calculate the work done by the body in 1st second?

$\Rightarrow$ $V = \frac{dx}{dt} = \frac{\cancel{3}t^2}{\cancel{3}}$

$\boxed{V = t^2}$

$a = \frac{dv}{dt} = \boxed{2t}$

$\boxed{a = 2t}$

$t = 2\,sec \quad M = 2\,kg$

$F = ma$

$F = 2 \times 2t$

$F = 4t$

$W = \int_{x_1}^{x_2} F\,dx$

$W = \int_0^2 4t\,(v\,dt)$

$W = \int_0^2 4t \cdot t^2 dt$

$W = \int_0^2 4t^3\,dt$

$W = 4\int_0^2 t^3$

$W = \cancel{4}\left[\frac{t^4}{\cancel{4}}\right]_0^2$

$W = (2)^4 - (0)^4$

$W = 16J$

$\boxed{\text{Work} \Rightarrow \text{Scalar quantity}}$

* Energy *

Ability to do work is called Energy.

$$\text{Work} = \text{Energy} = F \times S = [ML^2T^{-2}]$$

Like Work, Energy is a Scalar quantity.

S.I unit of Energy is Joule (J)

Types :->

Mechanical Energy

Kinetic Energy | Potential Energy

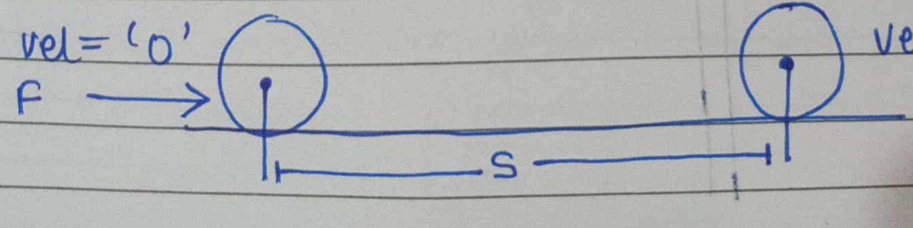
* KINETIC ENERGY *

The Energy possesed by a body by virtue of its motion is called Kinetic Energy.

Eg :- Moving hammer drives a nail into the wood.

* Expression for Kinetic Energy *

Suppose a constant force 'f' acts on a body produces displacement in its own direction



Let m = Mass of the body
u = Initial vel of the body
a = acceleration produce in the body
v = final velocity of the body

From 3rd equation of motion

$$v^2 - u^2 = 2as$$
$$v^2 - (0)^2 = 2as$$
$$\boxed{\frac{v^2}{2s} = a}$$

$$W = Fs$$
$$W = mas$$
$$W = m \times \frac{v^2}{2\cancel{s}} \times \cancel{s}$$

$$W = \frac{1}{2}mv^2$$

This work done is stored in the form of K.E.

$$K = \frac{1}{2}mv^2$$

2nd derivation next page

DOMS

• (Dot product) = Cos θ
× (Cross ") = Sin θ

* Kinetic Energy by Calculus method *

Consider a body of mass 'm' initially at rest. A force 'f' applied on the body produces a displacement 'ds' in its own direction ($\theta = 0$)
Then small work done is

$$dW = F.ds$$
$$= F ds \cos 0^\circ$$
$$dW = F ds$$
$$dW = ma\, ds$$
$$dW = m \frac{dv}{dt} ds \qquad [a = dv/dt]$$

$$dW = m.v\, dv$$

Integrating both sides

$$\int dW = \int_0^v mv\, dv$$

$$W = m \int_0^v v\, dv$$

$$W = m \left[\frac{v^2}{2}\right]_0^v$$

$$W = \frac{m}{2}\left[v^2\right]_0^v$$

$$W = \frac{m}{2}\left[v^2 - 0^2\right]$$

$$W = \frac{1}{2}mv^2$$

This work done is stored in the form of K.E

$$K = \frac{1}{2}mv^2$$

DOMS

Relation between Linear momentum & Kinetic Energy?

Linear momentum (P)

$$K = \frac{1}{2}mv^2 \qquad P = mv$$

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}\frac{mv^2}{m} \times m$$

$$K = \frac{1}{2m}m^2v^2$$

$$K = \frac{1}{2m}(mv)^2$$

$$\boxed{K = \frac{1}{2m}(P)^2}$$

$$\boxed{P = \sqrt{K2m}}$$

* WORK ENERGY THEOREM *

* It states that work done by the net force acting on a body is equal to the change produced in the kinetic Energy of the body.

PROOF (Expression) of work Energy theorem for a constant force

Suppose a constant force f acting on a body of mass 'm' produces acceleration 'a' in it. After covering distance 's' Suppose a velocity of the body changes from u to v.

From 3rd equation of motion

$$v^2 - u^2 = 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

$$W = Fs = mas$$

$$= m\left(\frac{v^2 - u^2}{2\cancel{s}}\right)\cancel{s}$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\boxed{W = K_f - K_i}$$ (Work done = final K.E − initial K.E)

Proof of Work Energy Theorem by a variable force ***

Supose a variable (change) force 'f' acts on the body of mass 'm' and produces displacement ds in its own direction ($\theta = 0°$) then small work done

$$dW = F.ds$$

$$dW = F ds \cos 0°$$

$$dW = F ds$$

$$dW = m a ds$$

$$dW = m \frac{dv}{dt} ds \qquad \left[\because a = \frac{dv}{dt}\right]$$

$$dW = m . v dv \qquad [v = ds/dt]$$

Integrating both sides

$$\int dW = \int_u^v m v dv$$

$$\int dW = m \int_u^v v dv$$

$$\int dW = m \left(\frac{v^2}{2}\right)_u^v$$

$$W = \frac{m}{2}(v^2 - u^2)$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\boxed{W = K_f - K_i}$$

H/W

Pg - 6.10 - 13, 14, 15, 16, 17, 21, 22

V.V. Imp

Potential Energy

It is the energy stored in a body or a system by virtue of its position in a field of force by its configuration (shape & size).

Eg :-

Due to position

① A body lying on the roof of a building.

② Water stored in a dam.

Due to configuration

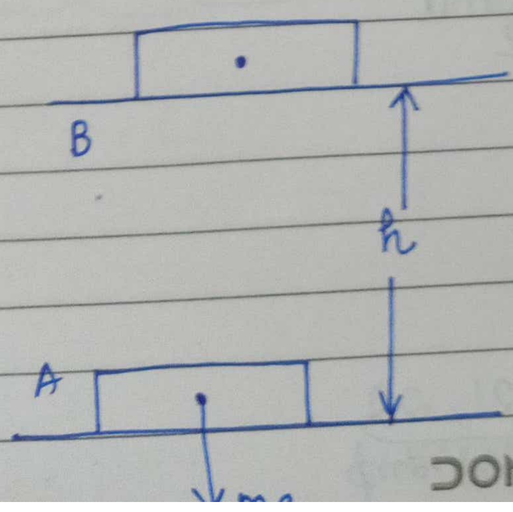
① In a toy car the ~~to~~ wound spring has potential energy which moves the toy car.

~ G.P.E (Gravitational Potential Energy) ~

It is the energy possesed by the body by virtue of its position above the surface of the earth.

Expression

Consider a body of mass 'm' lying on the surface of the earth



$F =$ weight of the body

$F = mg$

$W = FS$

$W = Fh$ (h is displacement here)

$$\boxed{W = mgh}$$

* This work done is stored in the form of G.P.E *

$$G.P.E = mgh$$

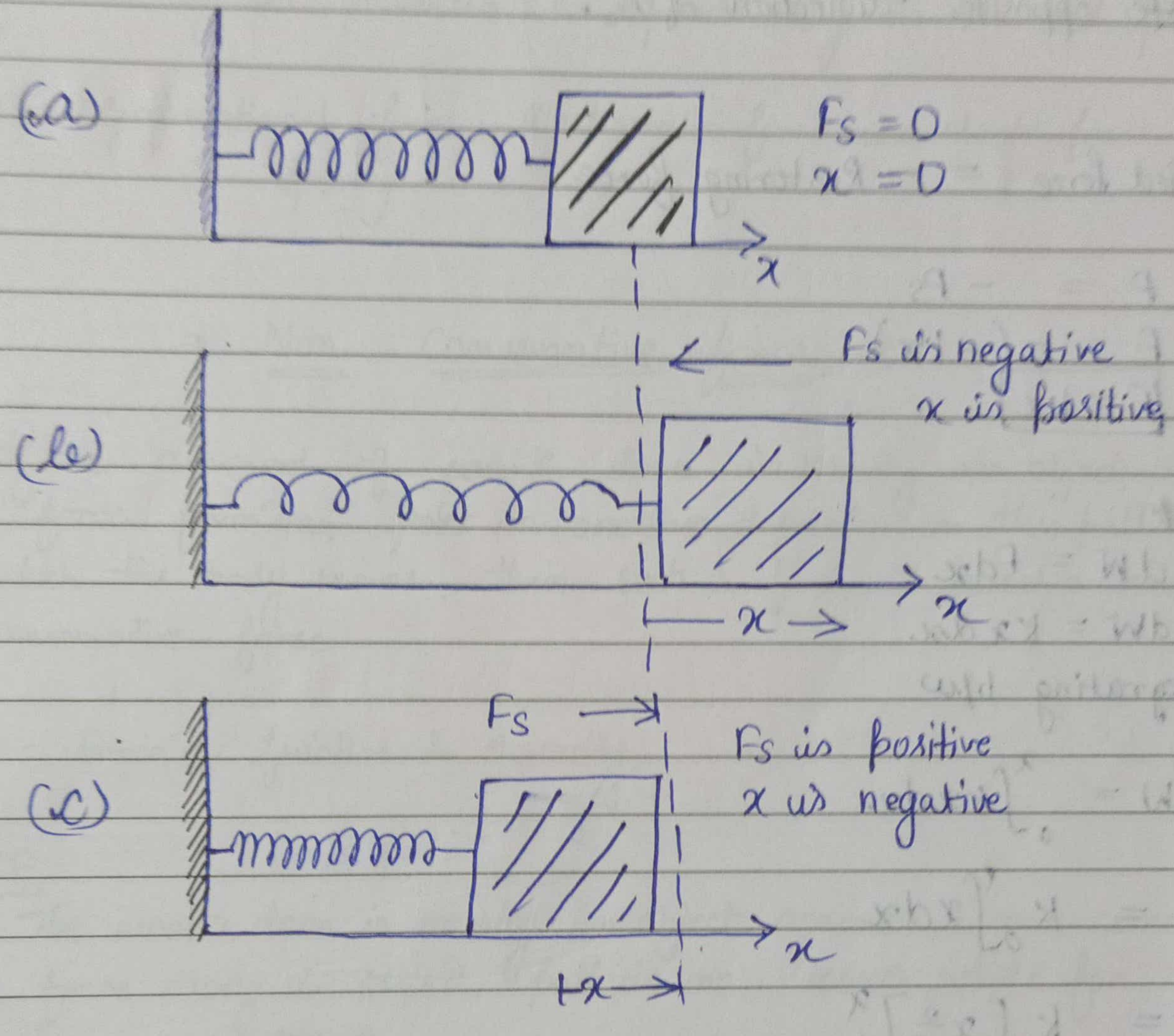
At the surface

$h = 0$

$$G.P.E = 0$$

SPRING

Potential Energy of a Spring



$x=0$

Consider an elastic spring of negligibly small mass with its one end attached to a rigid spot. Its other end is attached to a block of mass 'm' which can slide over a smooth horizontal surface.

If the block is pulled through distance x from equilibrium position a restoring force Fs is set up in the spring.

According to Hooke's law

$$F_s \propto x \qquad F_s = -kx$$

where k is the spring constant, the $-ve$ sign shows F_s acts in ~~diffe~~ opposite direction of x.

Now

Applied force $= -$ Restoring force

$$F = -F_s$$

$$F = -(-kx)$$

$$\boxed{F = kx}$$

Again

$$dW = F dx$$

$$dW = kx\,dx$$

Integrating b/w

$$\int dW = \int_0^x kx\,dx$$

$$W = k\int_0^x x\,dx$$

$$W = k\left[\frac{x^2}{2}\right]_0^x$$

$$W = \frac{kx^2}{2} \text{ or } \frac{1}{2}kx^2$$

This work done is stored as the elastic P.E (Potential Energy) (U)

$$\boxed{U = \frac{1}{2}kx^2}$$

* Conservative forces *

A Force is conservative if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particles & depends only on the end points.

Eg :- Gravitational force, Electrostatic force and elastic force of a spring.

* Non Conservative force *

If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force

Eg :- Force of friction & viscocity

NOTE

The work done in moving an object against a non-conservative force along a closed path is not zero while for " force is zero.

~~Conservations~~ Conservative and non-conservative forces

आप होते

DOMS

* Conservative Nature of Gravitational force *

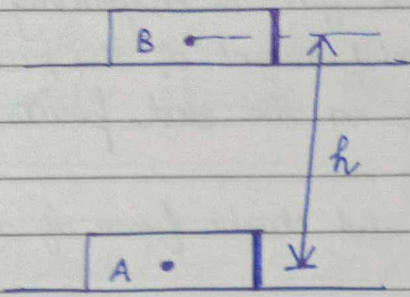


Figure A

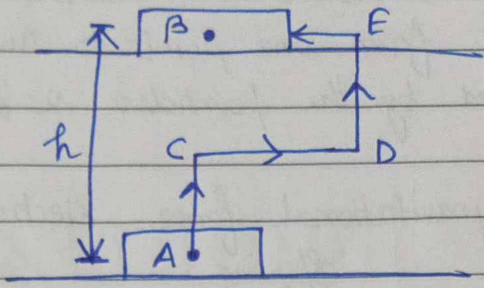


Figure B

Suppose a body of mass 'm' is raised to a height 'h' vertically upward from position A – B (figure A) Then work done against gravity

$$\boxed{W = mgh}$$

Now from figure B, body is taken from A-B along path A, C, D, E, B. During horizontal path CD and EB the force of gravity is perpendicular to the displacement, So work done is zero.

$$W = W_{AC} + W_{CD} + W_{DE} + W_{EB}$$

$$W = mg \times AC + 0 + mg \times DE + 0$$

$$W = mg(AC + DE)$$

$$\boxed{W = mgh}$$

Pg – 6.22 – Eg 35, 36, 37, 38

Power ~ ताकत~

It is defined as rate of doing work.
Power → Scalar quantity

Power $[ML^2T^{-3}]$

* Instantenous power *

It is defined as the limiting value of the average power as the time interval approaches to zero.

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$P = \frac{dW}{dt}$$

Power as Dot product

$$P = \frac{dW}{dt}$$

$$P = \frac{F \cdot ds}{dt}$$

* $\boxed{P = F \cdot V}$ Important (another formula of power.

Pg - 6.29 Eg:- 50, 51, 55,

__Conservation__ :- which do not depend on path

* Power Continue *

~ Collisions ~

A collision is said to occur b/w two bodies, either if they physically collide againt each other or if the path of one is affected by the force exerted by the other. For collision to take place, the actual physical contact is not necessary.

__Types__

① __Elastic collision__

If there is no loss of K.E during a collision, its is called an elastic collision.

__Characterstics__

1. The momentum is conserved
2. Total Energy is conserved
3. The K.E " "
4. Forces involved during the collision ~~is~~ are conservative
5. The mechanical energy is not converted into heat, light etc.

Eg :- Collision b/w subatomic particles, Collision b/w glass ball, etc.

② __Inelastic Collision__

If there is a loss of K.E during a collision, it is called an inelastic collision

Characteristics

1. The momentum is conserved
2. Total Energy " "
3. The K.E is not "
4. Some or all of the forces involved are non-conservative.
5. A part of the mechanical energy is ~~conver~~ converted into heat, light, sound etc.

Eg :- Collision b/w two vehicles, collision b/w a ball & floor.

(3) Perfectly inelastic collision

If two bodies stick together after the collision and move as a single body with a common velocity, then the collision is said to be perfectly inelastic collision.
In such collisions, momentum is conserved, but the loss of K.E is maximum.

Eg :- Mud thrown on a wall and sticking to it, a man jumping into a moving trolley, a bullet fired into a wooden block & remaining embedded in it, etc.

(4) Superelastic or explosive collision

In such a collision, there is an increase in K.E. This occurs if there is a release of P.E on an impact.

Eg :- Bursting of a cracker when it hits the floor forcefully, the collision of trolley with another may release a compressed spring & thereby releasing the energy stored in the spring

⑤ Head - on or one - dimensional collision

It is the collision in which the colliding bodies move along the same straight path before & after the collision.

Eg:- Collision b/w two railway compartments.

⑥ Oblique collision or two - dimensional collision

If two bodies do not move along the same straight line path but lie in the same plane before and after the collision, the collision is said to be oblique or 2 dimensional collision.

Eg - Collision b/w two carram coins

S test

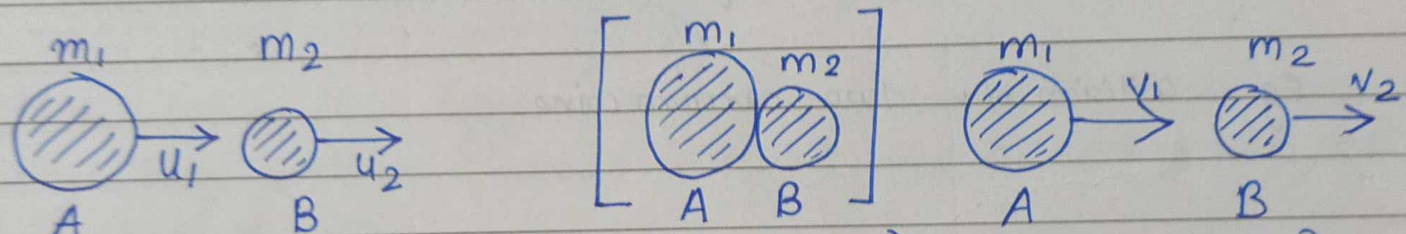
v.v. Imp

Elastic Collision in One Dimension

Consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively.

Let $u_1 > u_2$

After some time, the two bodies collide head-on and continue moving in the same direction with velocities v_1 and v_2 respectively. The two bodies will seperate after the collision if $v_2 > v_1$.



(Before collision) (During collision) (After collision)

As linear momentum is conserved in any collision,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{——— (1)}$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{——— (2)}$$

As K.E is also conserved

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\cancel{\frac{1}{2}}(m_1u_1^2 + m_2u_2^2) = \cancel{\frac{1}{2}}(m_1v_1^2 + m_2v_2^2)$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$
$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad \text{— (3)}$$

Dividing eq^n (3) by (2), we get

$$\frac{\cancel{m_1}(u_1+v_1)\cancel{(u_1-v_1)}}{\cancel{m_1(u_1-v_1)}} = \frac{\cancel{m_2}(v_2+u_2)\cancel{(v_2-u_2)}}{\cancel{m_2(v_2-u_2)}}$$

$$u_1 + v_1 = v_2 + u_2$$
$$u_1 - u_2 = v_2 - v_1 \quad \text{— (4)}$$ — यहाँ तक 3 marks का आता है।

Relative velocity of approach = Relative velocity of seperation.

From equation (4) we get

$$v_2 = u_1 - u_2 + v_1$$

Put in eq^n (1)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 - u_2 + v_1)$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 - m_2u_2 + m_2v_1$$
$$m_1u_1 + m_2u_2 + m_2u_2 - m_2u_1 = m_1v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

$$\frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2} = v_1$$

$$V_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2 u_2}{(m_1 + m_2)} \quad \text{--- (5)}$$

Similarly

$$V_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1 u_1}{m_1 + m_2} \quad \text{--- (6)}$$

यहाँ तक 5 marks का आता है

Special cases

(1) when $m_1 = m_2 = m$

From eq 5,

$$V_1 = \frac{(0)u_1}{2m} + \frac{2\cancel{m}u_2}{2\cancel{m}}$$

$$\boxed{V_1 = u_2}$$

From eq 6,

$$V_2 = \frac{(0)u_2}{2m} + \frac{2\cancel{m}u_1}{2\cancel{m}}$$

$$\boxed{V_2 = u_1}$$

(2) when $m_1 = m_2 = m$ & $u_2 = 0$ (stationary)

From eq 5,

$$V_1 = \frac{0 u_1}{2m} + \frac{2m(0)}{2m}$$

$$\boxed{V_1 = 0}$$

From eq 6

$$V_2 = \frac{(0)(0)}{2m} + \frac{2\cancel{m}u_1}{2\cancel{m}}$$

$$\boxed{V_2 = u_1}$$

(3) When a light body collides against a massive stationary body

$m_1 << m_2$, $u_2 = 0$

Neglecting m_1

From eq 5

$$V_1 = -\frac{\cancel{m_2}u_1}{\cancel{m_2}} + \frac{0}{m_2}$$

$$\boxed{V_1 = -u_1}$$

From eq 6

$$V_2 = \frac{m_2(0)}{m_2} + \frac{2 m_1 u_1}{m_2}$$

$$\boxed{V_2 \simeq 0}$$ (m_2 is very large)

(4) When a massive body collides against a light stationary body.

$m_1 >> m_2$, $u_2 = 0$ Neglecting m_2

From eq 5

$$V_1 = \frac{\cancel{m_1}u_1}{\cancel{m_1}} + 0$$

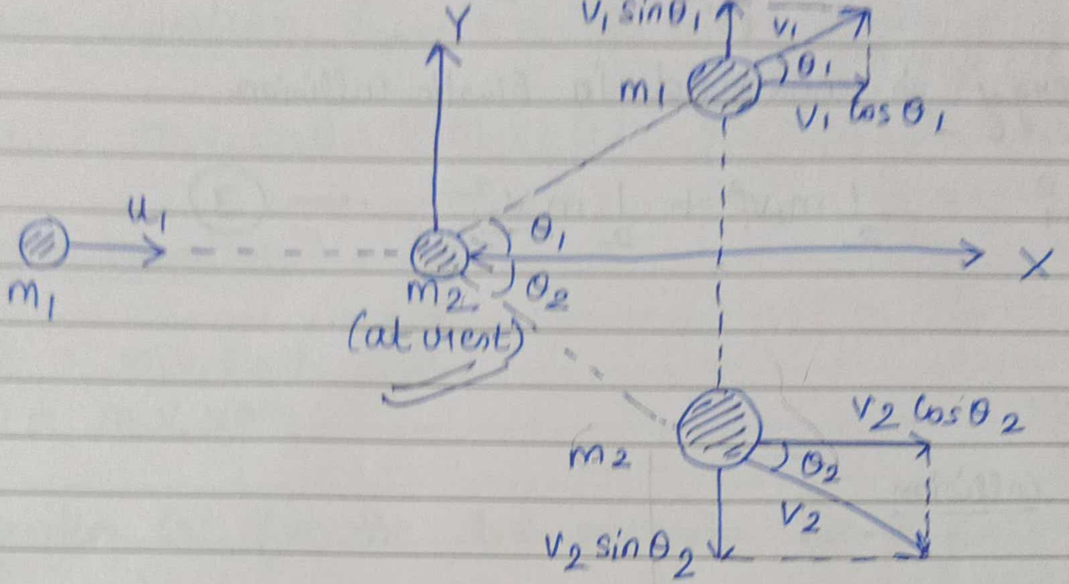
$$\boxed{V_1 = u_1}$$

From eq 6

$$V_2 = \frac{0}{m_1} + \frac{2\cancel{m_1}u_1}{\cancel{m_1}}$$

$$\boxed{V_2 = 2u_1}$$

Elastic Collision in Two Dimensions



Suppose a particle of mass m_1 moving along X-axis with velocity u_1 collides with another particle of mass m_2 at rest. After the collision, let the two particles move with velocities v_1 & v_2, making angles θ_1 and θ_2 with x-axis.

After the collision, the rectangular components of m_1 ~~of the mom~~

(I) $m_1 v_1 \cos\theta_1$, along +ve X-axis
(II) $m_1 v_1 \sin\theta_1$, along +ve Y-axis

Now for m_2

(I) $m_2 v_2 \cos\theta_2$, along +ve X-axis
(II) $m_2 v_2 \sin\theta_2$, along −ve Y axis

According to law of conservation of momentum along x-axis

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{—— (1)}$$

Now, applying law of conservation of linear momentum along Y-axis

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \quad \text{—} \; (2)$$

cause its -ve Y axis

As kinetic Energy is conserved in Elastic Collision

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \text{—} \; (3)$$

Special cases

(1) Glancing collision

$\theta_1 \simeq 0^\circ \quad \theta_2 \simeq 90^\circ$

From equation (1) and (2)

(1) $m_1 u_1 = m_1 v_1 (1) + 0$

$\cancel{m_1} u_1 = \cancel{m_1} v_1$

$u_1 = v_1$

(2) $0 = m_1 v_1 (0) - m_2 v_2 (1)$

$0 = -m_2 v_2$

$0 = v_2$

Kinetic Energy of target particle $= \frac{1}{2} m_2 v_2^2 = 0$

(2) Head-on collision

$\theta_2 = 0°$

From equation 1,

$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2$ (1)

From equation 2,

$0 = m_1v_1\sin\theta_1 - 0$

$0 = m_1v_1\sin\theta_1$

Equation (3) for the K.E remains unchanged. (क्योंकि θ है ही नहीं)

(3) Elastic Collision of two identical particles $(m_1 = m_2 = m)$

According to law of conservation of linear momentum

$m\vec{u_1} = m\vec{v_1} + m\vec{v_2}$

$\vec{u_1} = \vec{v_1} + \vec{v_2}$ — ① (mass common & cancelled)

According to conservation of K.E

$\frac{1}{2}m\vec{u_1}^2 = \frac{1}{2}m\vec{v_1}^2 + \frac{1}{2}m\vec{v_2}^2$

$\vec{u_1}^2 = \vec{v_1}^2 + \vec{v_2}^2$ — ②

$\vec{u_1}\cdot\vec{u_1} = (\vec{v_1}+\vec{v_2})\cdot(\vec{v_1}+\vec{v_2})$ (Puting value of $\vec{u_1}$ from ①)

$\vec{u_1}\cdot\vec{u_1} = \vec{v_1}\cdot\vec{v_1} + \vec{v_1}\cdot\vec{v_2} + \vec{v_2}\cdot\vec{v_1} + \vec{v_2}\cdot\vec{v_2}$

$\vec{u_1}^2 = \vec{v_1}^2 + \vec{v_2}^2 + 2\vec{v_1}\cdot\vec{v_2}$

$\vec{u_1}^2 = \vec{u_1}^2 + 2\vec{v_1}\cdot\vec{v_2}$ (from ②)

$$0 = 2\vec{V_1} \cdot \vec{V_2}$$

$$0 = \vec{V_1} \cdot \vec{V_2}$$

$$\vec{V_1} \perp \vec{V_2} \qquad \theta = 90^\circ$$

<u>Coefficient of restitution or coefficient of resilience</u>

The coefficient of restitution gives a measure of the degree of restitution of a collision and is defined as ratio of the magnitude of relative velocity of seperation after collision to the magnitude of " " of approach before collision. It is given by

$$e = \frac{|v_1 - v_2|}{|u_1 - u_2|} = -\frac{v_1 - v_2}{u_1 - u_2}$$

The value of e depends on the materials of the coiliding bodies. For two glass balls, $e = 0.95$ and for the lead balls, $e = 0.20$.

The coefficient of restitution can be used to distinguish between the different types of collisions as follows:

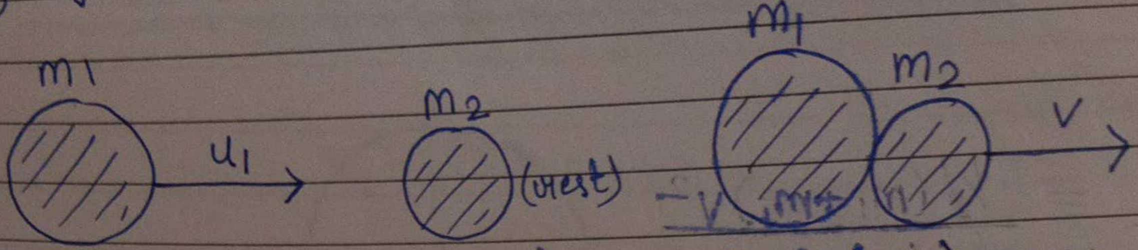
(i) For a perfectly elastic collision, $e = 1$ i.e, relative velocity of seperation is equal to the relative velocity of approach.

(ii) For an inelastic collision, $0 < e < 1$ i.e, the relative velocity of seperation is less than relative velocity of approach

(iii) For a perfectly inelastic collision, $e = 0$ i.e, the relative velocity of seperation is zero. The two bodies move together with a common velocity.

(iv) For a superelastic collision. $e > 1$ i.e., the kinetic energy incre

* Perfectly Inelastic Collision in One Dimensions *



Before collision ([illegible]) After ([illegible]) Collision

Consider a body of mass m_1 moving with velocity u_1 collides head-on with another body of mass m_2 at rest. After the collision, the two bodies move together with a common velocity v.

As linear momentum is conserved

$$m_1 u_1 + m_2 \times 0 = (m_1 + m_2) v$$

$$\boxed{\frac{m_1 u_1}{m_1 + m_2} = v}$$

The loss of Kinetic Energy

$$\Delta K = K_i - K_f = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) v^2$$

$$\Delta K = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u_1}{m_1 + m_2}\right)\left(\frac{m_1 u_1}{m_1 + m_2}\right)$$

$$\Delta K = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}\frac{(m_1^2 u_1^2)}{m_1 + m_2}$$

$$\Delta K = \frac{1}{2} m_1 u_1^2 \left(1 - \frac{m_1}{m_1 + m_2}\right)$$

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} u_1^2$$

This is a +ve quantity. The K.E is lost mainly in the form of heat and sound.

Also

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1+m_2)v^2}{\cancel{\frac{1}{2}m_1u_1^2}}$$

$$= \left(\frac{m_1+m_2}{m_1u_1^2}\right)\left(\frac{m_1u_1}{m_1+m_2}\right)\left(\frac{m_1u_1}{m_1+m_2}\right)$$

$$\frac{K_f}{K_i} = \left(\frac{m_1}{m_1+m_2}\right)$$

which is $K < 1$ i.e. K.E after collision is less than K.E before collision

Special case

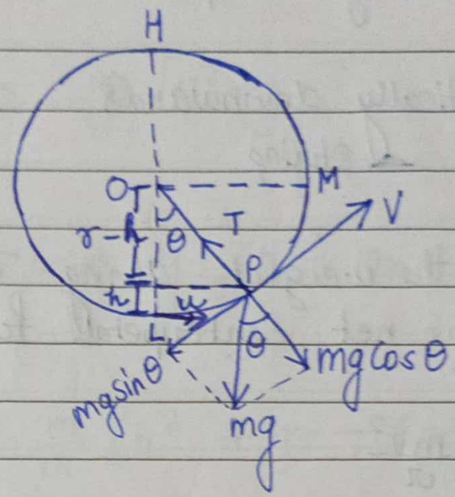
When $m_2 >> m_1$ then,

$$\frac{K_f}{K_i} \simeq 0 \quad \text{i.e.} \quad K_f \simeq 0$$

V. V. V. Imp Topic

Ex? __/__/__

* Motion in a Vertical Circle *



Consider a body of mass m tied to one end of a string and rotating in a vertical circle of radius 'r',

Velocity at any point

Suppose the body passes through lowest point L with velocity u and through any point P with velocity v. In moving from L to P, it has moved up through a vertical height LN = h

According to the law of conservation of energy

$$(K.E + P.E) \text{ at } L = (K.E + P.E) \text{ at } P$$

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgh$$

$$u^2 = v^2 + 2gh$$

$$v^2 = u^2 - 2gh$$

$$v = \sqrt{u^2 - 2gh} \quad \text{—— (1)}$$

This eq gives velocity v of the body at any point.

* Tension along the string at any point *

The forces acting on the body at point P are

① Weight mg acting vertically downwards
② Tension T along the string

The component $mg\cos\theta$ of the weight along the string acts opposite to T, so that the net centripetal force is

$$T - mg\cos\theta = \frac{mv^2}{r}$$

$$T = mg\cos\theta + \frac{mv^2}{r} \quad — (2)$$

From right angled ΔOPN

$$\cos\theta = \frac{ON}{OP} = \frac{r-h}{r} \quad — (3)$$

Using eq (1) and (3), eq (2) becomes

$$T = mg\,\frac{r-h}{r} + \frac{m}{r}(u^2 - 2gh)$$

$$T = \frac{m}{r}(gr - gh + u^2 - 2gh)$$

$$T = \frac{m}{r}(u^2 + gr - 3gh) \quad — (4)$$

This eq gives tension along the string at any point of the circle

—/—/—

(a) Tension in the string at the bottom and the top

At the lowest point L, $h = 0$, hence the tension in the string is

$$T_L = \frac{m}{r}[u^2 + gr] \quad — (5)$$

At the highest point H, $h = 2r$, hence the tension in the string is

$$T_H = \frac{m}{r}(u^2 + gr - 6gr)$$

$$= \frac{m}{r}(u^2 - 5gr)$$

Now

$$T_L - T_H = \frac{m}{r}[(u^2 + gr) - (u^2 - 5gr)] = 6mg$$

Thus the difference in tensions at the lowest and the highest points is equal to 6 times the weight of the revolving body.

(b) Minimum velocity of projection at the lowest point for looping the loop

The body will be able to cross the highest point H without any slackening of the string if T_H is +ve i.e,

$$T_H \geq 0 \quad \text{or} \quad \frac{m}{r}(u^2 - 5gr) \geq 0$$

$$u^2 \geq 5gr \quad \text{or} \quad u \geq \sqrt{5gr}$$

Hence $\sqrt{5gr}$ is the minimum velocity which the body must possess at the bottom of the circle so as to go round the circle completely i.e, for looping in the loop

Ⓒ <u>Minimum velocity at the top</u>

If V is the velocity which the body possesses at highest point H in just the case of no slacking of the string, then

$$V^2 = u^2 - 2g \cdot 2r \quad [\because h = 2r \text{, at the highest point}]$$

$$= 5gr - 4gr \quad [\because u = \sqrt{5gr}]$$

or $V = \sqrt{gr}$

This gives the minimum or critical velocity at the highest point.