

## CHAPTER - 4

# LAWS OF MOTION

### FORCE :

It may be defined as an agency which changes or tends to change the state of rest or of uniform motion, or direction of a body motion

### EFFECTS PRODUCED BY A FORCE :

1. Force can change speed of an object
2. Force can change the direction of an object
3. Force can change shape of an object

### INERTIA :

The inherent property of a body by virtue of which it cannot change by itself its state of rest or of uniform motion in a straight line is called Inertia

### TYPES OF INERTIA :

1. Inertia of Rest
2. Inertia of Motion
3. Inertia of direction

### ▷ INERTIA OF REST :

The tendency of a body to remain in its position of rest is called Inertia of

Eg: A person standing in a bus falls backward when the bus suddenly stops. When the bus starts moving, the lower part of the body begins to move along the bus while the upper part of the body remains at rest due to inertia of rest. That's why a person falls backward when the bus starts.

### ➤ Inertia of Motion:

The tendency of a body to remain in its state of uniform motion in a straight line is called Inertia of Motion.

E.g. When a moving bus suddenly stops, a person sitting in it falls forward.

As the bus stops, the lower part of its body comes to rest along with the bus, while the upper part of the body remains in motion due to Inertia of motion, falls forward.

### ➤ Inertia of Direction:

The inability of a body to change by itself its direction of motion, is called inertia of direction.

E.g. When a bus takes a sharp turn, a person sitting in a bus experiences the force acting away from the centre of the curve path due to his tendency to move in original direction. He has to hold on a support to prevent himself from swaying away in turning bus.

Weight is a Vector quantity

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NOTE: Mass is the measure of Inertia

MOMENTUM: ( $P$ ) = mass  $\times$  velocity

It is equal to the product of mass and velocity of the body

$$\vec{P} = m \vec{V}$$

Unit of  $P$  =  $\text{Kg ms}^{-1}$

It is a vector quantity

Momentum =  $[MLT^{-1}]$

### NEWTON'S LAW OF MOTION

1<sup>st</sup> law of Motion:

Everybody continues in its state of rest or of uniform motion in a straight line unless it is compelled by some external force to ~~state~~ change that state

2<sup>nd</sup> law of Motion: linear

The rate of change of momentum of a body is directly proportion to the applied force and the change takes place in the direction of applied force.

3<sup>rd</sup> Law of Motion:

To every action, there is always an equal & opp. direction

NOTE: Newton's 1<sup>st</sup> law defines force

## MEASUREMENT OF FORCE FROM NEWTON'S 2<sup>nd</sup> LAW OF MOTION

If a body of mass ( $m$ ) is moving with velocity ( $v$ ) then its linear momentum is

$$P = mv$$

Differentiating b/s w.r.t time

$$\frac{dp}{dt} = \frac{d(mv)}{dt}$$

$$\frac{dp}{dt} = m \frac{dv}{dt}$$

$$\frac{dp}{dt} = ma$$

Acc<sup>n</sup> to Newton's 2<sup>nd</sup> law

$$F \propto \frac{dp}{dt}$$

$$F \propto ma$$

$$F = kma$$

If  $k=1$

$F = ma$

## IMPULSIVE FORCE :-

A large force acting for a short time to produce a finite change in momentum is called impulsive force.  
E.g.

Force exerted by a bat while hitting a ball

## IMPULSE OF A FORCE :-

Impulse is defined as the product of force and time for which it acts and is equal to total change in momentum

$$\text{Impulse} = \text{Force} \times \text{time} = \text{Total change in momentum}$$

## DERIVATION :-

Suppose a force ( $\vec{F}$ ) acts for a small time ( $dt$ ) then impulse is

$$d\vec{J} = \vec{F} dt$$

If we consider a finite interval of time from  $t_1$  to  $t_2$

$$d\vec{J} = \vec{F} dt$$

$$\int d\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$$

$\vec{F}_{av}$  is the average force

$$\vec{J} = \vec{F}_{av} (t)_{t_1}^{t_2}$$

$$\vec{J} = \vec{F}_{av} (t_2 - t_1)$$

$$\boxed{\vec{J} = \vec{F}_{av} \Delta t}$$

## Impulse-Momentum Theorem

According to Newton's 2<sup>nd</sup> law of Motion  
 $F = \frac{dp}{dt}$

$$F dt = dp$$

Integrating b/s with proper limits

$$\int_0^t f dt = \int_{p_1}^{p_2} dp$$

$$J = \left[ p \right]_{p_1}^{p_2}$$

$$J = p_2 - p_1$$

Impulse = Change in Momentum

FOR NUMERICALS

Area under force-time graph gives impulse

Application of Impulse

Impulse =  $F \times t$  = Change in momentum

For two different forces producing the same impulse

$$I = F_1 t_1 = F_2$$

## Applications of the Concept of Impulse

(i) A cricket player lowers his hands while catching a ball & when the ball is caught, the impulse received by the hands is equal to the product of the force exerted by the ball & the time taken to complete the catch. By moving the hands backwards, the cricketer increases the time of catch. The force exerted on his hands becomes much smaller and it does not hurt him.

(ii) A person falling from a certain height receives more injuries when he falls on a cemented floor than when he falls on a heap of sand. In both cases, the impulse or the total change in momentum is same. On the cemented floor, the person is stopped abruptly. So the cemented floor exerts a large force of reaction causing him severe injuries. When the person falls on a heap of sand, the sand yields under his weight. The person takes longer time to stop.



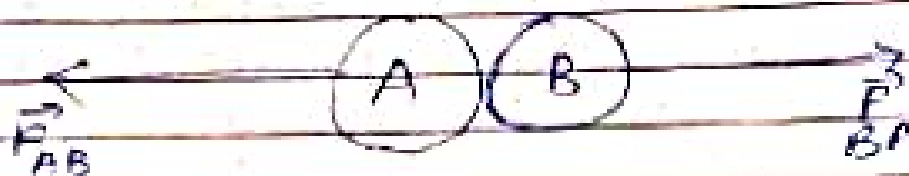
(iii) Automobiles (car, buses etc) are provided with shockers. When a vehicle moves on an uneven road, it receives a jerk. The shocker increases the time of Jerk & hence reduces the force. This makes journey comfortable and saves the automobile from damage due to bumps.

iv) Buffers are provided b/w the bogies of a train. Buffers increase the time of Jerk during shunting. This decreases the force of impact b/w the bogies. The bogies are thus prevented from receiving severe jerks.

v) ~~Chin~~ Chinawares are packed in straw paper before packing. The straw paper b/w the chinawares increases the time of experiences the jerks during transportation. Hence they strike against each other with a lesser force and are less likely to be damaged.

# Newton's 3<sup>rd</sup> Law of Motion

To every action there is always equal and opposite reaction



of  $\vec{F}_{BA}$  is the force exerted by a body A on B and  $\vec{F}_{AB}$  force exerted by a body B on A.

According to Newton's 3<sup>rd</sup> Law:

$$\vec{F}_{AB} = - \vec{F}_{BA}$$

Some Important points about Newton's Third Law of Motion

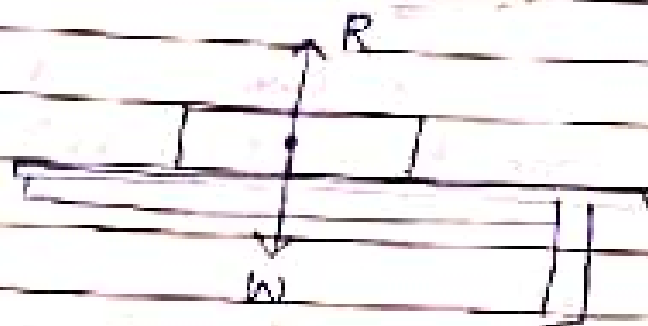
1. Action & Reaction always act on different bodies
2. It is applicable irrespective of the nature of the forces
3. No action can ~~be~~ <sup>occur</sup> in absence of Reaction
4. Forces of action & reaction cannot cancel each other this is because action & reaction ~~is~~ <sup>are</sup> though equal & opp. always act on diff. bodies so cannot balance each other

1.

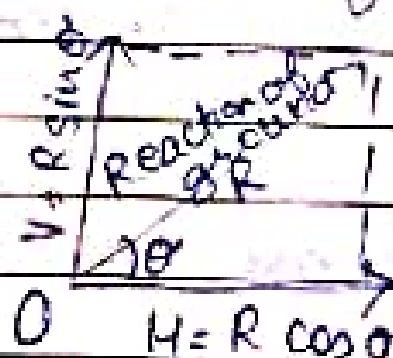
Book kept on a table: Consider a book of weight  $w$  resting on a table top. The book exerts a downward force (action) on the table equal to its own weight  $w$ . According to Newton's third law, the table also exerts an equal and upward force  $R$  (reaction) on the book such that

$$\vec{R} = -\vec{w}$$

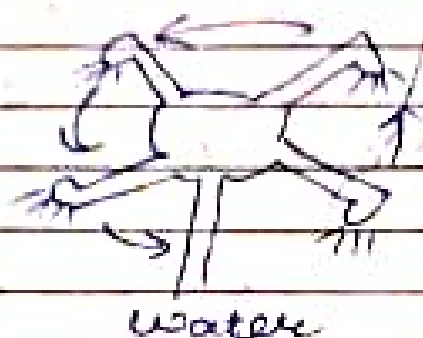
As the book is under the action of two equal and opposite forces, it remains in equilibrium.



2) While walking, we press the ground (action) with our feet slightly slanted in the backward direction. The ground exerts an equal and opp. force on us. The vertical component of the force of reaction balances our weight and the horizontal component enables us to move forward, as shown



- ③. Rotatory lawn sprinkler: The action of a rotatory lawn sprinkler is based on third law of motion. As water issues out of the Nozzle, it exerts an equal and opp. force in the backward direction, causing the sprinkler to rotate in the opp. direction. Thus water is scattered in all direction.



Pg. 5.16 from Book (other & r.)

## Law of Conservation of Linear Momentum

It states that when No external force acts on a system of several interacting particles<sup>net</sup>, the total linear momentum of the system is conserved.

### Derivation of Law of Conservation of Linear Momentum from Newton's 2<sup>nd</sup> Law of Motion

Consider an isolated system of  $n$  particles. Suppose the  $i$  particles have masses  $M_1, M_2, M_3, \dots, M_n$  and velocity  $V_1, V_2, \dots, V_n$  resp.

Then The total linear momentum

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 \dots \dots \vec{P}_n$$

$$P = m_1 v_1 + m_2 v_2 + m_3 v_3 \dots \dots + m_n v_n$$

If  $F$  is the external force acting on the system then According to Newton's 2<sup>nd</sup> law of Motion

$$\vec{F} = \frac{d\vec{P}}{dt}$$

$$0 = \frac{d\vec{P}}{dt}$$

$$P = \text{constant}$$

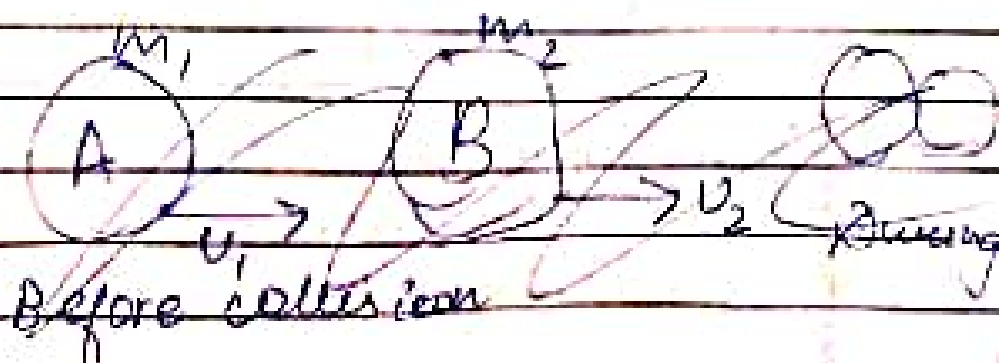
$$P_1 + P_2 + \dots \dots + P_n = \text{constant}$$

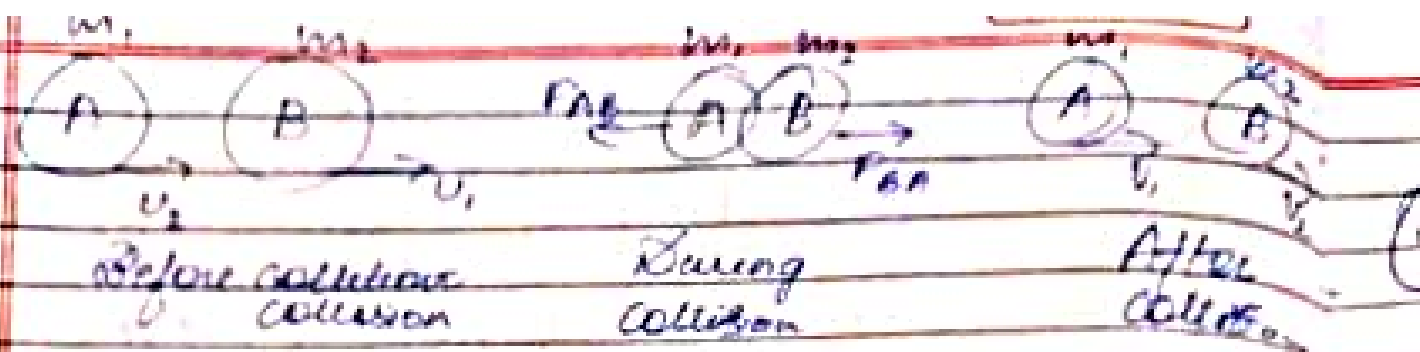
Derivation of Law of Conservation of Linear momentum from Newton's 3<sup>rd</sup> Law

Consider two bodies A and B of masses ' $m_1$ ' and ' $m_2$ ' moving in the same direction along a straight line with velocities  $u_1$  and  $u_2$  keep ( $u_1 > u_2$ )

They collide for time  $\Delta t$

Let velocity =  $v_1$  &  $v_2$





According to Newton's 3<sup>rd</sup> law :  
 $(F_{AB} = -F_{BA})$

where  $F_{AB}$  is the force Body 'A' exerts force on body B

Impulse of  $F_{AB}$

$$\text{Impulse of } \vec{F}_{AB} = \vec{F}_{AB} \times \Delta t = \text{Change in momentum of A} \\ = m_1 v_1 - m_1 u_1$$

$$\text{Impulse of } \vec{F}_{BA} = \vec{F}_{BA} \Delta t = \text{Change in momentum of B} \\ = m_2 v_2 - m_2 u_2$$

Now,

$$F_{AB} \Delta t = -F_{BA} \Delta t$$

$$m_1 v_1 - m_1 u_1 = -(m_2 v_2 - m_2 u_2)$$

$$m_1 v_1 - m_1 u_1 = -m_2 v_2 + m_2 u_2$$

$$\boxed{m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2}$$

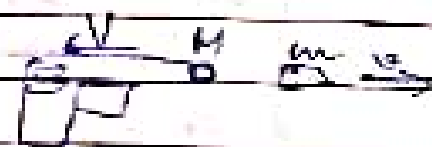
Total linear momentum after collision = Total L.M before collision

## Examples :

- ① Recoiling of a gun: Let 'M' be the mass of the mass of the gun and 'm' be the mass of the bullet. Before firing, both the gun & bullet are at rest. After firing, both the gun & the bullet are at rest. After firing, the bullet moves with velocity  $\vec{v}$  and the gun moves with velocity  $\vec{V}$ . As no external force acts on the system, so according to the principle of conservation of Momentum, Total momentum before firing = Total momentum after firing
- $$0 = m\vec{v} + M\vec{V}$$

$$M\vec{V} = -m\vec{v}$$

$$\vec{V} = -\frac{m}{M}\vec{v}$$



The negative sign shows that  $\vec{V}$  &  $\vec{v}$  are in opp. direction i.e. the gun gives a kick in the backward direction or the gun recoils with velo. of  $\vec{V}$ . Further as  $M \gg m$  so  $V \ll v$  i.e. the recoil velo. of the gun is much smaller than the forward velo. of the bullet.



- (ii) While firing a Bullet, the gun should be held tight to the shoulder. The recoiling gun can hurt the shoulder. If the gun is held tightly against the shoulder, then the body and the gun together will constitute one system. Total mass becomes large and recoil the velo. becomes small.
- (iii) When a man jumps out of a boat to the shore, the boat slightly moves away from the shore. Initially the total momentum of the boat & the man is zero. As the man jumps from the boat to the shore, he gains a momentum in forward dir. To conserve mom., the boat also gains an equal momentum in the opp. direction. So the boat slightly moves back.
- (iv) An astronaut in open space, who wants to return in the spaceship, throws some object in a direction opp. to the direction of motion of the spaceship. By doing so, he gains a momentum equal & opp. to that of the thrown object & so he moves toward the spaceship.

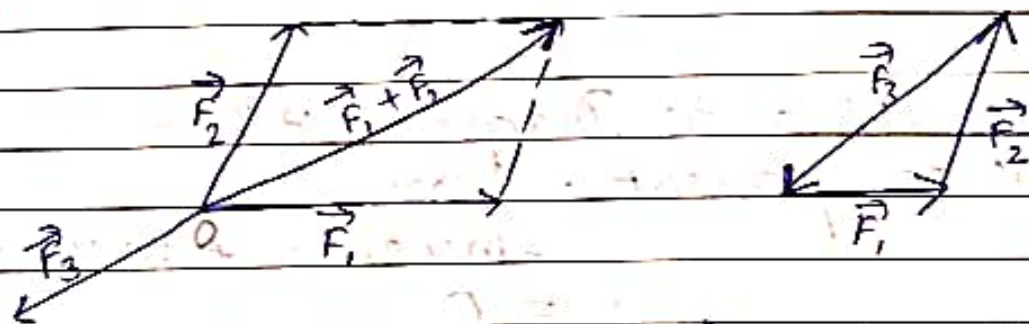


## Equilibrium of Concurrent Forces

Force acting at the same point <sup>on</sup> of a body are called concurrent forces. When a number of forces act on a body at a same ~~time~~ point and the net unbalance force is zero, the body will continue in its state of rest or of uniform motion along a straight line <sup>and</sup> are said to be in Equilibrium.

### Expression

Consider three concurrent forces  $\vec{F}_1, \vec{F}_2, \vec{F}_3$  acting at a same point at O as shown in fig.



Resultant of  $\vec{F}_1$  and  $\vec{F}_2 = \vec{F}_1 + \vec{F}_2$

OR If the third force  $\vec{F}_3$  acts on the body

such that  $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$

then, the body will be in Equilibrium

$$\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

For, no. of forces acting at same point then

$$\vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = 0$$

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2<sup>nd</sup> Law is the Real Law of motion  
because we can derive 1<sup>st</sup> & 3<sup>rd</sup> law from  
2<sup>nd</sup> law of motion

# 1<sup>st</sup> law is contained in 2<sup>nd</sup> law?  
According to Newton's 2<sup>nd</sup> law of Motion

$$F = ma$$

In the absence of Force

$$F = 0$$

$$ma = 0$$

$$m \neq 0 \quad a = 0$$

$$\frac{v-u}{t} = 0$$

$$v - u = 0$$

$$\boxed{v = u}$$

# 3<sup>rd</sup> law is contained in 2<sup>nd</sup> law?

Consider an isolated system of 2 bodies A & B.  
Suppose two bodies interact mutually with  
each other. According to Newton's 2<sup>nd</sup> law of  
motion

Force exerted by A on B

$$\vec{F}_{BA} = \frac{d\vec{P}_B}{dt}$$

Force exerted by B on A

$$\vec{F}_{AB} = \frac{d\vec{P}_A}{dt}$$



$$\vec{F}_{BA} + \vec{F}_{AB} = \frac{d}{dt} (\vec{P}_B + \vec{P}_A)$$

In the absence of any external force

$$\frac{d}{dt} (\vec{P}_B + \vec{P}_A) = 0$$

$$\vec{F}_{BA} + \vec{F}_{AB} = 0$$

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

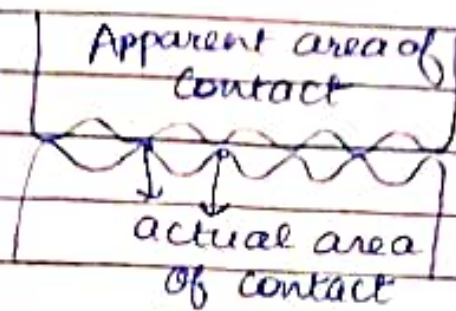
Action = - Reaction

## # Friction:

When a body moves or tends to move over a surface of another body a force comes into play which acts parallel to the surface of contact and opposes the relative motion this opposing force is called friction



## Cause of Friction :-



The force of friction is due to the atomic or molecular force of friction attraction b/w the two ~~for~~ surfaces at the pt. of actual contact. Due to the surface irregularities, the actual area of contact is much smaller than the apparent area of contact. The pressure at the point of contact is very large. Molecular bonds are ~~forced~~ formed at these point. Force sliding one body over the surface of other these bond has to be broken. The force required to break such bond is called force of friction.



## STATIC FRICTION, LIMITING FRICTION, Kinetic friction

Consider a wooden block placed over a horizontal table, the block is attached to a string which passes over a frictionless pulley and carries a scale pan at <sup>the</sup> free end. If we place a small weight in scale pan, a horizontal force act on the block and the block does not move. The force of friction comes into plane which balances the applied force.

➤ "The force of friction which comes into plane b/w two bodies for one body actually start moving over the other is called static friction"

(1/5)

As the applied force ~~or the applied force~~ on the block is increased the applied static friction also increases to balance the applied force and the block does not move. Once the applied force is increased beyond a certain limit, the block just begins to move. At this stage the static friction is maximum.

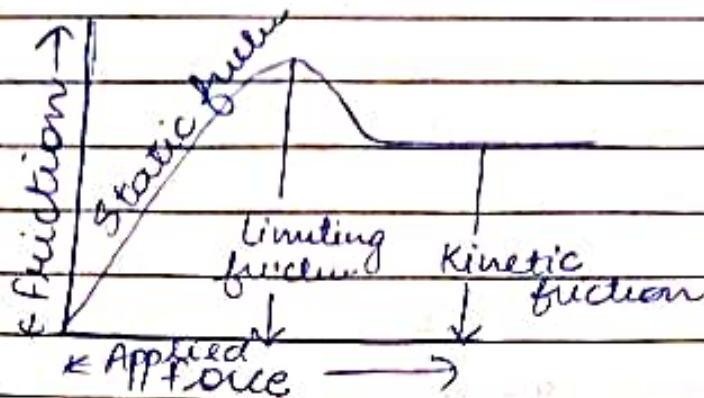
➤ The maximum force of static friction which comes into plane when a body just start moving over the surface of another body is called limiting friction ( $f_s^{\max}$ )

Once the motion has begun the force of friction decreases the smaller force is now necessary to maintain uniform motion.



- 22 The force of friction which comes into play when a body is in the state of steady motion over the surface of another body is called Kinetic or dynamic friction ( $f_k$ )

Graph



Acc<sup>n</sup> to Graph as the applied force increases the static friction increases accordingly to balance it. This shows that static friction is a self adjusting force

$$f_k < f_s^{\text{max.}}$$

## Laws of Limiting friction

1. The limiting friction depends on the nature of the surfaces in contact and their state of polish.
2. The limiting friction acts tangential to the two surfaces in contact and in a direction opp. to direction of motion of the body.
3. The value of limiting friction is independent of the area of the surface in contact so long as the normal reaction remains the same.
4. The limiting friction ( $f_s^{\max}$ ) is directly proportional to the normal reaction  $R$  between the two surfaces i.e.

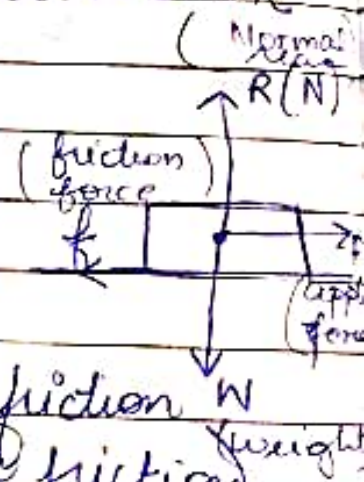
$$f_s^{\max} \propto R$$

$$f_s^{\max} = \mu_s R$$

where  $\mu_s$  is coefficient of limiting friction (Static friction)

It is defined as the ratio of limiting friction to the Normal reaction is called coeff. of limiting friction

$$\frac{f_s^{\max}}{R} = \mu_s$$





# Friction is necessary evil

## Advantages & disadvantages of friction

### Advantage

1. It is due to friction b/w ground and feet that we are able to ~~move~~ walk.
2. It is due to friction we can write.
3. The brakes of vehicle cannot work without friction.
4. Various part of machine are able to rotate because of belt and pulley.
5. The tyre of vehicle are ~~to~~ made rough to increase friction.

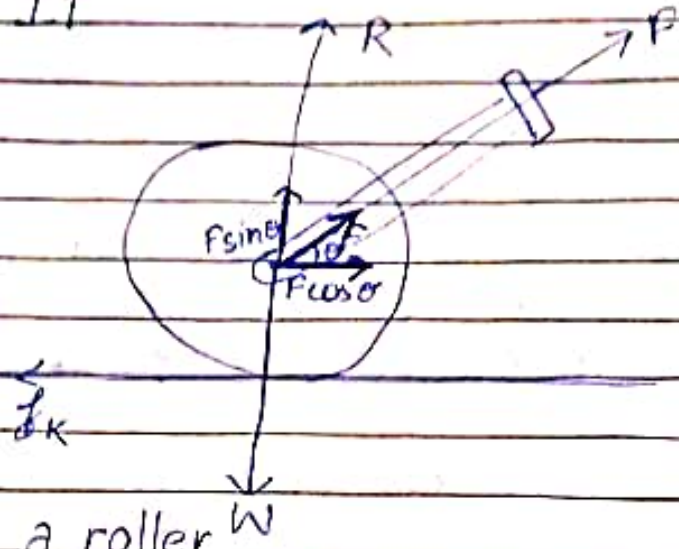
### Disadvantage

1. Wear & Tear of the machinery is due to friction.
2. A large amount of power is wasted in overcoming friction and the efficiency of the machine ~~of machines~~ decreases considerably.
3. Excessive friction b/w rotating parts of a machine produces enough heat and causes damage to the machinery.

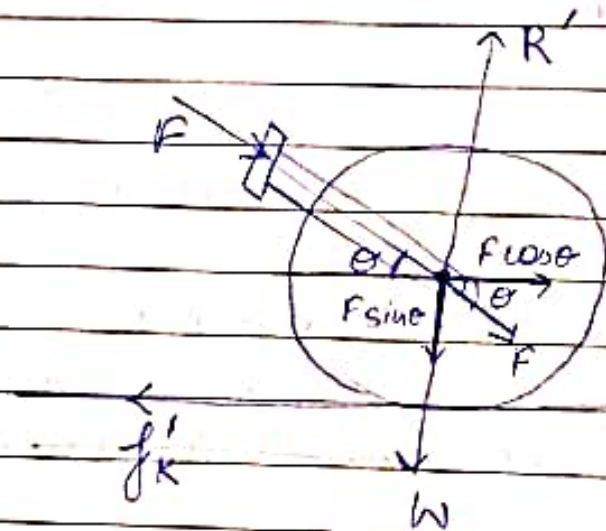
As friction has both advantage & disadvantage we can say that friction is necessary evil.



IT IS EASIER TO PULL A LAWN ROLLER THAN TO PUSH IT



(a) Pulling a roller



(b) Pushing a roller

Suppose a force ( $F$ ) is applied to pull a lawn roller of weight ( $W$ ).

$F$  has two rectangular component :

1. Horizontal component ( $F \cos \theta$ ) help to move roller forward
2. Vertical component ( $F \sin \theta$ ) <sup>acts</sup> ~~helps~~ <sup>helps</sup> in upward (pulling) and downward (pushing) <sup>pulling</sup> <sup>pushing</sup>

In case of pulling:

$$R + F \sin \theta = W$$

$$R = W - F \sin \theta$$

$$f_k = \mu_k R = \mu_k (W - F \sin \theta) \quad \text{--- (1)}$$

In case of pushing:

$$R' = W + F \sin \theta$$

$$f_k' = \mu_k R' = \mu_k (W + F \sin \theta) \quad \text{--- (2)}$$

Comparing (1) and (2)

$$f_k < f_k'$$

So it is easier to pull a lawn roller than to push it.

## ROLLING FRICTION :

The force of friction that comes into play when a body rolls over the surface of another body is called rolling friction. E.g. when a wheel rolls over a road rolling friction comes into the plane.

~~Whereas~~



## ROLLING FRICTION IS SMALLER THAN SLIDING FRICTION.

When a wheel rolls without slipping over a non-plane surface, the surfaces at contact do not rub each other. The relative velocity of the point of contact of the wheel with respect to the plane is zero if there is no slipping. There is no sliding or static friction in such an ideal situation. We need to overcome rolling friction only which is smaller than sliding friction.

### LAWS OF ROLLING FRICTION :

Experiments show that -

1. Rolling friction is directly proportional to the normal reaction.

$$f_r \propto R$$

2. Rolling friction is inversely proportional to the radius of the rolling cylinder or wheel.

$$f_r \propto \frac{1}{R}$$

Combining above two :-

$$f_r \propto \frac{R}{R}$$

$$f_r = \mu_r \frac{R}{R}$$

Where  $\mu_r$  is coefficient of rolling friction



# METHODS OF REDUCING FRICTION

1. By polishing the bumps and depression b/w surfaces in contact get minimised
2. Lubrication - It fills up depression b/w the surfaces in contact & hence reduces friction
3. Stream lining - Friction due to air is considerably reduced by streamlining shape of the body

## CENTRIPETAL FORCE $\Rightarrow$

$$F = ma$$

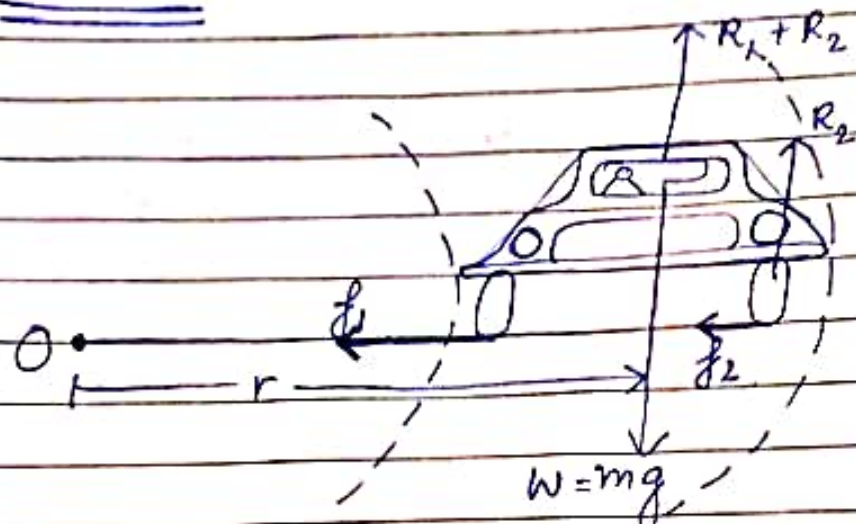
$$= \frac{mv^2}{r}$$

$$\text{or } = m\omega^2 r \quad \left( a = \frac{v^2}{r} = \omega^2 r \right)$$

A force required to make a body move along a circular path with uniform speed is called centripetal force.

It always acts along the radius and towards the centre of the circular path

## CAR ON A CIRCULAR LEVEL ROAD



$$R_1 + R_2 = mg$$

When a car negotiates a curved level road, the force of friction b/w the road & the tyres provides the necessary centripetal force required to keep the car in motion along the curve.

Consider a car of weight  $W = mg$  going around a circular level road of radius  $r$  with constant speed  $v$ .

$$\text{Now, } f_1 = \mu R_1$$

$$f_2 = \mu R_2$$

$$f = f_1 + f_2$$

$$= \mu R_1 + \mu R_2$$

$$= \mu (R_1 + R_2)$$

$$\underline{f = \mu mg}$$



For a car to stay on the road, <sup>the</sup> max. force of friction must be equal to or greater than centripetal force

$$\mu mg \geq \frac{mv^2}{r}$$

$$\mu \geq \frac{v^2}{rg}$$

$$v^2 \leq \mu rg$$

$$v \leq \sqrt{\mu rg}$$

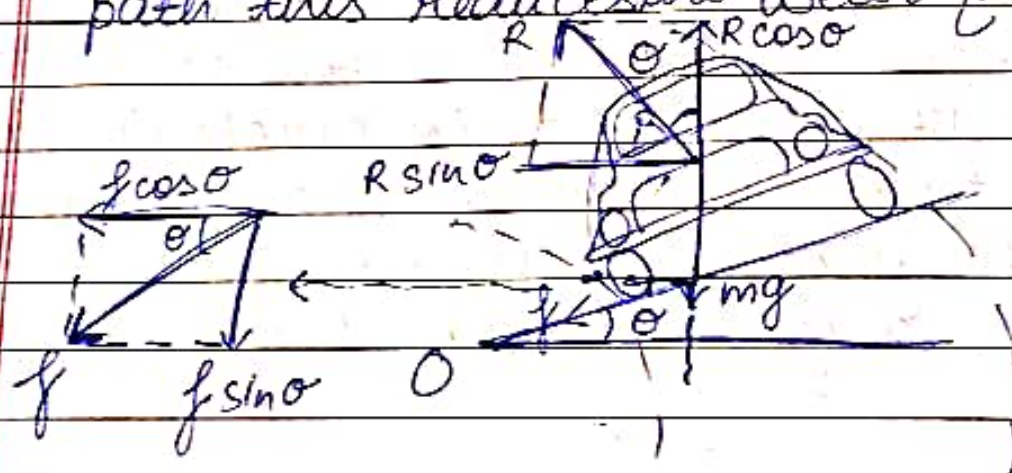
$$v_{\max} = \sqrt{\mu rg}$$

NOTE: If the speed exceeds  $v_{\max}$  the car will skid and go off the road in circle of radius greater than  $R$ . This is because the max available friction is insufficient to provide the necessary centripetal force.

## BANKING OF THE CURVED ROAD :-

The system of raising the outer edge of a curved road above its inner edge is called banking of the curved road. The angle through which the outer edge of the curved road is raised above the inner edge is called angle of banking.

When the circular road is bend, the hor. component of normal reaction of the road provides the necessary centripetal force for the vehicle to move it along the curved path this reduces the wear & tear of tyre.



### Derivation -

Consider a car of weight  $mg$  going along a curved path of radius ' $r$ ' with speed ' $v$ ' on a road. Bend at an angle  $\theta$

Weight  $mg$  acting vert. downward

Normal  $R_x \approx R$  acting at  $\angle \theta$  with vertical



$$R \sin \theta + f \cos \theta = \frac{mv^2}{r} \quad (1)$$

$$mg + f \sin \theta = R \cos \theta$$

$$R \cos \theta - f \sin \theta = mg \quad (2)$$

Dividing (1) by (2)

$$\frac{R \sin \theta + f \cos \theta}{R \cos \theta - f \sin \theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\frac{R \sin \theta + f \cos \theta}{R \cos \theta - f \sin \theta} = \frac{v^2}{rg}$$

Dividing Numerator & Denominator of LHS by  $R \cos \theta$

$$\frac{R \sin \theta + f \cos \theta}{R \cos \theta} = \frac{v^2}{rg}$$

$$\frac{R \sin \theta + f \cos \theta}{R \cos \theta} = \frac{v^2}{rg}$$

$$\frac{\tan \theta + \frac{f}{R}}{1 - \frac{f \tan \theta}{R}} = \frac{v^2}{rg}$$

$$\frac{\tan \theta + \mu}{1 - \mu \tan \theta} = \frac{v^2}{rg}$$

$$v^2 = rg \left( \frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right)$$

$$v = \sqrt{rg \left( \frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right)}$$

Special cases : When  $\mu = 0$  (when there is no friction)

$$\star \left( v = \sqrt{rg \tan \theta} \right) \star$$

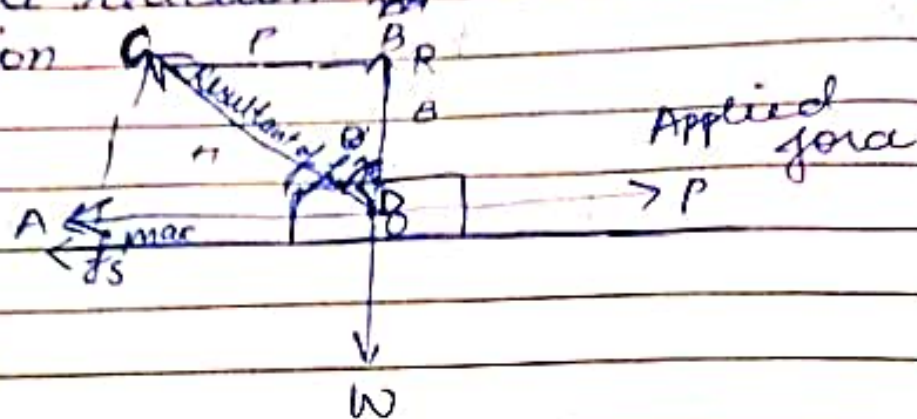
$$\text{Sq. b/s} \Rightarrow v^2 = rg \tan \theta$$

$$\frac{v^2}{rg} = \tan \theta \quad \theta \text{ is angle of banking}$$



## Angle of Friction :

It is defined as the angle which the resultant of the limiting friction and normal reaction makes with the normal reaction.



$$\tan \theta = \frac{BC}{DB}$$

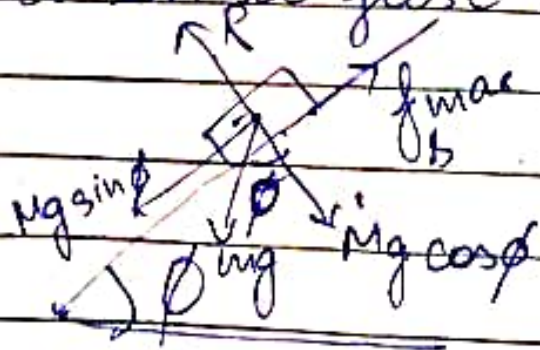
$$\tan \theta = \frac{BA}{DB}$$

$$\tan \theta = \frac{f_s^{\max}}{R}$$

$$\boxed{\tan \theta = \mu_s}$$

## Angle of Repose

It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide.



$$f_s = mg \sin \phi \quad (1)$$

$$R = mg \cos \phi \quad (2)$$

$$(1) \div (2)$$

$$\frac{f_s^{\max}}{R} = \frac{mg \sin \phi}{mg \cos \phi}$$

$$\boxed{\mu_s = \tan \phi}$$

NOTE:  $\mu_s = \tan \theta = \tan \phi$   
 $\theta = \phi$

### LAWS OF KINETIC FRICTION:

1. Kinetic friction opposes the relative motion and has a constant value depending on the nature of two surfaces in contact.
2. The value of the kinetic friction is independent of normal  $R$  remain the same.
3. The  $f_k$  does not depend on velocity provided the velocity is neither too large nor too small.
4.  $f_k$  is directly proportional to Normal  $R$  b/w 2 surfaces.  

$$f_k \propto R$$

$$f_k = \mu_k R$$

$$\boxed{\mu_k = \frac{f_k}{R}}$$

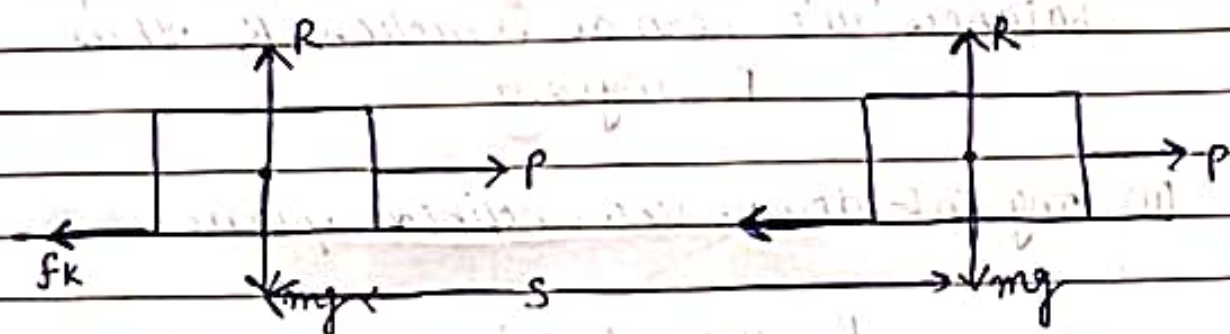


## LAWS OF MOTION

### # Work Done against Friction:

- ✓ work done in sliding a body over a horizontal surface.

Consider a body of weight  $mg$  resting on a rough horizontal surface.



work done against friction in moving the body through distance  $S$  will be

$$W = f_k \times S$$

But  $f_k = \mu_k R = \mu_k \cdot mg$

$$W = \mu_k \cdot mg \cdot S.$$

- ✓ work done in moving a body up an inclined plane

Suppose a body of weight  $mg$  is placed on an inclined plane, a force  $P$  is applied on the body so that it just begins to slide up the inclined plane.

- \_ / \_ / \_
- (ii) when lift moves downwards with acceleration  $a$ :  
Net downward force on man is

$$mg - R = ma$$

Apparent weight,  $R = m(g - a)$

$\therefore$  when lift accelerates downwards, apparent weight of man decreases.

- (iii) when the lift is at rest or moving with uniform velocity  $v$  downward/upward.  
The acceleration  $a = 0$ . Net force on the man is

$$R - mg = m \times 0 = 0$$

$$R = mg$$

$\therefore$  Apparent weight = Actual weight

- (iv) when the lift falls freely.  
If the supporting cable of the lift breaks, the lift falls freely under gravity.  
When  $a = g$ . The net downward force on the man is

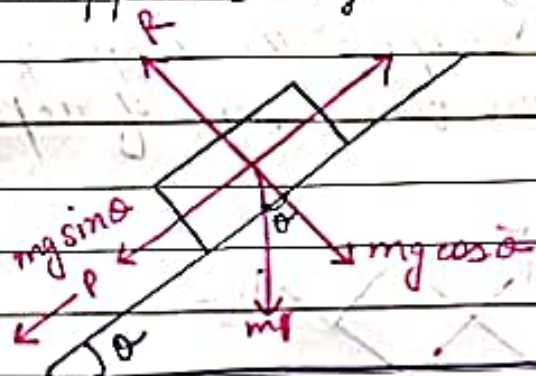
$$R = m(g - g) = 0$$

$\therefore$  Apparent weight = 0



### ✓ Work done in moving a body down an inclined plane.

Suppose a body of weight  $mg$  on an inclined plane. A force  $P$  is applied to just slide the body down the inclined plane.



The weight  $mg$  of the body has 2 components:

- (i)  $mg \cos \theta$  perpendicular to the inclined plane. It balances the normal reaction  $R$ . Thus,

$$R = mg \cos \theta$$

- (ii)  $mg \sin \theta$  down the inclined plane.

$$P = f_k - mg \sin \theta$$

$$f_k = \mu_k R = \mu_k mg \cos \theta$$

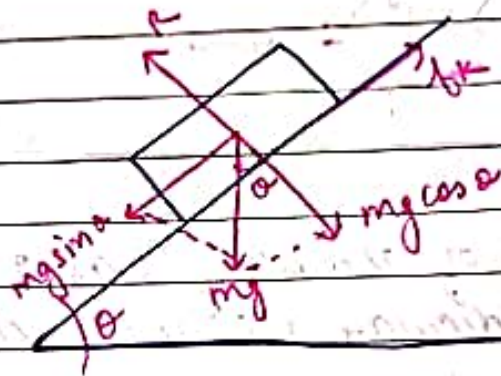
$$P = \mu_k mg \cos \theta - mg \sin \theta$$

$$= mg(\mu_k \cos \theta - \sin \theta)$$

$$W = P \times s = mg(\mu_k \cos \theta - \sin \theta)s$$

## # ACCELERATION OF A BODY SLIDING DOWN A ROUGH INCLINED PLANE:

Consider a body of weight  $mg$  placed on an inclined plane. Suppose the angle of inclination  $\theta$  be greater than the angle of repose.



The weight  $mg$  has 2 rectangular components:

- (i)  $mg \cos \theta$  perpendicular to the inclined plane. It balances the normal  $R$ . Thus,

$$R = mg \cos \theta$$

- (ii)  $mg \sin \theta$  down the inclined plane.

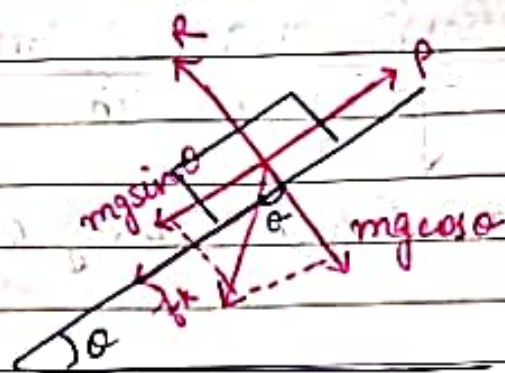
$$F = mg \sin \theta - f_k$$

$$f_k = \mu_k R = \mu_k mg \cos \theta$$

$$ma = mg \sin \theta - \mu_k mg \cos \theta$$

$$\text{Hence } a = g(\sin \theta - \mu_k \cos \theta).$$





The weight  $mg$  of the body has two components:

- (i)  $mg \cos \theta$  perpendicular to the inclined plane. It balances the normal reaction  $R$ . Thus

$$R = mg \cos \theta$$

- (ii)  $mg \sin \theta$  down the inclined plane:

$$P = mg \sin \theta + f_k$$

But  $f_k = \mu_k R = \mu_k mg \cos \theta$

$$P = mg \sin \theta + \mu_k mg \cos \theta$$

$$= mg (\sin \theta + \mu_k \cos \theta)$$

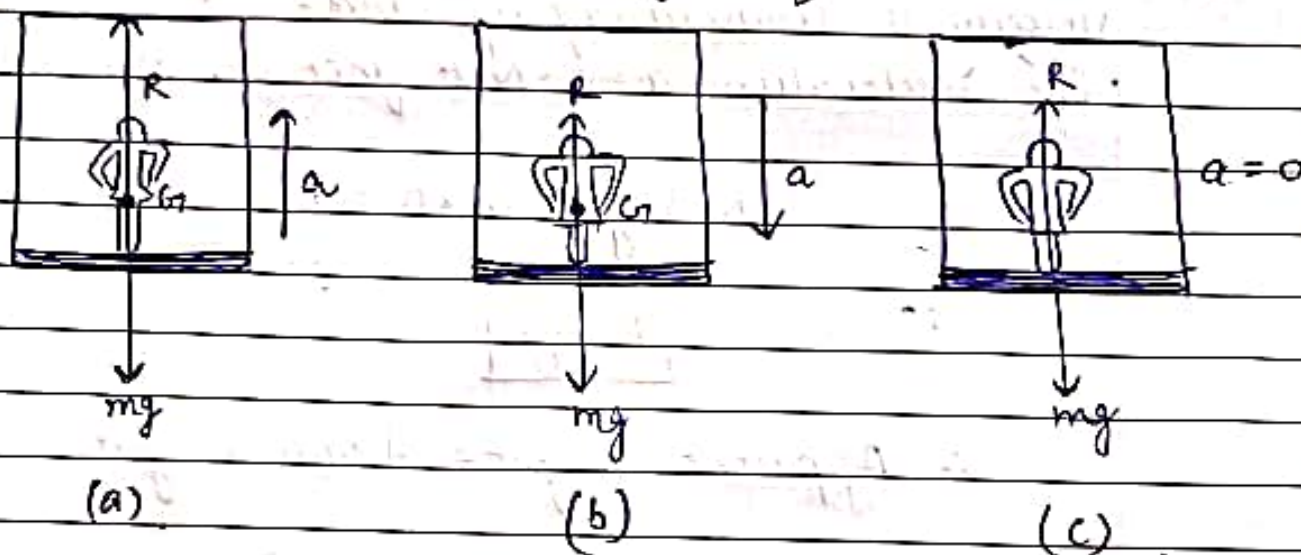
Work done in pulling the body through distance  $S$  up the inclined plane is

$$W = P \times S = mg (\sin \theta + \mu_k \cos \theta) S$$

# Apparent weight of a man in an elevator/lift:

Consider a man of mass  $m$  standing on a weighing machine placed in a lift. The actual weight of the man is  $mg$ .

The weighing machine reads the reaction  $R$  which is the force experienced by the man. So  $R$  is the apparent weight of the man.



- (1) when the lift moves upwards with acceleration, net upward force on the man is

$$R - mg = ma$$

$$\therefore \text{Apparent weight, } R = m(g+a)$$

$\therefore$  Apparent weight of man increases if lift accelerates upwards.