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UNIT - 2 [KINEMATICS]
CHAPTER - 3
MOTION IN A STRAIGHT LINE

• Kinematics
It is a branch of mechanichs that is the study of motion of objects without considering the ~~motion~~ cause of motion.

(1) Rest :- An object is said to be at rest if it does not change its position with respect to its surrounding with the passage of time. Ex:- A book lying on a table.

(2) Motion :- An object is said to be in motion if it changes its position with respect to its surrounding with the passage of time. Ex :- a train moving on rails

Rest and motion are relative terms :-
A passenger sitting in a moving train is at rest with respect to ~~its~~ his fellow passengers but he is in motion with respect to the objects outside the train. Thus an object may be in motion relative to another object Hence rest and motion are relative terms.

[Path length]

Distance	Displacement
• Distance is the length of the actual path traversed by a body, irrespective of its motion.	• Displacement is the shortest distance between the initial and final position of body in a given direction.
• Distance between two points may be same or different for different paths chosen.	• Displacement between two given points is always same.
• It is a scalar quantity.	• It is a vector quantity.
• Distance ~~ma~~ covered may be positive or zero.	• Displacement covered may be positive, negative or zero.

Speed	Velocity
• It is the distance travelled by a body per unit time in any direction.	• It is the distance travelled by a body per unit time in a fixed direction.
• Scalar quantity	• Vector quantity.
• Speed may be positive or zero but never negative.	• Velocity may be positive, negative or zero.
Speed = $\frac{\text{Distance}}{\text{time}}$	Velocity = $\frac{\text{~~Distance~~ Displacement}}{}$

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AVERAGE SPEED :-

For an object moving with a variable speed, the average speed is the total distance travelled by the object divided by the total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{Total Distance Travelled}}{\text{Total time taken}}$$

Instantaneous speed :-

The speed of an object at any particular instant of time or at a particular distance point of its path is called the instantaneous speed of the object.

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

AVERAGE VELOCITY :-

For an object moving with variable velocity, average velocity, average velocity is defined as the ratio of its total ~~distance~~ displacement to the total time interval in which that displacement occurs.

$$\text{Average Velocity} = \frac{\text{Total displacement}}{\text{Total Time}}$$

$$V_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

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Instantaneous velocity :-
The velocity of an object at a particular instant of time or at a particular ~~time~~ point of its path is called its instantaneous velocity.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

UNIFORM ~~SPEED~~ MOTION :-
An object is said to be in uniform motion if it covers equal distance in equal interval of time, however small these time intervals may be, in the same fixed direction.

NON- UNIFORM MOTION :-
A body is said to be in non-uniform motion if its velocity changes with time.
for ex :- The motion of a bicycle along a zig-zag path is a non-uniform motion.

FRAME OF REFERENCE :-
The coordinates $[x, y, z]$ of an object describe the position of the objects with respect to their coordinate system. To measure time, we position a clock in this system, this coordinate system along with a clock constituents of frame of reference

Rate = $\frac{\text{—}}{\text{time}}$

Non Uniform = Variable

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The point of intersection of three axis is called origin and ~~there are~~ ~~are~~ ~~x~~ serve as reference point.

ACCELERATION :
The rate of change of velocity of an object with time is called acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

SI unit :- ms^{-2}
Dimensional formula :- $[LT^{-2}]$

Uniform acceleration :
The acceleration of an object is said to be uniform acceleration if its velocity changes by equal amounts in equal interval of time, however small these time intervals may be.

Variable acceleration :-
The acceleration of an object is said to be variable acceleration if its velocity changes by unequal amounts in equal interval of time.

Average acceleration :-
It is the ratio of the total change in velocity of object to the total time interval taken.

$$a_{av} = \frac{V_2 - V_1}{\Delta t} = \frac{\Delta V}{\Delta t}$$

Instantaneous acceleration :-
The acceleration of an object at a given instante of time or at a given point of its motion is called its instantaneous acceleration.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$v = \frac{dx}{dt}$$

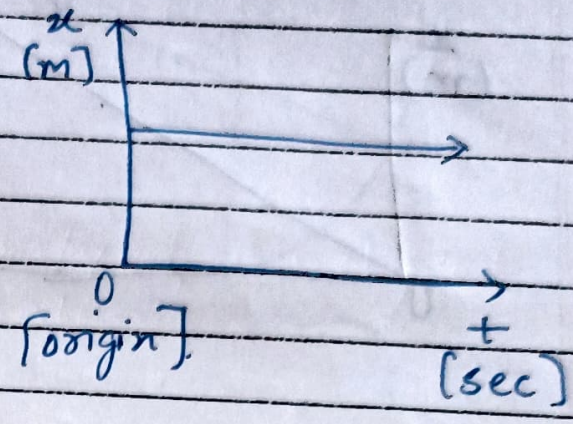
$$a = \frac{d}{dt}\left[\frac{dx}{dt}\right] = \frac{d^2x}{dt^2}$$

Positive acceleration :-
If the velocity of an object increases with time, its acceleration is positive

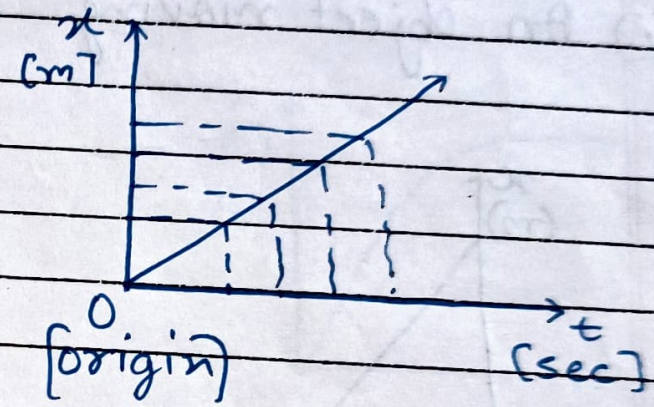
Negative acceleration :-
If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called retardation & deceleration.

POSITION TIME GRAPH :

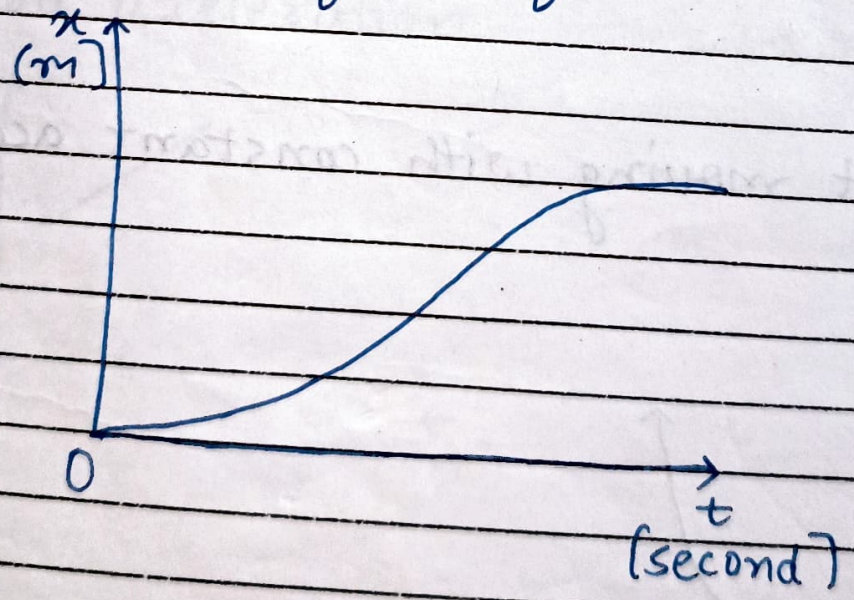
(a) Stationary Object :-



(b) Uniform motion :-



(c) Position time graph of a car :-



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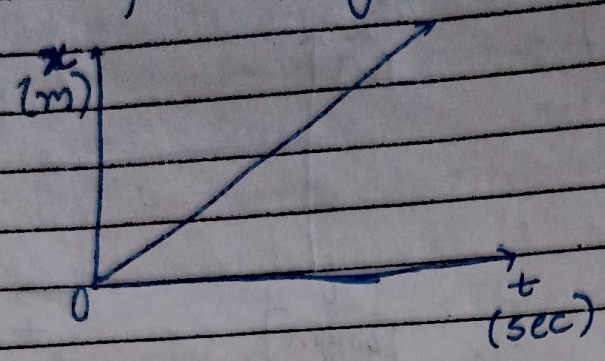
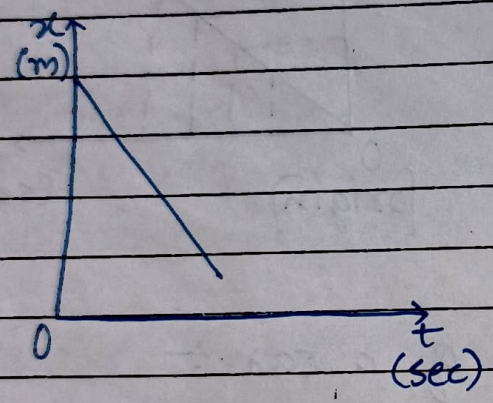
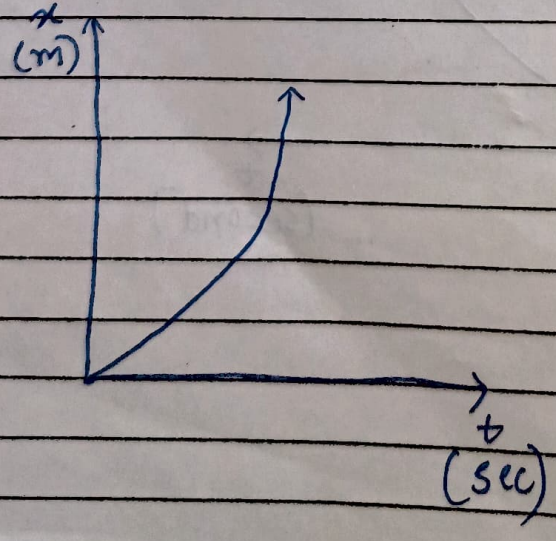
(d) An object moving with positive velocity.



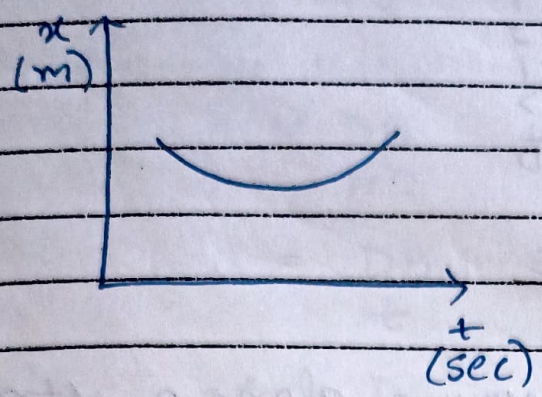
(e) An object moving with negative velocity.

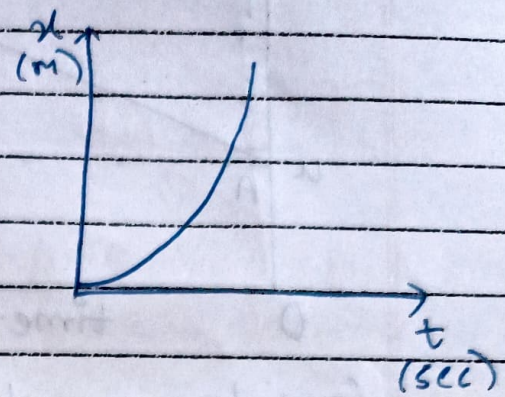


(f) An object moving with constant acceleration.

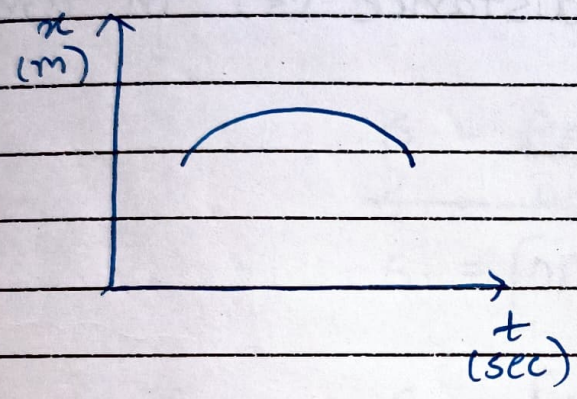


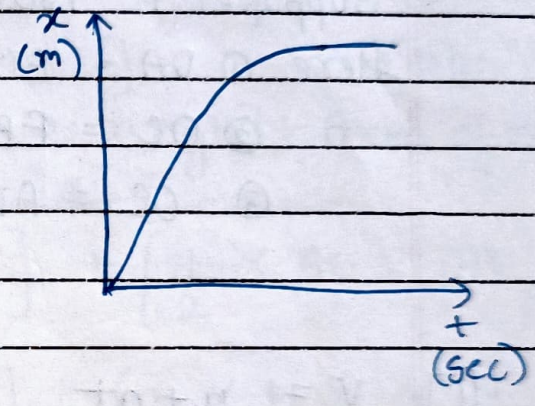
(g) Positive acceleration :-



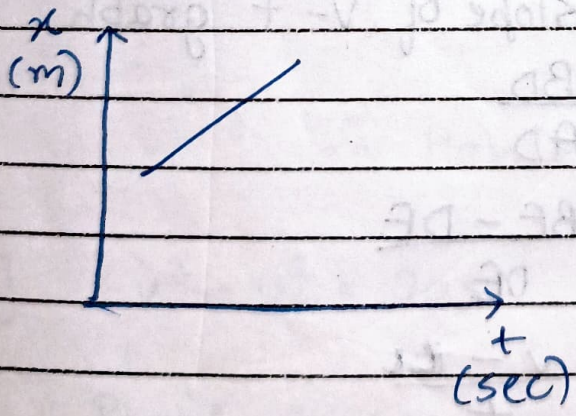


(h) Negative acceleration :-





(i) zero acceleration.



Slope of position time graph → velocity
Slope of velocity time graph → Acceleration.

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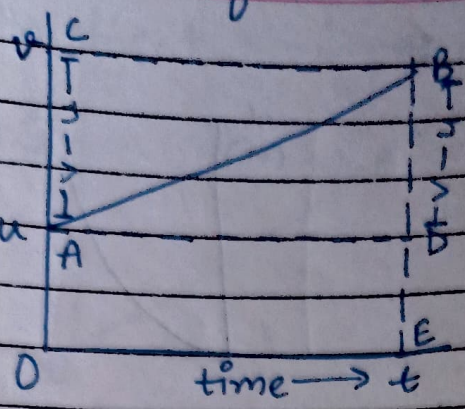
Equation of motion by



Consider an object moving along a straight line path with initial velocity (u) and uniform acceleration (a).
Suppose it travels distance (s) in time (t)

Here ① OA = ED = u
② OC = EB = v
③ OE = AD = t

[1] $v = u + at$

Proof : We know
Acceleration = Slope of v-t graph

$$a = \frac{BD}{AD}$$

$$a = \frac{BE - DE}{OE}$$

$$a = \frac{v - u}{t}$$

$$\boxed{at = v - u}$$

∴ Hence Proved $[v - u = at]$

Area of trapizium = $\frac{1}{2}$ [Sum of parallel sides] × height

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[2] $s = ut + \frac{1}{2}at^2$

Proof :- We know

$$a = \frac{DB}{AD}$$

$$a = \frac{DB}{t}$$

$$at = DB$$

Also, Distance travelled = Area under v-t graph

$$S = \underset{[ADED]}{\text{Area of rectangle}} + \underset{[ABD]}{\text{Area of trian}}$$

$$\cancel{S = ADED + ABD}$$

$$S = [OA \times OE] + \left[\frac{1}{2} \times AD \times DB\right]$$

$$S = [u \times t] + \left[\frac{1}{2} \times t \times at\right]$$

$$\boxed{S = ut + \frac{1}{2}at^2}$$

∴ Hence Proved

[3] $v^2 - u^2 = 2as$

Proof :- We know, Distance travelled = Area under v-t graph

$$S = \text{Area of trapizium [OABE]}$$

$$S = \frac{1}{2}[EB + OA] \times OE$$

Also,

$$a = \frac{DB}{AD} = \frac{EB - ED}{OE} = \frac{v - u}{t\,[OE]}$$

$$OE = \frac{v - u}{a}$$

Now,

$$S = \frac{1}{2}\left[EB + EDA\right] \times \left[\frac{v - u}{a}\right]$$

$$S = \frac{1}{2}\left[EB + ED\right] \times \left[\frac{v - u}{a}\right]$$

$$S = \frac{1}{2}\left[v + u\right] \times \left[\frac{v - u}{a}\right]$$

$$S = \frac{1}{2}\left[\frac{v^2 - u^2}{a}\right]$$

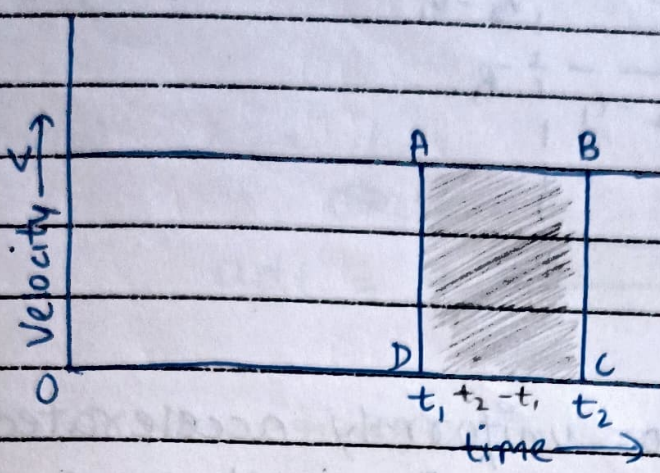
$$S = \frac{v^2 - u^2}{2a}$$

$$\boxed{2as = v^2 - u^2}$$

∴ Hence Proved.

Velocity Time Graph

[1]

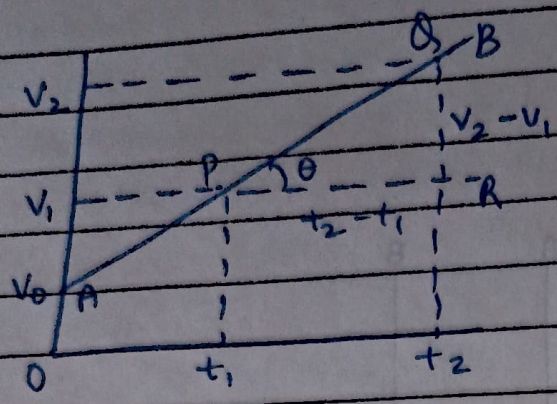


The v-t graph for uniform motion is a straight line parallel to time axis.

Area under v-t graph = Area of rectangle [ABCD]
= AD × DC
= $v \times [t_2 - t_1]$
= Velocity × time
= Distance / Displacement

Hence, Area under v-t graph gives displacement

[2] Velocity time graph for uniformly accelerated motion



v-t graph for uniformly accelerated motion is a straight line inclined to the time axis.

Slope of v-t graph $= \tan\theta$

$$= \frac{QR}{PR}$$

$$= \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{\text{Change in velocity}}{\text{time Interval}}$$

$= \text{acceleration}$.

Hence, slope of v-t graph gives acceleration.

① $\int x^n dx = \frac{x^{n+1}}{n+1}$ ② $\int 1 \cdot dx = x$.

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Equations of motion by Calculus Method

① $v = u + at$

We know $a = \frac{dv}{dt}$

$$adt = dv \quad \text{——— ①}$$

When time = 0 initial velocity = u
time = t final velocity = v

Integrating both side with proper limits

$$\int_u^v dv = \int_0^t adt$$

$$\int_u^v dv = a\int_0^t dt$$

$$[v]_u^v = a[t - 0]$$

$$v - u = at$$

$$\boxed{v = u + at}$$:- Hence Proved.

② $s = ut + \frac{1}{2}at^2$

We know $v = \frac{ds}{dt}$.

$vdt = ds$.

$\therefore v = [u + at]$

$\therefore [u + at]\,dt = ds$. ——— ①

When time = 0 Distance = 0.
time = t Distance = s.

Integrating eq ① b/s with in proper limit

~~$\int [u + at$~~

$\int ds = \int [u + at]\,dt$.

$\int_0^s ds = \int_0^t (u + at)\,dt$

$\int_0^s ds = \int_0^t u\,dt + \int_0^t at\,dt$

$[s]_0^s = u[t]_0^t + a\left[\frac{t^{1+1}}{1+1}\right]_0^t$

$s = u[t] + a\left[\frac{t^2}{2}\right]^t$.

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$$\boxed{s = ut + \frac{1}{2}at^2}$$

③ $v^2 - u^2 = 2as.$

We Know $a = \frac{dv}{dt} \times \frac{ds}{ds}$

$a = \frac{dv}{ds} \times v.$

$ads = vdv$ ——— ①

Integrating eq ① and ② in proper limits.

when time = 0 distance = 0 velocity = u
time = t distance = s velocity = v

$$\int_0^s ads = \int_u^v vdv$$

$$a\int_0^s ds = \int_u^v vdv$$

$$a[s]_0^s = \left[\frac{v^2}{2}\right]_u^v$$

$$a[s-0] = \frac{1}{2}[v^2 - u^2]$$

$$\boxed{2as = v^2 - u^2}$$

$S_n - S_{n-1} = S_{nth}$

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(4) $S_{nth} = u + \frac{a}{2}(2n-1)$.

Distance travelled in n^{th} second.

We know

$$v = \frac{ds}{dt}$$

$$ds = v\,dt$$

$$ds = [u+at]\,dt \quad \text{——— (1)} \quad [v = u+at]$$

When time = $[n-1]$ second, distance = S_{n-1}

time = n second, distance = S_n

Integrating both side with in proper limits

$$\int_{S_{n-1}}^{S_n} ds = \int_{n-1}^{n} [u+at]\,dt$$

$$\left[S\right]_{S_{n-1}}^{S_n} = \int_{n-1}^{n} u\,dt + \int_{n-1}^{n} at\,dt$$

$$\left[S\right]_{S_{n-1}}^{S_n} = u\int_{n-1}^{n} dt + a\int_{n-1}^{n} t\,dt$$

$$S_n - S_{n-1} = u\left[t\right]_{n-1}^{n} + a\left[\frac{t^2}{2}\right]_{n-1}^{n}$$

$$S_{nth} = u[n-n+1] + \frac{a[n^2 - n^2 + 2n - 1]}{2}$$

$$S_{nth} = u + \frac{a}{2}[2n-1]$$

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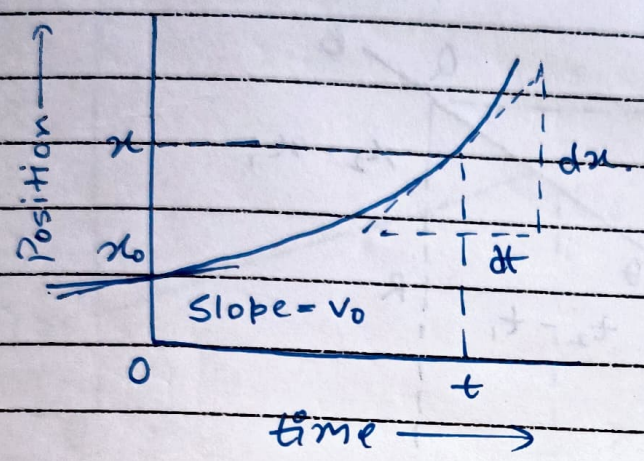
POSITION TIME GRAPH FOR UNIFORM ACCELERATION

The position time graph for uniformly accelerated motion is a parabola

$[s = ut + \frac{1}{2}at^2]$

Where highest degree is 2 [Parabola]



Slope of position time graph

$= \frac{\text{Small change in vertical coordinate}}{\text{Small change in Horizontal Coordinate}}$

$= \frac{dx}{dt}$

= velocity at instant [t]

Thus, slope of position time graph gives instantaneous velocity.

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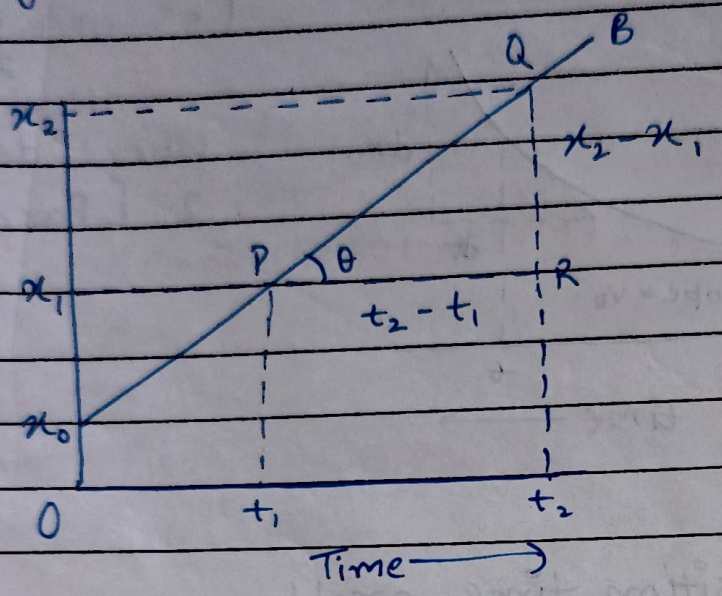
Position time graph for uniform motion

The position time graph for uniform motion is a straight line inclined to time axis.



Slope of Position time graph.

$$= \tan\theta = \frac{QR}{PR} = \frac{x_2 - x_1}{t_2 - t_1}$$

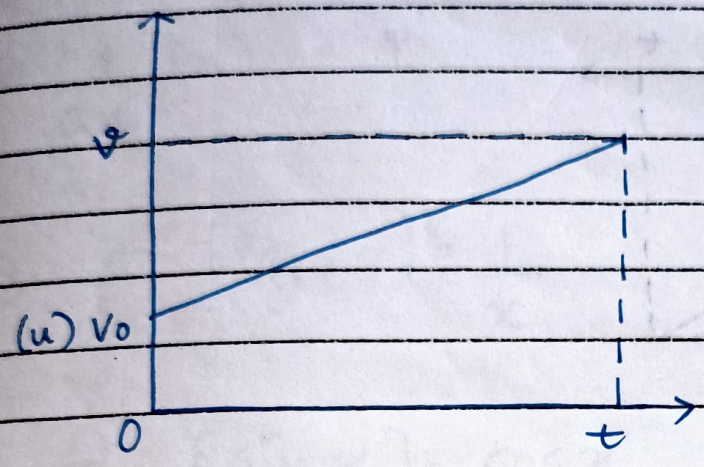
$$= \frac{\text{Displacement}}{\text{time}}$$

$$= \text{velocity } [v].$$

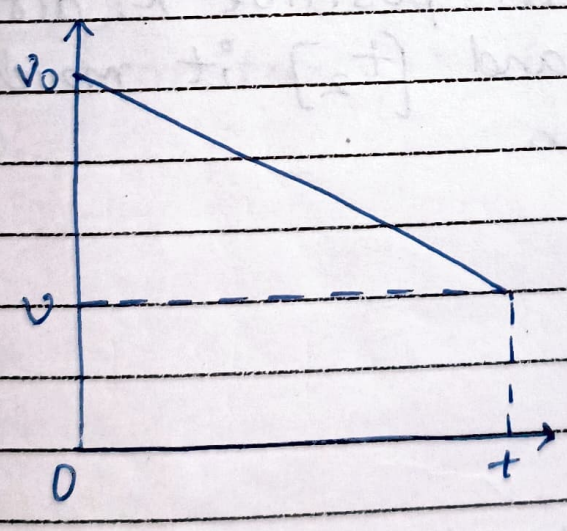
Hence, Slope of x-t graph gives velocity.

Velocity Time Graphs :-

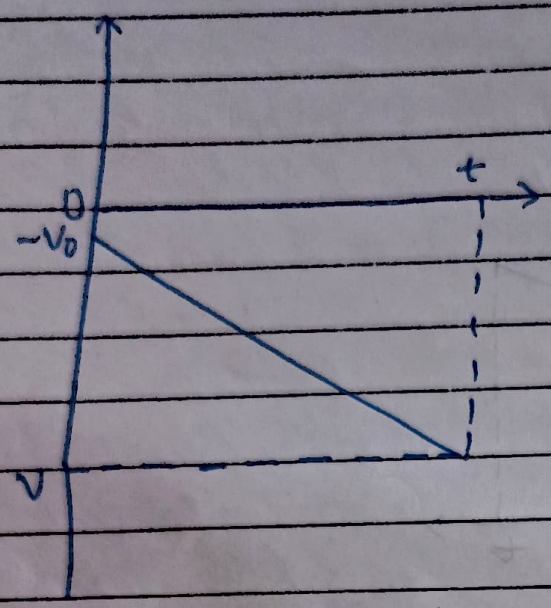
Motion In Positive Direction With Positive Acceleration :-



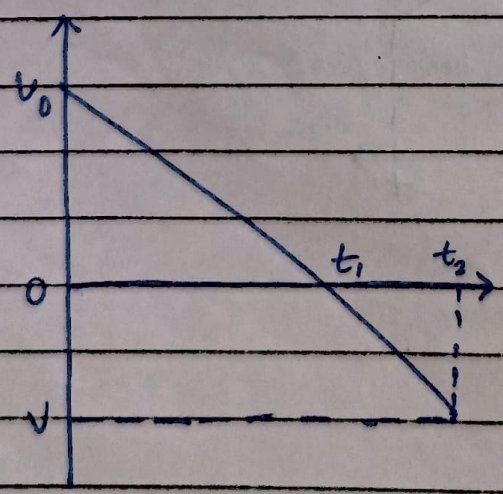
Motion In Positive Direction With Negative Acceleration



Motion in Negative Direction with negative acceleration.



Motion of an object with negative acceleration that changes direction at time $[t_1]$. $[0]$ to $[t_1]$ it moves in positive x. direction and between $[t_1]$ and $[t_2]$ it moves in the opposite direction.



DIFFERENCIATION

① $\frac{d[x^n]}{dx} = nx^{n-1}$

② $\frac{d}{dx}(x) = 1$

③ $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

④ $\frac{d}{dx}[\sin x] = \cos x$

⑤ $\frac{d}{dx}[\cos x] = -\sin x$

⑥ $\frac{d}{dx}[\text{constant}] = 0.$