Third task

- Download the SPT fgas data
- Fit the data to f0(1 + f1z) where f0 and f1 are unknown constants
- Determine the best fit values of f0 and f1 including 68% and 90% credible intervals using emcee and corner.py
- The priors on f0 and f1 should be 0<f0 <0.5 and −0.5<f1 <0.5.
- Use the same likelihood as in equation 6 of https://arxiv.org/pdf/2001.08340.pdf (radial acceleration relation for galaxy cluster
- download emcee and python corner module and look up https://emcee.readthedocs.io/en/stable/tutorials/line/ which shows how to fit a model to a straight line

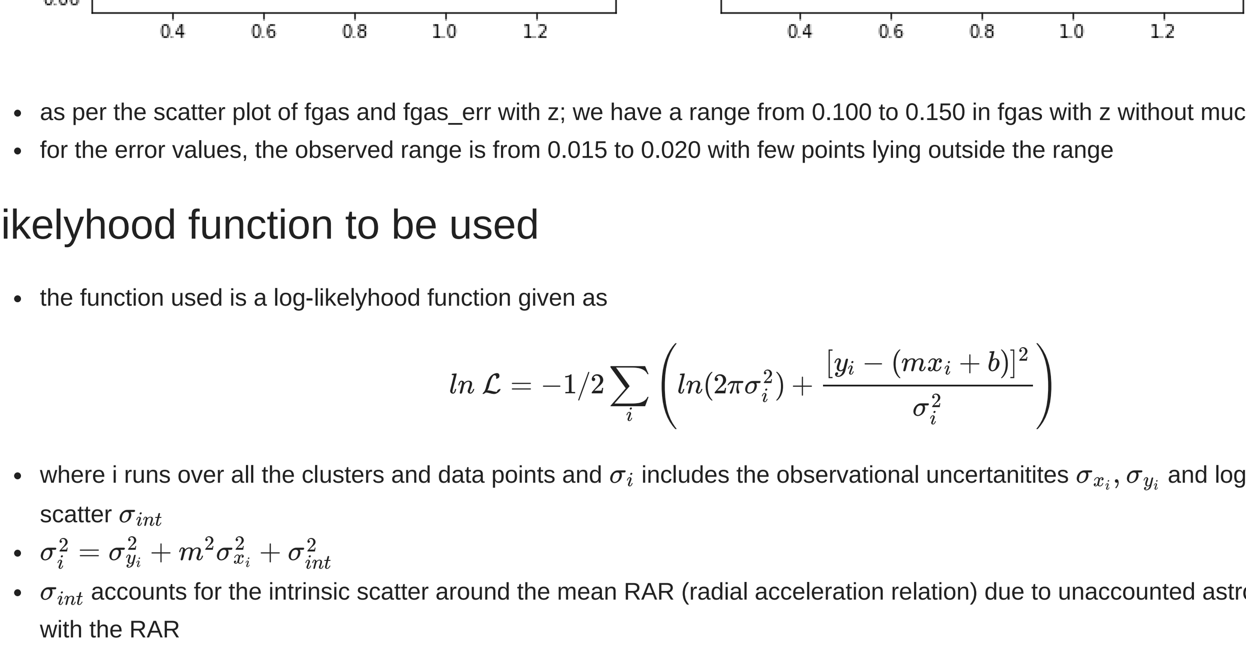
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy as spy
```

In [2]:
```
fname = np.loadtxt('fgas_spt.txt', delimiter= " ") #loading the given data
print('the structure is z, fgas, fgas_error, ignore', fname[0]) #checking for the structre of the element
```

```
the structure is z, fgas, fgas_error, ignore [0.2777     0.09661017 0.01488267 0.        ]
```

In [3]:
```
# arrays as given in the data

z = [] #red shift
fgas = [] #hot gas fraction
fgas_err = [] #corresponding error in the observations
ignore = []

for i in fname:
    z.append(i[0])
    fgas.append(i[1])
    fgas_err.append(i[2])
    ignore.append(i[3])

x = np.array(z)
y = np.array(fgas)
yerr = np.array(fgas_err)
```

In [4]:
```
fig, ax = plt.subplots(1,2, figsize=(12,6))
ax[0].scatter(x , y)
ax[1].scatter(x,yerr, color='r')
ax[0].set_ylabel('fgas')
ax[1].set_ylabel('fgas_err')

plt.show()
```



- as per the scatter plot of fgas and fgas_err with z; we have a range from 0.100 to 0.150 in fgas with z without much deviation
- for the error values, the observed range is from 0.015 to 0.020 with few points lying outside the range

likelyhood function to be used

- the function used is a log-likelyhood function given as

$$ln\,\mathcal{L} = -1/2 \sum_i \left(ln(2\pi\sigma_i^2) + \frac{[y_i - (mx_i + b)]^2}{\sigma_i^2} \right)$$

- where i runs over all the clusters and data points and σ_i includes the observational uncertanitites $\sigma_{x_i}, \sigma_{y_i}$ and lognormal intrinsic scatter σ_{int}
- $\sigma_i^2 = \sigma_{y_i}^2 + m^2\sigma_{x_i}^2 + \sigma_{int}^2$
- σ_{int} accounts for the intrinsic scatter around the mean RAR (radial acceleration relation) due to unaccounted astrophysics associated with the RAR

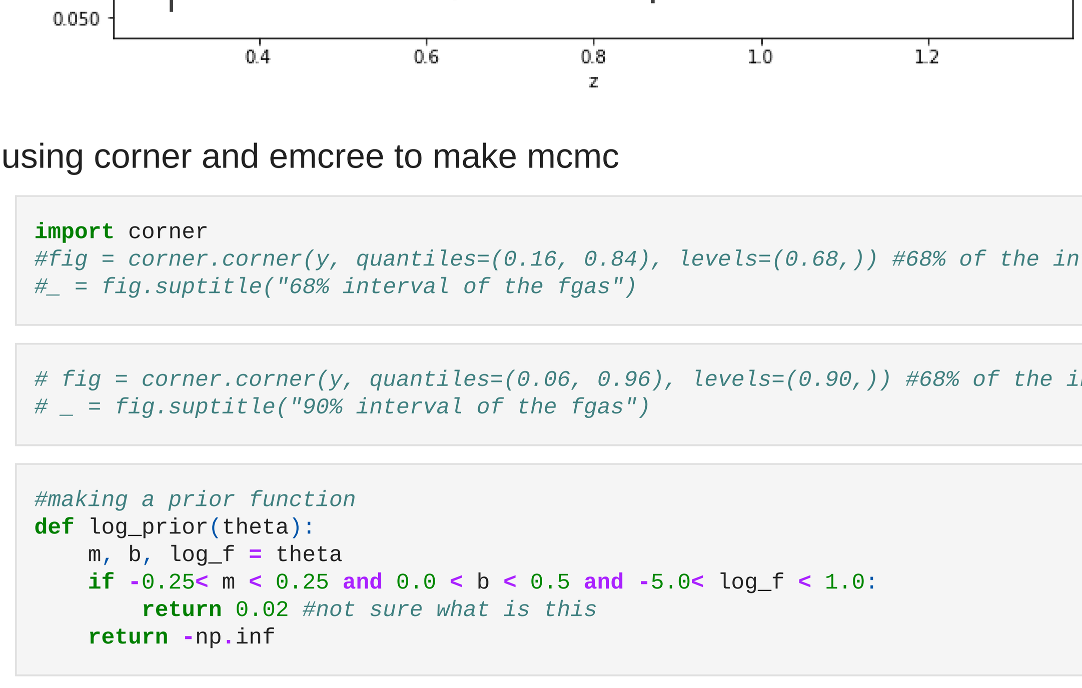
In [5]:
```
#coding the likelyhood function
def log_likelihood(theta, x, y, yerr):
    m = theta[0]
    b = theta[1]
    log_f = theta[2]

    model = m*x + b
    sigma2 = yerr**2 + model**2 * np.exp(2 * log_f)
    return -0.5 * np.sum((y - model) ** 2 / sigma2 + np.log(2*np.pi*sigma2))
```

- as per the standard equation and the given model $f0 * (1 + f1 * z)$; we have $m = f0 * f1$ and $b = f0 \implies f0 = b \,\&\, f1 = \frac{m}{f0}$ with

$$0 < b < 0.5 \; and - 0.25 < m < 0.25$$

In [7]:
```
from scipy.optimize import minimize

# choosing the true parameters
m_true = -0.012
b_true = 0.125
f_true = np.std(ignore) #fractional amount of underestimation

nll = lambda *args: -log_likelihood(*args)
initial = np.array([m_true, b_true, np.log(f_true)]) #making array of true params
soln = minimize(nll, initial, args=(x, y, yerr))#minimized the -ve likelihood function
m_ml, b_ml, log_f_ml = soln.x #output of the values

f0 = b_ml
f1 = m_ml/b_ml

print("Maximum likelihood estimates:")
print("m = {0:.3f}".format(m_ml))
print("b = {0:.3f}".format(b_ml))
print("f = {0:.3f} \n".format(np.exp(log_f_ml)))

print('the best fit values from maximum likelihood is')
print("f0 = {0:.3f}".format(b_ml))
print("f1 = {0:.3f}".format(m_ml/b_ml))

plt.figure(figsize=(10,6))
plt.errorbar(x, y, yerr=yerr, fmt=".k", capsize=0)
plt.plot(x, m_true * x + b_true, "r", alpha=0.5, lw=3, label="truth") #true linear plot
#plt.plot(x, np.dot(np.vander(x, 2), w), "--k", label="LS")
plt.plot(x, np.dot(np.vander(x, 2), [m_ml, b_ml]), ":r", label="ML") #maximum likelihood function estimation
plt.legend(fontsize=14)

#plt.xlim(0, 10)
plt.xlabel("z")
plt.ylabel('fgas')
plt.title('maximum likelihood estimation of the parameters')
plt.show()
```

```
Maximum likelihood estimates:
m = -0.008
b = 0.121
f = 0.137

the best fit values from maximum likelihood is
f0 = 0.121
f1 = -0.069
```



using corner and emcree to make mcmc

In [75]:
```
import corner
#fig = corner.corner(y, quantiles=(0.16, 0.84), levels=(0.68,)) #68% of the interval
#_ = fig.suptitle("68% interval of the fgas")
```

In [76]:
```
# fig = corner.corner(y, quantiles=(0.06, 0.96), levels=(0.90,)) #68% of the interval
# _ = fig.suptitle("90% interval of the fgas")
```

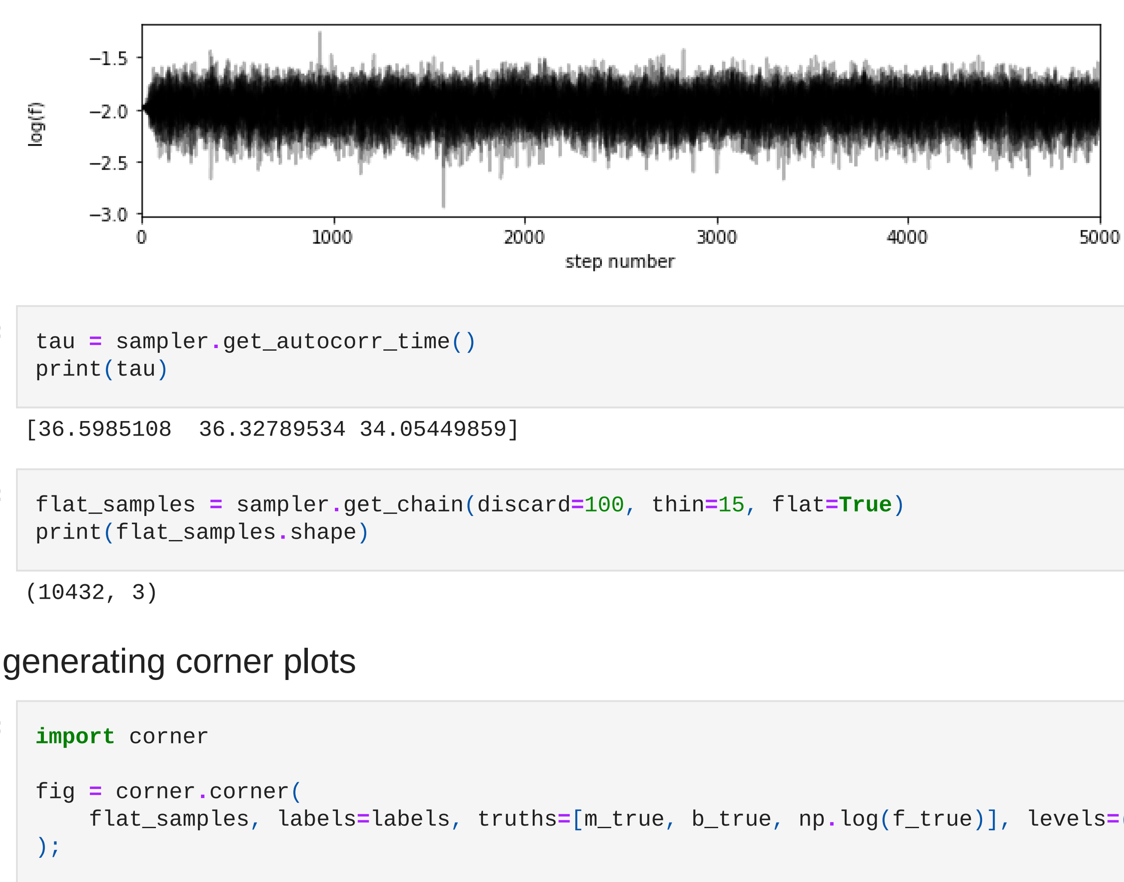
In [77]:
```
#making a prior function
def log_prior(theta):
    m, b, log_f = theta
    if -0.25< m < 0.25 and 0.0 < b < 0.5 and -5.0< log_f < 1.0:
        return 0.02 #not sure what is this
    return -np.inf
```

In [78]:
```
#log_probability function
def log_probability(theta, x, y, yerr):
    lp = log_prior(theta)
    if not np.isfinite(lp):
        return -np.inf
    return lp + log_likelihood(theta, x, y, yerr)
```

In [79]:
```
import emcee

pos = soln.x + 1e-4 * np.random.randn(32, 3)
nwalkers, ndim = pos.shape

sampler = emcee.EnsembleSampler(
    nwalkers, ndim, log_probability, args=(x, y, yerr)
)
sampler.run_mcmc(pos, 5000, progress=True);
```

```
100%|██████████████████████████████████████████| 5000/5000 [00:06<00:00, 716.61it/s]
```

In [80]:
```
fig, axes = plt.subplots(3, figsize=(10, 7), sharex=True)
samples = sampler.get_chain()
labels = ["m", "b", "log(f)"]
for i in range(ndim):
    ax = axes[i]
    ax.plot(samples[:, :, i], "k", alpha=0.3)
    ax.set_xlim(0, len(samples))
    ax.set_ylabel(labels[i])
    ax.yaxis.set_label_coords(-0.1, 0.5)

axes[-1].set_xlabel("step number");
```



In [81]:
```
tau = sampler.get_autocorr_time()
print(tau)
```

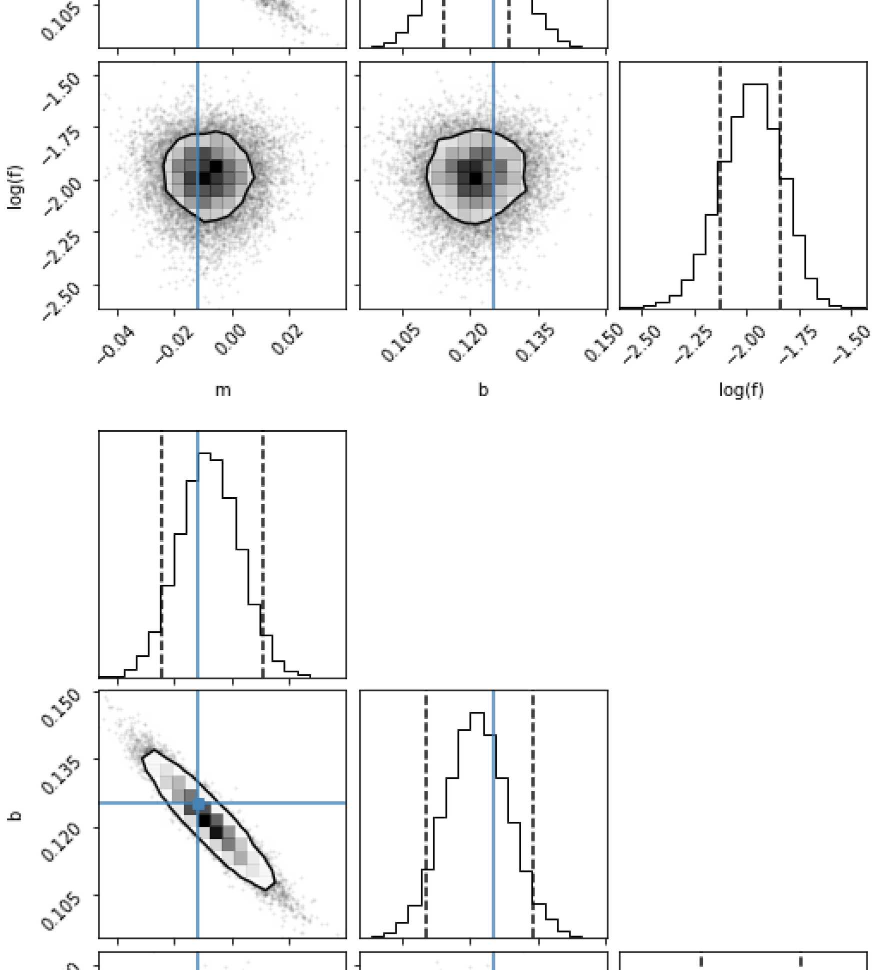
```
[36.5985108  36.32789534 34.05449859]
```

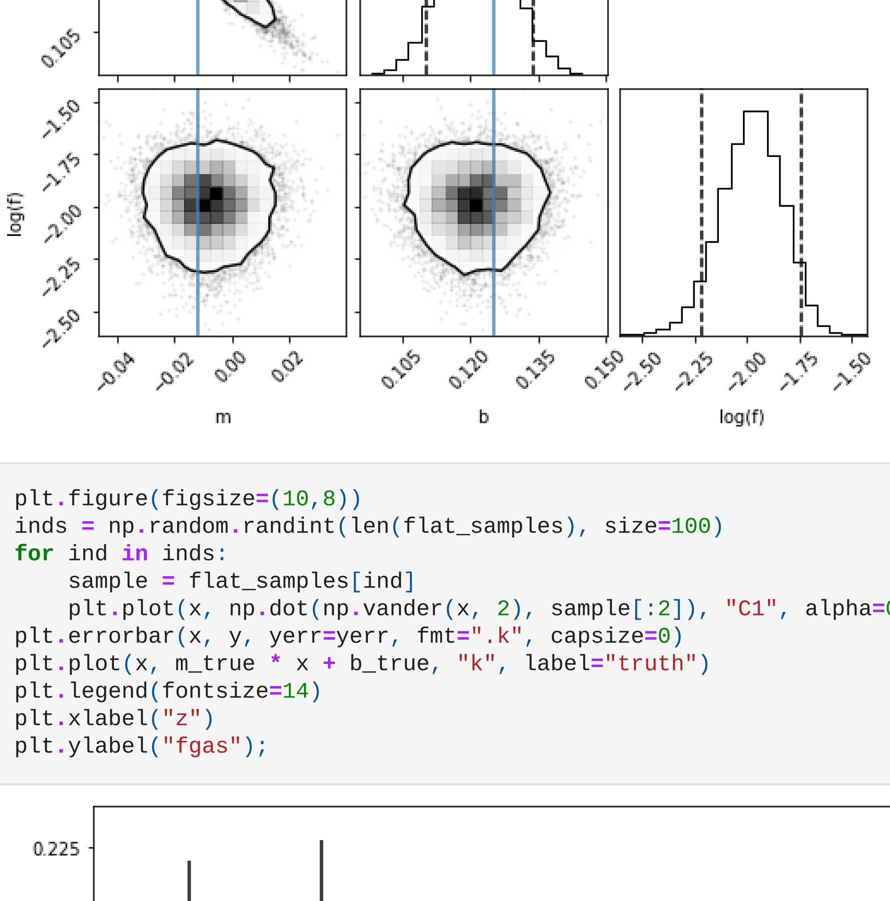
In [82]:
```
flat_samples = sampler.get_chain(discard=100, thin=15, flat=True)
print(flat_samples.shape)
```

```
(10432, 3)
```

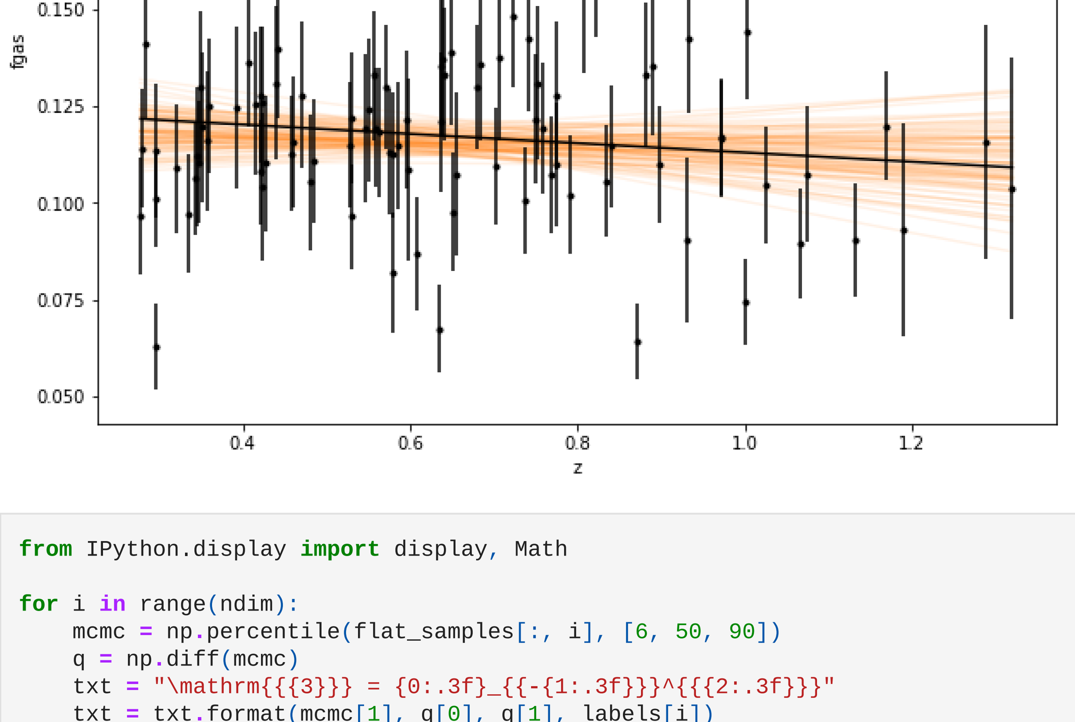
generating corner plots

In [90]:
```
import corner

fig = corner.corner(
    flat_samples, labels=labels, truths=[m_true, b_true, np.log(f_true)], levels=(0.68,), quantiles=(0.16, 0.8
);

fig = corner.corner(
    flat_samples, labels=labels, truths=[m_true, b_true, np.log(f_true)], levels=(0.90,), quantiles=(0.06, 0.9
);
```





In [91]:
```
plt.figure(figsize=(10,8))
inds = np.random.randint(len(flat_samples), size=100)
for ind in inds:
    sample = flat_samples[ind]
    plt.plot(x, np.dot(np.vander(x, 2), sample[:2]), "C1", alpha=0.1)
plt.errorbar(x, y, yerr=yerr, fmt=".k", capsize=0)
plt.plot(x, m_true * x + b_true, "k", label="truth")
plt.legend(fontsize=14)
plt.xlabel("z")
plt.ylabel("fgas");
```



In [92]:
```
from IPython.display import display, Math

for i in range(ndim):
    mcmc = np.percentile(flat_samples[:, i], [6, 50, 90])
    q = np.diff(mcmc)
    txt = "\mathrm{{{3}}} = {0:.3f}_{{-{1:.3f}}}^{{{2:.3f}}}"
    txt = txt.format(mcmc[1], q[0], q[1], labels[i])
    txt1 = "\mathrm{{{3}}} = {0:.3f}_{{-{1:.3f}}}^{{{2:.3f}}}"
    display(Math(txt))
```

$$\mathrm{m} = -0.008_{-0.016}^{0.014}$$

$$\mathrm{b} = 0.121_{-0.011}^{0.009}$$

$$\mathrm{log(f)} = -1.977_{-0.242}^{0.173}$$

In [93]:
```
print("f0 = {0:.3f}".format(-0.008))
print("f1 = {0:.3f}".format(-0.008/0.121))
```

```
f0 = -0.008
f1 = -0.066
```

- the best fit values are same as the ones from maximum likelihood function