

Interest Rates: APR and EAR

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Last Time

Time Value of Money

- Intuition, tools and discounting
- Compounding
- Useful shortcuts
- Taxes
- Inflation





This Time

Interest Rates

- Interest rate quotes
- Non-annual cash flows and compounding




APR & EAR

Current 5-Year Jumbo CD Rates

Institution	APY	Rate
 ★★★★★	2.40% Tue Dec 16	2.37% Compounded daily
Maximize growth and savings. Member FDIC. Apply Now!		
 ★★★★★	2.25% Tue Dec 16	2.23% Compounded daily
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Difference between
“Rate” and “APY”?

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Rate = APR or Annual Percentage Rate

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Rate = APR or Annual Percentage Rate

Measures amount of simple interest earned in a year

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Rate = APR or Annual Percentage Rate

Simple interest = interest earned *without* compounding

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Many bank quotes are in terms of APR

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APR typically *not* what we earn or pay

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APY = Annual
Percentage Yield

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APY = EAR or
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APY = **EAR** or
Effective Annual Rate

EAR measures actual
amount of interest
earned/paid in year

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HOW ARE DIFFERENT RATES RELATED?

Lesson: EAR is a discount rate

EAR is what matters for computing interest and discounting cash flows

Lesson: APR is *not* a discount rate.

APR is a means to an end. We use it to get a discount rate (e.g., EAR)

How do we get from an APR to an
EAR (and vice versa)?

Lesson: The relation between APR and EAR is:


$$\begin{aligned} EAR &= \left(1 + \frac{APR}{k} \right)^k - 1 \\ &= (1 + i)^k - 1 \end{aligned}$$

Lesson: The relation between APR and EAR is:

$$EAR = \left(1 + \frac{APR}{k} \right)^k - 1$$
$$= (1 + i)^k - 1$$

k is the number of
compounding periods
per year

Lesson: The relation between APR and EAR is:

$$EAR = \left(1 + \frac{APR}{k}\right)^k - 1$$
$$= (1 + i)^k - 1$$


i is the periodic interest rate, or
periodic discount rate

Example

- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

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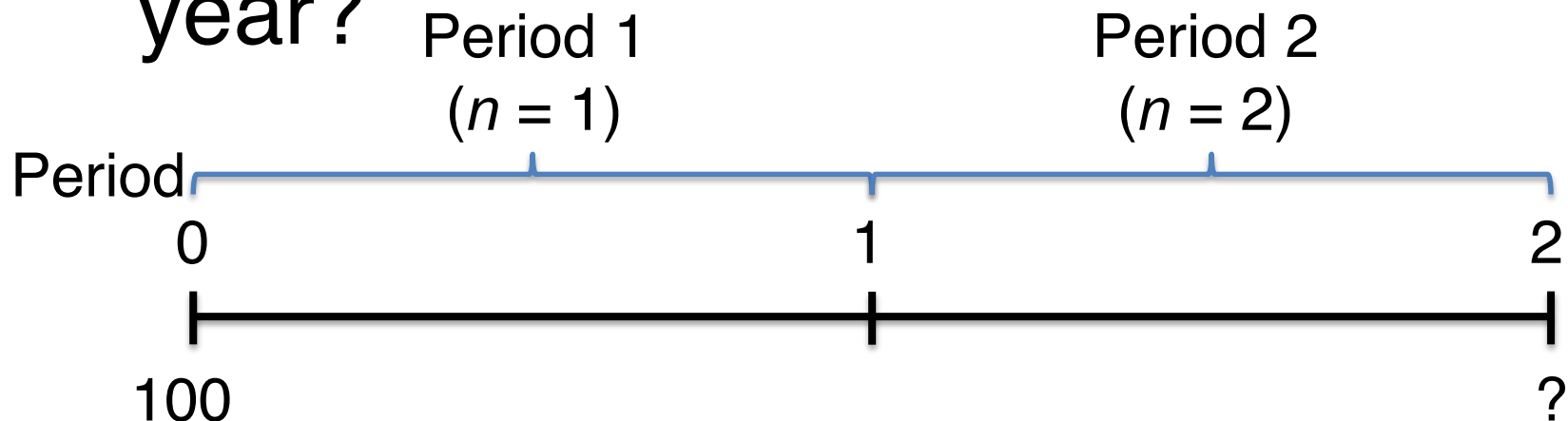
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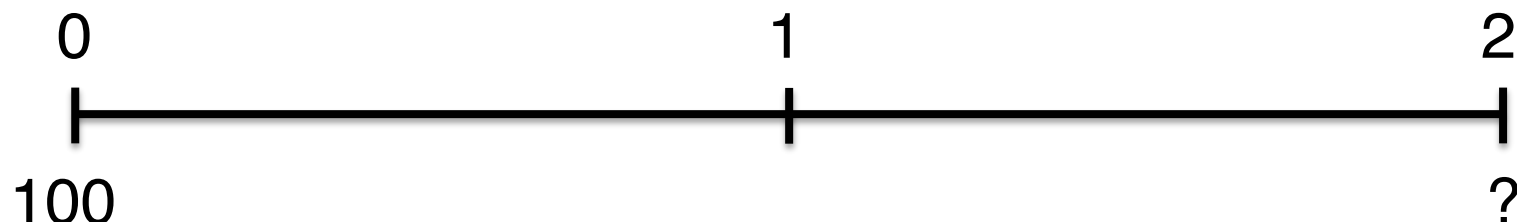


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$$i = \frac{APR}{k} = \frac{5\%}{2} = 2.5\%$$

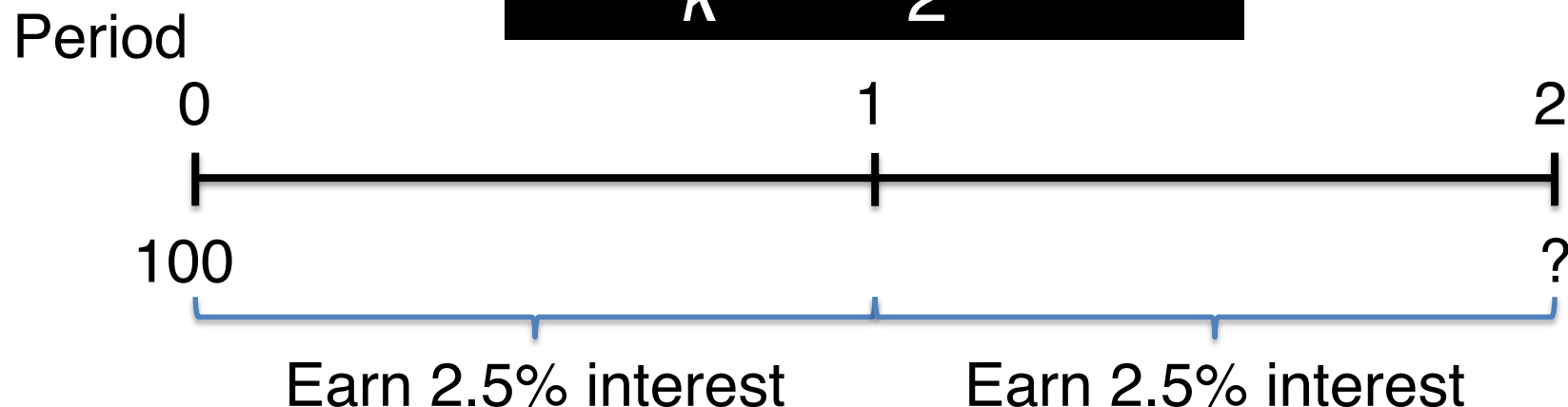
Period



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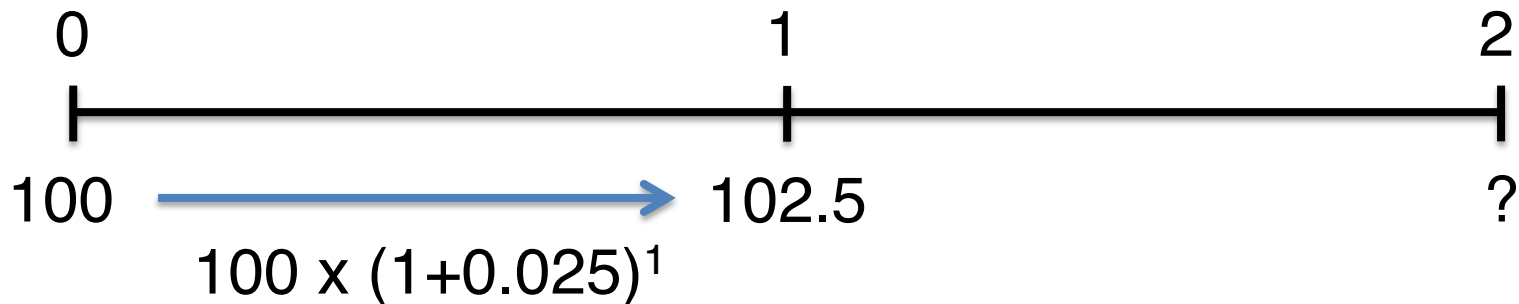


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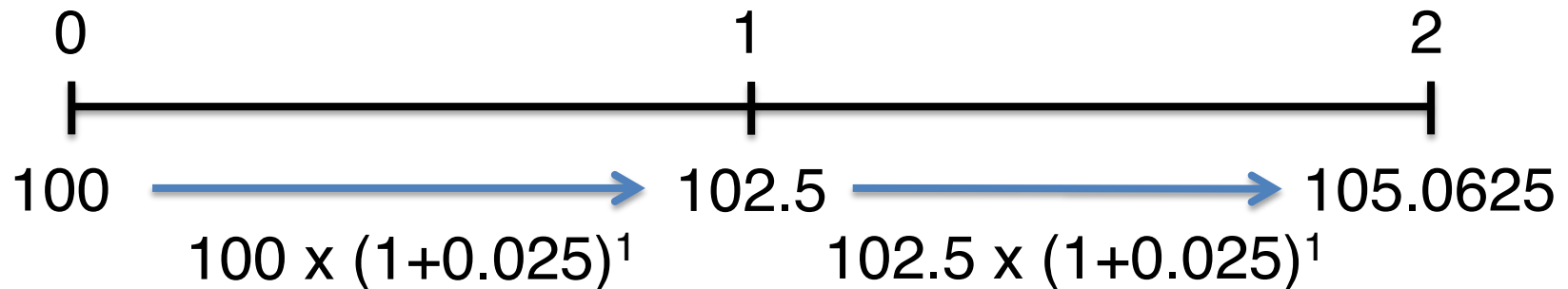


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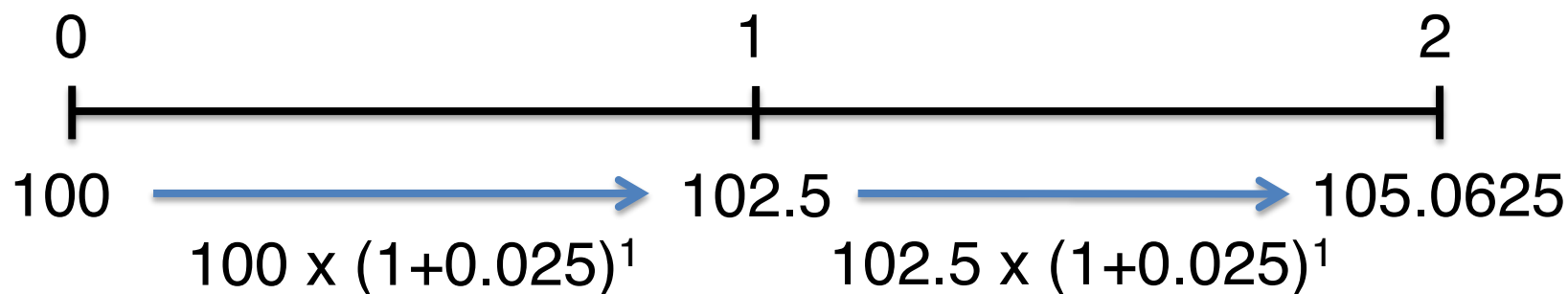


Example

- Invest \$100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

$$FV_2(100) = 100 \times (1 + 0.025)^2 = 105.0625$$

Period



Example

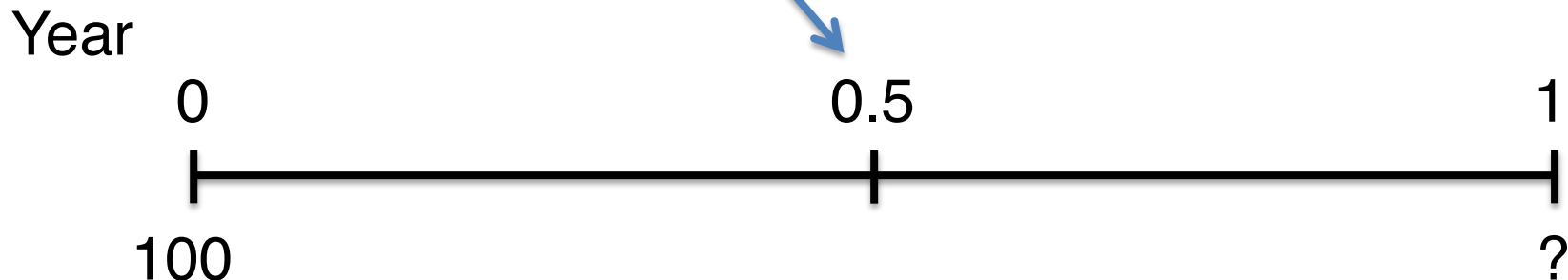
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Year



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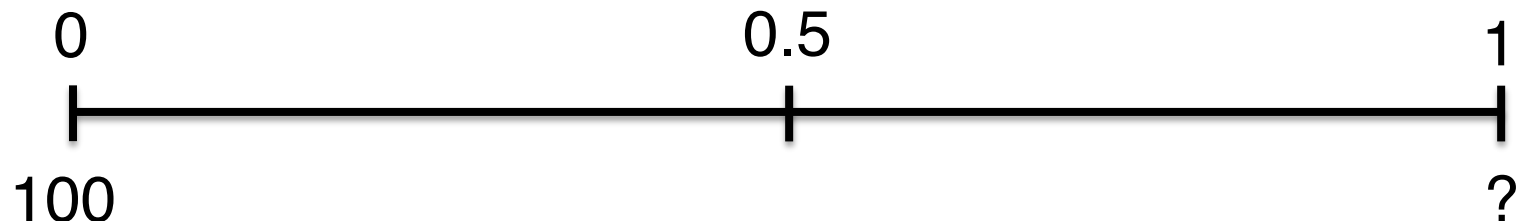


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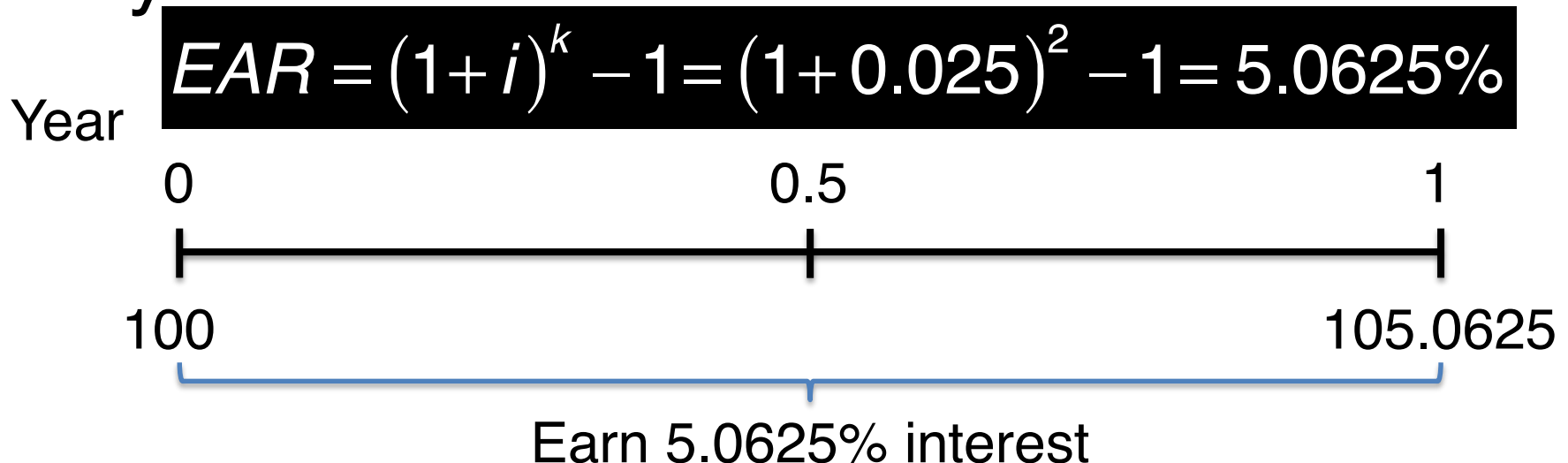
$$EAR = (1+i)^k - 1 = (1+0.025)^2 - 1 = 5.0625\%$$

Year



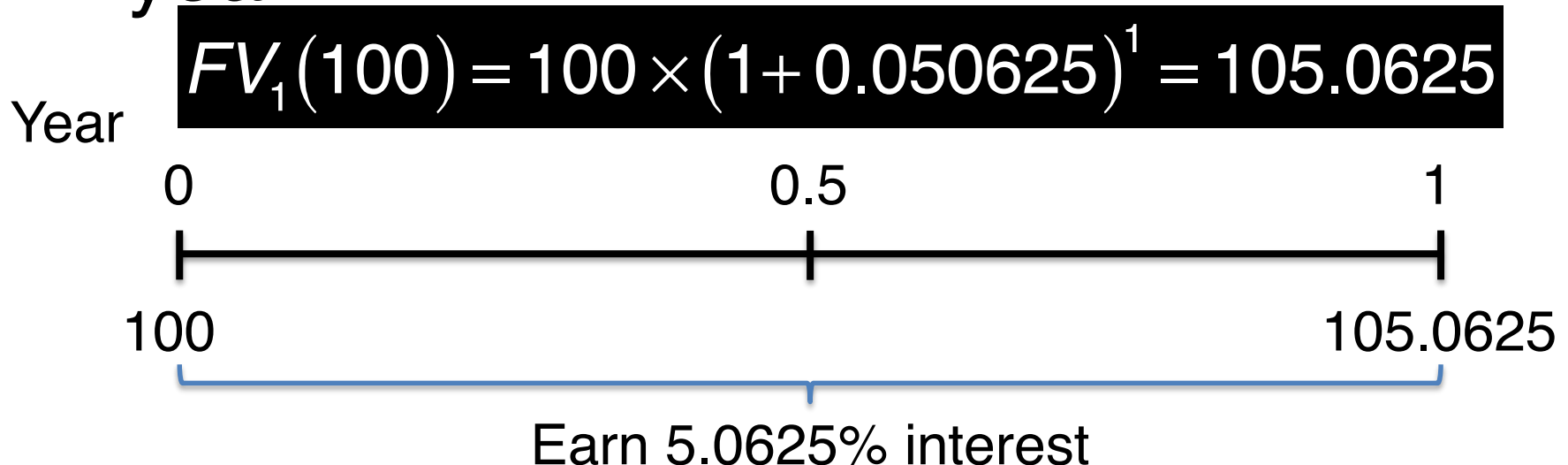
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Lesson: If you discount cash flows using EAR, then measure time in years. If you discount cash flows using periodic interest rate, then measure time in periods.

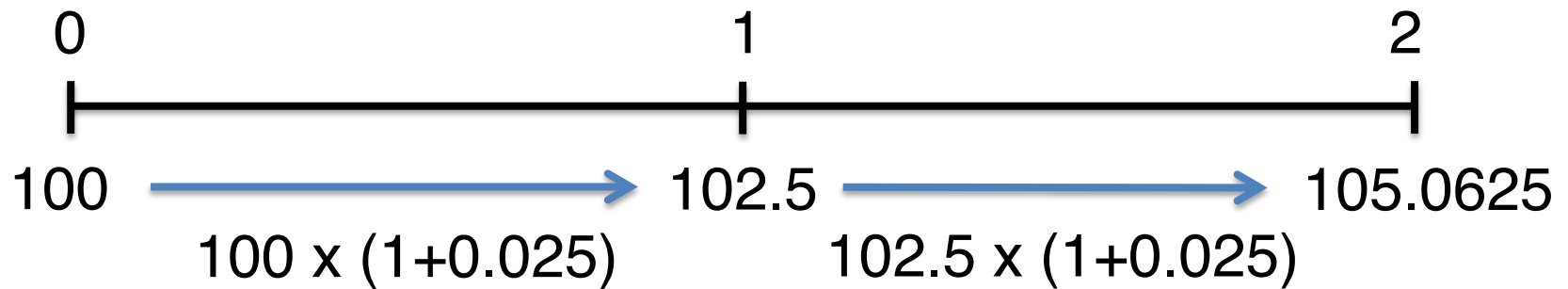
Proof

$$\begin{aligned}(1 + EAR)^T &= \left(1 + (1 + i)^k - 1\right)^T \\&= \left((1 + i)^k\right)^T \\&= (1 + i)^{kT} \\&= (1 + i)^N\end{aligned}$$

where $N = kT = \#$ of periods

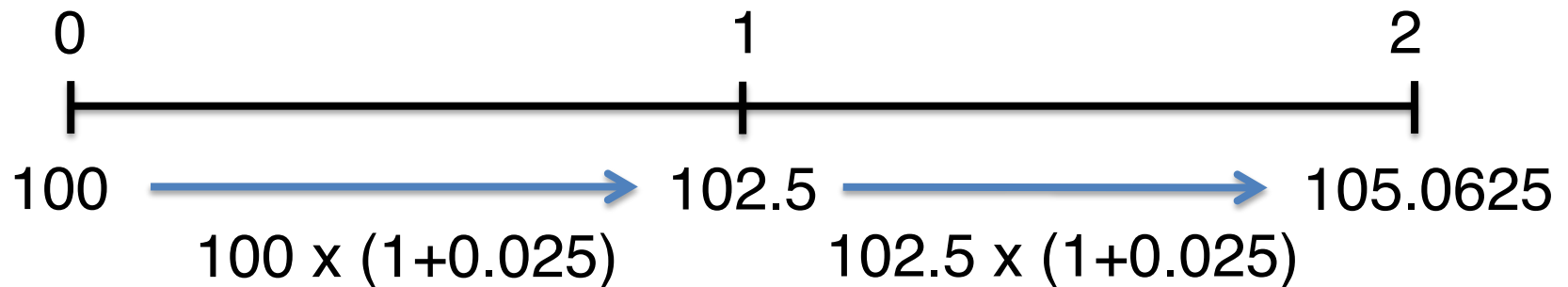
Periods vs Years

Periods

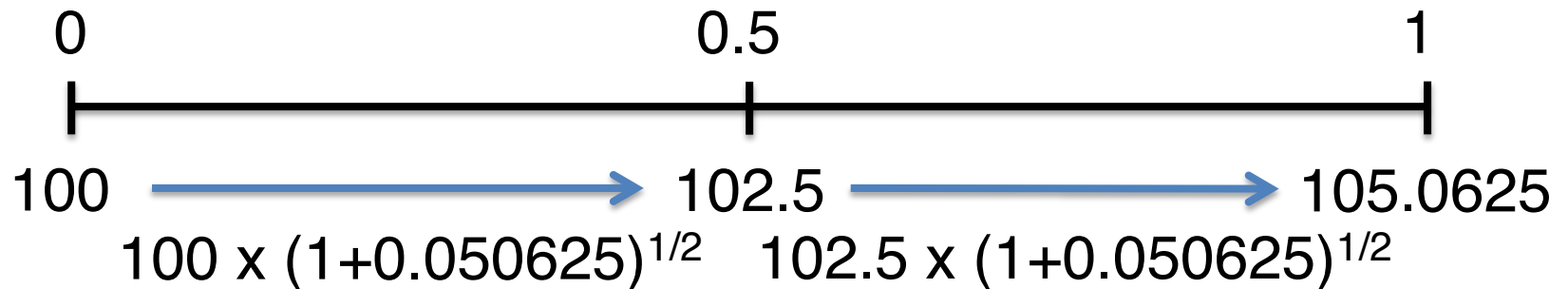


Periods vs Years




Periods



Years







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APR = 2.37%

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$APR = 2.37\%$

$k = 365$ (or 360, 252)

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$$\text{APR} = 2.37\%$$

$$k = 365 \text{ (or 360, 252)}$$

$$\begin{aligned} \rightarrow i &= 2.37\% / 365 \\ &= 0.006714\% \end{aligned}$$

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$$k = 365 \text{ (or 360, 252)}$$

$$\begin{aligned} \rightarrow i &= 2.37\% / 365 \\ &= 0.006714\% \end{aligned}$$

$$\begin{aligned} \rightarrow \text{EAR} &= \\ &= (1 + 0.006714\%)^{365} - 1 \\ &= 2.398\% \end{aligned}$$

Summary

Lessons

- **EAR** is a discount rate
 - Measures cash flows in years
- **Period interest rate, i ,** is a discount rate
 - Measures cash flows in periods
- **APR** is not a discount rate

Lessons

- Moving between **EAR** and **APR**

$$\begin{aligned} EAR &= \left(1 + \frac{APR}{k} \right)^k - 1 \\ &= (1 + i)^k - 1 \end{aligned}$$

where

$i = APR / k$ and $k = \#$ of periods per year

Coming up next

- Interest Rates
 - Term Structure of interest rates and the yield curve