Manav Madan
Stefan Möhrle
Vanshaj Taxali

AutoML
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Assignment 6

Exercise 1

(a)

$$\sigma_n^{-2}\phi_\star^T A^{-1}\Phi\mathbf{y} = \phi_\star^T\Sigma_p\Phi(\Phi^T\Sigma_p\Phi + \sigma_n^2 I)^{-1}\mathbf{y} \tag{1}$$
$$\sigma_n^{-2}A^{-1}\Phi = \Sigma_p\Phi(\Phi^T\Sigma_p\Phi + \sigma_n^2 I)^{-1} \tag{2}$$
$$\sigma_n^{-2}\Phi = A\Sigma_p\Phi(\Phi^T\Sigma_p\Phi + \sigma_n^2 I)^{-1} \tag{3}$$
$$\sigma_n^{-2}\Phi(\Phi^T\Sigma_p\Phi + \sigma_n^2 I) = A\Sigma_p\Phi \tag{4}$$
$$\sigma_n^{-2}\Phi(\Phi^T\Sigma_p\Phi + \sigma_n^2 I) = (\sigma_n^{-2}\Phi\Phi^T + \Sigma_p^{-1})\Sigma_p\Phi \tag{5}$$
$$\sigma_n^{-2}\Phi\Phi^T\Sigma_p\Phi + \sigma_n^{-2}\Phi\sigma_n^2 I = \sigma_n^{-2}\Phi\Phi^T\Sigma_p\Phi + \Sigma_p^{-1}\Sigma_p\Phi \tag{6}$$
$$\sigma_n^{-2}\Phi\Phi^T\Sigma_p\Phi + \Phi = \sigma_n^{-2}\Phi\Phi^T\Sigma_p\Phi + \Phi \tag{7}$$

1. Equivalence to show
2. Devide by $\phi_\star^T$ and $\mathbf{y}$
3. Multiply A from the left
4. Multiply $\Phi^T\Sigma_p\Phi + \sigma_n^2 I$ from the right
5. Replace A
6. Remove braces by expanding
7. Simplify

(b)

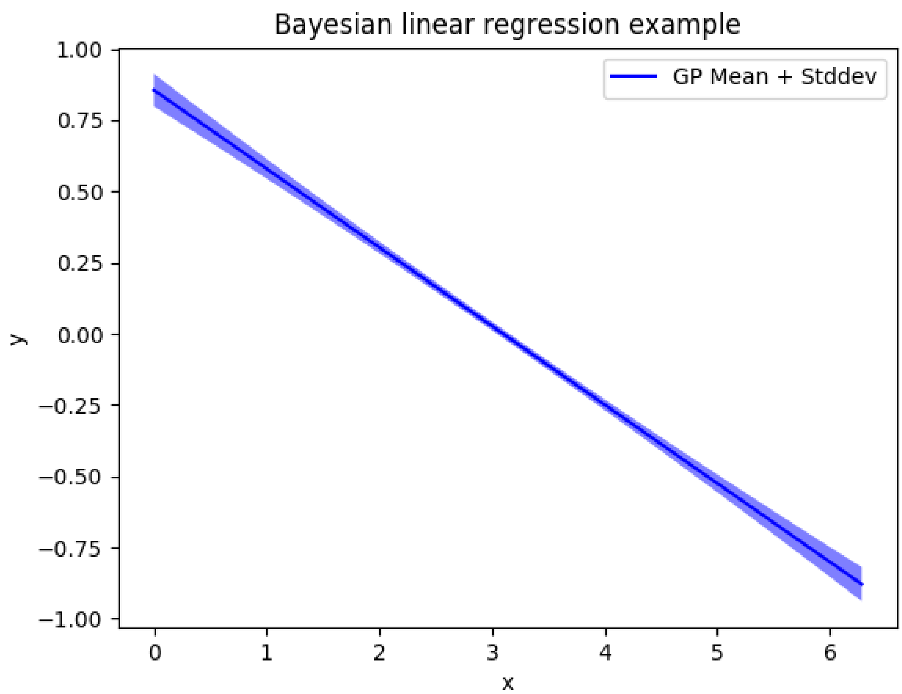

Figure 1: Mean and variance of the GP for a feature space of size $N = 2$

(c)

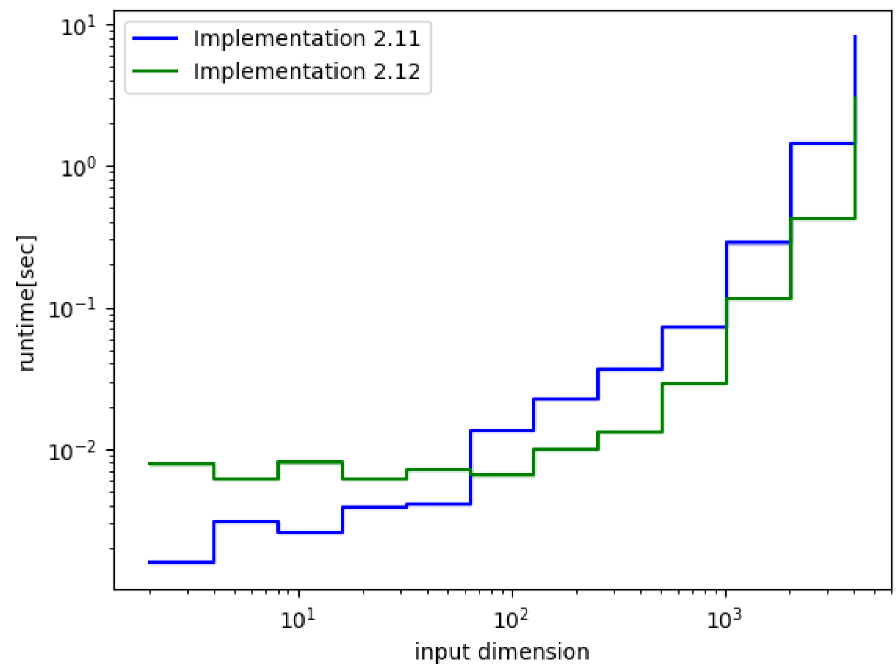

Figure 2: Mean and variance of the runtime of the different implementations. Variance is only barely visible

Feedback

The assignment took 1.5 days.

By doing the proof and implementing the different equations, we learnd a lot about bayesian linear regression and GPs.