Gateway Classes

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303) - By Gulshan Sir

B.TECH

(SEM III) THEORY EXAMINATION 2024-25

Model Paper -1

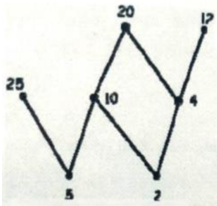
Time: 3 Hours **Total Marks: 70**

SECTION A

1. Attempt all Sections. If require any missing data; then choose suitably **2 x 7 = 14**

a. *Differentiate complemented lattice and distributed lattice.*

b. *Find the Maximal elements and minimal elements form the following Hasse's diagram.*



c. *State De Morgan'slaw and Absorption Law.*

d. *What are the contrapositive, converse, and teh inverse of the conditional statement*

"The home team wins whenever it is raining?"

e. *State Universal Modus Ponens and Universal Modus Tollens laws.*

f. *Define Ring and Field.*

g. *Define Chromatic number and Isomorphic graph.*

SECTION B

2. Attempt any three of the following: **7 x 3 = 21**

a. *Identify whether the each of the following relations defined on the set* $X = \{1, 2, 3, 4\}$

are reflexive, symmetric, transitive and/or antisymmetric?

$(i)\quad R_1 = \{(1, 1), (1, 2), (2, 1)\}$

$(ii)\quad R_2 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$

$(iii)\quad R_3 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$

b. *Justify that for any sets A, B and C :*

$(i)\ (A - (A \cap B)) = A - B$

$(ii)\ (A - (B \cap C)) = (A - B) \cup (A - C)$

c. (i) *Express Converse, Inverse and contrapositive of the following statement*

"$If\ x + 5 = 8\ then\ x = 3$"

(ii) $Show\ that\ the\ statements\ \ P \leftrightarrow Q\ and\ (P \wedge Q) \vee (\text{y}P \wedge \text{y}Q)\ are\ equivalent$

d. $What\ do\ you\ mean\ by\ cosets\ of\ a\ subgroup?\ Consider\ the\ group\ Z\ of\ integers$

$under\ addition\ and\ the\ subgroup$

$H = \{\dots\dots, -12, -6, 0, 6\ 12, \dots\dots\}\ considering\ of\ multiple\ of\ 6$

$(i)\ Find\ the\ cosets\ of\ H\ in\ Z$

$(ii)\ What\ is\ the\ index\ of\ H\ in\ Z.$

e. $Define\ the\ following\ with\ one\ example.$

$(i)\ Bipartite\ Graph.$

$(ii)\ Complete\ Graph$

$(iii)\ How\ many\ edges\ in\ K_7\ and\ K_{3,\ 6}.$

$(iv)\ Planar\ Graph.$

SECTION C

3. Attempt any one part of the following: **7 x 1 = 7**

a. $Define\ Modular\ Lattice. Justify\ that\ if\ 'a'\ and\ 'b'\ are\ the\ elements\ in\ a\ bounded$

$distributive\ lattice\ and\ 'a'\ if\ \ has\ complement\ a'.then$

$(i)\ a \vee (a' \wedge b) = a \vee b$ $\qquad$ $(i)\ a \wedge (a' \vee b) = a \wedge b$

b. (i) $Justify\ that\ (D_{36}, \backslash)\ is\ lattice.$

$(ii)\ Let\ L_1\ be\ the\ lattice\ defined\ as\ D_6\ and\ L_2\ be\ the\ lattice\ (P(S), \leq, where\ P(S)\ be\ the$

$power\ set\ defined\ on\ set\ S = \{a, b\}. Justify\ that\ the\ two\ lattices\ are\ isomorphic.$

4. Attempt any one part of the following: **7 x 1 = 7**

a. $Simplify\ the\ following\ Boolean\ expressions\ using\ Boolean\ algebra$

(i) $\quad xy + x'z + yz$

(ii) $\quad C(B + C)(A + B + C)$

(iii) $\quad A + B(A + B) + A(A' + B)$

(iv) $\quad XY + (XZ)' + XY'Z(XY + Z)$

b. $(\boldsymbol{i})$ $Let\ R = \{(1, 2)(2, 3)(3, 1)\} defined\ on\ A = . Find\ the\ transitive\ \ closure\ of\ R\ using$

$Warshall's algorithm.$

$(\boldsymbol{ii})$ $Justify\ that$ If f:A→B and g:B→C be one-toone onto functions, then gof is also one to one onto and $(gof)^{-1}=f^{-1}o\ g^{-1}$".

5. Attempt any one part of the following: **7 x 1 = 7**

a. $What\ is\ a\ tautology, contradiction\ and\ contingency?\ Show\ that\ (p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ $is\ a\ tautology, contradiction\ or\ contingency.$

b. $Show\ that\ the\ premises$ It is not sunny this afternoon and it is colder than yesterday, $We\ will\ go\ swimming\ only\ if\ it\ is\ sunny, If we\ do\ not\ go\ swimming, then\ we\ will\ take\ a$ $canoe\ trip.$ and $If\ we\ take\ a\ canoe\ trip, then\ we\ will\ be\ home\ by\ sunset$ lead to the conclusion. $"We\ will\ be\ home\ by\ sunset."$

6. Attempt any one part of the following: **7 x 1 = 7**

a. $Let\ G = \{1, -1, i, -i\}\ with\ the\ operation\ of\ ordinary\ multiplication\ on$

$G\ be\ an\ algebric\ structure, where\ i = \sqrt{-1}.$

$(i)\ Determine\ whether\ G\ is\ Abelian.$

$(ii)\ Determine\ the\ order\ of\ each\ element\ in\ G.$

$(iii)\ Determine\ whether\ G\ is\ a\ cyclic\ group, if\ G\ is\ a\ cyclic\ group, then$

$determine\ the\ generator/generators\ of\ the\ group\ G.$

$(iv)\ Determine\ a\ subgroup\ of\ the\ group\ G.$

b. $Let\ (G,*)\ and\ (G',*')\ be\ any\ two\ groups\ and\ let\ e\ and\ e' be\ their\ respective$ $identities. If\ f\ is\ a\ homomorphism\ of\ G\ into\ G', the\ prove\ that$

$(i)\ f(e) = e'$

$(ii)\ f(X^{-1}) = [f(X)]^{-1}, \forall\ X \in G$

7. Attempt any one part of the following: **7 x 1 = 7**

a. $Define\ and\ Explain\ any\ two\ the\ following:$

$(i)\ Euler\ Graph$

$(ii)\ Adjacency\ matrix\ of\ a\ graph.$

b. $Explain\ the\ following\ terms\ with\ example:$

$(i)\ Justify\ that$ "$In\ a\ undeirected\ graph\ the\ total\ number\ of\ odd\ degree\ vertices\ is\ even$".

$(ii)\ Justify\ that$ "$The\ maximum\ number\ of\ edges\ in\ a\ simple\ graph\ is\ n(n-1), 2$"

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Model Paper -2

Time: 3 Hours **Total Marks: 70**

SECTION A

1. Attempt all Sections. If require any missing data; then choose suitably **2 x 7 = 14**

a. *Draw the Hasse's diagram of the POSET* $(L, \subseteq)$, *where*

$L = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$, *where the sets are given by*

$S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$ $S_3 = \{a, b, c, e\}$

$S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$, $S_7 = \{a\}$

b. *Define various types of functions.*

c. *Show that a lattice with* 5 *elements is not a boolean algebra.*

d. *Show that* $\neg (p \vee q)$ *and* $\neg\ p \wedge \neg q$ *are logically equivalent.*

e. *Write the contra positive of the implication*: "*If it is Sunday then it is a holiday*"

f. *Define normal subgroup.*

g. *Explain pigeonhole principle with example.*

SECTION B

2. Attempt any three of the following: **7 x 3 = 21**

a. *Prove that in any lattice the following distributive inequalities hold.*

$(i)\ a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

$(ii)\ a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

b. *If* $f: R \rightarrow R, g: R \rightarrow R$ *and* $h: R \rightarrow R$ *defined by* $f(x) = 3x^2 + 2, g(x) = 7x - 5$ *and* $h\ (x) = 1/x$.

Compute the following composition function.

$(i)\ (fogoh)(x)$ $(ii)\ (gog)(x)$

$(iii)\ (goh)(x)$ $(iv)\ (hogof)\ (x)$

c. *Show that* $((p \vee q)\ \wedge \sim (\sim q \vee (\sim q \vee \sim r))) \vee (\sim p \wedge \sim q) \vee (\sim p \vee qr)$ *is a tautology*

without using truth table.

d. *State and prove Lagrange theorem for group.*

e. $Define\ planar\ graph. Prove\ that\ for\ any\ connected\ planar\ graph, v - e + r = 2$

$where\ v, e, r\ is\ the\ number\ of\ vertices, edges, and\ regions\ of\ the\ graph\ respectively.$

SECTION C

3. Attempt any one part of the following: **7 x 1 = 7**

a. $Show\ that\ the\ inclusion\ relation\ \subseteq is\ a\ partial\ ordering\ on\ the\ power\ set\ of\ a\ set\ S.$

$Draw\ the\ Hasse\ diagram\ for\ inclusion\ on\ the\ set\ P(S), where\ S = \{a, b, c, d\}.$

$Also\ Determine\ whether\ (P\ (S), \subseteq) is\ a\ lattice.$

b. $State\ Principle\ of\ Duality. Let\ A\ denote\ the\ set\ of\ real\ numbers\ and\ a\ relation\ R\ is$ $defined\ on\ A\ such\ that\ (a, b)R\ (c, d)\ if\ and\ only\ if\ a^2 + b^2 + c^2 + d^2. Justify\ that\ R\ is$ $an\ equivalence\ relation.$

4. Attempt any one part of the following: **7 x 1 = 7**

a. $Solve\ the\ following\ Boolean\ finctions\ using\ K - map$:

$(i)\ F\ (A, B, C, D) = \Sigma(m_0, m_1, m_2, m_4, m_5, m_6, m_8, m_9, m_{12}, m_{13}, m_{14})$.

$(ii)\ F\ (A, B, C, D) = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$

b. $Define\ Boolean\ algebra. Show\ that\ a'. [(b' + c)' + b.c] + [(a + b')'. c] = a'. b\ using$ $rules\ of\ Boolean\ Algebra. Where\ a' is\ the\ complement\ of\ an\ element\ a.$

5. Attempt any one part of the following: **7 x 1 = 7**

a. $(a)\ Prove\ the\ validity\ of\ the\ following\ argument. If\ Mary\ runs\ for\ office, She\ will\ be\ elected.$ $If\ Mary\ attends\ the\ meeting, she\ will\ run\ for\ office. Either\ Mary\ will\ attend\ the\ meeting\ or$ $she\ will\ go\ to\ India. But\ Mary\ cannot\ go\ to\ India.$ Thus Mary will be elected".

b. $Convert\ the\ following\ two\ statements\ in\ quantified\ expressions\ of\ predicate\ logic$

$(i)\ For\ every\ number\ there\ is\ a\ number\ greater\ than\ that\ number.$

$(ii)\ Sum\ of\ every\ two\ integer\ is\ an\ integer.$

$(iii)\ Not\ Every\ man\ is\ perfect.$

$(iv)\ There\ is\ no\ student\ in\ the\ class\ who\ knows\ Spanish\ and\ German$

6. Attempt any one part of the following: **7 x 1 = 7**

a. $Define\ the\ binary\ operation * on\ Z\ by\ x * y = x + y + 1\ for\ all\ x, y\ belongs$ $to\ set\ of\ integers. Verify\ that\ (Z,*)\ is\ an\ abelian\ group?$

$Discuss\ the\ properties\ of\ abelian\ group.$

b. (i) $Justify\ that$ "$The\ intersection\ of\ any\ two\ subgroup\ of\ a\ group\ (G,*)\ is$ $again\ a\ subgroup\ of\ (G,*)$".

(ii) $Justify\ that$ "$If\ a, b\ are\ the\ arbitrary\ elements\ of\ a\ group\ G\ then\ (ab)^2 = a^2b^2$ $if\ and\ only\ if\ G\ is\ abelian.$

7. Attempt any one part of the following: **7 x 1 = 7**

a. $Find\ the\ number\ between\ 1\ to\ 500\ that\ are\ not\ divisible\ by\ any\ of\ the\ integers\ 2\ or$ $3\ or\ 5\ or\ 7.$

b. $Acollection\ of\ 10\ electric\ bulbs\ contain\ 3\ defective\ ones.$

(i) $In\ how\ many\ ways\ can\ a\ sample\ of\ four\ bulbs\ be\ selected?$

(ii) $In\ how\ many\ way\ can\ a\ sample\ of\ 4\ bulbs\ be\ selected\ which\ contain\ 2\ good\ bulbs\ and\ 2$ $defective\ ones?$

(iii) $In\ how\ many\ ways\ can\ a\ sample\ of\ 4\ bulbs\ be\ selected\ so\ that\ either\ the\ sample\ contain\ 3$ $good\ ones\ and\ 1\ defectives\ ones\ or\ 1\ good\ and\ 3\ defectives\ ones?$