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Ans1. Consider a random sample $(x_1, x_2, x_3, \dots, x_n)$

$\mu = \theta_1$ (mean) and $\sigma^2 = \theta_2$ (Variance)

Likelihood function $(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(-\frac{(x_i-\theta_1)^2}{2\theta_2}\right)}$

To maximise, take log ~~no~~ on both sides

$$\Rightarrow \ln[L(\theta_1, \theta_2)] = \sum_{i=1}^{n} \left[\frac{-1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2}\right]$$

(i) Diff. w.r.t. θ_1

for θ_1: $$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta_1 = \sum_{i=1}^{n} \frac{x_i}{n} \text{ [mean]}$$

(ii) Diff. w.r.t. θ_2

for θ_2: $$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2}\right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2 \text{ [variance]}$$

Ans2.

Binomial distribution of $B(n, \theta)$ $\quad p = \theta, q = 1-\theta$

pmf $$f(x; n, \theta) = {}^{n}C_{x}\, \theta^{x} (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

Taking log on both sides

$$\ln[L(\theta)] = \sum_{i=1}^{n} [\ln {}^{n}C_{x_i} + x_i \ln\theta + (n-x_i)\ln(1-\theta)]$$

Diff. w.r.t. θ.

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta}\right] = 0$$

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Find θ

$$\sum_{i=1}^{n}\left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta}\right] = 0$$

$$\sum_{i=1}^{n}\left[\frac{(1-\theta)\,x_i - \theta(n-x_i)}{\theta(1-\theta)}\right] = 0$$

$$\sum_{i=1}^{n} (1-\theta)x_i - (n-x_i)\theta = 0$$

$$\theta\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot m.$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}} \longrightarrow \text{MLE}$$