ECN-300 Lab Based Project

# Implementing Energy Efficiency Optimization using BRB Algorithm

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***Abstract*— The Global Optimization of Energy Efficiency metrics of wireless networks in the presence of multi-user interference is challenging because the numerator of the EE becomes a non-concave function of the transmit power. Optimization methods, which have been widely used in recent communication and networking systems designs, face a significant hurdle in such non-convexity of many optimization formulations that arise from practical systems. To address these issues, mathematicians captured the monotonic structures and properties of these problems jointly with fractional programming theory to propose a new branch-reduce-and-bound procedure. We implemented this algorithm based on our problem-specific bounds for energy-efficiency global maximization that allows for faster convergence, generally with exponential complexity in the number of network links. Although the complexity is still exponential in the number of variables, this Monotonic Framework is much lower than other available global optimization frameworks.**

***Keywords*— Energy Efficiency, BRB Algorithm, Monotonic Framework.**

## I. INTRODUCTION

The global data traffic is increasing rapidly with each passing day. Energy management is crucial for the sustainability of future wireless communication networks, whose energy efficiency (EE) is predicted to increase by a factor of 2000 as compared to present networks. Since limiting the ICTs is not very realistic, developments are made in approaches that enhance the communication and networking system performances. For us, the most promising answer is to maximize the energy efficiency, the amount of information reliably transmitted per Joule of consumed energy.

Most recent advances in optimization techniques rely on the convexity of the problem formulation. Nonetheless, many problems encountered in practical engineering systems are nonconvex in their original forms and cannot be equivalently transformed to convex ones by any existing methods. However, most of these problems encountered in communication and networking systems exhibit monotonic or hidden monotonic structures. Recall that searching for an optimal global solution of a non-convex optimization problem involves examining every feasible point in the entire feasible region. If the objective function $f(z)$: $R^n \rightarrow R$ to be maximized is increasing, then once a feasible point **z** is known, one can ignore the whole cone $z-R^n_+$, because no better feasible solution exists in this cone. Whereas, if we have an increasing function $g(z)$**:** $R^n \rightarrow R$ in a constraint like $g(z) \leq 0$, then once a point **z** is known to be infeasible, the whole cone $z+R^n_+$ can be discarded. The monotonic nature, thus allows us to limit the global search to a much smaller region of the feasible set and simplifies the problem.

The performance of multiuser systems is challenging to measure correctly and to optimize. Most resource allocation problems are nonconvex and NP-hard, even under simple assumptions such as perfect channel, homogeneous channel properties, and easier power constraints. The implemented branch-reduce-and-bound (BRB) algorithm and the overall algorithm for EE optimization algorithm under interference limited channels that involve fractional programming are presented in the Report. The BRB algorithm is computationally costly, but it shows better convergence than the previously proposed outer Polyblock approximation algorithm.

## II. SIGNAL MODEL AND PROBLEM FORMULATION

Consider k mutually interfering links communicating over the channel of bandwidth B. All links are single-source transmitter which transmits the signal at a power $p_k$, constraint to $0 < p_k < P_{kmax}$ (where $P_{kmax}$ is the maximum power feasible to transmit). Let P be the power vector containing all the power as $P = [p_1, p_2, \ldots\ldots p_k]$. Now we wish to define signal to noise plus interference ratio for our problem. Before we write any particular expression for SINR we take an important assumption about achievable rate $R_k$ which constraints our expression for SINR.

*Assumption 1* - The function $\gamma_k(p)$: $R^K_+ \rightarrow R_+$ (SINR for each link) is, for all k, such that the achievable rate $R_k(p)$ of link k can be expressed as the difference of two non-negative functions-

$$R_k(p) = B*\log_2 (1 + \gamma_k(p)) = q^+_k(p) - q^-_k(p)$$

with $q^+_k(p)$, $q^-_k(p)$: $R^K_+ \rightarrow R_+$, $q^+_k(p)$ and $q^-_k(p)$ are assumed to be monotonic functions to establish monotonic optimization framework. From now we assume assumption 1 in general holds for all these three cases of SINR discussed below.

1) The typical form of SINR for a general multi-link system can be written in the following form-

$$\gamma_k(p) = p_k *\alpha_k / (\sigma^2 + \sum p_i\beta_{i,k}) \ i=1 \text{ to } K,\ i\neq k$$

where $\sigma^2$ is the power of the noise due to receiver, $\alpha_k$ is the channel gain over the link k, $\beta_{i,k}$ is the term the accounts for the interference due to presence of multiple links.

2) The above expression doesn't consider the possible self-interfering term which may distort the signal received. Including the self-interfering term, the SINR expression can be modified as follows-

$$\gamma_k(p) = p_k * \alpha_{k\,/}\, (\sigma^2 + p_k\varphi_k + \sum p_i\beta_{i,k})\ i=1 \text{ to } K,\ i\neq k$$

where $\varphi_k$ is the coefficient of self-interference that depends on propagation channel and system parameters.

3) In the third possible SINR expression we assume vector channel with Least mean square error at the receivers.

In the considered interference network scenario, the EE (measured in bit/Joule) of link k is considered as the ratio of the achievable rate and the total power consumption:

$$EE_k(p) = B*\log_2 (1 + \gamma_k(p))/\ \mu_k p_k + \Psi_k$$

wherein $\mu_k = 1/\eta$, with $\eta$ the efficiency of the transmit power amplifier, while $\Psi_k$ includes the power dissipated in all other circuit blocks of the transmitter and receiver. The circuit power is a fixed power cost which depends neither on the transmit power p, nor on the communication rate R**.** Our approach is to stick to the physical meaning of energy efficiency as benefit-cost ratio, where now the benefit and cost should be related to the whole network, rather than to individual link-

$$\mathrm{GEE}(\mathbf{p}) = \frac{\sum_{k=1}^{K} B \log_2(1+\gamma_k(\mathbf{p}))}{\sum_{k=1}^{K} \mu_k p_k + \Psi_k}$$

- The GEE is the ratio between the total benefit produced by the network and the total incurred cost. However, while having a clear and strong physical interpretation, the GEE does not allow to tune the individual energy efficiencies of the different links.

$$\mathrm{WSEE}(\mathbf{p}) = \sum_{k=1}^{K} w_k \frac{B \log_2(1+\gamma_k(\mathbf{p}))}{\mu_k p_k + \Psi_k}$$

- This choice results in the point where the hyperplane is tangent to the Pareto boundary. Unlike the GEE, the WSEE depends on the individual energy efficiencies, which can be assigned different priorities through the choice of the weights.

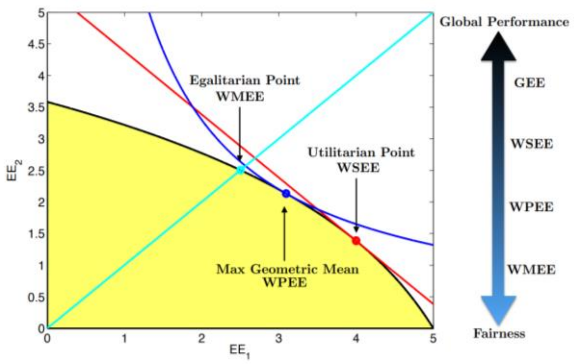


**Figure 1 Energy-efficient Pareto region with the target points on the boundary achieved by the different multi-objective metrics [IV]**

## III. Mathematical Background

This section begins with a mathematical preliminary to understand the algorithm-

- (Increasing functions) A function f: $R^n_+ \to R$ is increasing if $f(x) \le f(y)$ when $0 \le x \le y$.
- (Boxes) If $a \le b$, then box [a, b] is the set of all $x \in R^n$ satisfying $a \le x \le b$. A box is also referred to as a hyper-rectangle.
- (Normal sets) A set $G \subset R^n_+$ is normal if for any point $x \in G$, all other points x' such that $0 \le x' \le x$ is also in set G. In simple words, $G \subset R^n_+$ is normal if $x \in G \Rightarrow [0, x] \subset G$.
- (Conormal sets) A set H is conormal if $x \in H$ and $x' \ge x$ implies $x' \in H$. The set is conormal in [0, b] if $x \in H \Rightarrow [x,b] \subset H$. Clearly, conormal and normal sets are complementary sets in [0, b].
- (Upper boundary) A point x' of a normal closed set G is called an upper boundary point of G if $G \cap \{x \in R^n_+ \,|x > x'\} = \emptyset$. The set of all upper boundary points of G is called its upper boundary and denoted by $\partial^+G$.

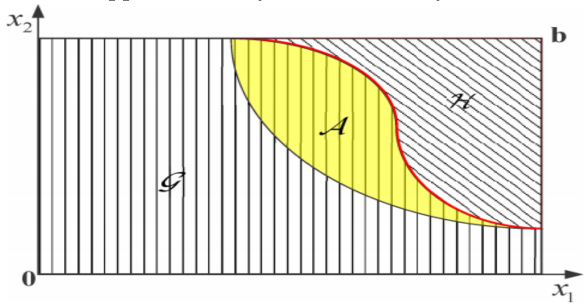


**Figure 2 Illustration of boxes, normal set, conormal set, normal hull, upper boundary[IV]**

Here, the rectangle represents box [0, b]. Set H is a conormal set in box [0, b]. Its complement is set G that is obviously a normal set. The red curve is the upper boundary of G, denoted by $\partial^+G$.

### A. *Canonical Monotonic Optimization Formulation*

Monotonic Optimization is concerned with problems of the following form:

$$\max\{f(x)|x \in G \cap H\}$$

where f(x): $R^n_+ \to R$ is an increasing function, $G \subset [0, b] \subset R^n_+$ is a compact normal set with nonempty interior, and H is a closed conormal set on [0, b]. Sometimes, H is not present in the formulation, and the problem becomes:

$$\max\{f(x)|x \in G\}$$

In this case, we can assume that the conormal set H in this Equation is box [0, b] itself. In the remaining of this monograph, we assume that the problem considered is feasible, i.e., $G \cap H \neq \emptyset$. Sets G and H often result from constraints involving increasing functions.

| **PROPOSITION:** |
|---|
| For any increasing function g(x) on $R^n_+$, the set $G = \{x \in R^n_+ \vert g(x) \le 0\}$ is normal and it is closed if g(x) is lower semi-continuous. Similarly, for any increasing function h(x) on $R^n_+$, the set $H = \{x \in R^n_+ \vert h(x) \ge 0\}$ is conormal and it is closed if h(x) is upper semi-continuous. |

Here f(x) may correspond to some system performance, g(x) may correspond to some scarce resources that have limited availability, and h(x) may correspond to users' satisfaction which has to reach a certain level. Problems with hidden monotonicity can be also converted to the canonical form.

### B. *Fractional Programming*

Fractional programming theory is the branch of optimization theory that is concerned with the properties and optimization of fractional functions. The energy efficiency of a communication system is always expressed through fractional functions and the energy-efficient resource allocation problem is naturally casted as a fractional program. Let $D \subseteq R^N$ and consider the functions $f_k$: $D \rightarrow R$ and $g_k$: $D \rightarrow R{+}{+}$, with $k = 1$ to K. A generalized fractional program is defined as-

$$\underset{\mathbf{x}}{\text{maximize}} \quad \min_{k=1,\dots,K} \frac{f_k(\mathbf{x})}{g_k(\mathbf{x})} \quad \text{s.t.} \quad \mathbf{x} \in \mathcal{D}.$$

Dinkelbach's algorithm [1] allows us to solve the above generalized equation.

## IV. Monotonic Optimization

Given this background the problem we have can written as-

$$\text{Maximize } u(p)$$
$$\text{s.t. } p \text{ belongs to } P$$

where u(p) is the function to be optimized (either GEE or WMEE) and P is our feasible set given by-

$$P = p \in R^K{}_+;\ p_k \leq P_{max,k}\ ,\ c_k(p) \geq 0\ \forall k \in \{1 \dots K\}$$

Where $c_k(p)$ introduces all the possible constraints that may arise due to the system. Note that we take another assumption about $c_k(p)$ similar to what we did with SINR.

*Assumption 2* -The function $c_k(p)$ can be expressed $\forall k$ as the difference of two non-negative functions, namely:

$$c_k(p) = c_k^+(p) - c_k^-(p)$$

with $c_k^+(p)$ and $c_k^-(p)$: $R^K{}_+ \rightarrow R_+$. As we will see further $c_k(p)$ will be manipulated in such a way to design the required normal and conormal sets for defining monotonic optimization framework. But the EE optimization lacks a monotone structure for direct implementation of this algorithm. We can do certain manipulation and make assumptions to overcome this challenge.

*Assumption 3* - The functions $q_k^+(p)$ and $q_k^-(p)$, and the functions $c_k^+(p)$ and $c_k^-(p)$ are monotonic functions $\forall k \in \{1 \dots K\}$ as stated in signal formulation section. GEE maximization belongs to a single fraction fractional programming framework. Hence by Dinkelbach's algorithm, the problem can be stated as-

$$\underset{\mathbf{p}}{\text{maximize}} \sum_{k=1}^{K} B \log_2(1+\gamma_k) - \lambda_j \left(\mu_k p_k + \Psi_k\right) \quad \text{s.t. } \mathbf{p} \in \mathcal{P}$$

The above structure is not a monotonic optimization problem in canonical form. As the objective function is not a monotonic function and the constraint doesn't guarantee to be the intersection of normal and conormal set. Using mathematical formulations and introducing an auxiliary variable **t** we converted the main problem to a monotonic form-

$$\text{maximize}_{(t,\, p)}\ q^+(p) + t$$
$$\text{s.t. } (t, p) \in P \cap Q$$
$$Q = \{(t, p): 0 \leq t + q^-(p, \lambda_j) \leq q^-(p_{max}, \lambda_j)$$
$$0 \leq t \leq q^-(p_{max}, \lambda_{j)} - q^-(0_K, \lambda_j)\}$$

The problem is not yet monotonic optimization as the constraint function $c_k(p)$ is expressed as a difference of two monotonic function. The overcome this problem we write $c_k(p) \geq 0$ for all $k = 0$, ……K as $\min_k[c_k^+(p) - c_k^-(p)] \geq 0$ which is a single constraint. We can now introduce an auxiliary variable **s** and change the form easily, similarly like the above manipulation. Now to write the feasible set as an intersection of two normal-conormal set-

$$\mathcal{S} = \Big\{(t, s, \mathbf{p}) : \mathbf{p} \preceq \mathbf{p}_{\max}, t + q^-(\mathbf{p}, \lambda_j) \leq q^-(\mathbf{p}_{\max}, \lambda_j),$$
$$s + c^-(\mathbf{p}) \leq c^-(\mathbf{p}_{\max})\Big\}$$

$$\mathcal{S}_c = \left\{(t, s, \mathbf{p}) : \mathbf{p} \succeq \mathbf{0}_K, t \geq 0, s + c^+(\mathbf{p}) \geq c^-(\mathbf{p}_{\max})\right\} .$$

Now all the constraints are monotonic and continuous and one can state from the preposition that GEE optimisation problem can be stated as monotonic optimization problem in canonical form. Hence, Proved.

## V. Branch-Reduce-Bound Algorithm

In this section, we aim at solving the monotonic optimization problem formulated in the previous section using the famous BrB Algorithm with Global convergence. The algorithm maintains a set ***N*** of non-overlapping hyper rectangles that surely covers the parts of the robust performance region *R* where the optimal solutions lie (the solution might be non-unique). Iteratively, we split certain hyper rectangles and try to improve a lower bound $f_{min}$ and an upper bound $f_{max}$ on the optimal set. To aid this process, a local feasible point $g_M$ and a local upper bound β(M) are stored for each box M ∈ *N*. The algorithm proceeds until $f_{max} - f_{min} < €$, for a predefined solution accuracy €.
Initially, $N = \{M_0\}$ for a box $M_0 = [0, b_0] \subseteq R^{Kr}{}_+$, where $b_0$ is based on some suitable upper bound that guarantees $R \subseteq M_0$. The initial upper bound is $f_{max} = f(b_0)$, while the lower bound is initialized as $f_{min} = f(0)$.

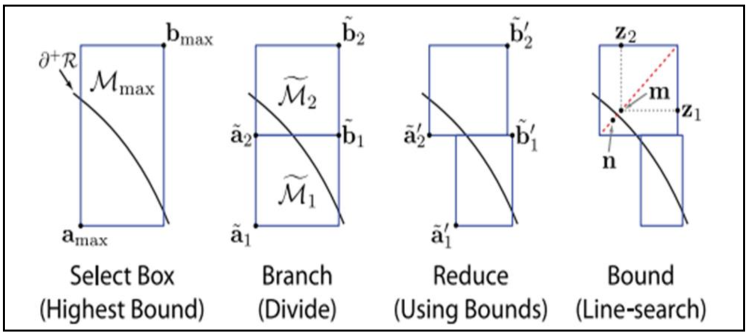


**Figure 3 An iteration of the BRB algorithm: A box is selected and branched into two new boxes. These are reduced based on the current bounds on the optimal value. Finally, line search between the lower and upper corners of the outermost box is applied to improve the bounds. [VII]**

Each iteration of the BRB algorithm consists of three steps.

A. Branch: Divide a box $M_{max} \in N$ into two new boxes. First, $M_{max} = [a_{max}, b_{max}]$ which contains the current upper bound $f_{max}$, is divided into two disjoint boxes $M_1$, $M_2$. $M_{max}$ is bisected along its longest side. The local feasible points and upper bounds of $M_1$, $M_2$ can then be selected. The (local) upper bounds over these new boxes are also based on $M_{max}$:

$$\beta(\widetilde{\mathcal{M}}_1) = \min\left(\beta(\mathcal{M}_{\max}), f(\mathbf{b}_{\max}\right.$$

$$\beta(\widetilde{\mathcal{M}}_2) = \beta(\mathcal{M}_{\max}).$$

B. Reduce: Remove parts of these new boxes that cannot contain optimal solutions. The new boxes $M_L = [a_L, b_L]$ (for L = 1,2) are reduced by removing parts that cannot contain the optimal solution; that is, parts that either give performance below the lower bound $f_{min}$ or above the (local) upper bound $\beta(M_L)$. The lemma 2.19[V] from shows how to replace $M_L$ with a (potentially) smaller box $[a'_L, b'_L]$. The reduction procedure is a two-step procedure: first, the lower point $a'_L$ is updated then it is used to update the upper point $b'_L$.

C. Bound: Apply the bounding procedure in Lemma 2.14 [V] to one of the new boxes, to improve local and global bounds. Each iteration ends with a bounding step where we search for feasible solutions in the new box with the largest (local) upper bound (i.e. $\beta(M_1) < \beta(M_2)$). These solutions are used to improve $f_{min}$, $f_{max}$ and $\beta(M_2)$. We check if there are any feasible points in $M_L = [a_L, b_L]$, or if $M_L \cap R = \emptyset$. If the lemma concludes $M_L \cap R = \emptyset$, then $M_L$ is deleted from $N$. Finally, the global lower bound is updated as $f_{min} = \mathrm{Max}_{M \in N} f(g_M)$ and the global upper bound is updated as $f_{max} = \mathrm{Max}_{M \in N} \beta(M)$ among the remaining boxes. The stopping criterion $f_{max} - f_{min} < \varepsilon$ is checked at the end of each iteration.

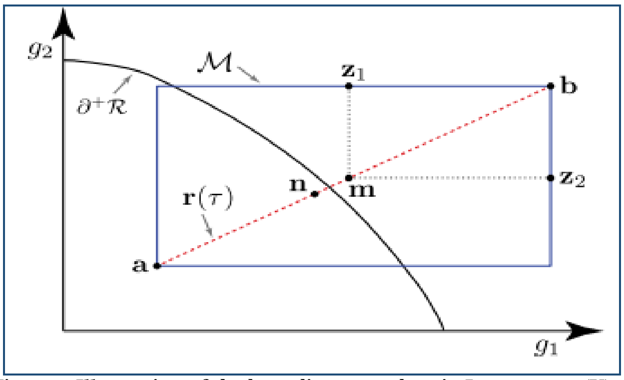


**Figure 4 Illustration of the bounding procedure in Lemma 2.14[V]**

## VI. Results

We applied the above discussed General optimization framework for resource allocation in multi-cell MISO downlink systems considering WMEE as our main objective function. The output of our Matlab code is shown in the figure. The output procedure of the Matlab code is the same as we expected in the monotonic case. The output procedure of the Matlab code is the same as we have assumed in the monotonic structure section. GEE Formulation and WMEE implementation had some remarkable differences:

- In our problem we considered single transmitter whereas in the code we assumed the situation of multi-input single-output system, with multiple transmitter antennas.
- Our problem was a single objective function whereas in the code we considered weighted sum which is a multi-objective case.
- In the code the interference was not explicitly mentioned in the mathematical formulation unlike our problem.

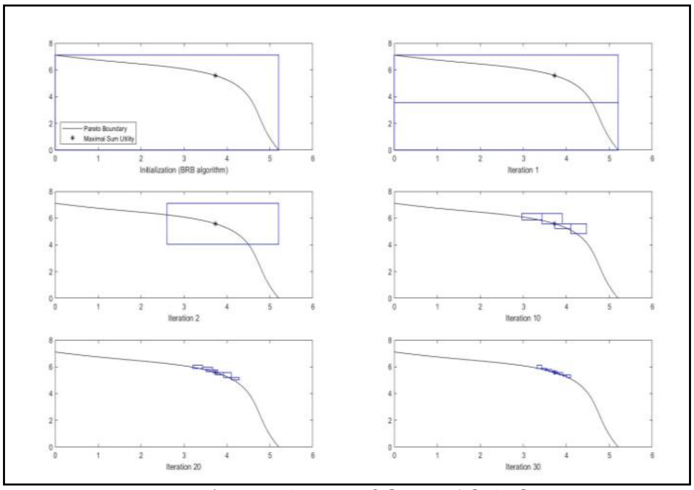


**Figure 5 Output of the Matlab Code**

## VII. Conclusion

Throughout the project we discussed the interference limited channel and formulated the function with each user's performance as input. We studied various forms of energy efficiencies and derived the formula for canonical Monotonic structure of GEE, merging the tools of monotonic optimization and fractional programming. We then studied the famous BrB Algorithm in its general form to solve various multi-objective functions in multi-cell scenario. We applied the Algorithm using Matlab code to optimize WMEE. In each iteration, a line-search is performed in the robust performance region—a quasiconvex fairness-profile optimization problem that can be solved efficiently. Since most multiuser resource allocation problems are nonconvex and NP-hard, the BRB algorithm is mainly suitable for computing benchmarks due to high computational complexity. This algorithm provides far better convergence than the previously known outer polyblock approximation algorithm. With minor changes the Algorithm can also be applied to solve our initial single objective function GEE.


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