

POC-3

Q1 a) Define $V, \Sigma, P \& S$

$\Sigma = \{a, b\}$

$V = \{S, A\}$

start symbol: S

Production (P):

$S \rightarrow aaS / aaA$

$A \rightarrow bA / \epsilon$

S generates an even no. of a 's (at least 2) & any no. of b 's

b) string: $aaaabb$
define & Derivation

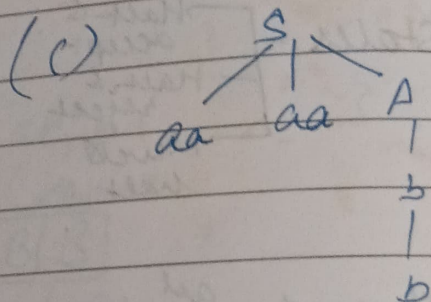
$S \rightarrow aaS$

$\rightarrow aa aaA$

$\rightarrow aa aa bA$

$\rightarrow aa aabbA$

$\rightarrow aa aabb$



Leaves

$aa aa bb \rightarrow "aaaabb"$

(d) Regular expression for L :

$(aa)^+ b^*$

Since this RE exists, L is a Regular language and every regular language is also Context free.

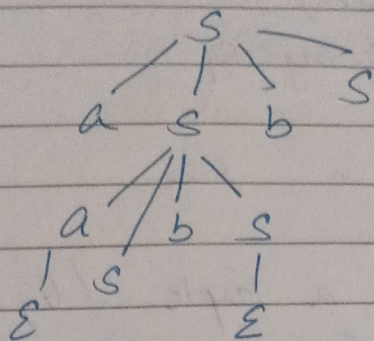
Q2 Derive $aabb$

$S \rightarrow aSbS$

$\rightarrow a(aSbS)bS$

$\rightarrow aaSbSbS \rightarrow aabb$

→ Parse Tree



Leaves (left to right): $aabb \rightarrow aabb$

- (b) A grammar is ambiguous if there exists at least one string in the language that has two distinct parse trees (equivalently two different leftmost or rightmost derivations)

Claim: The grammar $S \rightarrow aSbS / \epsilon$ is unambiguous

Proof:-

- Any nonempty string generated by this grammar must begin with a (because the only production introducing terminal with a) & therefore must be produced by one application of $S \rightarrow aS_1bS_2$

- In any derivation of a non-empty string w , the first a in w must match some b later in w , let that matching b at posⁿ k . The grammar forces that matching to be the b produced in the same production

aS_1bS_2 that produces the first a . that uniquely splits w into three parts

- a (the first terminal)
- substring generated by S_1
- b
- substring " " S_2

Q3. Define the PDA as a tuple $(Q, \Sigma, \Gamma, \delta, q_0, z_0, f)$
 Let $M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, f)$ where

$$Q = \{q_0, q_1, q_f\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{z_0, x\} - z_0$$

q_0 is start state

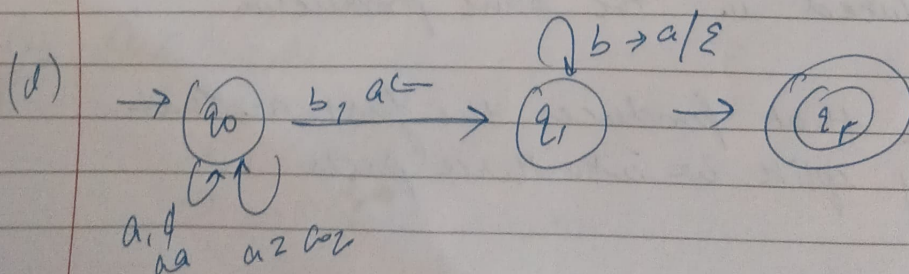
z_0 is initial state

$$f = \{q_f\}$$

- b)
- $\delta(q_0, a, z_0) = \delta(q_0, x, z_0)$
 - $\delta(q_0, a, x) = \gamma(q_0, x, x)$
 - $\delta(q_0, b, x) = \gamma(q_1, \epsilon)$
 - $\delta(q_1, b, x) = \gamma(q_1, \epsilon, z_0)$
 - $\delta(q_1, \epsilon, z_0) = \gamma(q_f, z_0)$
 - $\delta(q_0, \epsilon, z_0) = \gamma(q_f, z_0)$

(c) start: $(q_0, aaabbb, z_0)$

- $(q_0, aaabbb, z_0)$
- $(q_0, aabbb, xz_0)$
- $(q_0, abbb, xxz_0)$
- $(q_0, bbb, xxxz_0)$



Q. a) There is a pumping length $p \geq 1$ such that for every string s with $|s| \geq p$

$$s = uv^kxy$$

Satisfying

- 1) $|uv| \leq p$
- 2) $|v| > 0$
- 3) $i \geq 0$

b) Choose string $s \in L$ where $|s| \geq p$
Let p be the pumping length

$$s = a^p b^p c^p \in L$$

$$|s| = 3p \geq p$$

c) show some pumped string uv^iwx $\in L$

$$s = a^p b^p c^p \in L$$

$$|s| = 3p \geq p$$

Let $s = a^p b^p c^p$ since $|vwx| \leq p$, the substring vwx lies fully inside one block.

If vwx is within one block, pumping adds/removes symbols.

If vwx spans two blocks, pumping disturbs their balance while the third block stays unchanged.

d) Because the pumping lemma for CL is necessary property of all context-free languages.

Q5 $L(G) = \{a^m b^n \mid m > 0, n \geq 0\}$

To find: Context free grammar G that generates this (language)

Grammar:

Let $G = (V, \Sigma, P, S)$ where

$V = \{S, T\}$

$\Sigma = \{a, b\}$

S is the start symbol

Production P

$S \rightarrow aS \mid aT$

$T \rightarrow bT \mid \epsilon$

2) Explanation

S ensures at least one 'a' is produced since every derivation starts with a

T generates zero or more b s

→ eg derivations

$S \rightarrow aS \rightarrow aaT \rightarrow aabT \mid aabT \mid aabbb$

Generated string: $aabbb \in L(G)$

$L(G) = \{a^m b^n \mid m > 0, n \geq 0\}$

Q6

$S \rightarrow aS \mid aSbS \mid c$

Two different parse trees for $aacbc$

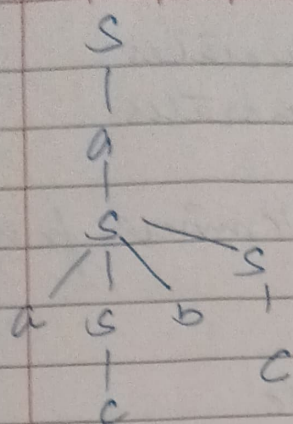
Derivation A

1 $S \rightarrow aS$

2 $aS \rightarrow a(aSbS)$

3 $c(aSbS) \rightarrow a(cacSbS)$

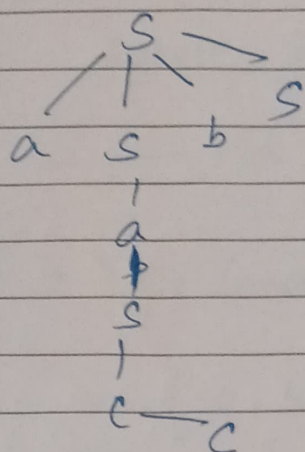
4 $a(cacSbS) \rightarrow a(cacbc) = aacbc$



Derivation

1. $S \rightarrow acbs$
2. $asbs \rightarrow a(as)bs$
3. $a(as)(bs) \Rightarrow a(ac)bs$
4. $a(ac)bs + a(ac)bc = aacbc$

Parse trees B (diff shape)



(right most $S \rightarrow c$)

The two trees are structurally different
 \therefore the grammar is ambiguous

→ A grammar is ambiguous some doing string has two diff parse trees

Q Given:

$G \rightarrow (T, S, A, B) \subseteq \{a, b, c\}^*$ with
 $PS \rightarrow B \rightarrow C$
 $A \rightarrow aB$

1) Find non terminals that generates terminal

$B \rightarrow C \rightarrow B$ generate

$A \rightarrow aB$ & B generate $\rightarrow A$ generates
 $S \rightarrow A$ & A generates $\rightarrow S$ generates

Step-2: Find reachable non terminal from start S :

' S ' is start

from $S \rightarrow A \rightarrow A$ reachable

from $A \rightarrow aB \rightarrow B$ reachable