

P0C-3

Q1 a) Define V, E, P & S

- $\Sigma = \{a, b\}$

$V = \{S, A\}$

Start symbol: S

- Production (P):

$$S \rightarrow aas / aaA$$

$$A \rightarrow bA / \epsilon$$

S generates an even no. of a's (at least 2) & j
any no. of b's

b) string: aaaaabb

derivation & Derivation

$$S \rightarrow aas$$

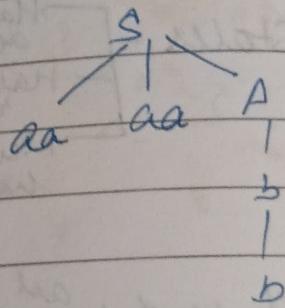
$$\rightarrow aa \ aAA$$

$$\rightarrow aa \ aa bA$$

$$\rightarrow aa \ aabbA$$

$$\rightarrow aa \ aabb$$

(c)



Leaves

$$aa \ aa \ bb \rightarrow "aaaaabb"$$

(d) Regular expression for L:

$$(aa)^* + b^*$$

Since this RE exists, L is a regular language
and every regular language is also context
free.

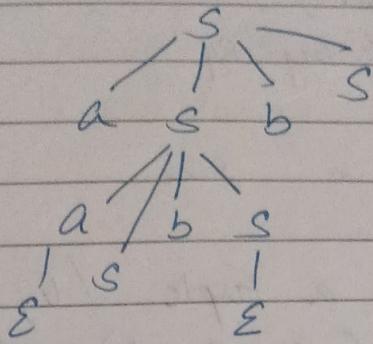
Q2 Derive aabb

$$S \rightarrow asbs$$

$$\rightarrow a(asbs)bs$$

$$\rightarrow aa\epsilon b\epsilon b\epsilon = aabb$$

7) Parse Tree



Leaves (left to right): $aabb \rightarrow aabb$

(b) A grammar is ambiguous if there exists at least one string in the language that has two distinct parse trees (equivalently two different leftmost or rightmost derivations)

Claim: The grammar $S \rightarrow aSbS \mid \epsilon$ is unambiguous

Proof:-

- Any nonempty string generated by this grammar must begin with a a because the only production introducing terminal with a) & therefore must be produced by one application of $S \rightarrow aS_1bS_2$
- For any derivation of a non-empty string w . the first a in w must match some b earlier in w . Let that matching b at posⁿ k . The grammar forces that matching to be the b produced in the same production aS_1bS_2 that produces the first a . That uniquely splits w into three parts

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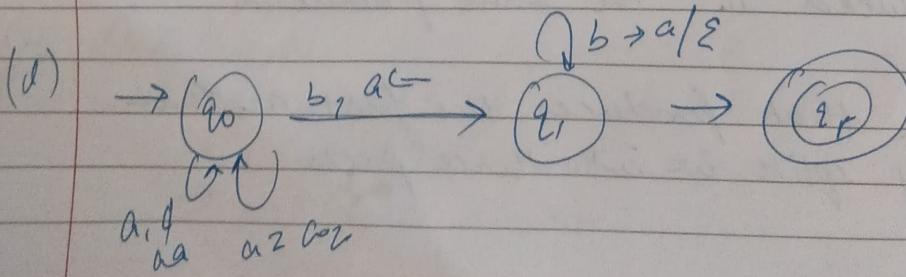
- a (the first terminal)
- substring generated by S_1
- b
- substring $u u s_2$

Q3. Define the PDA as a tuple $(\emptyset, \Sigma, \Gamma, S, q_0, z_0, f)$
 let $M = (\emptyset, \Sigma, \Gamma, S, q_0, z_0, f)$ where

$\emptyset = \{q_0, q_1, q_f\}$
 $\Sigma = \{a, b\}$
 $\Gamma = \{z_0, x\}$
 q_0 is start state
 z_0 is initial state
 $f = \{q_f\}$

- 1) $S(q_0, a, z_0) = S(q_0, xz_0) \cup$
- 2) $S(q_0, a, x) \cup S(q_0, x, z_0) \cup$
- 3) $S(q_0, b, x) \cup S(q_1, \epsilon) \cup$
- 4) $S(q_1, b, x) \cup S(q_f, z_0) \cup$
- 5) $S(q_1, \epsilon, z_0) \cup S(q_f, z_0) \cup$
- 6) $S(q_0, \epsilon, z_0) \cup S(q_f, z_0) \cup$

- (1) start: $(q_0, aaabb, z_0)$
- 1) $(q_0, aaabb, z_0)$
 - 2) $(q_0, aabb, xz_0)$
 - 3) $(q_0, abbb, xxz_0)$
 - 4) $(q_0, bbbb, xxxxz_0)$



Q. a) There is a pumping length $p \geq 1$ such that
for every string s over $\{a, b, c\}^*$
 $|s| \geq p$

Satisfying

- 1) $|vwx| \leq p$
- 2) $|vx| > 0$
- 3) $i \geq 0$

b) Choose string SEI where $|S| \geq p$
Let p be the pumping length

$$S = a^p b^p c^p EI$$

$$|S| = 3p \geq p$$

c) Show some pumped string leaves L

$$\text{Let } S = a^p b^p c^p EI$$

$$|S| = 3p \geq p$$

Let $S' = a^p b^p$ since $|vwx| \leq p$, the substring vwx lies fully inside one block

If vwx is within one block - pumping adds/ removes symbols

If vwx spans two block pumping disrupts their balance while the third block stays unchanged

d) Because the pumping lemma for CFL is necessary property of all context-free languages

Q5 $L(G) = \{a^m b^n \mid m \geq 0, n \geq 0\}$

To find: Context free grammar G that generates this language

Grammar:

Let $G = (V, \Sigma, P, S)$ where

$$V = [S, T]$$

$$\Sigma = \{a, b\}$$

S is the start symbol

Production P

$$S \rightarrow aS \mid aT$$

$$T \rightarrow bT \mid \epsilon$$

2) Explanation

S ensures at least one 'a' is produced since every derivation starts with a

T generates zero or more 'b's

\rightarrow Eg derivations

$$S \rightarrow aS \rightarrow aaT \rightarrow aabT \leftarrow aabT \leftarrow aabb$$

Generated string: aabb

$$L(G) = \{a^m b^n \mid m \geq 0, n \geq 0\}$$

Q6 $S \rightarrow aS \mid aSbS \mid C$

Two different parse trees for aabc

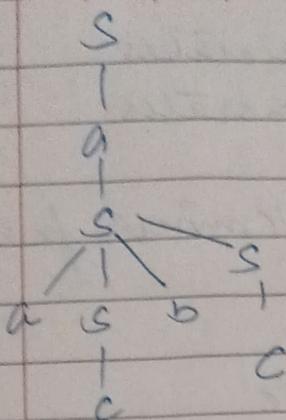
Derivation A

$$1 \quad S \rightarrow aS$$

$$2 \quad aS \rightarrow a(aSbS)$$

$$3 \quad a(aSbS) \rightarrow a(a(cacb))$$

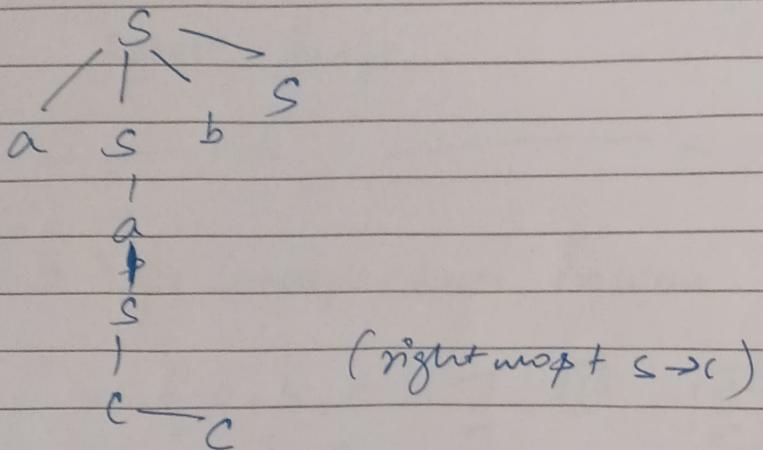
$$4 \quad a(a(cacb)) \rightarrow a(cacb) = aabc$$



Derivation

1. $S \rightarrow aSbS$
2. $aSbS \rightarrow a(as)bs$
3. $a(as)(bs) \Rightarrow a(ac)bs$
4. $a(ac)bs + a(ac)bc = aacb(c)$

Parse trees B (diff shape)



The two trees are structurally different
 \therefore the grammar is ambiguous

\rightarrow A grammar is ambiguous some doing using has two diff parse trees

Given:

$$G \rightarrow (TS, A, B) \quad \Sigma \{a, b, c\}^* \quad PS$$

$PS \rightarrow B \rightarrow C$
 $A \rightarrow aB$

1) Find non terminals that generates terminal

$$B \rightarrow C \rightarrow B \text{ generate}$$

$A \rightarrow aB$ & B generates $\rightarrow A$ generates
 $s \rightarrow A$ & A generates $\rightarrow s$ generates

Step-2 : Find reachable non terminal from start s :

' s ' is start

from $s \rightarrow A \rightarrow$ A reachable

from $A \rightarrow aB \rightarrow$ B reachable