unit 1

fourier Transform – The fourier tramform is an important image procening tool which is used to decompose an image into Sine & cosine components. The output of the transformation represents the image in fourier or frequency domain, while the input image is the spatial domain equivalent.

– The fourier transform is used in wide range of applications
- → Image analysis
- → Image filtering
- → Image reconstruction and image Compression
- → Image comprenion

fourier transform is a fundamental importance in Image procening tool which is used to decompose an image into its Sine and cosine components

fourier Transform

* Fourier Transform, named after Joseph fourier - employed transform signals between time and domain & frequency domain.

* It is tool that breaks a waveform into a alternate representation characterised by sine and cosines.

* fourier transforms is used in wide range of application such as image filtering, image compression.

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* **Fourier Transformation** (1-D Contineous Signal)

→ Let $f(x)$ is contineous function of some variable then the fourier transformation of $f(x)$ is $F(u)$.

$$\mathcal{F}\{f(x)\} = F(u) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi ux}\, dx$$

here $f(x)$ must be contineous & integralable.

* **Inverse fourier Transformation** (1-D Contineous Signal)

→ $F(u)$ is a fourier transform of signal $f(x)$ so after inverse fourier transformation of $F(u)$ we get $f(x)$

$$\mathcal{F}^{-1}\{F(u)\} = f(x) = \int_{-\infty}^{\infty} f(u)\, e^{-j2\pi ux}\, du$$

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fourier Transformation (2-D contineous signal)

forward fourier Transformation:

Let $f(x,y)$ is 2 dimensional signal with 2 variables

$$f(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

Inverse (Backward) fourier Transformation

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv$$

Types of fourier Transform

- Contineous Time fourier Transform (CTFT)
- Discrete Time fourier Transform (DTFT)
- Discrete fourier Transform (DFT)

Difference among CTFT, DTFT & DFT

	Time Domain	frequency Domain	Periodic in time domain	Periodic in frequency domain
CTFT	Contineous	continuous	NO	NO
DTFT	Discrete	Contineous	NO	Yes
DFT	Discrete	Discrete	Yes	Yes

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Discrete fourier tranform –

• working with fourier transform on a computer usually involves a form of transform known as the discrete fourier transform (DFT). A discrete transform is transforms is a tranform whose input and output values are discrete samples. These are two principles reason for using this form of the transform =

- The input and output of DFT are both discrete
- There is fast algorithm for computing the DFT known as fast fourier tranform (FFT).

⇒ The DFT is usually defined a discrete function $f(m,n)$ that is non zero only over the finite region $0 \le m \le M-1$ and $0 \le n \le N-1$. The two dimensional M-by-N DFT and inverse M-by-N DFT known as fast fourier transform (FFT)

$$F(p,q) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, e^{-j2\pi pm/M} e^{-j2\pi pm/M} e^{-j2\pi qn/N}$$

$$p = 0, 1 \ldots M-1$$
$$q = 0, 1 \ldots N-1$$

and

$$f(m,n) = \frac{1}{MN} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} F(p,q)\, e^{j2\pi pm/M} e^{j2\pi qn/N}$$

$$m = 0, 1, \ldots M-1$$
$$n = 0, 1 \ldots, N-1$$

⇒ Relationship to the fourier transform

The DFT coefficient $F(p,q)$ are samples of the fou transform $F(w_1, w_2)$.

$$F(p,q) = F[w_1, w_2] \quad w_1 = 2\pi p/M \quad w_2 = 2\pi q/N \qquad p = 0, 1, \ldots M-1 \quad q = 0, 1, \ldots N-1$$

Properties of fourier transform

① Linearity - Addition of two functions corresponding to the addition of two frequency ~~domain~~ spectrum is called linearity. If we multiply the a function by constant the fourier transform of sum or more function is the sum of fourier transformation of the function.

② Scaling - Scaling is the method that is used to change the range of the independent variables. If we stretch a function by the factor in the time domain then squeeze the fourier transform by some factor in frequency domain.

③ frequency Shift - frequency is shifted according to the co-ordinates. There is duality between the time and frequency domain and frequency shift affects the time shift.

④ Time-Shift - The time variable shift also affects the frequency domain. The time shifting property concludes that a linear displacement in time correspond to a linear phase factor in frequency domain.

Applications of fourier transforms

- Application of IBVP, circuit analysis, cell phones, Signal analysis, Image processing, image compression and others.

fast fourier Transform (FFT)

Sometimes need to transform image from the spatial to frequency (fourier) domain. The frequency domain is the basis of many images filters used to remove noise, sharpen image etc.

→ In frequency domain its x- and y axis frequency and its value is represented by amplitude.

→ The fast fourier Transform (FFT) commonly used to transform an image between the spatial and frequency domain. The FFT method preserves all original data.