Roll No. ....   Total Pages : 3

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MCA/DX

DISCRETE MATHEMATICAL STRUCTURES

Paper : MCA-103

Time : Three Hours]   [Maximum Marks : 80

Note : Attempt *five* questions. Question No. 1 is compulsory and attempt *one* question from each unit.

Compulsory Question

1. (a) Let (z, t) be the group of integers with respect to the operation addition and $(6z, t)$ be a subgroup of (z, t). Write all left cosets of $(6z, t)$ and describe their intersection.

(b) Write generating set of an element of a finite group.

(c) Draw complete group k_5 and k_4 respectively. Which graph represents an Euler circuit ?

(d) What is reachability matrix of digraph ? When a digraph is called strongly connected ?

(e) Prove that a field is an integral domain.

(f) Prove that $f(x) = x^3 + x + 1$ is reducible over z_3.

(g) Write V operation table for the lattice L = {1, 2, 3, 5, 30} under the relation divides.

(h) Show that the lattice L = {1, 2, 3, 5, 30} under the relation divides is not distributive lattice. 3×8

UNIT–I

2. (a) Let $(P_3, 0)$ be a permutation group over A = {1, 2, 3}. Express each permutation in terms of transpositions and prove that set of even permutations is a normal subgroup of (P_3, 0). 8

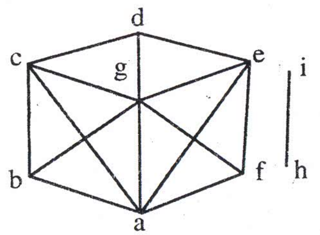
(b) Define group homomorphism and kernel. State and prove *two* properties of group homomorphism. 6

3. (a) Define grammar and its types. Find a regular grammar G which generates the language L consisting of all words on a and b with exactly one b. 7

(b) Let A = $\{a, b\}$. Find the regular expression for the language L = $\{b^m a b^n : m \text{ and } n \text{ positive}\}$. Draw finite state machine that accepts the given language. 7

UNIT–II

4. (a) Write algorithm for depth-first spanning tree. Implement the algorithm to the following graph :



(b) If G is a simple connected planner graph with more than one edge then prove the following :

(i) $2|E| \geq 3R$.

(ii) $|E| \leq 3|V| - 6$.

E is the set of edges and v is the set of vertices in G. 7

5. Let D = {V, E} be a directed graph such that V = {1, 2, 3, 4, 5, 6} and E = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (4, 3), (5, 4), (5, 6)}. Using matrix determine whether digraph is acyclic. Determine the number of directed paths of length 4 from a node to another node. 14

UNIT–III

6. Let D_{30} denote the set of all positive divisors of 30. Verify whether D_{30} under the relation divides is a Boolean algebra. 14

7. (a) Define atom of a Boolean algebra. Let A = {1, 2, 3}. Prove that power set of A under the relation $\subseteq$ (subset) is a Boolean algebra and atoms of the Boolean algebra generate P(A). 8

(b) Consider the Boolean expression

$$f(x_1, x_2, x_3) = (\overline{x_3} \wedge x_2) \vee (\overline{x_1} \wedge x_3) \vee (\overline{x_2} \wedge x_3)$$

on $[B_2, -, \vee, \wedge]$.

(i) Simplify f using basic Boolean algebra laws.

(ii) Draw the switching circuit and gate circuit diagram for the simplified f. 6

UNIT–IV

8. Find splitting field for the polynonimial

$f = x^2 + 2x + 2$ over z_3. 14

9. (a) Write an example of a finite integral domain. 7

(b) Prove that a polynomial of degree n over a field f has at most n roots. 7