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MCA/D-14
DISCRETE MATHEMATICAL STRUCTURES
Paper—MCA-103

10368

Time Allowed : 3 Hours]

[Maximum Marks : 80

Note : Attempt five questions in all, selecting at least one question from each Unit. Question No. 1 is compulsory.

Compulsory Question

1. Write short notes on the following :

- (a) Prove that subgroup of an abelian group is normal.
- (b) Find the order of each element of the group $(\mathbb{Z}_6, +_6)$.
- (c) Consider the language $L = \{a, b, c\}$ over $A = \{a, b\}$, Find L^3 .
- (d) What is a Regular Graph?
- (e) Define Partial ordering with suitable example.
- (f) What is Switching Circuit?
- (g) Explain briefly the concept of Irreducible Polynomial.
- (h) Prove that the Polynomial $x^4 + x + 1$ is irreducible over $(\mathbb{Z}_3, +, \cdot)$.

3x8=24

UNIT-I

- 2. (a) Let (S_3, \circ) be a permutation group over $A = \{1, 2, 3\}$. Find all the Subgroups of (S_3, \circ) and their generators. 7
- (b) Consider H and K be groups. Then define the direct group product $G = H * K$ of H and K . What is the identity element and order of $G = H * K$? 7
- 3. (a) Define Finite State Machines. Describe state table and state diagram of a 'finite, state machine. 7
- (b) What is Language? Briefly, explain the concept of Regular Languages. 7

UNIT-II

- 4. (a) What is a Graph? If a Graph contains two distinct paths from vertex t_i to vertex v , prove that G has a cycle. 7
- (b) Define Adjacency matrix. Write an algorithm for determining cycle in a Graph. 7
- 5. (a) What is a Tree? Consider the algebraic expression $E = (2x + y)(5a + b)^2$. Draw the tree T which corresponds to the expression E . 7
- (b) What is a Weighted Graph? Write an algorithm to find shortest path from the node a to node z in a Weighted Graph. 7

UNIT—III

- 6. Consider Boolean expression $E = xz' + y'z + xye$, then simplify E algebraically. Also draw the circuit diagram and switching circuits for E and the simplified E . 14
- 7. (a) Elaborate various laws of Boolean algebra. Write dual of $(a + b)(b + c) = ac + b$. 7
- (b) Describe the usage of Logic gates and circuits with examples: 7

UNIT—IV

8. Define Field and Splitting Field. Find splitting field of $x^3 + x + 1$ over (\mathbb{Z}_2, t^2, x^2) . 14
9. (a) Prove that the set \mathbb{R} of real numbers form a field with respect to the usual operations of addition and multiplication. 7
- (b) Define Integral domain. Show that a finite integral domain is a field. 7