MATHEMATICAL FOUNDATION - IV

Time : 3 Hours *Maxinum Marks : 90*

Note : Attempt five questions in all, selecting at least one question from each section and the question No. 1 which is compulsory.

(Compulsory Question)

1. a) If u be a homogeneous function of order n in x and y,

them $x\dfrac{\partial u}{\partial y}+y\dfrac{\partial u}{\partial y}=nu$.

b) Find the length of a loop of the curve $r = a\left(\theta^2 - 1\right)$

c) If $x = r\cos\theta$, Z=Z, then evaluate $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

d) To evaluate $\int_0^{\pi/2} \sin^p x.\cos^q x, p>-1, q>-1$

e) Find the equation of two tangent planes to the sphere $x^3+y^3+z^3 = 9$, Which pass through the line, $x+y=6$, $x-2z=3$.

2. a) If $y^3=3ax^2-x^3$ Prove that $\frac{d^2y}{dx^2}+\frac{2a^2x^2}{y^5}=0$

b) If $\theta = t^n e^{-r^2/4t}$, find the value of n for which

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right)=\frac{\partial \theta}{\partial t}$$

3. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

SECTION-II

4. a) Show that $\int_0^{\pi}\frac{\sin nx}{\sin x}dx = 0$ or π, according as n is even or odd positive integer.

b) Obtain a reduction formula for $\int \frac{x^m dx}{\left(x^3-1\right)^{1/3}}$ and find the value of $\int x^8\left(x^3-1\right)^{-1/3}dx$.

5. a) Find the perimeter of the loop of the curve $9ay^2=(x-2a)(x-5a)^2$.

b) Find the intrinsic equation of the cardioid $r=a(1-\cos\theta)$.

Section-III

6. a) Find the area common to the circle $x^2+y^2=4$ and the ellipse $x^2+4y^2=9$.

b) Find the area of the curved surface generated by the revolution of the cycloid $x = a(\theta+\sin\theta)$, $y = a(1-\cos\theta)$,

$y = a(1-\cos\theta)$ about its base.

7. i) Verify that $\int_1^2 \int_3^4 \left(xy + e^y\right) dy\, dx$

$$\int_3^4 \int_1^2 \left(xy + e^y\right) dx\, dy$$

ii) Evaluate $Evaluate \iint \sqrt{a^2 - x^2 - y^2}\, dx\, dy$ over the semi circle $x^2+y^2=ax$ in the positive quadrant.

SECTION-IV

8. a) To show that $B(m,n) = \dfrac{m-1.n-1}{m+n-1}$ *if* m, n are positive integers.

b) Prove that $\sqrt{\left(\frac{1}{2}\right)} = \sqrt{\pi}$

9. a) Evaluate $\int_0^a \dfrac{\log(1+ax)}{1+x^2}\, dx$

b) Prove that the equation $7x^2 + 2y^2 + 2z^3 - 10zx + 10xy + 26x - 2z - 17 = 0$ represents a cone whose vertex is $(1, -2, 2)$.