Mathematicsl Foundations-I

Time : Three Hours Maximum Marks : 80

Note : Total *five* questions are to be attempted. Selecft *one* question from each section. Question No. 1 is compulsory.

Compulsory Question

1. (a) Differentiate x^x w.r.t. x. 3

(b) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$. 3

(c) Prove that f is a subset of every set. 3

(d) Find the complement of each element of the lattice D_{30}. 3

(e) Write down all subset {a, b, c}. 2

(f) Define Permutation and Combination. 2

SECTION-I

2. (a) Prove that $A-(B \cap C)=(A-B) \cup (A-C)$. 8

(b) Which of the following lattices are Boolean algebra?
(i) D_{70} (ii) D_{30} (iii) D_6 (iv) D_{210}

3. (a) In how many ways can 5 boys and 5 girls be seated at a round table ? So that no two girls are set together. 8

(b) Find the numbers of arrangements that can be made out of the letter of the word "MATHEMATICS". In how many of these vowels occure together. 8

(b) Using t – d definition, prove that |x| is a continuous function. 8

SECTION-II

4. (a) Show that $\lim_{x \to 0} \frac{e^{1/x}+1}{e^{1/x}-1}$ does not exist. 8

(b) If $x^y + y^x = 0$. Find $\frac{dy}{dx}$. 8

5. (a) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, $x \neq y$. Find $\frac{dy}{dx}$. 8

(b) Using t – d difinition, prove that |x| is a continuous function. 8

SECTION-III

6. (a) Find the differential equation of the system of circles touching y-axis at the origin. 8

(b) Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$. 8

7. (a) Solve $(x^3+3xy^2)dx + (3x^2y+y^3)dy = 0$. 8

(b) Solve $(y \log x - 1)y\,dx = dy$. 8

SECTION-IV

8. (a) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x.$ 8

(b) Solve $\frac{d^2y}{dx^2} + y = xe^x \sin x.$ 8

9. (a) Solve $\frac{d^2y}{dx^2} + y = \text{cosec } x.$ 8

(b) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xc^{4x}$ 8