MATHEMATICAL FOUNDATION-IV

Time : Three Hours Maximum Marks : 80

Note : Attempt ***five*** questions are to be attempted. Selecft ***one*** question from each unit. Question No. 9 (Unit-V) is compulsory.

UNIT-I

1. (a) If $u = e^{xyz}$, show that $\frac{\partial^2 u}{\partial x\,\partial y\,\partial z} = e^{xyz}(1 + 3xyz + x^2y^2z^2)$. 8

(b) Examine for maximum and minimum values of the function, $xy + \frac{a^3}{x} + \frac{a^3}{y}$. $a > 0$. 8

2. (a) If u and v are functions of x and y defined by $x = u + e^{-v} \sin u$ and $y = v + e^{-v} \cos u$.

find $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$. 8

(b) Find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. 8

UNIT-II

3. (a) Show that $\int_0^{\pi} \frac{\sin nx}{\sin x}\, dx = 0$ or π according as n is even or odd positive integer. 8

(b) Find the length of the loop of the curve $3ay^2 = x(x - a)^2$. 8

4. (a) Find the intrinsic equation of the parabola $y^2 = 4ax$. 8

(b) Find the whole lengtg of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$. 8

UNIT-III

5. (a) Find the area common to the parabola $y^2 = 4x$ and $x^2 = 4y$. 8

(b) Evalue $\iint x^2 y^2 dx\, dy$ over $x^2 + y^2 \leq 1$. 8

6. (a) Evaluate $\int_0^{\infty}\int_0^{x} x e^{-x^2/y} dy\, dx$ 8

(b) Find the surface area of a sphere of radius a. 8

UNIT-IV

7. (a) If $\alpha^2 < 1$, prove that

$$\int_0^{\pi/2} \log(1 - \alpha^2 \sin^2\theta)\, d\theta = \pi \log\left(\frac{1+\sqrt{1-\alpha^2}}{2}\right).$$ 8

(b) Prove that $B(m, m) = 2^{1-2m}B(m, 1/2)$. 8

8. (a) Find the equation of right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. 8

(b) Find the equation of the sphere which touches the plane $3x + 2y - z = 0$ at the point $(1, -2, 1)$ and cuts

UNIT-V

Compulsory Question

9. Attempt all the following :

(i) Find the equation of the cylinder with genetrator parallel to X-axis and passing through the curve $x^2 + y^2 + 2z^2 = 12$ and $x + y + z = 1$. 2

(ii) Show that the *two* spheres

$$x^2 + y^2 + z^2 + 6y + 2z + z = 0, \text{ and}$$
$$x^2 + y^2 + z^2 + 6x + 8y + 4z = -20$$

cut orthogonally.

(iii) Find the centre of the section of the sphere $x^2 + y^2 + z^2 = 25$ by the plane $2x + y + 2z = 9$. 2

(iv) If $u = f(y/x)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial u} = 0$. 2

(v) If $u = x(1 - y)$ $v = xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$. 2

(vi) Evalute $\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{1-x^2}\sqrt{1-y^2}}$. 2

(vii) Prove that $\mu(n) = (n - 1)\,\mu(n - 1)$, $n > 1$, where μ is gamma function. 2

(viii) Define Right circular cone. 2