

BCA (14-15)/D-15

MATHEMATICAL FOUNDATION-I

Paper-BCA-113

Time allowed : 3 hours]

[Maximum marks : 80

Note : Attempt **five** questions in all, selecting **one** question from each section. **Question No. 1 is compulsory.**

(Compulsory Question)

1. (a) If $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$,
 $C = \{2, 6, 7, 10\}$ and $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
then verify that $A \cup (B \cap C) = (A \cup B) \cap C$ 4
- (b) Prove that $(2n)! = 2^n \cdot n!$. [1.3.5 -- $(2n-1)$] 3
- (c) Evaluate : $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$ 3
- (d) Show that : $x^2 + 4y = 0$ is a solution of

$$\left(\frac{dy}{dx}\right)^2 + x \left(\frac{dy}{dx}\right) - y = 0$$
 3
- (e) Differentiate x^x w.r.t x . 3

Section-I

2. (a) In a recent survey of 5000 people, it was found that 2800 read Indian Express and 2300 read 'The Tribune', while 400 read both the papers.
 - (i) How many read neither Indian Express nor Tribune ?
 - (ii) How many read Indian Express only ?
 - (iii) How many read Tribune only ?

(2)

- (b) If R is an equivalence relation on a set A , show that R^{-1} is also an equivalence relation on A . 8
3. (a) Find the value of n if ${}^nP_4 : {}^{n-1}P_3 = 9 : 1$ 8
- (b) There are 15 points in a plane, no three of which are collinear except 6 of them which are all on a line. How many
- (i) straight line
- (ii) triangles can be formed by joining them. 8

Section-II

4. (a) Using ϵ - δ definition Prove that $\lim_{x \rightarrow a} \cos x = \cos a$ where $a \in \mathbb{R}$. 8
- (b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ show that
- $$\frac{dy}{dx} = -\frac{1}{(1+x)^2} \quad 8$$
5. (a) Differentiate: $\sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$ 8
- (b) If $y = (\sin^{-1} x)^2$; prove that $(1-x^2)y_2 - xy_1 = 2$. 8

Section-III

6. (a) Find the differential equation of all circles touching the x -axis at the origin. 8
- (b) Solve the differential equation
- $$\cos^2 x \frac{dy}{dx} + y = \tan x. \quad 8$$

7. (a) Solve the differential equation

$$(1 + e^{x/y}) dx + e^{x/y} \left(1 - \frac{x}{y} \right) dy = 0 \quad 8$$

- (b) Solve the differential equation

$$(e^y + 1) \cos x dx + e^y \sin x dy = 0 \quad 8$$

Section-IV

8. (a) Solve the differential equation

$$\frac{d^2 y}{dx^2} + a^2 y = \sin ax \quad 8$$

- (b) Solve the differential equation

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + y = e^x \sin 2x \quad 8$$

9. (a) Solve the differential equation

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4 \quad 8$$

- (b) Solve the differential equation

$$(x+1)^2 \frac{d^2 y}{dx^2} + (x+1) \frac{dy}{dx} = (2x+3)(2x+4) \quad 8$$