Mathematical Foundations - I

Time : Three Hours] [Maximum Marks : 90

1. (a) Define power set and find power set of the set $\{\phi\}$
   (b) Which of the following lattices are boolean Algebra :
   (i) D_{70} (ii) D_{99}
   (c) Find the n-th derivative of sin x sin 2x.
   (d) Write complementary function for the equation $(D^4+a^4)\, y = \sin ax$
   (e) Define integrating factor and find integrating factor of equation $ydx - ydx = 0$. 18

UNIT - I

2. (a) Prove that $A\Delta B = (A \cup B) - (A \cap B)$ 9
   (b) Show that the relation "is congruent to" on the set of all triangles in a plane is an equivalence relation. 9
3. (a) Define Boolean Algebra as an algebraic structure. 9
   (b) In how many ways can 5 boys and 5 girls be seated at a round table, so that no two girls are together? 9

UNIT - II

4. (a) Prove that limit of $\frac{1}{x^3}$ does not exist as x tends to zero. 9

   (b) $If\ y = \cos^{-1}\left[\frac{2\cos x + 3\sin x}{\sqrt{13}}\right], show\ that\ \frac{dy}{dx} = 1$ 9

5. (a) Find the n-th derivative of $e^{2x} \sin^3 x$.

   (b) $If\ x\sqrt{1+y} + y\sqrt{1+x} = 0$, show that $\frac{dy}{dx} = -\frac{1}{(1+x^2)}$ 9

UNIT - III

6. (a) Find the differential equation of all circles of given radius.
   (b) Solve the differential equation :
   $(y \log x - 1)\, ydx = dy$ 9
7. (a) Solve the differential equation:
   $(x + 2y)(dx - dy) = dx + dy$ 9

(b) Solve the differential equation :
$(x^2 + y^2 + 2x)\,dx + 2y\,dy = 0$ 9

UNIT - III

6. (a) Find the differential equation of all circles of given radius.
   (b) Solve the differential equation :
   $(y \log x - 1)\,ydx = dy$ 9
7. (a) Solve the differential equation:
   $(x+2y)\,(dx-dy) = dx + dy$ 9
   (b) Solve the differential equation:
   $(x^2+y^2+2x)\,dx + 2y\,dy = 0$ 9

UNIT - IV

8. Solve the differential equation :
   (a) $(D^3+1)y=3 + e^{-x}$ 9

   (b) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 5y = e^{2x}\sin x$

9. Solve the differential equation :

   (a) $x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + y = \dfrac{1}{(1-x^2)}$ 9

   (b) $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 3y = x^2\log x$ 9

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