

MATHEMATICAL FOUNDATIONS-1

Note : Attempt five questions in all, selecting one question from each unit in addition to Q. No. 1 which is compulsory.

(Compulsory Question)

1. (a) Define Proper subset and give an example to explain it.
 (b) Find the complement of each element of lattice D_{35} .

(c) Differentiate $\frac{x^2}{1+x^2}$

(d) Solve the differential equation

$$2\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 2y = 0.$$

(e) Show that $x^2 + 4y = 0$ is a solution of $\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$

18

UNIT-I

2. (a) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. 9
 (b) If R is a relation from N to N defined by $(a,b) R (c,d)$ and iff $ad = bc$ then show that R is an equivalence relation. 9
 3. (a) From a class of 12 boys and 10 girls, 10 students are to be chosen for a competition, including at least 4 boys and 4 girls. The two girls who won the prizes last year should be included. In how many ways can the selection be made?

9

- (b) Let D_m be the set of all positive factors of m ($m > 2$). If m is the product of distinct prime numbers, then prove that D_m is a Boolean algebra. 9

UNIT - II

4. (a) Using $\varepsilon - \delta$ definition, prove that

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \frac{3}{2}$$

- (b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \quad 9$$

5. (a) If $x^p y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$ 9

- (b) Find the n th derivative of $\sqrt{ax+b}$. 9

UNIT - III

6. (a) Find the differential equations of all parabolas whose axes are parallel to y -axis. 9

- (b) Solve $(1+y^2) dx + (1+x^2) dy = 0$, given that $y=1$ when $x=0$ 9

7. (a) Solve the differential equation $(1+y^2) dx = (\tan^{-1} y - x) dy$. 9

- (b) Verify that the differential equation.

$$x dx + y dy = \frac{a^2 (x dy - y dx)}{x^2 + y^2}$$

is exact, and solve it.

9

8. (a) Solve the differential equation

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = e^{3x}$$

9

- (b) Solve the differential equation.

$$\frac{d^2 y}{dx^2} + y = \sin x \sin 2x$$

9

9. (a) Solve the differential equation.

$$(x^2 D^2 - 3xD + 4)y = x^m.$$

9

- (b) Solve the differential equation

$$(x+a)^2 \frac{d^2 y}{dx^2} - 4(x+a) \frac{dy}{dx} + 6y = x$$

9