

MCA/D08
Discrete Mathematical Structures
MCA -103

Time : 3 Hours

MM:50

Note:- Attempt Five questions by selecting One Question from each unit, and Question no 1 is compulsory.

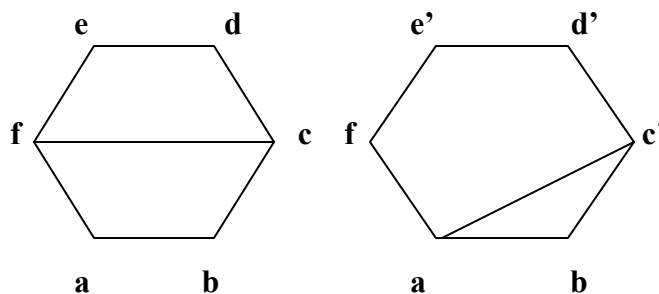
- 1(a) Write an example of a cycle group
- (b) Define finite state machine and write an example for it.
- (c) What is meant by edge connectivity of a graph? Prove that edge connectivity of graph is less than or equal to $(2e/n)$.
- (d) Prove that an Acyclic diagram can be topologically sorted.
- (e) Define partial order relation. Write the relation set for the relation 'divides' in D_{50} where D_{50} is the set of all positive divisors of 50.
- (f) Write join and meet operation tables for the relation 'divides' in D_{50}
- (g) Verify whether the polynomial x^2+1 is reducible over (\mathbb{Z}_2+2) and (\mathbb{R}_1+1) respectively or not.
- (h) Write definition for field. Is \mathbb{Z} , the set of integers, a field or not? Give reason in support of your answer.

UNIT-II

- 2(a) Let $n \geq 2$. Prove that the set of even permutations is S_n (a symmetric group on $\{1,2,3,\dots,n\}$) is a proper subgroup of S_n and also a normal subgroup, and the order of this subgroup is $n!/2$.
 - (b) Prove that Kernel of group homomorphism is a normal subgroup.
 - (c) Find a subgroup of $(\mathbb{Z}_7, +)$ where $\mathbb{Z}_7 = \{0,1,2,3,4,5,6\}$ is the set of integers modulo 7 and addition modulo 7 is the operation.
- 3(a) Define regular expression and regular language respectively. Prove that $L = \{a^m b^n : m \text{ and } n \text{ positive}\}$ the language over $A = \{a,b\}$ is a regular language.
 - (b) Define a grammar. Can we find a grammar for the language $L = \{a^n b^n : n > 0\}$ over $A = \{a,b\}$ explain.

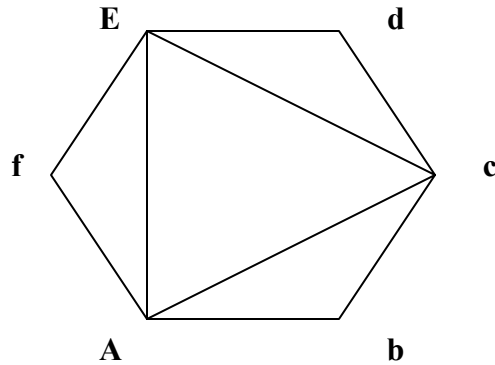
UNIT-III

- 4(a) Determine whether the following graphs are isomorphic or not.

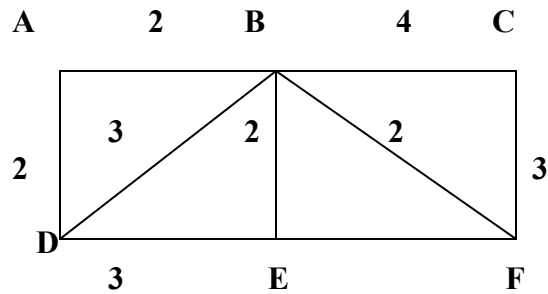


- (b) Define planar graph and state Euler's formula. Which complete graphs and complete bi-partite graphs are planar?

- 5(a) Find all directed paths of length 3 in the following diagram.



- (b) Find shortest path in the following weighted diagram from A to f



UNIT-III

- 6(a) Define Lattice. Verify whether $L = \{1, 2, 3, 4, 5, 6, 7, 8\}$ w.r.t. the relation divides is a lattice or not.
- (b) Define Boolean algebra. Verify whether $B = \{1, 3, 5, 6, 10, 15, 30\}$ w.r.t. the relation divides is a Boolean algebra or not.

- 7 Consider the Boolean function:

$$F(x_1, x_2, x_3, x_4) = x_1 + (x_1 + x_4) + x_3 \cdot (x_2 + x_4)$$

- (a) Simplify f algebraically
- (b) Draw the switching circuit for f and for simplified f .
- (c) Draw the circuit (gate) diagram for f and for simplified f .

UNIT-IV

- 8 Define Integral domain. Verify whether Z_{10} the set of integers modulo 10 w.r.t. the operations addition modulo 10 and multiplication modulo 10 is an Integral domain or not. When Z_n is an Integral domain?
- 9(a) Write an example of a finite field

(b) Prove that a polynomial of degree n over a field F has at most n roots.