

**BCA / M-19**  
**MATHEMATICAL FOUNDATION-II**  
**Paper-BCA-123**

*Time allowed : 3 hours]**[Maximum marks : 80*

*Note : Attempt five questions in all. Question No. 1 is compulsory. Attempt four more questions selecting exactly one question from each unit. All questions carry equal marks.*

**Compulsory Question**

1. Explain following :

8×2=16

- (a) Truth table
- (b) Mathematical induction
- (c) Group
- (d) Cosets
- (e) Singular matrix
- (f) Rank of a matrix
- (g) Eigen vector
- (h) Skew-Hermitian matrix.

**Unit-I**

2. (a) Show that  $\sim(p \Leftrightarrow q) \equiv p \Leftrightarrow \sim q$  8
- (b) Using the principle of mathematical induction, prove that for all  $n \in \mathbb{N}$ ,  $11^{n+2} + 12^{2n+1}$  is divisible by 133. 8

3. State and prove the laws of logic.

16

### Unit-II

4. (a) If  $(G, .)$  be a group; then solve the equation  
a.  $x. a = b$  in  $G$ . 8

(b) Let  $H$  be a subgroup of group  $G$  and define  
 $N(H) = \{\alpha \in G : \alpha H = H\alpha\}$ . Prove that  $N(H)$  is a  
subgroup of  $G$ . 8

5. (a) Let  $H = \{5x : x \in \mathbb{Z}\}$  be a subgroup of  $\mathbb{I}$ . Prepare the  
composition table for  $\mathbb{Z}/H$ . 8

(b) Let  $D$  be an integral domain and  $F$  be a field. Define a  
mapping  $\psi : D \rightarrow F$  such that

$\psi(\alpha) = (\alpha, 1)$  for all  $\alpha \in D$ . Then show that  $\psi$  is an  
isomorphism of  $D$  into  $F$ . 8

### Unit-III

6. Find  $A^{-1}$ , where  $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ . Hence, solve the

system of linear equations :

$$x + 2y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

16



(3)

7. Solve the following system of equations :

$$x - y + 2z - 3w = 0$$

$$3x + 2y - 4z + w = 0$$

$$4x - 2y + 9w = 0$$

16

#### Unit-IV

8. (a) Prove that the eigen values of a triangular matrix are the diagonal elements of the matrix. 8
- (b) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a Hermitian matrix are orthogonal. 8

9. Diagonalize, if possible, the matrix  $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & -4 \\ 9 & 1 & 3 \end{bmatrix}$  16