11/5/18

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BCA/M-18 1906

MATHEMATICAL FOUNDATION-II

BCA-123

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) If p and q be any statements, then construct the truth table of $\sim p \vee \sim q$.

(b) If every element of a group is its own inverse then show that the group is abelian.

(c) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 - xA + yI = 0$.

(d) If A and B are Hamilton matrices show that AB – BA in skew-Hermitian.

(e) Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ by definition.

Section I

2. (a) Define contradiction and tautology propositions. Use truth-table to establish contradiction and tautology from the following properties :

   (i) $\left[(\sim q) \wedge p\right] \wedge q$

   (ii) $\left[p \wedge (\sim q)\right] \wedge \left[(\sim p) \vee q\right]$

   (b) Identify the quantifiers and write negative of the statement :

   (i) there exists a number which is equal to its square

   (ii) some diseases are curable and not infections.

3. (a) Using principle of mathematical induction, prove that for all $n \in \mathrm{N}$

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots\ldots.+\frac{1}{n.(n+1)}=\frac{n}{n+1}.$$

   (b) Show that $\sim\left[(\sim p) \wedge q\right]$ and $p \vee (\sim q)$ are logically equivalent.

Section II

4. (a) Let S = {0, 1, 2, 3, 4} and $+_5$ is binary operation defined by $a +_5 b$ is the remainder obtained on dividing $a + b$ by 5, then prove that S is a group w.r.t. $+_5$.

(b) Show by an example that union of two subgroups is not necessarily a subgroup.

5. (a) Define ring, ideal and field with an example.

(b) Prove that the set of matrices $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subring of 2×2 matrices with integral elements.

Section III

6. (a) Define adjoint of a matrix. If $A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$ then find its adjoint and verify (A) (Adj. A) = $|A|I_3$.

(b) Find inverse of $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Also show that $A^{-1} = A^2$.

7. (a) Find the rank of $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 3 & -2 \\ -3 & 1 & 2 & 5 \end{bmatrix}$.

(b) For what value of λ will the equation $3x - y + \lambda z = 1$; $2x + y + z = 2$; $x + 2y - \lambda z = -1$, fail to have a unique solution ? Will the equations have any solution for thin value of λ.

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Section IV

8. (a) Find the eigen value of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.

Also find the eigen vector corresponding to any *one* of eigen value.

(b) Prove that the eigen values of a real symmetric matrix are real.

9. (a) Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Also find A^{-1}.

(b) Diagonalize $\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$ if possible.