# Mathematical Foundations-III

## Compulsory Question

1. (a) Differentiate $\sin^3$ w.r.t. $\cos^3 x$. 2

(b) At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x-axis? 2

(c) State Taylor's theorem with Largrange's form of remainder after n terms. 2

(d) Evaluate $\lim_{x \to b} \frac{x^b - b^x}{x^x - b^b}$. 2

(e) How we can obtain asymptotes parallel to x-axis and y-axis for a given equation of a curve? 2

(f) Prove that the curvature of a circle at any point on it is constant and is equal to the reciprocal of the radius of the circle. 2

(g) Define Node. 2

(h) What is the shape of parabola $y^2 = -4ax$? Write its axis and coordinates of focus. 2

(i) Prove that for the curve $P = f(r), P = r.\frac{dr}{dp}$

## SECTION-A

2. (a) If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \infty}}}$, 9

prove that $(2y-1)\frac{dy}{dx} = \frac{1}{x}$

(b) If $e^x + e^y = e^{x+y}$, prove that $\frac{dy}{dx} = -e^{y-x}$. 9

3. If $y = (\sin^{-1} x)^2$, prove that

(a) $(1-x^2)\, y_2 - xy_1 - 2 = 0.4$

(b) $(1-x^2)y_{x+2} - (2n+1)\, xy_{x+1} - n^2 y_x = 0$. 5

(c) Deduce that $\lim_{x \to 0} \frac{y_{n+2}}{y_n} = n^2$ and find $y_n(0)$.

## SECTION-B

4. (a) Find the point on the curve $y = be^{-x/a}$ at which the tangent makes an angle $\tan^{-1}\left(\frac{-b}{a}\right)$ with x-axis. 9

   (b) Find the angle of intersection of the curves.
   $r = a\cos\theta$ and $r = a(1-\cos\theta)$ 9

5. (a) If $f(x) = x^3 + 2x^2 - 5x + 11$, find the value of $f\left(\frac{9}{10}\right)$ with the help of Taylor's series for f (x+h). 9

   (b) State and prove Maclaurin's theorem with Lagranges form of remainder after n terms. 9

## SECTION-C

6. (a) Find all the asymptotes of the curve 9
   $x(y-x)^2 - x(y-x) - 2 = 0$

   (b) Show that the four asymptotes of the curve
   $xy(x^2-y^2) + 25y^2 + 9x2 - 144 = 0$
   cut it again in eight points on an ellipse whose eccentricity is $\frac{4}{5}$. 9

7. (a) Find the points of inflexion of the curve $y = (x-2)^6 (x-3)^5$.

   (b) Find the position and the nature of the double points on the curve. 9

## SECTION-D

8. (a) If $p_1$ and $p_2$ are the radii of curvature at the extremities of a focal chord of a parabola whose semilatus-rectum is $e$, prove that $(p_1)^{-2/3} + (p_2)^{-2/3} = (e)^{-2/3}$. 9

   (b) If $c_x$, $c_y$ be the chord of curvature parallel to co-ordinate axes at any point of the curve $y = c\cosh\left(\frac{x}{c}\right)$; prove that
   $4c^2\left(c_x^2 + c_y^2\right) = c_y^4$. 9

9. (a) Trace the curve $r = a(1+\cos\theta)$. 9

   (b) Trace the curve $x = t^2$, $y = t - \frac{1}{3}t^3$ 9