MATHEMATICAL FOUNDATION-III

Time : 3 Hours *Maximum Marks : 90*

Note : A candidate will be required to answer five question in all, selecting one question from each unit in addition to compulsory Question No. 1. All question carry equal marks.

1. i) Differentiate $\frac{x^2}{1+x^2} w - r - t - x^2$

ii) At what points on the curve x2 + y2 - 2x -4y+1=0, the tangent is Parallel to y-axis.

iii) Find the n^{th} derivative of $\text{Sin}^3 x$-

iv) Evalute Lt $x \to 0$ $\underset{x\to 0}{Lt} \dfrac{\left(\tan^{-1} x\right)^2}{\log(1)}$ by

L' Hospital rule

v) Show that the curve y2-4ax = 0 has no asymptotes.

vi) Define cus p.

vii) What is the shape of a Parabola $x^2 = 4$ ay write its axis and Coordinates of focus.

viii)Show that for the curve

$$S= \sqrt{8ay}, P = 4a\sqrt{1-\frac{y}{2a}}$$

ix) At what points of the curve $y2=2x^3$ is the slope of the targent equal to 3?

UNIT-I

2. If y = $\sqrt{\cos x + \sqrt{\cos x + \sqrt{Cos\, x + ...\infty}}}$

i) Show that $(2y-1)\frac{dy}{dx}+Sin\,x=0$

ii) If $\sqrt{1-x^2}+\sqrt{1-y}^{2}=a(x-y)$ Prove that

$$\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

3. i) State and Prove the LEIBNITZ'S Theorem-

ii) If y = Sin (m $Sin^{-1}x$) $|x|<1$, Prove that

$(1-x^2)_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$

UNIT-II

4. i) Tangent are drawn from the origine to the curve y = Sin x : Prove that theiri point of the contact lie on the curve $x^2 y^2 = x^2 - y^2$

ii) Find the condition that the curves $ax^2 + by^2=1$ and $a'x^2+b'y^2=1$ may cut orthogonally.

5. i) State and Prove the taylor's theorem with Lagrange's form of remainder after n Terms.

ii) Show that $\underset{x \to 0}{Lt} + \dfrac{(1+x)^{1/x} - e + \dfrac{e^x}{2}}{x^2} = \dfrac{11e}{2y}$

UNIT-III

6. i) Find the cubic which has the same asymptotes as the curve.

$x^3-6x^2y+11xy^2-6y^3+x+y+1=0$

And which touches the axis of y at the origine and passes through the point (3,2).

ii) Find the asymptotes of

$r^n f_n(\theta)+r^{n-1}(\theta)+ \dots\dots\dots + f0(\theta)=0$

7. i) Show that the curve $y^2 = bx \, Sin \dfrac{x}{a}$ has a node or a conjugate point at the origine according as a and b have like or Unlike signs.

ii) Find the points of inflesion on the curve.

$x=a\left(2\theta - Sin\theta\right), y = a\left(2 - Cos\theta\right)$

UNIT-IV

8. (i) If P1, P2 be radii of curvature at the extremities of a pair of semiconjugates diameters of an ellipse, Prove that

$$[(P_1)^{2/3}+(P_2)^{2/3}]\,(ab)^{2/3}=a^2+b^2$$

(ii) Find the radius of curvature at the origine for the curve.

$$2x^4+4x^3y+xy^2+6y^3-2xy+y^2-4x=0$$

9. i) Trace the curve $r = a\,(1+\mathrm{Sin}\,\theta)$

ii) Trace the curve $x^2y^2 = x^2 - a^2$

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