MATHEMATICAL FOUNDATIONS-II

Time : 3 Hours *Maximum Marks : 80*

1. a) Find the a characteristics roots of the matrix.

$$A = \begin{Bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{Bmatrix}$$

b) If a matrix has 24 elements, what are the possible orders it can have?

c) Identify the quantifier and state its type in the statement : There exists a capital you every state in India.

d) Define rank of a matrix and give example.
e) Define order of an element of a group.
f) Check whether the sentence "May God bless you" is a statement or not, give reason.

SECTION-I

2. a) Prove that 32^{n+2}-8n-9 is a multiple of 64.
   b) Construct the truth table of the following statements:
   i) $\sim(p \wedge \sim q)$ ii) $\sim p \Rightarrow \sim q$.
3. a) Shown that biconditional is both commutative and associative.
   b) State and prove distributive laws.

Section-II

4. a) Show that the set of all non-zero rational numbers forms an abelian group under the operation of multiplication of rational numbers.
   b) Prove that $(\{0, 1, 2, 4, 4\}) +_5, X_5)$ is a field.
5. a) Show that the inter section of any two left ideals of a ring is a left ideal of the ring.
   b) Prove that the set {0, 1, 2, 3, 4, 5} with addition modulous 6 and multiplication modulous 6 as compositions is a ring.

SECTION-III

6. $Let\ A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Show that $I + A = (I - A)\begin{bmatrix} Cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

   b) If A and B are two non-singular materices of the same order, show n that AB is invertible and $(AB)^{-1} B^{-1} A^{-1}$.

7. a) Express $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

as an product of elementary matries.

b) Check whether the following system of equations is consistent or not. Solve if it is consistent :

$2x - y + 3z = 3$

$x + 2y - z - 5w = 4$

$x + 3y - 2z - 7w = 5$

SECTION-IV

8. a) Prove that the absolute value of each characteristics root of unitary matrix is unity.

b) Determine the eigen values and eigen vectors of the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

9. a) Diagonalise, if possible the matrix $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 7 & -4 \\ 9 & 1 & 3 \end{bmatrix}$

b) If λ is an eigen value of a square matrix A, then prove that λ is an eigen value of $A\theta$ and conversely.

❑❑❑