Paper : VI
Mathematical Foundation

Time : 3 Hours *M.M. : 100*

1. (a) Find the total derivative $\frac{du}{dt}$ if

$u = 3x - 2xy + 5y$ where
$x = 3t^2 + 2t,\ y = 5t + 7.$ 4

(b) If $u_n = \int \cos n\theta \operatorname{cosec} \theta \, d\theta;$

prove that $u_n - u_{n-2} = \frac{2\cos(n-1)\theta}{n-1}$

(c) Find the surface are of a sphere of radius a. 4

(d) To show that $B(m, n) = B(n, m)$. 4

(e) Find the points in which the line, $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ cuts the surface, $11x^2 - 5y^2 + z^2 = 0$. 2

SECTION–I

2. (a) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$,

prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. 8

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that :

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

(ii) $x^2 - \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u (1 - 4 \sin^2 u)$. 10

3. (a) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(b) Examine for the minimum and maximum values of the function $\sin x + \sin y + \sin (x + y)$. 9

UNIT–II

4. (a) Find a reduction formula for

$$\int \cos^m x \sin nx \, dx.$$ 9

(b) Find the length of arc $x^2 + y^2 - 2ax = 0$ in the first quadrant. 9

5. (a) If $u_n = \int_0^{\pi/2} \theta \cdot \sin^n \theta \, d\theta, (n > 1)$:

prove that $u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$ and evaluate $u_5 = \frac{149}{225}$. 9

(b) Find the length of the curve $x^2(a^2 - x^2) = 8a^2y^2$. 9

UNIT–III

6. (a) Show that the surcace are of the solid obtained by revolving the ellipse $b^2x^2 + a^2y^2 = a^2b^2$, about x-axis is $2\pi ab \left[\sqrt{1-e^2} + \frac{1}{e}\sin^{-1} e\right]$, where e is the eccentricitly. 9

(b) Find the area common to the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 4ax$. 9

7. (a) Show that the area of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is 6π. 9

(b) Evaluate $\iint \sqrt{a^2 - x^2 - y^2}\, dx\, dy$ over semicircle $x^2 + y^2 = ax$ in the positive quadrant. 9

SECTION–IV

8. (a) State an prove Duplication formula. 9

(b) Evaluate :

$$\int_0^{\pi/2} \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$$ 9

9. (a) Find the centres of the two spheres, which touch the plame $x + 2y + 2z - 5 = 0$ at the point $(1, 1, 1)$ and the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z - 11 = 0$. 9

(b) To show that the equation of the right circular come whose vertex is origin, axis the z-axis and semi-verticle angle α is $x^2 + y^2 = z^2 \tan^2 \alpha^2$. 9