**Roll No. .....**

**MCA/D09** **5418**

**Discrete Mathematical Structures**

**Paper: MCA-I03**

Time: Three Hours Maximum Marks: 80

**Note:-** Attempt **FIVE** questions in all Question No.**1** is compulsory and attempt **FOUR** more questions by selecting **ONE** question from each unit.

1. (a) Prove that subgroup of an abelian group is normal.
(b) Find order of each element of the group $(Z_6, t_6)$.
(c) Draw complete bipartite graphs $K_{2,3}$, $K_{3,3}$, $K_{2,4}$. Which graph contains Hamiltonian circuit?
(d) Define isomorphism of graphs. What are conditions that the graphs must hold?
(e) Let A = {a, b, c} and P (A) be power set of A. Prove that is a partial order relation on P (A).
(f) Let $(L, \vee, \wedge)$ be a complemented and distributive lattice, then prove that complement a of an element a € L is unique.
(g) Show that the polynomial $x^4 + x + 1$ is redouble over $(Z_3, X_3, t_3)$.
(h) Show that $(Z_8, t_8, X_8)$ has zero divisors. 3*8

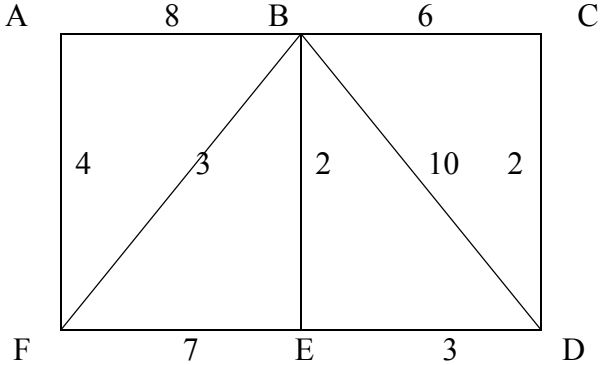
**UNIT-I**

2 (a) Let $(S_3, 0)$ be a permutation group over A = {1, 2, 3}. Find all subgroups of $(S_3, 0)$ and their generators also. 10
(b) Write cosets of (5Z, t) in (Z, t). 4

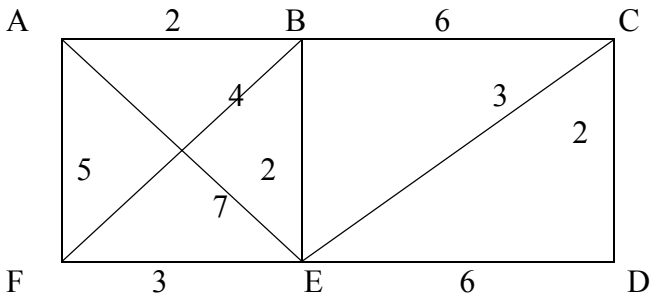
3 (a) Find the language L (G) over A= {a, b, c} generated by the grammar G with productions:-
S → aSb, aS → Aa, Aab→ c. Determine the type of grammar. 6
(b) Find a content free grammar G which generates the language L consists of all words of the form $a^r b^s c^t$, r, s, t>0. Draw the diagram of the finite state machine for the language, if possible. 8

**UNIT-II**

4 (a) State and prove Euler's formula for a connected planar graph G. Which complete graph is a planar graph? 8
(b) What is minimal spanning tree? Write an algorithm for finding a minimal spanning tree. Explain it for the following graph: 6



5 Write an algorithm for finding shortest path from the node a to the node z in a weighted digraph D. Find shortest path from A to D in the digraph.




Determine source and sink in this digraph. 14

## UNIT-III

6 (a) State basic Boolean algebra laws and principle of duality. 6
(b) Define Boolean algebra. Verify whether B = {1, 2, 4, 6, 8, 12, 24} w.r.t. the relation divides is a Boolean algebra or not. 8

7 Consider the Boolean expression E = xz'+y'z+xyz'.
(i) Simplify E algebraically.
(ii) Draw the switching circuit for E and for the simplified E.
(iii) Draw the circuit (gate) diagram for E and for simplified E. 14

## UNIT-IV

8 Find splitting field of polynomial $x^3+x+1$ over $(Z_2,t_2,x_2)$. 14

9 (a) Prove that a finite integral domain is a field. 7
(b) Prove that (Z,t,x) is an integral domain but not a field. 7