4/12/14 December-2014

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BCA / D–14

MATHEMATICAL FOUNDATION–I

Paper–BCA–113

Time allowed : 3 hours]   [Maximum marks : 80

Note : *Attempt **five** questions in all, selecting **one** question from each section. **Question No. 1** is compulsory.*

1. (a) Verify that $(A \cap B)' = A' \cup B'$, where

     $A = \{2, 3, 4, 5, 6\}$ and $B = \{3, 6, 7, 8\}$

     are subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ 4

    (b) Prove that $(2n)! \cdot (n-1)! = 2(n)!(2n-1)!$ 3

    (c) Evaluate :

     $$\lim_{n \to \infty} \frac{\Sigma n^3}{x^4}$$ 3

    (d) Find order and degree of diff. equation

     $$\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$$ 3

    (e) Differentiate $\dfrac{x^2}{1+x^2}$ w.r.t x^2 3

Section–I

2. (a) In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many like both coffee and tea.   8

(b) In the set of integers, let a relation R be defined as aRb if and only if a - b is even. Prove that R is an equivalence relation.　　8

3. (a) Find the value of n such that $^nP_5 = 42\ ^nP_3$, $n > 4$.　　8

(b) How many words each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE.　　8

Section–II

4. (a) Using $\in$-S definition prove that

$$\underset{x \to 1}{Lt}\ \frac{x^3 - 1}{x^2 - 1} = \frac{3}{2}$$　　8

(b) If $y = \sqrt{\dfrac{1-x}{1+x}}$, prove that $(1 - x^2)\dfrac{dy}{dx} + y = 0$　　8

5. (a) If $x = \cos^{-1}\dfrac{1}{\sqrt{1+t^2}}$ and $y = \sin^{-1}\dfrac{t}{\sqrt{1+t^2}}$; find $\dfrac{dy}{dx}$　　8

(b) If $y = (m \sin^{-1} x)$; Prove that

$$(1 - x^2)\,y_2 - xy_1 + m^2 y = 0$$　　8

Section–III

6. (a) Find the differential equations of all parabolas whose axes are Parallel to y-axis.　　8

(b) Solve the differential equation

$$(1 + x^2)\frac{dy}{dx} + 2\,yx - 4\,x^2 = 0$$　　8

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7. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$ 8

(b) Solve the differential equation

$$\left(x^4 - 2xy^2 + y^4\right)dx - \left(2x^2y - 4xy^3 + \sin y\right)dy = 0$$ 8

Section–IV

8. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 4y = e^x + \sin 2x.$$ 8

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = x \sin x$$ 8

9. (a) Solve the differential equation

$$x\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x}.$$ 8

(b) Solve the differential equation

$$(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$ 8

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