MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE-I

Time : Three Hours   Maximum Marks : 100

Note : Attempt *five* questions are to be attempted. Selecft *one* question from each section. All questions carry equal marks.

UNIT-I

1. (a) Find the differential equations of the family of curves $y = Ae^{3x} + Ve^{5x}$, where A, B are arbitrary constants.

    (b) Solve $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

2. (a) Solve $y^2 dx + (xy + x^2) dy = 0$.

    (b) Solve $\cos^2 x \dfrac{dy}{dx} + y = \tan x$.

3. (a) Solve $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$.

    (b) Solve $\dfrac{d^2 y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$.

4. (a) Solve $\dfrac{d^2 y}{dx^2} - 7\dfrac{dy}{dx} + 6y = e^{2x}$. Given that $y = 0$ when $x = 0$.

    (b) Solve $x^2 \dfrac{d^2 y}{dx^2} - 2x\dfrac{dy}{dx} - 4y = x^4$.

UNIT-II

5. (a) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(b) Let A, B and C be finite sets with $|A| = 6, |B| = 8, |C| = 6, |A \cap B| = 3, |A \cap C| = 2, |B \cap C| = 5$ and $|A \cup B \cup C| = 11$. Find $|A \cap B \cap C|$.

6. (a) Let A be a set with n elements and $1 \le r \le n$. Find the number of sequence of length r that can be formed from elements of A, allowing repetitions.

(b) How many different seven peson committees can be formed each containing 3 women from a set of 20 women and 4 men from a set of 30 men?

7. (a) Is the following a true statement?
$3 > 2$ if and only if $0 < 3 - 2$.

(b) Prove by induction that $3 \mid n^3 - n$ for every positive integer n.

UNIT-III

8. (a) Let G be a group and let a and b be elements of G. Prove that $(ab)^{-1} = b^{-1}a^{-1}$.

(b) Let G be a group. Show that the function $f : G \circledR G$ defined by $f(a) = a^2$ is a homomorphism if and only if /G is Abelian.

9. (a) Let A and B are Hermitian; prove that $AB + BA$ is Hermitian and $AB - BA$ is skew-Hermitian.

(b) Find the inverse of the matrix :

$$A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

10. (a) Determine the characteristic roots of the matrix :

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

(b) Show that the matrices A and $C^{-1}AC$ have the same characteristic roots.