MATHEMATICAL FOUNDATION-III

Time Allowed : 3 Hours *Maximum Marks : 80*

Note : Attempt five questions in all, selecting **one** question from each unit in addition to compulsory **Question No. 1.** All questions carry equal marks.

Compulsory Question

1. (a) Differentiate $\sqrt{\sec\sqrt{x}}$ w.r.t. x. 2

(b) Find the derivative of $y = \log(\sin x)$ w.r.t. x. 2

(c) Fine for the curve $r = a(1 - \cos\theta)$. 2

(d) Define Taylor's theorem with Cauchy's form of remainder. 2

(e) Show that the function $f : R^2 \to R$ defined by

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} xy & , \quad (x,y) \neq 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$ is continuous

at (0, 0). 2

(f) Define Concavity and Convexity. 2

(g) Define Symmetry about origin. 2

UNIT-I

2. (a) Find $\frac{dy}{dx}$. where $y = x^x + (x)^{\sin x}$. 8

(a) Differentiate $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$ w.r.t. $\sin^{-1} \frac{2x}{1+x^2}$.

3. (a) Find nth derivative of $\sin^3 x$.

(b) If $y = (\sin^{-x} x)^2$

Prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + n^2 y_n = 0$

UNIT-II

4. (a) Show that the two circles $x^2 + y^2 + 2ax + c = 0$

and $x^2 + y^2 + 2by + c = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$. 8

5. (a) Show that $\left(x + \sqrt{1+x^2}\right)^n = 1 + nx + \frac{n^2 x^2}{2!}$

$$+ \frac{n\left(n^2 - 1^2\right)x^3}{3!} + \frac{n\left(n^2 - 2^2\right)x^4}{12} + \ldots\ldots$$

(b) Evaluate the following :

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\tan^2 x}\right).$$ 8

UNIT-III

6. (a) Find all asymptotes to the curve
$x^2y - xy^2 + xy + y^2 + x - y = 0$ 8

(b) Find asymptote of the curve $r = a\sec\theta + b\tan\theta$.

7. (a) Find the position and nature of double points of the curve $x^3 + x^2 + y^2 - x - 4y + 3 = 0$ 8

(b) Find point of inflexion on the curve $x = a(2\theta - \sin\theta)$, $y = a(2 - \cos\theta)$. 8

UNIT-IV

8. (a) If ρ_1 and ρ_2 are radii of curvature at the extremities of a focal chord of a parabola whose latus rectum is 1, prove that $(\rho_1)^{\frac{-2}{3}}$ + and $(\rho_2)^{\frac{-2}{3}}$

$= (1)^{\frac{-2}{3}}$

(b) Find the radius of curvature at the origin of the two branches of the curve given by $x = 1 - t^2$, $Y = t - t^3$.
8

9. (a) Trace the curve $x^4 + y^4 = a^2xy$. 8

(b) Trace the curve $e = a(1 - \sin\theta)$. 8