MATHEMATICAL FOUNDATION-II

Time : Three Hours Maximum Marks : 80

Note : Attempt ***five*** questions in all. Selecft ***one*** question from each section. Question No. 1 is compulsory.

(COMPULSORY QUESTION)

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1. (a) Prove that $\sim[p \vee (\sim p)]$ is a contradiction. 3

(b) Define Group. 3

(c) If $a^2 = e$, $\forall\ a \in G$, then G is an abelian group. 3

(d) Find the inverse of matrix $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. 3

(e) Evaluate $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[1\ \ 2\ \ 4\ \ 5]$. 2

(f) Define Prime Ideals. 2

UNIT-I

2. (a) Using P.M.I., prove that for all $n \in N$,

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots\dots\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$ 8

(b) Prove by Truth table
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. 8

3. (a) Construct the Truth table of
(i) . $(\sim p \wedge \sim q) \vee r$. (3 +5)

(b) Using P.M.I., prove that $2^n < 3^n$, $\forall\, n \in N$. 8

UNIT-II

4. (a) If a group has four elements, show that it must be abelian. 8

(b) Show that a subgroup H of a group G is normal if and only if $xHx^{-1} = H$, $\forall\, x \in G$. 8

5. (a) Defined 'Field'. 8

(b) Let $f : R - R'$ be homomorphism, then f is an isomorphis m of R into R' iff ker $f = 0$. 8

UNIT-III

6. (a) Find the adjoint of matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 3 & 2 & 6 \\ 0 & 1 & 0 \end{bmatrix}$ and verify

$(\text{adj } A)\, A = A(\text{adj } A) = |A|\, I_3$. 8

(b) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^3 - 3A - 2I = 0$ and

hence find A^{-1}. 8