Mathematical Foundations - II

Paper-BCA-124

Time : Three Hours] [Maximum Marks : 90

1. (a) Define Subgroup; Normal Subgroup; Principal Ideal. 3

   (b) Construct 3×3 matrix whose elements are given by $a_{ij} = j - i$. Also name this matrix. 3

   (c) Find the rank of the matrix $\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix}$ where w is a cube root of unity. 3

   (d) If $x^2 = e$ for all $x \in G$ where G is a group. Then prove that G is an ablian group. 3

(e) Construct the truth table of $(p \wedge q) \vee \sim(p \vee q)$. 3

(f) Prove that a group of order 2 is an abelian. 3

SECTION-I

2. (a) Prove that $5^n > 3^n$ for all $n \in N$. 6

(b) Prove that $\left[(p \Leftrightarrow q) \wedge \{(q \Rightarrow r) \wedge r\}\right] \Rightarrow r$ is a tautology. 6

(c) Prove that $(1+x)^n \geq 1+nx$ for all $x \in N$. 6

3. (a) Prove that $x^n - y^n$ is divisible by $x - y$; $x \neq y$. 6

(b) Identify the quantifiers and write the negation of the statements : 12

(i) There exists a capital for every state in India.

(ii) Some real numbers are rotational.

(iii) Every child is naughty or intelligent.

(iv) Any integer is either positive or negative

SECTION-II

4. (a) Prove that set of Natural number is not a RING. 9

(b) Prove that the set G = {1, -1, i, -i} is an abelian group under multiplication. Also find all its subgroups. 9

5. (a) Let G = {1, 2, 3, 4, 5, 6} find the orders of elements of the group G under binary operation multiplication Modulo 7. 9

(b) Show that the Set $\left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} / a, b \in z \right\}$ is a right ideal but not a left ideal in the ring of all 2 × 2 matrices, whose elements are integers. 9

SECTION-III

6. (a) If $A=\begin{bmatrix}3 & 5\\2 & 7\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$;

then show that $(AB)^{-1} = B^{-1} A^{-1}$. 9

(b) Prove that the rank of the product of two matrices can not exceed the rank of either matrix. 9

7. (a) For what values of λ, μ, the system of equations

$x + y + z = 6$; $x + 2y + 3z = 10$; $x+2y+\lambda z = \mu$ has

(i) no solution.

(ii) a unique solution

(iii) an infinite number of solution

(b) Solve by matrix number of solutions. 9

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$$

$$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$$

SECTION-IV

8. (a) Prove that the characteristic roots of a real symmetric matrix are all real. 9

(b) Varify Caylay-Hamilton theorem for matrix 9

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

Also find A^{-1}.

9. Diagonalize the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ if possible. 18

□□□