

# MATHEMATICAL FOUNDATIONS-I

Time : 3 Hours

Maximum Marks : 80

## (Compulsory Question)

1. Attempt all the following :

a) Write down all the subsets of  $\{1, 2, 3\}$ .

b) Find the complement of each element of the lattice  $D_{35}$ .

c) In how many ways can 5 beads of different colours form a necklace?

d) Solve  $\frac{d^3 y}{dx^3} - 7 \frac{dy}{dx} - 6y = 0$

e) Differentiate  $(e^{x^r})$  w.r.t.  $x$ .

### SECTION - I

2. a) Prove that  $A - (B \cap C) = (A - B) \cap (A - C)$ .

b) In how many ways can a selection be made out of 4 red, 2 white and 2 blue identical balls?

3. a) Which of the following lattices are Boolean algebra.

i)  $D_{15}$       (ii)  $D_{70}$       iii)  $D_{99}$       iv)  $D_{30}$

b) Find the number of arrangements that can be made out of the letters of the word "MATHEMATICS". In How many of these vowels occur together?

### SECTION-II

4. (a) Show that

$$\lim_{x \rightarrow 0} \frac{e^{1/x}}{e^{1/x} + 1} \text{ does not exist.}$$

b) If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ . Find  $\frac{dy}{dx}$

5. a) If  $y = x^x + x^{\sin x}$ , find  $\frac{dy}{dx}$

- b) Find nth derivative of  $\frac{1}{2x+3}$ .

### SECTION-III

6. a) Find the differential equation of the system of circles touching x-axis at the origin.

b) Solve  $y - x \frac{dy}{dx} = a \left( y^2 + \frac{dy}{dx} \right)$

7. a) Solve  $(x+2y^2) \frac{dy}{dx} = y$

b) Solve  $x^2 y dx - (x^3 + y^3) dy = 0$

### Section-IV

8. (a) Solve  $\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} = 1 + x^2$

b) Solve  $\frac{d^2 y}{dx^2} + 4y = x \sin x$

9. a) Solve  $\frac{d^2 y}{dx^2} + y = x e^x \cos x$ .

b) Solve  $x^2 \frac{d^2 y}{dx^2} - 2y = x^2 + \frac{1}{x}$