BCA/D-16   **855**

MATHEMATICAL FOUNDATION – I
Paper : BCA-113

Time : Three Hours]   [Maximum Marks : 80

Note : Attempt *five* questions in all. Select *one* question from each section in addition to Question No. 1 which is compulsory.

Compulsory Question

1. (a) Prove that the following sets are equal :

Set A = $\{x : x - 5 = 0\}$

Set B = $\{x : x$ is a positive integral root of

$$x^2 - 2x - 15 = 0\}. \qquad 3$$

(b) Define Supremum and Infimum of a set.   3

(c) Prove that $\underset{\theta \to 0}{\mathrm{Lt}} \dfrac{3 \sin \theta - \sin 3\theta}{\theta^3} = 4.$ 3

(d) If $y = \dfrac{2bt}{1+t^2}$, $x = \dfrac{a(1-t^2)}{1+t^2}$, find $\dfrac{dy}{dx}$.   4

(e) Find the value of n if

$$^nP_4 : {}^{n-1}P_3 = 9 : 1. \qquad 3$$

SECTION–I

2. (a) A survey shows that 63% of Indians like cheese whereas 76% like apples. If $x\%$ of Indians like both cheese and apples, find the value of x. 8

(b) Define Boolean Algebra as an algebraic structure. 8

3. (a) (i) How many different numbers of six digits can be formed with the digits 3, 1, 7, 0, 9, 5 ? 8

(ii) How many of them are divisible by 10 ? 8

(b) A man has 7 friends. In how many ways can he invite one or more of them to a party ? 8

SECTION–II

4. (a) Find the value of a if function

$$f(x) = \begin{cases} 2x-1, & x < 2 \\ a, & x = 2 \\ x+1, & x > 2 \end{cases}$$

is continuous at $x = 2$. 8

(b) If $\sin y = x \sin (a + y)$, prove that $\dfrac{dy}{dx} = \dfrac{\sin^2 (a+y)}{\sin a}$. 8

5. (a) Find nth derivative of $e^{ax} \sin (bx + c)$. 8

(b) If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1 + \log x)^2}$. 8

SECTION–III

6. (a) Find Differential equation of family of curves $(x - h)^2 + (y - k)^2 = a^2$, where h and k are arbitrary constants. **8**

(b) Solve the Differential equation
$$\frac{dy}{dx} = (4x+y+1)^2.$$
8

7. (a) Solve $\frac{dy}{dx} - 2y\,\tan x = y^2\,\tan^2 x.$ **8**

(b) Verify that the Differential equation
$$(x^4 - 2xy^2 + y^4)\,dx - (2x^2 y - 4xy^3 + \sin y)\,dy = 0$$
is exact and hence solve it. **8**

SECTION–IV

8. (a) Solve $\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} - 2y = x + \sin x.$ **8**

(b) Solve $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x.$ **8**

9. (a) Solve $x^2 \dfrac{d^2 y}{dx^2} + 8x\dfrac{dy}{dx} + 13y = \log x.$ **8**

(b) Solve $(x+1)^2 \dfrac{d^2 y}{dx^2} + (x+1)\dfrac{dy}{dx} = (2x+3)(2x+4).$ **8**