MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE-II

SECTION-A

1. (a) If $\sqrt{1-x^6}-\sqrt{1-y^6}=a^3\left(x^3-y^3\right)$ Then Prove that $\frac{dy}{dx}=-\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$ 7

(b) If $c^x+e^y=e^{x+y}$, show that $\frac{dy}{dx}+e^{y-x}=0$ 7

(c) If $x^p y^q=(x+y)^{p+q}$, Prove that $\frac{dy}{dx}=\frac{y}{x}$ 6

2. (a) Find nth derivative of $\frac{1}{x^2+x+1}$ 7

(b) Show that

$$\sin x+\cos x=1+x-\frac{x^2}{\lfloor 2}-\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}\left(\sin\theta x+\cos\theta x\right)$$

7

(c) Find the equation of Tangent at a joint on the curve $y=be^{-x/a}$ at which the Tangent makes an angle $\tan^{-1}\left(-b/a\right)$ with x-axis. 6

3. (a) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin^2 x}\right)$ 7

(b) Prove that the function f defined by

$$f(x,y)=\begin{cases}\dfrac{y\sin\dfrac{1}{x}}{0}, & x\neq 0\\ & x=0\end{cases}$$ is continuous at the origin. 7

(c) *If* $u=x\phi(y/x)+\psi(y/x)$, show that

$$x^2\frac{\partial^2 n}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x^2 y}+y^2\frac{\partial^2 u}{\partial y^2}=0$$ 6

4. (a) If u and v are functions of x and y defined by $x=u+e^{-v}\sin u,\ y=v+e^{-v}\cos u$ *find* $\frac{\partial u}{\partial y}$. 7

(b) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$ 7

(c) Caluculate the approximate value of $\sqrt{26}$ to three decimal places by Taylor's Expansion. 6

SECTION-II

5. (a) Examine the Extreme values of $xy+\frac{a^3}{x}+\frac{a^3}{y}$ 7

(b) Find the Asymptotes of the curve $y=x\frac{e^x-e^{-x}}{e^x+e^{-x}}$. 7

(c) Find the radius of curvature for the curve y=f(x). 6

6. (a) Prove that the curve y= log x is everywhere convex upwards. 5

(b) Discuss the nature of the origin for the curve

$y^3 = x^3 + ax^2$. 5

(c) Trace that curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ 10

7. (a) Show that the curve y=log x, the least value of ISI is $\frac{3\sqrt{3}}{2}$ where S is radius of curvayture. 10

SECTION-III

8. (a) If $I_n = \int_0^{\pi/2} \theta \sin^n \theta \, d\theta, \; n > 1,$ *prove that* 5

$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$, Hence find I_5. 10

(b) A sphere of constant radius r passes through the origin and meet the axes in A, B, C. Show that the locus of the foot of the perpendiclar from the origin to the plane ABC is the surface

$\left(x^2 + y^2 + z^2\right)^2 \left(x^{-2} + y^{-2} + z^{-2}\right) = 4r^2$ 10

9. (a) Evaluate $\iiint \left(x + y + z + 1\right)^{-3} \, dx \, dydz$ over the tetrahedran bounded by the co-ordinate planes and the plane x+y+z=1. 10

(b) Find the equation of the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9,\ x - y + z = 3$. 10

10. (a) Find the area between the curve $x^2y^2 = a^2\left(y^2 - x^2\right)$ and its asymptote. 10

(b) Find the equation of the cone whose vertex is (-1, 1, 2) and whose guiding curve is $3x^2 - y^2 = 1;\ z = 0$. 10