9/6/17

Roll No. .... Total Pages : 03

BCA/M-17 1891

MATHEMATICAL FOUNDATIONS-II

BCA-123

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **9** is compulsory.

Unit I

1. (a) Show that :

$[(\sim p) \wedge q] \wedge (q \wedge r) \wedge (\sim q)$ is a tautology. 8

(b) Prove that :

$5^n > 3^n$ by P.M.I. for all $n \in N$. 8

2. (a) Prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133 by using P.M.I. for all $n \in N$. 8

(b) Prove that :

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. 8

Unit II

3. (a) Prove that group G is abelian if and only if $(ab)^2 = a^2b^2$, $\forall\ a, b \in G$. 8

(b) Let G = {0, 1, 2, 3, 4}. Find the order of elements of group G under the binary operation addition modulo 5. 8

4. (a) Define field with an example. 8

(b) Prove that the intersection of any two ideals of a ring is again an ideal. 8

Unit III

5. (a) Find the matrix X such that : 8

$$\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

(b) Find the Rank of matrix $\begin{bmatrix} 3 & 4 & 1 & 2 \\ 3 & 2 & 1 & 4 \\ 7 & 6 & 2 & 5 \end{bmatrix}$. 8

6. (a) Solve :

$$\begin{aligned} 2x - 3y + z &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2 \end{aligned}$$

using rank method. 8

(b) Solve the system of equations : 8

$$\begin{aligned} 2x + 8y + 5z &= 5 \\ x + y + z &= -2 \\ x + 2y - z &= 2 \end{aligned}$$

2 8 5
1 1 1
1 2 -1

log 1000 = 10 log 10 = 1

Unit IV

7. Verify Cayley-Hamilton theorem for matrix

$$A = \begin{bmatrix} 2 & 6 & -1 \\ 0 & 1 & -6 \\ 3 & 4 & 2 \end{bmatrix}$$ and hence find A^{-1}. **16**

8. Find eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. **16**

Unit V

(Compulsory Question)

9. (a) Define ideals of a ring. **2**

(b) Define normal subgroup. **2**

(c) Define Hermitian matrix. **2**

(d) Define Binary operation. **2**

(e) Define minimal ideal. **2**

(f) If $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, find A^{-1}. **2**

(g) State Cayley-Hamilton theorem. **2**

(h) Define order of an element of a group. **2**