Paper-IV Mathematical Foundations - II

1. (a) If A is a singular matrix, prove that 0 is a latent root of the matrix. 3

   (b) Is multiplication of matrices necessarily commutative? If not, give example. 3

   (c) Identify the quantifier and write the negation of the statement : All cars are not fast and safe. 3

   (d) Find the inverse of the matrix : $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ 3

   (e) State Cayley-Hamilton Theorem. 3

   (f) If every element of a group is its own inverse, then show that the group is abelian. 3

SECTION-I

2. (a) Using principle of mathematical induction, prove that for all $n \in N$:

$$\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+........+\frac{1}{2^n}=1-\frac{1}{2^n}.$$

   (b) Construct the truth table of the following compound statements:

   (i) $(p \wedge q) \vee \sim(p \vee q)$ (ii) $\sim p \vee (q \wedge \sim r)$. 9

3. (a) Show that conditional connective is neither communitive nor associative.

   (b) Prove that $p \wedge (\sim p \vee \sim q)$ is neither a tautology nor a contradiction.

SECTION-II

4. (a) Let G = {0, 1, 2, 3, 4, 5}. Find the orders of elements of the group G under the binary operation addition modulo 6 (+6). 9

   (b) Prove that the set of all odd integers is not a ring. 9

5. (a) Show that the mapping $f : c \rightarrow C$, defined by $f(a+ib) = a - ib$ is homomorphism. 9

   (b) Give an exampe of a skew field which is not a field. 9

SECTION - III

6. (a) Every square matrix A can be expressed in one and only one way as P + iQ, where P and Q are hermitian matrices.

(b) If A $A=\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, then prove that for all $n \in N$, $(aI + bA)^n = a^n I + n\, a^{n-1} bA$. 9

7. (a) Find the rank of the matrix $A=\begin{bmatrix}9 & 0 & 2 & 3\\0 & 1 & 5 & 6\\4 & 5 & 3 & 0\end{bmatrix}$ by reducing it to normal form. 9

(b) If $a \neq b$ and x, y and z are not all zero and if :

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0,$$

prove that $a : b : c = 1 : 1 : 1$. 9

SECTION - IV

8. (a) Prove that the characteristic roots of a skew - hermitian matrix are either zero or purely imaginary. 9

(b) If $A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$, use Cayley-Hamilton theorem to express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A. 9

9. (a) Diagonalize, if possible, the matrix $\begin{bmatrix}-2 & 1\\-7 & 6\end{bmatrix}$. 9

(b) Prove that a square matrix A and its transpose A^t have the same set of eigen values. 9