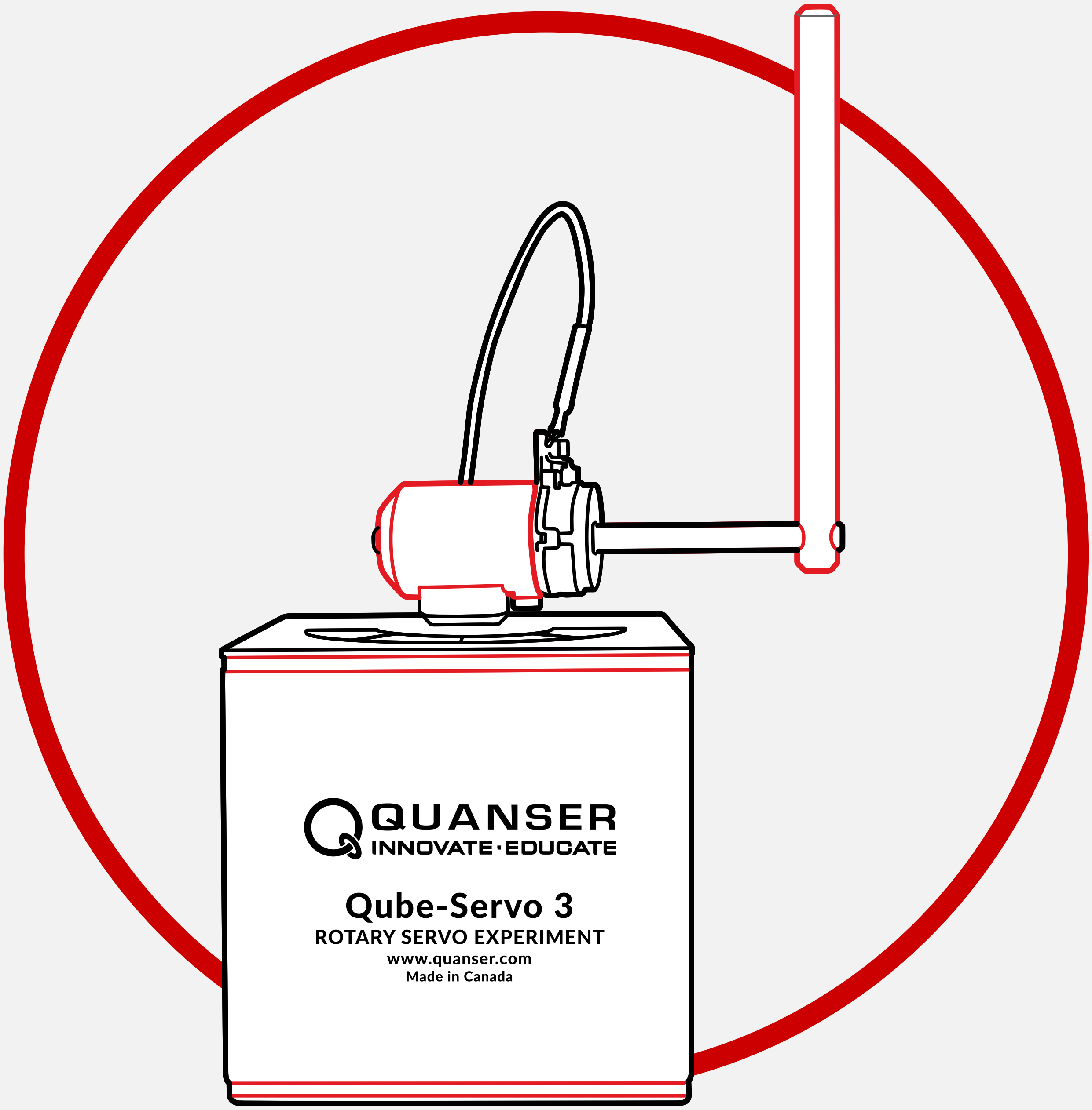

Qube-Servo 3

Optimal LQR Balance Control

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QUANSER
INNOVATE·EDUCATE

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Qube-Servo 3 – Application Guide
Optimal LQR Balance Control

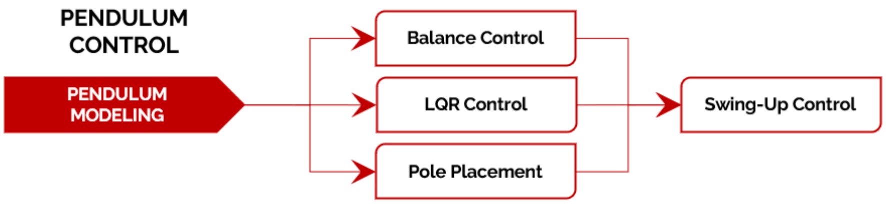
Why explore LQR Control?

Linear Quadratic Regulator (LQR) control is a powerful method for controlling systems with multiple inputs and outputs (MIMO) when the system can be described in state-space form and is operating in regions where linear approximations are valid. The approach finds an optimal feedback control law by minimizing a quadratic cost function that balances the tradeoff between state errors and control effort through user-defined weighting matrices. For pendulum systems and other applications, LQR effectively handles linearized dynamics around equilibrium points (including unstable ones) while allowing engineers to tune the system response characteristics through systematic adjustment of the cost function weights, provided the system is controllable.

Background

This lab is part of the Pendulum Control skills progression of the Qube-Servo 3. These labs are focused on understanding different ways to maintain balance of an inverted pendulum and finish off with creating an energy based controller to swing it up and then maintain the balance.

The lab progression is as follows:



Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator (LQR) theory is a technique that is ideally suited for finding the parameters of the pendulum balance controller.

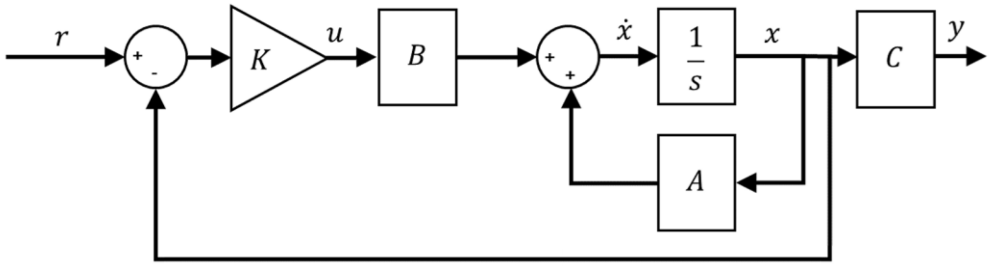
The standard state space representation of a multi input multioutput (MIMO) continuous linear time invariant (LTI) system with n state variables, r input variables, and m output variables is

$$\dot{x}(t) = Ax + Bu$$

$$y(t) = Cx(t) + Du(t)$$

Where:

- $x \in \mathfrak{R}^{n \times 1}$ is the vector of state variables
- $u \in \mathfrak{R}^{r \times 1}$ is the control input vector
- $y \in \mathfrak{R}^{m \times 1}$ is the output vector
- $A \in \mathfrak{R}^{n \times n}$ is the system matrix
- $B \in \mathfrak{R}^{n \times r}$ is the input matrix
- $C \in \mathfrak{R}^{m \times n}$ is the output matrix
- $D \in \mathfrak{R}^{m \times r}$ is the feed-forward matrix



The state feedback control of a system is shown in the figure above. If u is the state feedback control law, it has the form of

$$u = r - Kx$$

Without a reference signal: $u = -Kx$. Where $K \in \Re^{m \times n}$ is the feedback gain. Applying this to the state space equations gives the closed loop system

$$\dot{x}(t) = Ax + Bu = Ax - BKx = (A - BK)x$$

The state vector x of the rotary pendulum system is defined as

$$x = [\theta \quad \alpha \quad \dot{\theta} \quad \dot{\alpha}]^T$$

The reference signal is the desired rotary arm/base position and is defined as

$$x_{ref} = [\theta_r \quad 0 \quad 0 \quad 0]^T$$

The LQR algorithm computes a control law u such that the performance criterion or cost function below is minimized.

$$J = \int_0^\infty \left(x_{ref} - x(t)\right)^T Q \left(x_{ref} - x(t)\right) + u(t)^T R u(t) dt$$

The design matrices Q and R hold the penalties on the deviations of state variables from their setpoint and the control actions, respectively. When an element of Q is increased, therefore, the cost function increases the penalty associated with any deviations from the desired setpoint of that state variable, and thus the specific control gain will be larger. When the values of the R matrix are increased, a larger penalty is applied to the aggressiveness of the control action, and the control gains are uniformly decreased.

The control straregy used to balance the pendulum and track the rotary arm/base setpoint becomes

$$u = K\left(x_{ref} - x\right) = k_{p,\theta}(\theta_r - \theta) - k_{p,\alpha}\alpha - k_{d,\theta}\dot{\theta} - k_{d,\alpha}\dot{\alpha}$$

This control law is a state feedback control and is illustrated in the figure above. The structure is equivalent to the PD control used to balance the pendulum in the Balance Control lab.

Getting started

In this lab you will implement LQR control on the Qube-Servo 3 using the MATLAB *Control System Toolbox*. Using this toolbox, you will gain an intuitive understanding of how modifying the weighting matrices in LQR control affects the control gains calculated. Using this knowledge, you will then deploy the LQR controller to the Qube-Servo 3 and tune the weighting matrices to achieve certain performance criteria when balancing the pendulum.

Ensure you have completed the following labs

- **SP5_ Pendulum Modeling Labs**
- **Balance Control**

Before you begin this lab, ensure that the following criteria are met.

- If using a physical Qube-Servo 3, make sure it has been setup and tested. See the Qube-Servo 3 Quick Start Guide for details on this step.
- Make sure the pendulum attachment is set up and connected to the Qube-Servo 3 using the cable to the Encoder 1 port. Turn the plug to make sure the pendulum is centered around the front of the Qube at 0°. The resistance from the cable will help keep it in the desired position.
- If using the virtual Qube-Servo 3, make sure you have Quanser Interactive Labs open in the Qube 3 - Pendulum → Pendulum Workspace.
- You are familiar with the basics of Simulink. See the Simulink Onramp for more help with getting started with Simulink.