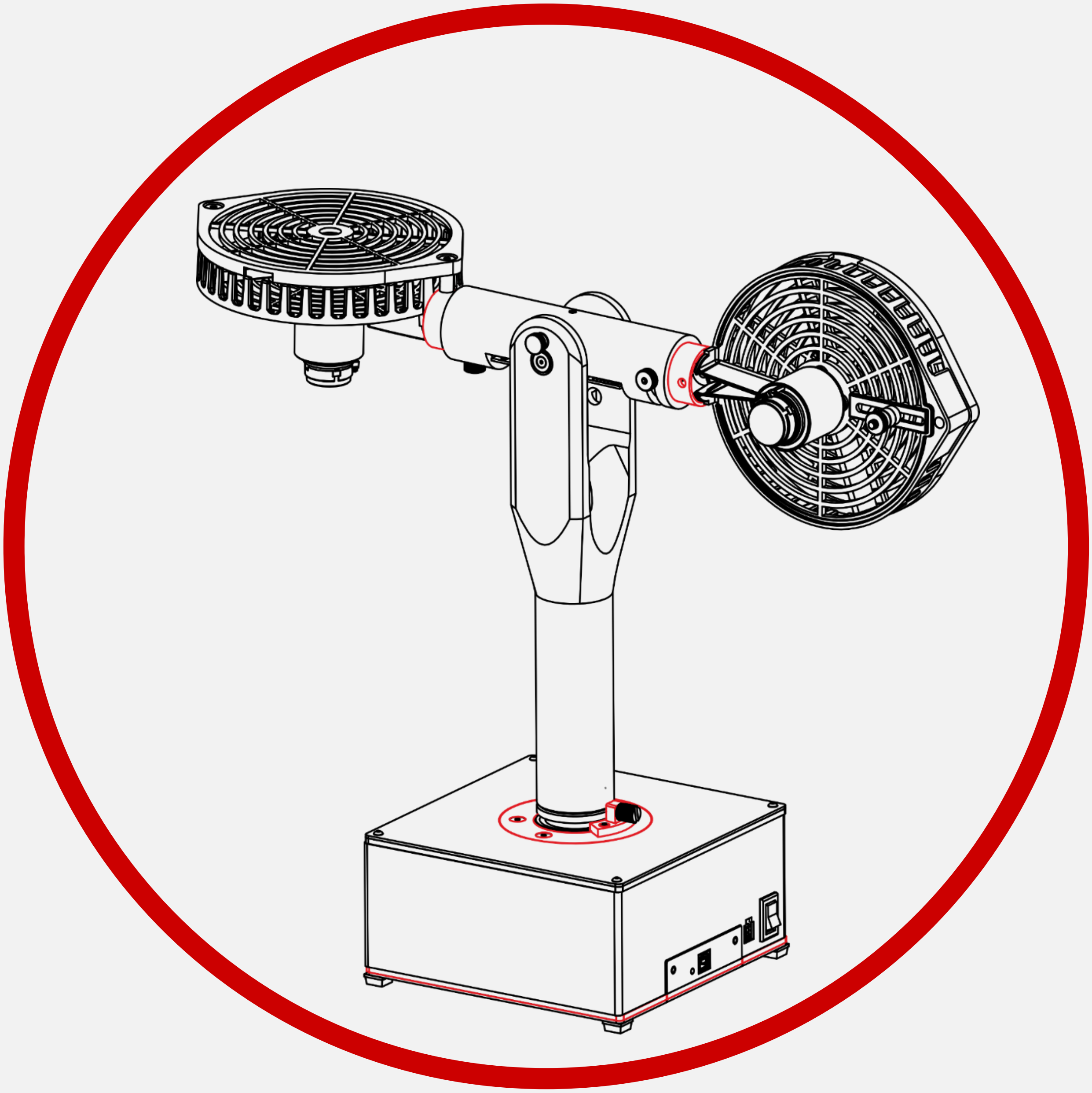

Aero 2

Pitch Parameter Estimation

V1.0 – 14th February 2025

Aero 2 – Application Guide

Pitch Parameter Estimation

Why Explore Parameter Estimation?

The modeling performed in the Rotor Step Response and Rotor Block Diagram labs modelled the rotor/thruster actuator on its own. The goal of this lab is to relate the rotor angular rate to the Aero 2's pitch angle using a transfer function model and identifying the parameters of that model. The model can then be used to develop a controller that will control the pitch angle .

This lab requires additional reading material to summarize key concepts.

1 DOF Pitch Parameter Estimation

In the Rotor Block Diagram Model – Lab Procedure, a model of the thruster/rotor was developed. However, the Aero 2 has two rotors - each of which is driven by a DC motor. The free-body diagram of the Aero 2 when configured as a 1 DOF pitch-only system is shown in Figure 1. The system parameters defined in Table *1*.

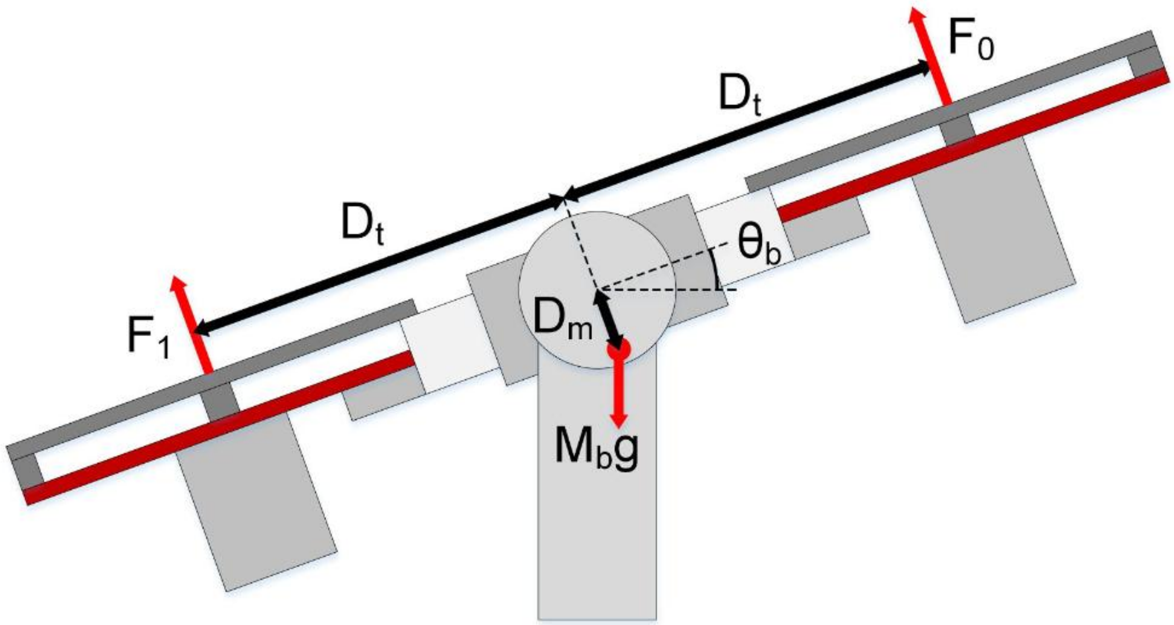

Figure 1 Free-body diagram of 1-DOF Aero 2

Note that the thrust force is only in the plane of the rotors.

System	Description	Value
M_b	Aero body mass	1.07 kg
D_t	Thrust displacement	0.167 m
D_m	Center of mass displacement	0.00240 m
J_p	Equivalent moment of inertia of pitch axis	0.0232 N-m/kg

Table 1 - Quanser Aero system parameters

The Aero 2 1 DOF pitch-only system can be represented by the following second-order equation of motion

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp}\theta = \tau_p = K_{pp}D_t\omega_m(t) \tag{1}$$

where θ is the pitch angle, J_p is the equivalent moment of inertia acting about the pitch axis, D_p is the viscous damping, K_{sp} is the stiffness, K_{pp} is the force thrust gain relative to the rotor speed, D_t is the distance from the pivot point to the center of the rotor, and ω_m and is the rotor propeller speed.

The corresponding open-loop transfer function of the Aero 2 1 DOF system is

$$P(s) = \frac{\Theta(s)}{\Omega_m(s)} = \frac{\frac{D_t K_{pp}}{J_p}}{s^2 + \frac{D_p}{J_p}s + \frac{K_{sp}}{J_p}} \tag{2}$$

Finding the Pitch Stiffness and Damping

The transfer function of a second-order system with a non-unity steady-state value is

$$\frac{Y(s)}{U(s)} = \frac{K_{ss}\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \tag{3}$$

where K_{ss} is the steady-state value when a step $1/s$ is applied, ω_n is the natural frequency, and ζ is the damping ratio.

Given the moment of inertia and measuring the natural frequency and damping ratio of the free-oscillation response, we can find the stiffness and damping coefficient model parameters using the equations

$$\begin{aligned} K_{sp} &= J_p\omega_n^2 \\ D_p &= 2J_p\zeta\omega_n \end{aligned} \tag{4}$$

Finding the Natural Frequency

The period of the oscillations in a system response can be found using the equation

$$T_{osc} = \frac{t_n - t_1}{n - 1} \tag{5}$$

where t_1 is the time of the first peak of the 1st oscillation, t_n is the time of the nth oscillation, and n is the number of oscillations considered. From this, the damped natural frequency (in radians per second) is

$$\omega_d = \frac{2\pi}{T_{osc}} \tag{6}$$

and the undamped natural frequency is

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \tag{7}$$

Finding the Damping Ratio

The damping ratio of a second-order system can be found from its response. For a typical second-order underdamped system, the subsidence ratio (i.e., decrement ratio) is defined as

$$\delta = \frac{1}{n} \ln \frac{O_1}{O_n} \tag{8}$$

where O_1 is the peak of the first oscillation and O_n is the peak of the nth oscillation. Note that $O_1 > O_n$, as this is a decaying response.

The damping ratio is defined

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}} \tag{9}$$

Finding the Thrust Gain

The thrust force gain can be found by applying a step to the pitch rotor and measuring the corresponding steady-state pitch angle. The transfer function when applying a step to the rotor with an amplitude of ω_0

$$\Theta(s) = \frac{\frac{D_t K_{pp}}{J_p}}{s^2 + \frac{D_p}{J_p} s + \frac{K_{sp}}{J_p}} \frac{\omega_0}{s} \tag{10}$$

The steady-state angle of this is

$$\theta_{ss} = \frac{D_t K_{pp}}{K_{sp}} \omega_0 \tag{11}$$

Solving for the thrust gain

$$K_{pp} = \frac{\theta_{ss} K_{sp}}{D_t \omega_0} \tag{12}$$