

# Aero 2

Pitch PID Control Design Laboratory  
for MATLAB/Simulink



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# Aero 2 Concept Review

## PID Control Design

### Why Explore PID Control Design?

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Tuning the PID controller manually as done in the [Qualitative PID Control – Lab Procedure](#) is an experimental heuristic way of finding suitable PID gains for a system. While this method does have its advantages (e.g., no model needed) and can yield a satisfactory response, it is also not practical or safe to be performed on certain systems. Model-based control design is the standard approach used in automotive and aerospace industries where the control gains are found analytically based on a set of specifications and a dynamic model of the system.

In the [Rotor PI Speed Control – Lab Procedure](#), a PI control was designed to control the speed of the rotor actuator based on second-order specifications. In this lab, a PID control will be designed to control the position of the pitch angle for the 1 DOF Aero 2 system based on third-order specifications.

## Control Design Overview

The block diagram shown in Figure 1 illustrates the full controller that will be implemented to control the pitch position. This is called a *cascade control* as it has two loops: an outer loop to control the pitch of the Aero 2 and an inner loop to control the speed of the rotor.

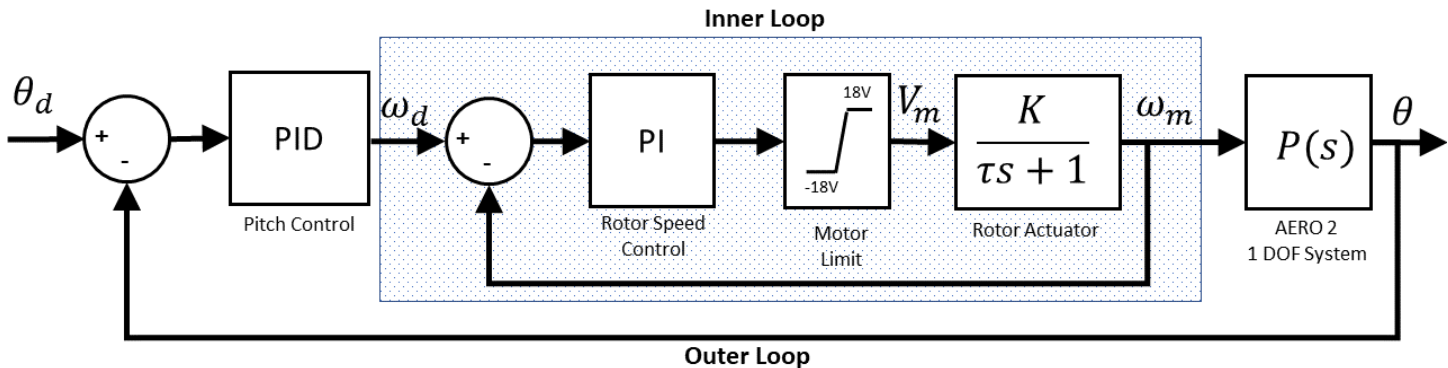


Figure 1 - Aero 2 Pitch Cascade Control

The inner rotor speed control design and implementation was performed in the [Rotor PI Speed Control](#) lab. We can treat the rotor as being *already controlled* and design the PID controller accordingly, i.e., assuming our control variable is the motor speed.

## Peak Time and Percent Overshoot

The standard second-order system transfer function of the form

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (1)$$

where  $\omega_n$  is the natural undamped frequency and  $\zeta$  is the damping-ratio. The shape of the response depends on the values of the  $\omega_n$  and  $\zeta$  parameters. A typical second-order response to a desired step signal of  $R_0$  at time  $t_0$  is shown in Figure 1.

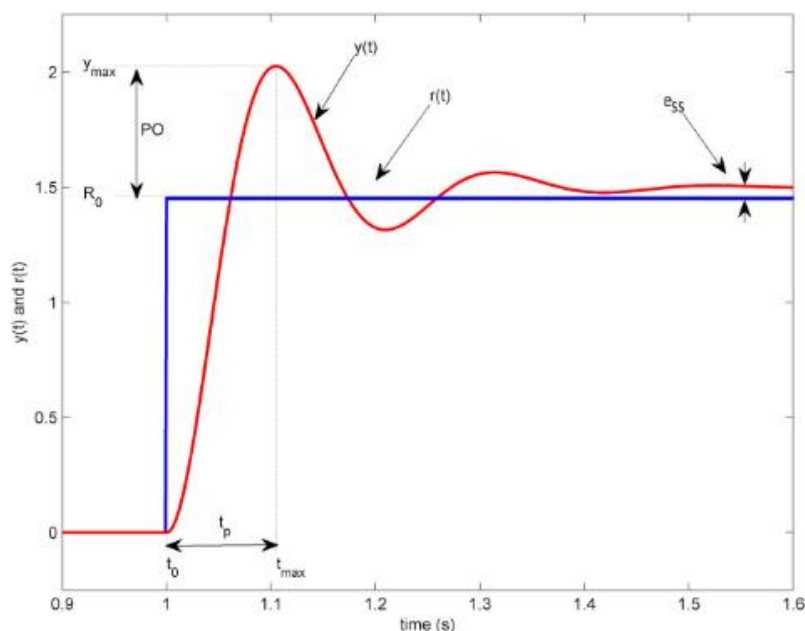


Figure 1: Standard second-order step response

The maximum value of the response is denoted by the variable  $y_{max}$  and it occurs at a time  $t_{max}$ . The *percent overshoot* is found using

$$PO = \frac{100(y_{mas} - R_0)}{R_0} \quad (2)$$

In a second order system, the amount of overshoot depends solely on the damping ratio parameter, and it can be calculated using the equation

$$PO = 100e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \quad (3)$$

From the initial step time  $t_0$ , the time it takes for the response to reach its maximum value is.

$$t_p = t_{max} - t_0 \quad (4)$$

This is called the *peak time* of the system and it depends on both the damping ratio and natural frequency of the system. It can be derived analytically as

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \quad (5)$$

The damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

## PID Control Design

The pitch transfer function was identified in the [Pitch Parameter Estimation – Lab Procedure](#)

$$P(s) = \frac{\Theta(s)}{\Omega_m(s)} = \frac{\frac{D_t K_{pp}}{J_p}}{s^2 + \frac{D_p}{J_p}s + \frac{K_{sp}}{J_p}} \quad (6)$$

where  $\theta$  is the pitch angle,  $J_p$  is the equivalent moment of inertia acting about the pitch axis,  $D_p$  is the viscous damping,  $K_{sp}$  is the stiffness,  $K_{pp}$  is the force thrust gain relative to the rotor speed,  $D_t$  is the distance from the pivot point to the center of the rotor, and  $\omega_m$  and is the rotor propeller speed.

The pitch of the Aero 2 system can be controlled using the following PID controller:

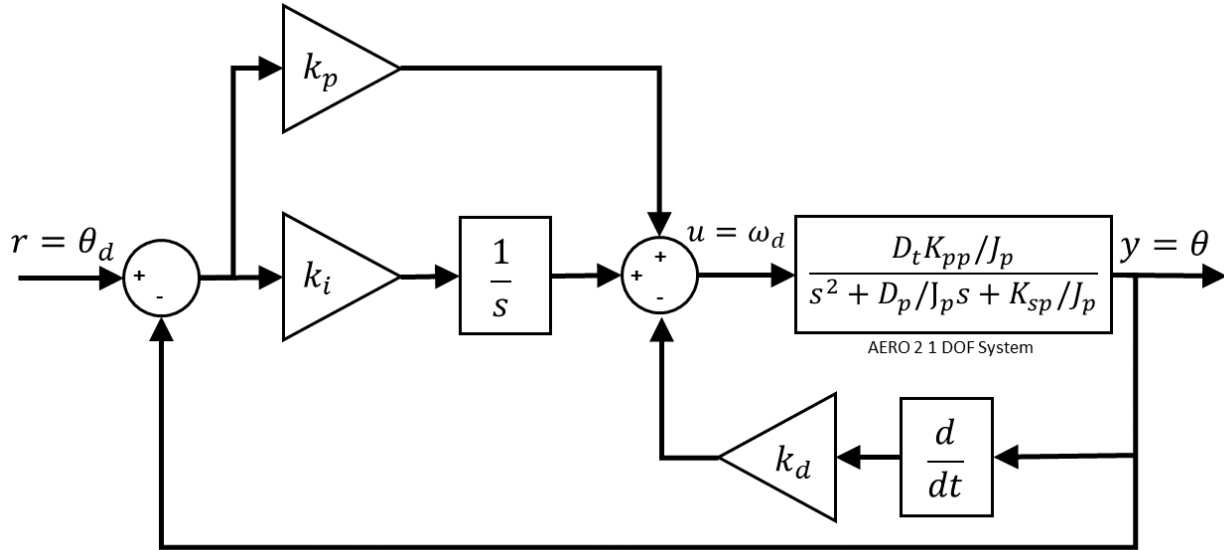


Figure 2 - Aero 2 Pitch PID Control

The time-domain equation of the PID controller in Figure 2 is

$$u = k_p(\theta_d - \theta) + k_i \int (\theta_d - \theta) dt - k_d \dot{\theta}$$

where  $k_p$  is the proportional gain,  $k_i$  is the integral gain,  $k_d$  is the derivative gain, and  $\theta_d$  is the desired pitch angle. Remark that only the measured velocity is used, i.e., instead of using the derivative of the error. Using the rate feedback directly is a slight variation of the more standard PID approach. Here the control variable is the desired rotor speed,  $u = \omega_d$ , that goes to the PI control

Applying this to the open-loop transfer function and solving for  $\Theta(s)/\Theta_d(s)$ , the closed loop transfer function is

$$G_{\theta,d}(s) = \frac{\Theta(s)}{\Theta_d(s)} = \frac{K_t D_t (k_p s + k_i)}{J_p s^3 + (D_p + D_t K_t k_d) s^2 + (K_{sp} + D_t K_t k_p) s + D_t K_t k_i} \quad (7)$$

The prototype third-order characteristic polynomial is

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)(s + p_0) = s^3 + (2\zeta\omega_n + p_0)s^2 + (\omega_n^2 + 2\zeta\omega_n p_0)s + \omega_n^2 p_0 \quad (8)$$

where  $\omega_n$  is the natural frequency,  $\zeta$  is the damping ratio, and  $p_0$  is the pole location.

The following PID gains that will make the characteristic equation in the closed-loop transfer function in Equation 7 match the desired characteristic equation in Equation 8:

$$\begin{aligned} k_p &= \frac{-K_{sp} + 2J_p p_0 \zeta \omega_n + J_p \omega_n^2}{D_t K_t} \\ k_i &= \frac{p_0 J_p \omega_n^2}{D_t K_t} \\ k_d &= \frac{-D_p + p_0 J_p + 2J_p \zeta \omega_n}{D_t K_t} \end{aligned} \quad (9)$$