Dynamic Parameters

1 Rigid Body Diagram & Parameters

The Quanser Arm's frame assignment (consistent with the Frame Assignments documentation) is provided below to a more realistic scale.

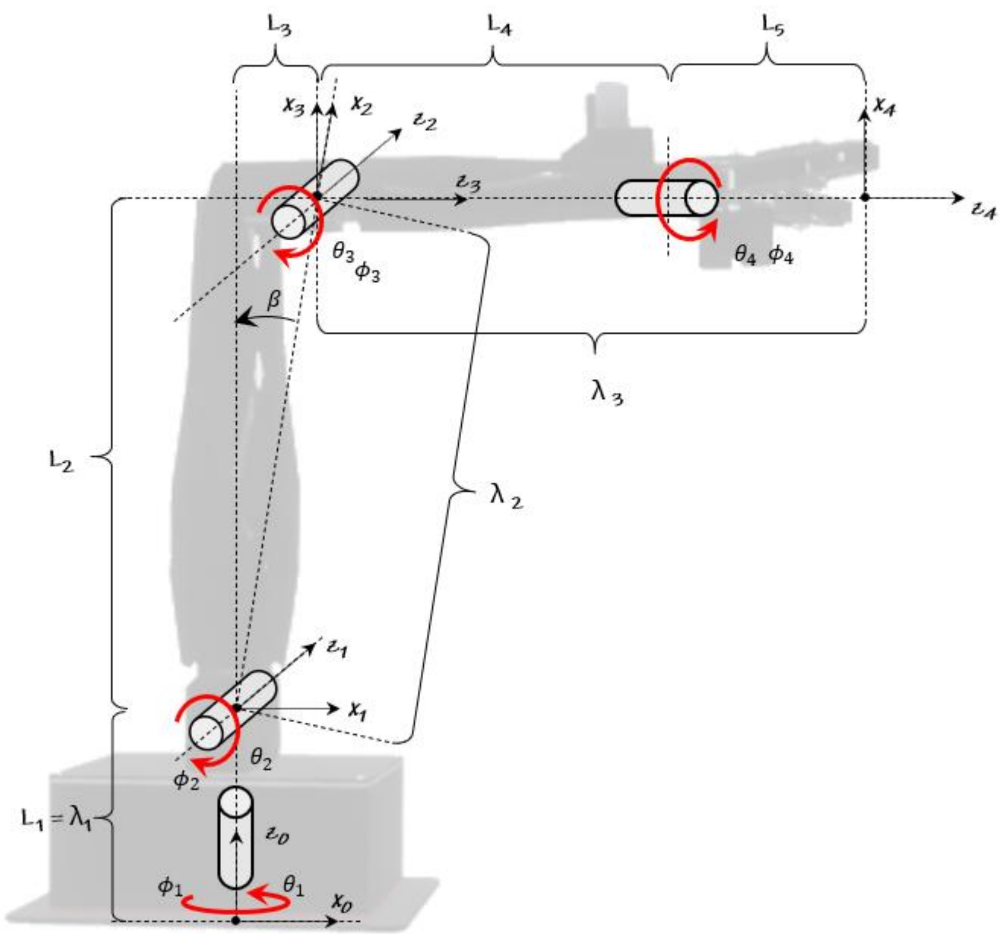

Figure 1. Frame diagram for the Quanser Arm manipulator

Maintaining frame assignments, the rigid body diagram is provided in Figure 2. The manipulator consists of 5 rigid bodies, B_0 to B_5. Note that B_0 is not shown in Figure 2 as it is stationary w.r.t the base frame and does not contribute to the dynamic equations. Also note that while the kinematic diagram in Figure 1 shows the x and z axis of each joint frame, the rigid body diagram lists the x, y or z axis for ease of notation. The third axis can be found using right-hand-rule.

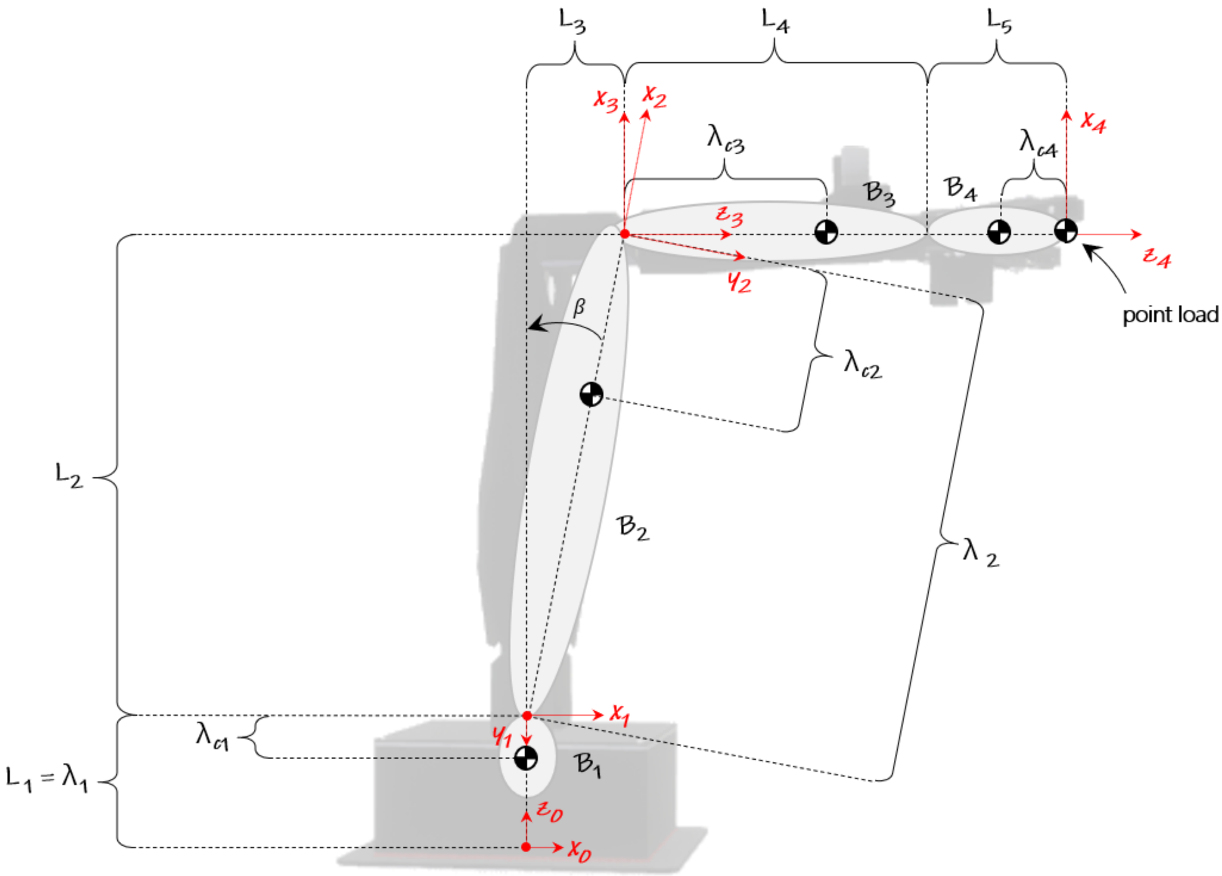

Figure 2. Rigid body diagram for the Quanser Arm manipulator

The centers of mass of the rigid bodies B_1 to B_4 are described in the local joint coordinate frame as follows,

$$ {}^{1}r_{c1} = \begin{bmatrix} 0 \\ \lambda_{c1} \\ 0 \end{bmatrix},\ {}^{2}r_{c2} = \begin{bmatrix} -\lambda_{c2} \\ 0 \\ 0 \end{bmatrix},\ {}^{3}r_{c3} = \begin{bmatrix} 0 \\ 0 \\ \lambda_{c3} \end{bmatrix},\ {}^{4}r_{c4} = \begin{bmatrix} 0 \\ 0 \\ -\lambda_{c4} \end{bmatrix} \tag{3} $$

The mass moments of inertia about the centers of mass are also provided in the local joint coordinate frames as,

$$ \begin{gathered} {}^{1}I_{c1} = \begin{bmatrix} I_{1L} & 0 & 0 \\ 0 & I_{1A} & 0 \\ 0 & 0 & I_{1L} \end{bmatrix},\ {}^{2}I_{c2} = \begin{bmatrix} I_{2A} & 0 & 0 \\ 0 & I_{2L} & 0 \\ 0 & 0 & I_{2L} \end{bmatrix}, \\ {}^{3}I_{c3} = \begin{bmatrix} I_{3L} & 0 & 0 \\ 0 & I_{3L} & 0 \\ 0 & 0 & I_{3A} \end{bmatrix},\ {}^{4}I_{c4} = \begin{bmatrix} I_{4L} & 0 & 0 \\ 0 & I_{4L} & 0 \\ 0 & 0 & I_{4A} \end{bmatrix} \end{gathered} \tag{3} $$

The parameters in equations 1 and 2 as well as the masses of the rigid bodies are provided in Table 1.

Parameter	Value	Parameter	Value
m_1	$0.7906\ kg$	m_2	$0.4591\ kg$
λ_{c1}	$0.0399\ m$	λ_{c2}	$0.1071\ m$
I_{1A}	$1.489\ \times 10^{-3}\ \ kg\ m^2$	I_{2A}	$1.922\ \times 10^{-4}\ \ kg\ m^2$
$n/a*$	-	$I_{2L}*$	$9.610\ \times 10^{-3}\ \ kg\ m^2$

Parameter	Value	Parameter	Value
m_3	$0.269\ kg$	m_4	$0.257\ kg$
λ_{c3}	$0.1561\ m$	λ_{c4}	$0.0998\ m$
I_{3A}	$2.679\ \times 10^{-4}\ \ kg\ m^2$	I_{4A}	$6.528\ \times 10^{-4}\ \ kg\ m^2$
$I_{3L}*$	$2.069\ \times 10^{-3}\ \ kg\ m^2$	$I_{4L}*$	$1.120\ \times 10^{-3}\ \ kg\ m^2$

Table 1. Dynamic parameters for the Qarm manipulator for the equations of motion provided
* see simplifications below

2 Raw Parameters

Some assumptions were made when deriving the model presented in the Dynamics – Lagrangian documentation. The first assumption was that the center of mass is located along the major axis in the local coordinate frame. This simplifies the dynamic model. The true values are provided below. The values highlighted in red are used in the simplified model.

$$^{1}r_{c1} = \begin{bmatrix} 0.0000 \\ 0.0399 \\ 0.0003 \end{bmatrix},\ ^{2}r_{c2} = \begin{bmatrix} -0.1071 \\ -0.0154 \\ -0.0001 \end{bmatrix},\ ^{3}r_{c3} = \begin{bmatrix} 0.0094 \\ 0.0020 \\ 0.1561 \end{bmatrix},\ ^{4}r_{c4} = \begin{bmatrix} -0.0116 \\ 0.0000 \\ -0.0998 \end{bmatrix}$$

Next, it was assumed that the bodies are uniform density cylinders with principal moments of inertia aligned with the local joint coordinate frames. This also assumes that the lateral components are equivalent. The raw parameters are provided below. When the lateral components were different, the higher value was used in the calculations. Values highlighted in red are used in the simplified model.

$$^{1}I_{c1} = \begin{bmatrix} 1.556 \times 10^{-3} & 0 & 0 \\ 0 & 1.489 \times 10^{-3} & 0 \\ 0 & 0 & 9.624 \times 10^{-4} \end{bmatrix},\ ^{2}I_{c2} = \begin{bmatrix} 1.922 \times 10^{-4} & 0 & 0 \\ 0 & 9.541 \times 10^{-3} & 0 \\ 0 & 0 & 9.610 \times 10^{-3} \end{bmatrix}$$

$$^{3}I_{c3} = \begin{bmatrix} 2.069 \times 10^{-3} & 0 & 0 \\ 0 & 1.976 \times 10^{-3} & 0 \\ 0 & 0 & 2.679 \times 10^{-4} \end{bmatrix},\ ^{4}I_{c4} = \begin{bmatrix} 1.120 \times 10^{-3} & 0 & 0 \\ 0 & 9.994 \times 10^{-4} & 0 \\ 0 & 0 & 6.528 \times 10^{-4} \end{bmatrix}$$