Concept Review

Trajectory Generation

Why Study Trajectory Generation?

It's important to understand different methods for generating commands for robotic platforms. Trajectory generation is used in applications of mobile robotics and robotic manipulation which must maintain certain geometric characteristics. Understanding different methods for trajectory generation can allow a user to design a trajectory which maintains kinematic constraints and optimize motion depending on a specific task.

Spline Trajectory Generation

Often, we want to command pulse or square waves for a devices' joints to move from point A to point B. Such motion involves a discontinuous jump from a setpoint to another. However, there are no such jumps in real physics. To achieve continuous motion between the setpoints, we must enforce two conditions that are always prevalent in real physical conditions – continuity in speed and position. This is realized in the form of four overall constraints, two at the start and two at the end of a trajectory traversal.

$$x|_{t=0} = x_0, \qquad x|_{t=T} = x_f, \qquad \dot{x}|_{t=0} = \dot{x}_0, \quad \dot{x}|_{t=T} = \dot{x}_f \tag{1}$$

Where $x(t)$ represents a task space position in meters as a function of time, $\dot{x}$ is the speed in m/s and t represents the time from 0 to T seconds. The terms x_0 and x_f are the initial and final setpoints (also referred to as the current and desired setpoints, respectively) with $\dot{x}_0$ and $\dot{x}_f$ being the corresponding initial and final speeds, as shown in Figure 1.

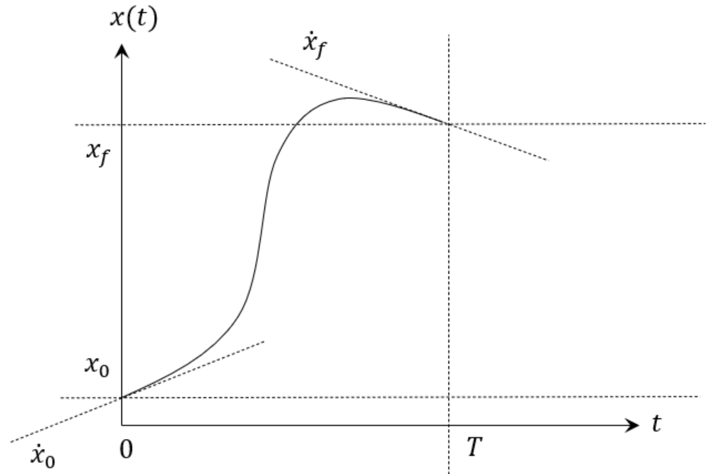

Figure 1: Initial and final conditions of position and speed

A trajectory function that satisfies four constraints is a cubic polynomial of the form,

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{2}$$

To solve for the coefficients in equation 2 in terms of the constraints in equation 1, first take the derivative of equation 2,

$$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 \tag{3}$$

Now evaluate equations 2 and 3 at the constraints in equation 1

$$\begin{gathered} x(t=0) = a_0 = x_0 \\ \dot{x}(t=0) = a_1 = \dot{x}_0 \\ x(t=T) = a_0 + a_1 T + a_2 T^2 + a_3 t^3 = x_f \\ \dot{x}(t=T) = a_1 + 2a_2 T + 3a_3 t^2 = \dot{x}_f \end{gathered} \tag{4}$$

The first two equations immediately provide solutions for a_0 and a_1, which, coupled with the third and fourth equations, yield a system of two linear equations in two variables.

$$\begin{gathered} a_2T^2 + a_3t^3 = x_f - x_0 - \dot{x}_0T \\ 2a_2T + 3a_3t^2 = \dot{x}_f - \dot{x}_0 \end{gathered} \tag{5}$$

Solving them yields,

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ \dot{x}_0 \\ 3x_f - 3x_0 - 2\dot{x}_0T - \dot{x}_fT \Big/ T^2 \\ \dot{x}_fT + \dot{x}_0T - 2x_f + 2x_0 \Big/ T^3 \end{bmatrix} \tag{6}$$

With these coefficients, a cubic spline can be created between the positions $x = x_0$ and $x = x_f$ with initial and final speeds $\dot{x} = \dot{x}_0$ and $\dot{x} = \dot{x}_f$, respectively.

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