Lab Procedure
State Space Modeling

Introduction

Ensure the following:

1. You have reviewed the Application Guide – State Space Modeling
2. The Qube-Servo 3 has been previously tested, is ON and connected to the PC.
3. Pendulum is attached and connected using the Encoder 1 connector to the Qube-Servo 3. If the pendulum is not centered on the Qube, you can turn it at the connection port to change its resting position.
4. Launch MATLAB and browse to the working directory that includes the Simulink models for this lab.

In this lab, we explore the development of a linear state-space model for a rotary pendulum system. By deriving and analyzing state-space equations, we aim to represent the system's dynamics in a linear framework. This lab provides a foundation for understanding how state-space models are used in control system design, enabling precise prediction and manipulation of system behavior.

Deriving the State Space Representation

The **Linear** Equations of Motion for the Rotary Pendulum are defined as:

$$J_r\ddot{\theta} + m_p l r\ddot{\alpha} = \tau - b_r\dot{\theta} \tag{1}$$

$$J_p\ddot{\alpha} + m_p l r\ddot{\theta} + m_p g l\alpha = -b_p\alpha \tag{2}$$

Use the following definitions for the state space model:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3}$$

$$y(t) = Cx(t) + Du(t) \tag{4}$$

The rotary pendulum system is defined as:

$$x(t) = [\theta(t) \quad \alpha(t) \quad \dot{\theta}(t) \quad \dot{\alpha}(t)]^T \tag{5}$$

$$y(t) = [\theta(t) \quad \alpha(t)]^T \tag{6}$$

Where:

$$x_1 = \theta(t),\ x_2 = \alpha(t),\ x_3(t) = \dot{\theta}(t),\ x_4(t) = \dot{\alpha}(t) \tag{7}$$

$$u(t) = \tau \tag{8}$$

1. Using the provided equations of motion *(1)* and (*2*), rearrange and solve to find explicit expressions for the angular accelerations $\ddot{\theta}$ and $\ddot{\alpha}$ in terms of the given parameters.

   Hint: Use MATLAB's symbolic toolbox and solve function to assist with solving the equations simultaneously.

2. Based on the output state $y(t)$ defined above in Equation (*6*), find the state space matrices C and D in Equation (*4*).

3. Using the solution from Question 1 and Equation (*5*), derive matrices A and B in Equation (*3*).

4. Complete the *A*, *B*, *C*, and *D* matrices in the file qs3_rotpen_ABCD_eqns_down.m. Use the qs3_rotpen_param.m file completed in the previous lab, which contains all the model parameters. After completing the matrices, run qs3_setup_rotpen_ss_model.m in the MATLAB Workspace to generate a state-space object for analysis.
   Note: In rotpen_ABCD_eqns_down.m the last few lines of code convert the model to be with respect to $u = v_m$, since it is defined as $u = \tau$ in Step 3.
   Note: In qs3_rotpen_param.m the base motor viscous damping coefficient, b_r, and the pendulum damping coefficient, b_p, were found experimentally to reasonably reflect the viscous damping of the system due to friction during a step response.

Model Validation

The Simulink model in Figure 1 applies a square wave voltage input to the Qube-Servo 3 and its state space model. The output scopes will display the responses of the base motor angle and pendulum angle of the Qube-Servo 3 in parallel with the calculated linear model response of the system.

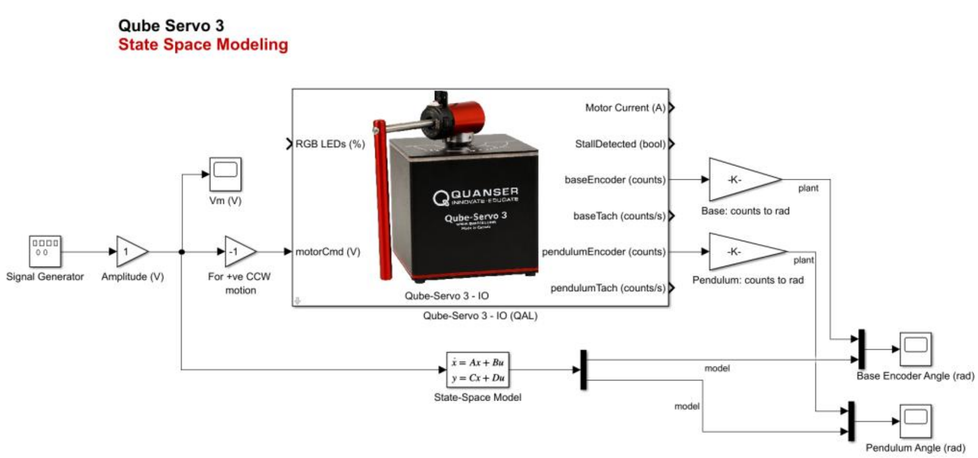

Figure 1: Simulink model comparing the response of the physical Qube-Servo 3 and the linear state space model.

5. Open qs3_pen_ss_model.slx, The model should apply a 1V, 1Hz square wave to the pendulum system and state space model.

6. Click **`Monitor & Tune`** in the **`Hardware`** or **`QUARC`** tab to deploy and run the model.

7. Capture a screenshot of the **`Base Encoder Angle (rad)`** scope and the **`Pendulum Angle (rad)`** scope. The response should look similar to Figure 2 below.

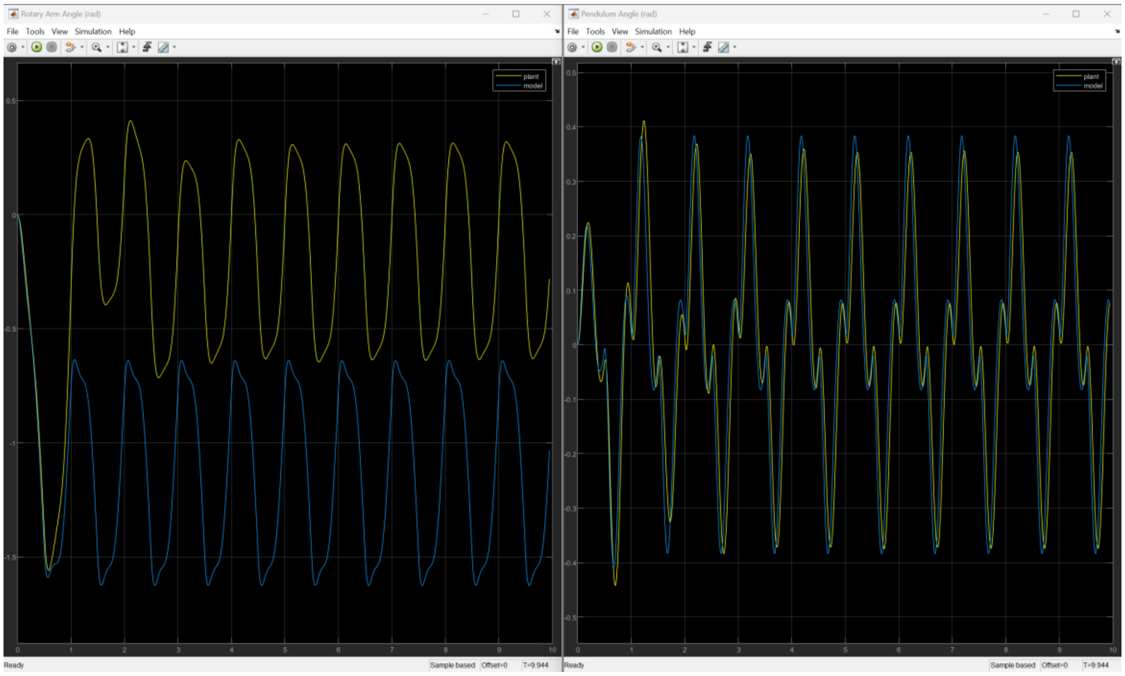

Figure 2: Sample response of the measured vs. modeled pendulum system.

8. The Simulink model should have logged data to the MATLAB workspace. Save this data for analysis later if necessary.

9. The viscous damping of each pendulum can vary slightly from system to system. If the model does not accurately represent the measured response, try modifying the damping coefficients b_r and b_p to obtain a more accurate response. If the model is significantly off, ensure the derivation of the state space is correct.

10. Take notes on whether the measured and modeled responses match well and any possible reasons why they wouldn't match.

11. Once the model has been validated, close your model. ensure you save a copy of the files for review later.

12. Turn OFF the Qube-Servo 3.