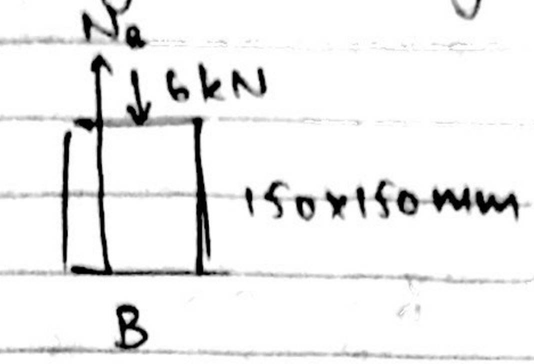
UES010 (Solids & Structures)

Tut-1

Bearing stresses by force at A, B & C



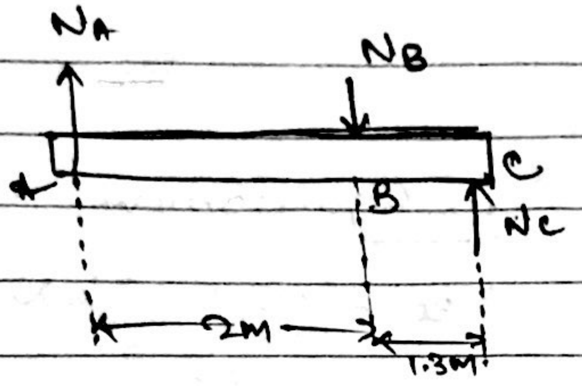
$N_B = 6\,kN$

$\sigma = \frac{P}{A}$.

$\sigma = \frac{6\times10^3}{150\times150\times10^{-6}}$

$\sigma_B = 0.266\times10^6\ N/m^2$

$= 0.266\ MPa$.



$N_A + N_C = N_B$

$N_A + N_C = 6\,kN$.

Moment abt A:

$2N_B = 3.3N_C$

$\frac{2(6kN)}{3.3} = N_C$

$N_C = \cancel{8}\,3.636\ kN$.

Also

$N_A + N_C = 6\,kN$

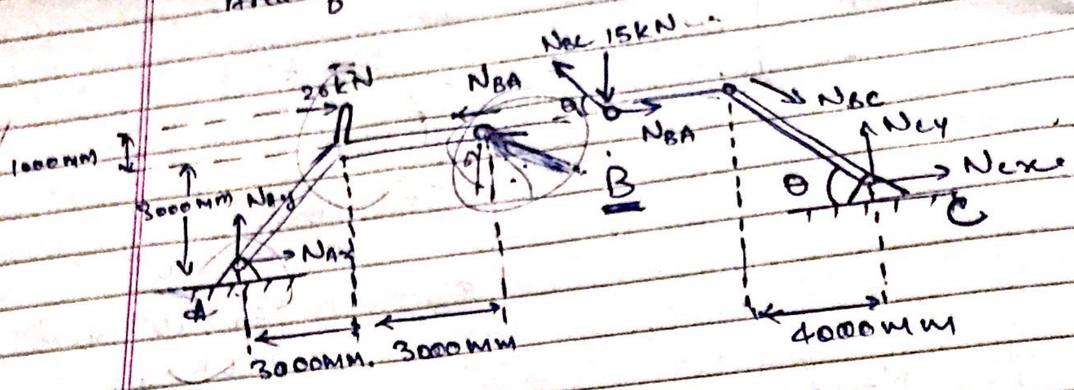
$\therefore N_A = 6 - N_C$

$= 6 - 3.636$

$= 2.36\ kN$

$\curvearrowleft$ ~~20 kNm~~ 10 kNm

(2.) Intermediate hinge at B
reaction at A, B, C and at B.
FBD of AB & BC, Normal stress in BC
Area of cross-section of BC = 400 mm^2



$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = 36.86$

$\sin\theta = 0.6$

$\cos\theta = 0.8$.

Force Equilibrium at B

$N_{BC} \sin\theta = 15\,kN$

$N_{BC} = \frac{15 \times 10^3}{0.6}$

$N_{BC} = 25\,kN$

Also $N_{BC} \cos\theta = N_{BA}$

$25 \times 0.8 = N_{BA}$

$N_{BA} = 20\,kN$

Torque Balance?

Force Equilib. At AB

$N_{AX} + 20 = N_{BA}$

$N_{AX} = 0$

~~$N_{AY} = 0$~~

Force Eq at BC

$N_{BC}\cos\theta + N_{CX} = 0$

$N_{CX} = -25 \times 0.8$

$= -20\,kN$

$N_{BC}\sin\theta = N_{CY}$

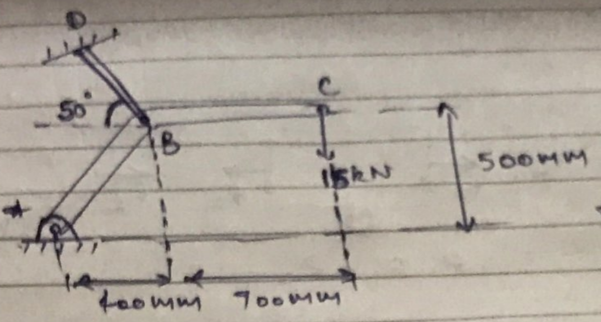
$N_{CY} = 25 \times 0.6$

$= 15\,kN$

Normal stress in BC $= \dfrac{25 \times 10^3}{400 \times 10^{-6}}$

$= 62.5$ MPa

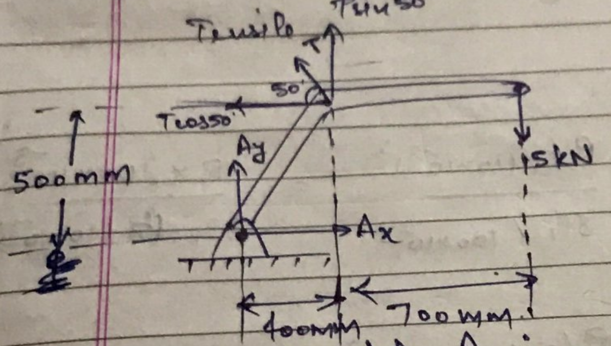
(3.)



Tensile stress on BD?
shearing stress in pin at A

Diam. (cable) = 20mm
——— (pin) = 10mm

FBD of ABC,



Force equilibrium:

$A_y + 0.766T = 15$ kN

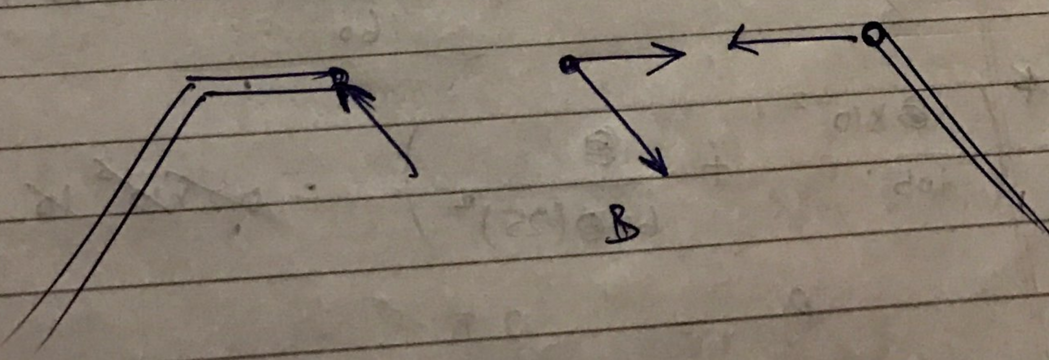
$A_x = T(0.64)$

Moment abt. A :

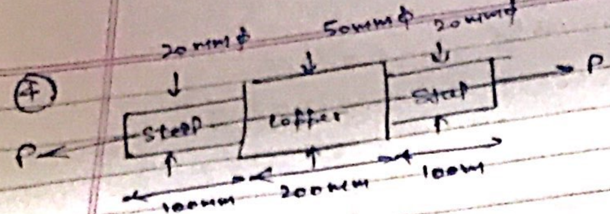
$\cancel{500 \times 10^{-3}}(0.64T) + (0.766T)(\cancel{400 \times 10^{-3}}) = 15 \times 10^3 (\cancel{1100 \times 10^{-3}})$

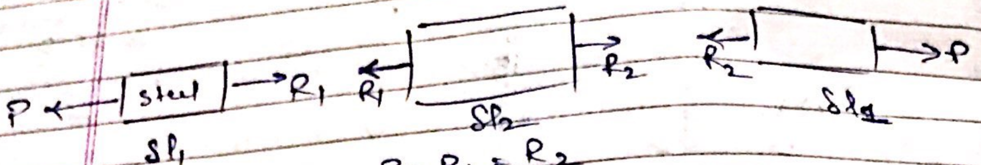
$3.2T + 3.064T = 165$ kN

$T = 26.34$ kN



④





$\therefore \quad P = R_1 = R_2$

$E_{steel} = 210 \text{ GPa}$

$E_{copper} = 120 \text{ GPa}$

$$210 \times 10^9 = \frac{P/\pi(10\times10^{-3})^2}{\delta l_1/100\times10^{-3}} = \frac{P\times100\times10^{-3}}{\delta l_1\,(\pi(10\times10^{-3})^2)}$$

$$120\times10^9 = \frac{P/\pi(25\times10^{-3})^2}{\delta l_2/200\times10^{-3}} = \frac{P\times200\times10^{-3}}{\delta l_2\,(\pi(25\times10^{-3})^2)}$$

$$2\,\delta l_1 + \delta l_2 = 0.2\text{ mm}$$

$$\cancel{2}\left(\frac{P\times\cancel{10^{-1}}\,10^{-6}}{\underset{105}{\cancel{210}}\times\cancel{10^{9}}(\pi)\cancel{(10)^{-4}}}\right) + \frac{P\times\cancel{2}\times10^{-1-9+6}}{\underset{60}{\cancel{120}}\times\cancel{10^{9}}(\pi)(25\times\cancel{10^{-3}})^2} = 0.2\times10^{-3}$$

$$P\,\frac{\cancel{10^{4}}}{\pi}\left(\frac{\cancel{8}\times10^{-2}}{105} + \frac{\cancel{8}}{60\cancel{8}(25)^2}\right) = \cancel{0.2\times10^{-3}}\ \cancel{10}\ 2$$

$$P = \frac{2\pi}{\frac{10^{-2}}{105} + \frac{1}{625\times60}} = 51.541\text{ kN}$$

⑤ $P = 10$ kN

$E = 205$

60 mm

60−

5

0.12 m

δl_1

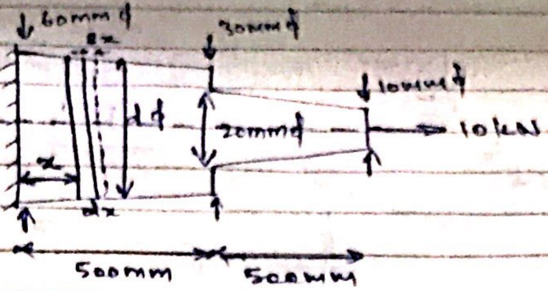
$\times 100\times10^{-3}$ / $(\pi(10\times10^{-3})^2)$

$\times 200\times10^{-3}$ / $\pi(2.5\times10^{-3})^2$

$= 0.2\times10^{-3}$

(6.) P = 10 kN

E = 205 GPa



$$\frac{60-30}{500} = \frac{60-d}{x}$$

$$\left(\frac{30}{500}\right)x = 60-d$$

$$d = 60 - \frac{3x}{50}$$

$$205\times10^9 = \frac{P/\pi(d/2\times10^{-3})^2}{\frac{\delta x}{dx}}$$

$$\frac{\delta x}{dx} = \frac{P}{\pi\left(30-\frac{3x}{100}\right)^2\times10^{-6}\times205\times10^9}$$

$$\delta = \frac{4PL}{\pi E d_1 d_2}$$

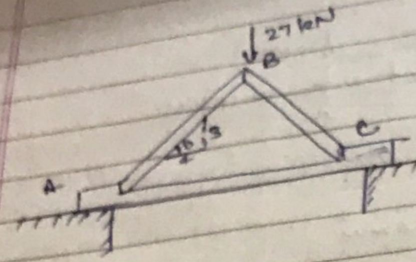
$$\delta_{net} = \delta_1+\delta_2$$

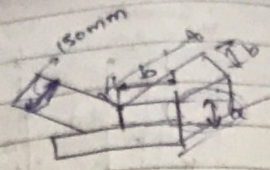
$$= \frac{4\times10\times10^3\times500\times10^{-3}}{\pi\times205\times10^9\times10^{-6}}\left(\frac{1}{60\times30}+\frac{1}{30\times10}\right)$$

$$0.12\,mm = \frac{2}{3}\times\frac{1}{205\pi}\left(\frac{7}{60}\right) = \frac{4\times10\times500\times10^{-3}}{\pi\times205}\times\frac{1}{30}\left(\frac{1}{60}+\frac{1}{10}\right)$$

(6.) Allowable direct bearing = 6 N/mm^2

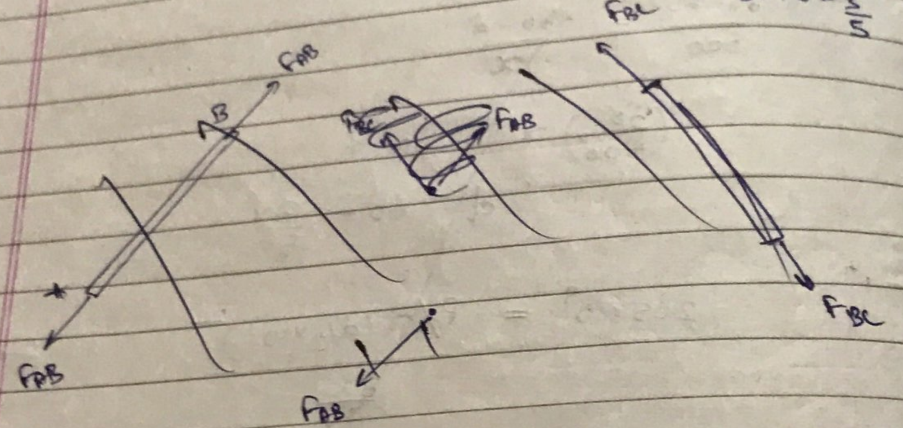
shear = 1.2 N/mm^2

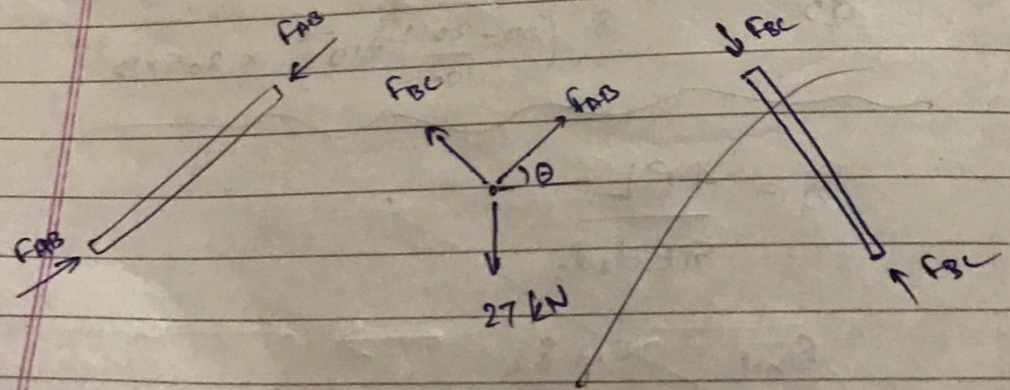




$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$





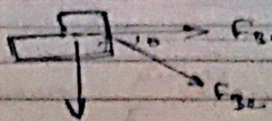
$$27 = F_{AB}\sin\theta + F_{BC}\sin\theta \qquad F_{AB}\cancel{\cos\theta} = F_{BC}\cancel{\cos\theta}$$

$$2F_{BC}\,\frac{\cancel{3}}{5} = \cancel{27}\,9 \qquad F_{AB} = F_{BC}$$

$$F_{BC} = 22.5\text{ kN.}$$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$



$F_{BC}\cos\theta = 22.5 \times \frac{4}{5} = 18\,kN$

$F_{BC}\sin\theta = 13.5\,kN$

$6 = \frac{18 \times 1000}{150 \times b}$

$b = \frac{13.5 \times 1000}{150 \times x}$

$b = 20\,mm$

$1.2 = \frac{13.5 \times 1000}{150 \times a}$

$a = \frac{13500\cancel{0}}{15\cancel{0} \times 12}$

Test Sheet-2

① Young's Mod = $\frac{\text{Stress}}{\text{strain}}$

$$= \frac{\frac{25\times10^{3}}{\pi(8\times10^{-3})^{2}}}{\frac{300\times10^{-6}}{500\times10^{-3}}}$$

$$= \frac{25\times10^{3}}{\pi\times64\times10^{-6}} \times \frac{5\cancel{00}\times10}{3\cancel{00}\times10^{-6}}$$

$$= 207.23\ 10^{12-3}\ N/m^{2}$$

$$= 207.23\times10^{9}\ N/m^{2}$$

Poisson's Ratio $= \frac{\frac{2.4\times10^{-6}}{16}}{\frac{300\times10^{-6}}{500}} = 0.25 = \nu$

Shear Modulus (G) $= \frac{E}{2(1+\nu)} =$

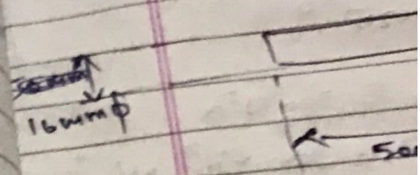
① 500 mm long

ϕ 16 mm

$\Delta l = 300\,\mu m$

$\Delta\phi = 2.4\,\mu m$

Axial load =



Mod. of

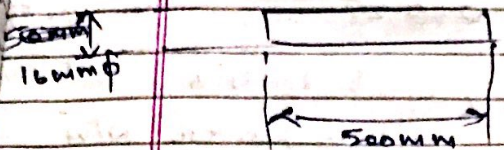
Tut Sheet - 2

(1) 500 mm long
ϕ 16 mm
$\Delta l = 300 \mu m$
$\Delta \phi = 2.4 \mu m$
Axial load = 25 kN

Tensile Mod.
Elastic Mod.
Young's Mod.

Mod. of Elas ?
Poisson's Ratio ?
Shear Mod. ?

$$\text{Poisson's Ratio} = \frac{-e_{contraction}}{-e_{extension}} = \frac{-e_{lateral}}{-e_{longitu.}}$$



$$\text{Mod. of Elasti.} = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{\dfrac{25 \times 1000}{\pi \times (8 \times 10^{-3})^2}}{\dfrac{2.4}{16}}$$

$$= \frac{25 \times 1000}{\pi \times 64 \times 10^{-6}} \times \frac{16}{2.4}$$

$$= 828.93 \times 10^{6} \, N/m^2$$

$$E = 0.828 \, GPa$$

$$\text{Poisson's Ratio} = + \frac{\dfrac{2.4 \times \cancel{10^{-6}}}{16}}{\dfrac{300 \times \cancel{10^{-6}}}{500}}$$

$$\nu = \frac{2.4}{16} \times \frac{500}{300} = 0.25$$

$$\text{Shear Modulus, } G \Rightarrow 2G(1+\nu) = E = 3K(1-2\nu)$$

$K \downarrow$ Bulk Mod

$$G = \frac{E}{2(1+\nu)} = \frac{0.828}{2(1.25)} = 0.3312 \, GPa$$

(2) 50 × 250 × 10 mm

Stresses along edges :

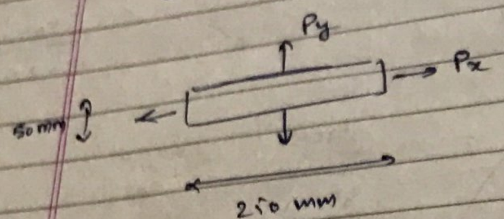
a) $P_x = 100$ kN } Change in thickness?

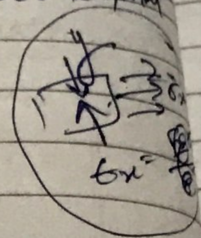
$P_y = 200$ kN

b) $P_x = ?$ for same change.

$E = 200$ GPa $\quad \nu = 0.25$

$E = 200$ GPa
$= 200 \times 10^9$ N/m^2
$= 200$ kN/mm^2





$$\epsilon_z = \frac{\sigma_z}{E} - \nu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$$

$$= 0 - \frac{0.25}{200 \times 10^3}\left(\frac{100 \times 10^3}{10 \times 50} + \frac{200 \times 10^3}{250 \times 10}\right)$$

$$= -\frac{0.25}{20 \times 50}\left(1 + \frac{2}{5}\right) = -3.5 \times 10^{-4}$$

$$(\Delta l)_z = -3.5 \times 10^{-4} \times 10 = -3.5 \times 10^{-3} \text{ mm}$$

$\nu = -\frac{\epsilon_y}{\epsilon_x}$

$= -\frac{\epsilon_y}{\sigma_x / E}$

$\epsilon_y = -\frac{\nu \sigma_x}{E}$

$$\epsilon_z = -\frac{\nu \sigma_x}{E}$$

$$\sigma_x = \frac{3.5 \times 10^{-4} \times 200 \times 10^3}{0.25 \times 10} = \frac{P_x}{50 \times 10}$$

$$P_x = \frac{3.5 \times 200 \times 50}{0.25} = 140 \text{ kN}$$

(3) $E = 210$ GPa

$\nu = 0.3$

$\sigma_x = 150$ MPa

$\epsilon_y = \frac{\sigma ...}{...}$

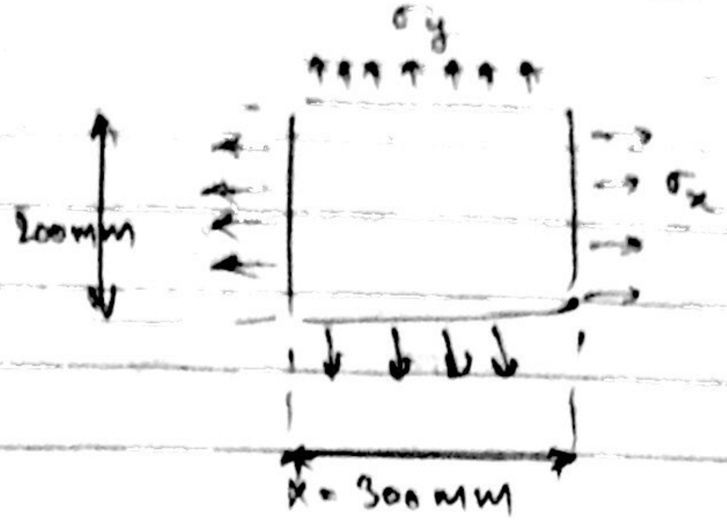
$\epsilon_y = 0 ...$

$\epsilon_x = ...$

(3) $E = 210\,GPa$ $\quad 300 \times 200 \times 10\ mm$

$\nu = 0.3$

$\sigma_x = 150\,MPa$



$$\epsilon_y = \frac{\sigma_y}{E} - \nu\left(\frac{\sigma_x}{E}\right)$$

$$\epsilon_y = 0 \Rightarrow \sigma_y = \nu\,\sigma_x$$

$$= 0.3\,(150)$$

$$= 45\,MPa$$

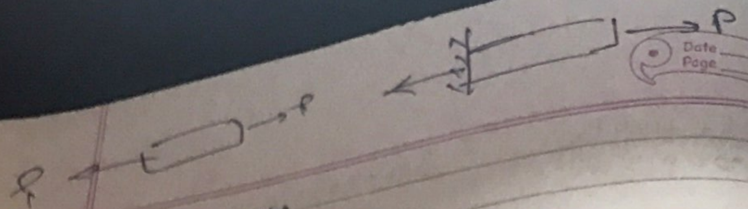
$$\epsilon_x = \frac{\sigma_x}{E} - \nu\left(\frac{\sigma_y}{E}\right)$$

$$= \frac{1}{210\times10^3}\left(150 - 0.3\,(45)\right) = 0.65\times10^{-3}$$

$$\Delta x = \epsilon_x \times x$$

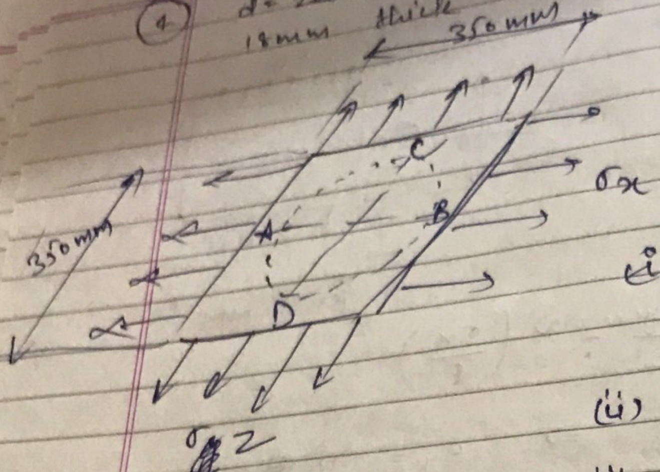
$$= 0.65\times10^{-3} \times 300\times10^{-3} = 0.195\ mm$$

$$x' = x + \Delta x$$



(4) d = 200mm

18mm thick



$\sigma_x = 85$ MPa

$\sigma_z = 150$ MPa

(i) $\Delta AB = ?$

$\Delta CD = ?$

(ii) thickness = ?

(iii) volume = ?

$E = 70$ GPa & $\nu = 1/3$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_z}{E}$$

$$= \frac{1}{E}\left(85 - \frac{1}{\cancel{3}} \times \overset{50}{\cancel{150}}\right)$$

$$= \frac{1}{\underset{2}{\cancel{70 \times 10^9}}\ 10^3} \times \overset{6}{\cancel{35 \times 10}}$$

$$\epsilon_x = 0.5 \times 10^{-3}\ m$$

$$= 0.5\ mm$$

→ P

$\sigma_x = 85$ MPa

$\sigma_z = 150$ MPa

AB = ?

D = ?

...ness = ?

...me = ?

$$\epsilon_z = \frac{\sigma_z}{E} - \nu\left(\frac{\sigma_x}{E}\right)$$

$$= \frac{150}{E} - \frac{1}{3}\times\frac{85}{E} \quad 28.33$$

$$= \frac{1}{E}\left(150 - 28.333\right)$$

$$= \frac{121.667}{70\times10^3}$$

$$= 1.73 \text{ mm}$$

$$\epsilon_y = -\nu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)$$

$$= \frac{-\frac{1}{3}\times(235)}{70\times10^3}$$

$$= -1.119 \text{ mm}$$

Volume of Plate $= ((1+\epsilon_x)350)((1+\epsilon_z)350)((1+\epsilon_y)18)$

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(5) 12.5 mm thick (4mm horizontal displ. of AB)
P = ?
G = 0.5 GPa

Shear Stress $= \frac{F}{A} = \frac{P}{0.54}$

Shear Strain $=$ ~~0.5×10^{9}~~ $\tan\theta = \frac{4\times10^{-3}}{0.6}$

$G = \frac{\text{Stress}}{\text{Strain}} = 0.5\times10^{9} = \frac{P/0.54}{4\times10^{-3}}\times 0.6$

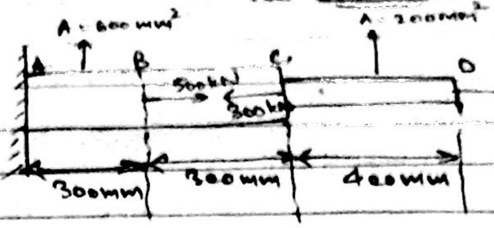
$0.9\times0.5\times10^{9}\times4\times10^{-3} = P$

$1.8\times10^{6}\ N = P$

Scanned by CamScanner

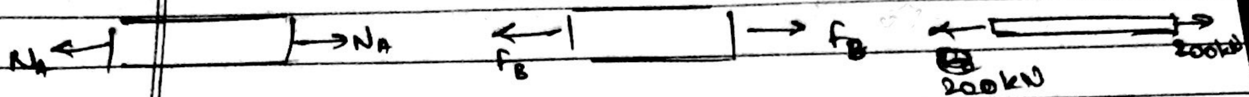
Tut Sheet - 3

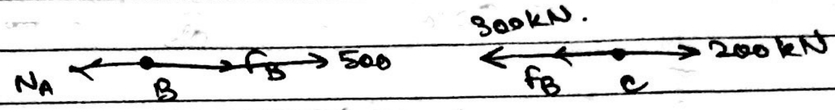
(1)



$E = 200\,GPa$

$E = \frac{Stress}{Strain}$



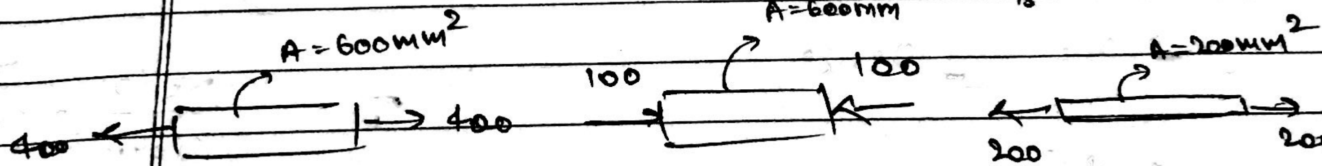


$N_A = 500 + F_B$

$N_A = 400\ kN.$

$300 + F_B = 200$

$F_B = -100\ kN$



$\Delta_1 = \frac{400 \times 10^3 \times 300}{600 \times 10^6 \times 200 \times 10^9}$

~~= 1 mm~~

$\Delta_2 = -\frac{100 \times 10^3 \times 300}{600 \times 10^6 \times 200 \times 10^9}$

$= -0.25\ mm$

$\Delta_3 = \frac{200 \times 400}{200 \times 200}$

$= 2\ mm$

$\Delta_{net} = \Delta_1 + \Delta_2 + \Delta_3 = \underline{2.75\ mm}$

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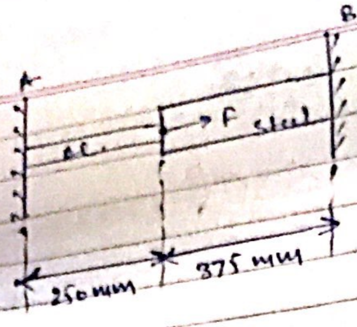
(2)



$A = 950\,mm^2$, $E = 68\,4N/m^2$ } Al

$A = 1300\,mm^2$, $E = 207\,4N/m^2$ } St

$\sigma_{Al} = 70\ MPa$

$\sigma_{steel} = ?$

$\cancel{\delta_{Al} = \delta_{steel}}$

$\cancel{\frac{F}{950 \times 68} \times 250 = \frac{F}{1300 \times 207} \times 375}$

$N_A \leftarrow \square \rightarrow N_A$ $\qquad$ $N_B \leftarrow \square \rightarrow N_B$

$N_A \leftarrow \bullet \rightarrow N_B\ F$

$N_A = N_B + F.$

$\delta_{Al} = \delta_{steel}$

$\frac{N_B + F}{950 \times 68} \times 250 = \frac{N_B}{1300 \times 207} \times 375$

$\sigma_{Al} = \frac{N_B + F}{950 \times 10^{-6}} = 70$

$(N_B + F) = 70 \times 950 \times \cancel{10}$

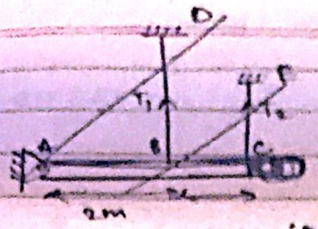
$\frac{70 \times \cancel{950} \times \cancel{10}}{\cancel{950} \times 68} \times 250 \times \frac{1300 \times 207}{375} = N_B$

$\sigma_{steel} = \frac{N_B}{1300 \times 10^{-6}}$

$\sigma_{steel} = \frac{70 \times \cancel{10}}{68} \times 250 \times \frac{\cancel{1300} \times 207}{375} \times \frac{1}{\cancel{1300} \times 10^{-6}}$

$= 142.05 \times 10^{6}$

$= 142.05\ MPa$



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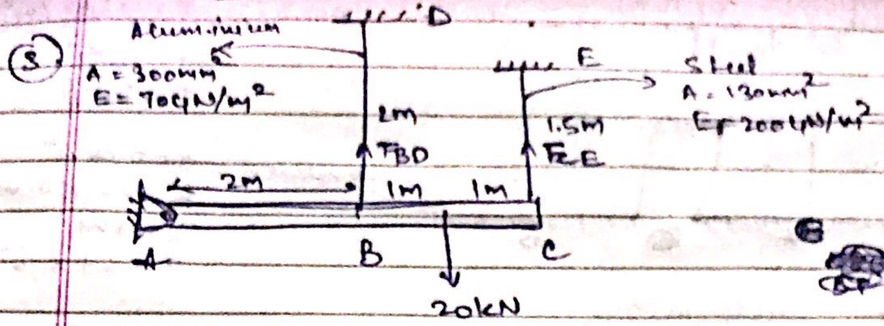


Moment abt. A: $2F_{BD} + 4F_{CE} = 3(20) = 60\,kNm$

$$\frac{\Delta BD}{2} = \frac{\Delta CE}{\cancel{4}\,2}$$

$$\Delta CE = 2\,\Delta BD$$

$$70\times10^9 = \frac{F_{BD}}{300\times10^{-6}} \times \frac{2}{\Delta BD}$$

$$200\times10^9 = \frac{F_{CE}}{130\times10^{-6}} \times \frac{\cancel{1.5}\,3}{2\Delta CE}$$

$\cancel{3F_{CE} = 60 - 2F_{BD}}$

$$3F_{CE} = 60000 - 2F_{BD}$$

$$200\times10^9\times130\times10^{-6}\times(2\Delta CE) = 3F_{CE}$$

$$70\times10^9\times300\times10^{-6}\times\Delta BD = 2F_{BD}$$

$$200\times10^9\times130\times10^{-6}\times4\,\Delta BD = 60000 - 70\times10^9\times300\times10^{-6}\times\Delta BD$$

$$\left((200\times130\times10^3\times4) + (70\times10^3\times300)\right)\Delta BD = 60000$$

$$\Delta BD = 4.8\times10^{-4}\ m$$

$$\Delta BD = 0.48\ mm$$

$$\Delta CE = 0.96\ mm$$

$$3F_{CE} = 200\times\cancel{10^9}\times130\times2\,(0.96)\times\cancel{10^{-3}}$$

$$F_{CE} = 16640\ N$$

$$\text{Stress} = \frac{16640}{130\times10^{-6}} = 128\,MPa$$

$$2F_{BD} = 70\times\cancel{10^9}\times300\times0.48\times\cancel{10^{-6}}$$

$$F_{BD} = 5040\ N$$

$$\text{Stress} = 16.8\ MPa$$

(4) Copper rod
12 mm ϕ
400 mm length

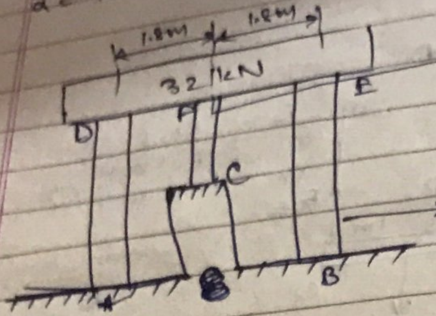
} in Alum. tube ext ϕ 20 mm
thickness 4 mm
same length

Heated from 20°C to 60°C

Cu
$E = 120\,GPa$
$\alpha = 18\times10^{-6}/°C$

Al
$E = 70\,GPa$
$\alpha = 23.7\times10^{-6}/°C$

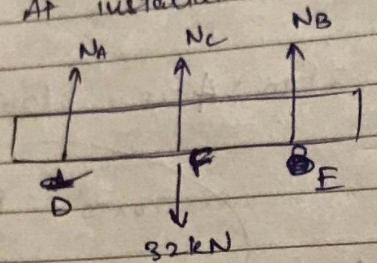
(5.)



Wood
$A = 0.04\,m^2$
$E = 12\,GN/m^2$
$L = 2\,m$
$\alpha = 6\times10^{-6}/°C$

Concrete
$A = 0.08\,m^2$
$E = 20\,GN/m^2$
$L = 4\,m$
$\alpha = 24\times10^{-6}/°C$

a.) At installation.



$N_A = N_B$

$N_A + N_B + N_C = 32\,kN$

$N_C + 2N_A = 32\,kN$

$E = \frac{F/A}{\delta L/L}$

$\delta L = \frac{F\times L}{A\times E}$

Moment abt A

$\cancel{1.8}\, N_C + \cancel{3.6}^{2} N_B = \cancel{1.8}(32)$

$N_C + 2N_B = 32$

$N_C + 2N_A = 32$

$N_C = 32 - 2N_A$

$\Delta_{CF} = \Delta_{AD}$

$\frac{N_C \times \cancel{2}}{\cancel{0.04}\times \cancel{12}_{3}} = \frac{N_A\times \cancel{4}\,\cancel{2}}{\cancel{0.08}\times\cancel{20}_{5}}$

$5N_C = 3N_A$

$N_C = \frac{3}{5}N_A$

$\frac{3}{5}N_A = 32 - 2N_A$

$3N_A = 160 - 10N_A$

$N_A = \frac{160}{13} = 12.30\,kN$

$N_C = 7.38\,kN$

$N_A = 12.30\,kN$

$\sigma_A = \frac{12.30\,kN}{0.08\,m^2}$

$= 153.75\,kPa$

$\sigma_C = \frac{7.38\,kN}{0.04\,m^2}$

$= 184.61\,kPa$

(b.) After a

$N_C + 2N_A =$

Also,

$N_C \times 2 \times$

$0.04 \times 12 \times 10$

$N_C \times 2 \times 10$

12×0.0

$N_C = 3$

σ_C

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(b) After a temp. decrease of 20°C

$N_c + 2N_A = 32\text{ kN} = 32000\text{ N}$

Also, $\delta_{CF} = \delta_{AD}$

$$\frac{N_c \times 2}{0.04 \times 12 \times 10^{9}/10^{3}} + 2 \times 6 \times 10^{-6} \times 20 = \frac{N_A \times 4}{0.08 \times 20 \times 10^{9}/10^{3}} + 4 \times 24 \times 10^{-6} \times 20$$

$$\frac{N_c \times 2 \times 10^{-3}}{12 \times 0.04} + 12 \times 20 = \frac{N_A \times 4}{20 \times 0.08} 10^{-3} + 4 \times 24 \times 20$$

$$\frac{2N_c \times 10^{-3}}{12 \times 0.04} - \frac{\cancel{4}\,2 N_A \times 10^{-3}}{20 \times \cancel{0.08}\,0.04} = 24 \times 4 \times 20 - 12 \times 20$$

$$\frac{2 \times 10^{-3}}{0.04}\left(\frac{N_C}{12} - \frac{N_A}{20}\right) = 84 \times \cancel{20}\ 10$$

$$\frac{N_C}{12} - \frac{N_A}{20} = 84 \times 10 \times 0.04 \times 10^{3}$$

$$\frac{5N_c - 3N_A}{60} = 84 \times 0.04 \times 10^{4}$$

$$5N_c - 3N_A = 20.16 \times 10^{5}$$

$N_c = 32000 - 2N_A$

$5N_c = 20.16 \times 10^{5} + 3N_A = 5(32000 - 2N_A)$

$20.16 \times 10^{5} + 3N_A = 160000 - 10N_A$

$13 N_A = -1856000$

$N_A = -142769.23\text{ N}$

$N_c = 32000 + (142769.23) \times 2$

$$\sigma_c = \frac{N_c}{0.04} = 7938461.5\text{ N/m}^2 = 7.938\text{ MPa}$$

$$\sigma_A = \frac{N_A}{0.08} = -1784615.38\text{ N/m}^2 = -1.784\text{ MPa}$$

(6.) **Bolt**

350 mm long

20 mm

2.4 mm pitch

$E_{steel} = 207\,GPa$

Alum:

int $\phi = 22$ mm

ext $\phi = 44$ mm

$E_{Al} = 67.5\,GPa$

$\Delta l = \frac{1}{3}(2.4)$ mm $= 0.8$ mm

Stress in tube & bolt?

$$\delta_{steel} + \delta_{Al} = 0.8 \times 10^{-3}$$

$$\frac{F \times 0.33}{\pi \times (10 \times 10^{-3})^2 \times 207 \times 10^9} + \frac{F \times 0.33}{\pi((22^2 - 11^2) \times 10^{-6}) \times 67.5} = 0.8 \times 10^{-3}$$

this would give F.

Now $\sigma_{steel} = \frac{F}{\pi \times (10 \times 10^{-3})^2}$

IIly $\sigma_{Alu} = \frac{F}{\pi(22^2 - 11^2) \times 10^{-6}}$

22/10/2017

$\frac{T}{J} = 7$

(?) 5 m long

stressed

Sha

Po

$$\frac{T}{J} = \frac{\tau_{max}}{c} = \frac{G\phi}{L}$$

Tut - 4

① 5 m long $\quad \phi = 4°$
stressed to 80 MPa $\quad G = 83$ GPa
Shaft diameter = ?
Power at f = 20 Hz ?

$$\frac{80\times10^6}{\rho} = \frac{83\times10^9\times4\times\frac{\pi}{180}\times10^3}{5}$$

$$\rho = \frac{80\times5\times180}{83\times4\times\pi\times10^3} = \frac{69.03}{10^3} = 0.069\ m$$

$= 69\ mm.$

$P = 2\pi f T = \omega T$

$$\frac{T}{J} = \frac{\tau}{\rho} = \frac{G\phi}{L}$$

$$T = \frac{83\times10^9\times4\times\frac{\pi}{180}}{5}\times\frac{\pi}{2}\times(0.069)^4$$

$T = 4.1337\times10^{-5+9}$ Nm

$P = 2\times\pi\times20\times4.1337\times10^4$

$= 5194614.79$ W

$= 5194$ kW.

② $\phi_{max} = 3°$

$L = 6m$

$T = 12$ kN-m

$\tau_{max} = ?$

$G = 83$ GPa

$$\frac{T}{J} = \frac{G\phi}{L}$$

$$J = \frac{\pi}{2}c^4 = \frac{TL}{G\phi}$$

$$c^4 = \frac{12\times10^3\times6\times10^{-6}}{83\times10^9\times3\left(\frac{\pi}{180}\right)}\times\frac{2}{\pi}$$

Solving

$d = 113.98$ mm

41.2 MPa

$\Rightarrow c = 0.075m$

$$\tau_{max} = \frac{83\times10^9\times3\frac{\pi}{180}\times0.075}{6} = 54.9\ MPa$$

$\frac{T}{J} = \frac{\tau_{max}}{c}$

(3) $P_{avg} = 100\,kW$

$\omega = 250\,rpm = \frac{250 \times 2\pi}{60}$ rad/sec

$d_o = ?$

$\tau_{max} = 75\,MPa$

$P_{max} = P_{avg}(1.3) = 1.3 \times 100\,kW$
$= 130\,kW$

$P = T\omega$

$T = P/\omega = \frac{130 \times 10^3}{250 \times 2\pi} \times 60 = 4965.63\,N$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$\frac{T}{J} = \frac{\tau_{max}}{c}$

$\frac{4965.63}{\frac{\pi}{2} c^3} = \frac{75 \times 10^6}{c}$

$c^3 = \frac{4965.63}{\frac{\pi}{2} \times 75} \times 10^{-6}$

$c = 0.0348\,m$

a) $d = 0.0696\,m = 69.6\,mm$

b) hollow shaft

$d_i = 0.8 d_o$ $\quad c_i = 0.8 c_o$

$\frac{T}{J} = \frac{\tau_{max}}{c_o}$

$\frac{4965.63}{\frac{\pi}{2}(c_o^4 - c_i^4)} = \frac{75 \times 10^6}{c_o} = \frac{4965.63}{\frac{\pi}{2}(c_o)^4 (1 - 0.8^4)}$

$\frac{75 \times 10^6}{c_o} = \frac{4965.63}{\frac{\pi}{2} c_o^3 \times 0.5904}$

$c_o^3 = \frac{4965.63}{\frac{\pi}{2} \times 75 \times 10^6 \times 0.5904}$ $\quad 71.89 \times 10^{-6}$

$d_o = 0.0702\,m$
$= 70.20\,mm$

$d_i = 56.16\,mm$

$\frac{d_o^2 - d_i^2}{d_{prev}^2} \times 1$

$(70.20)^2$

$(6$

% 8

$d_o =$

d_i

$\Rightarrow c$

$$\frac{d_o^2 - d_i^2}{d_{o\,solid}^2} \times 100$$

$$\frac{(70.20)^2 - (56.16)^2}{(69.6)^2} = 36.62\%$$

% Saving = 100 − 36.62 = 63.37%

$$\frac{\text{Area More} - \text{Area Less}}{\cancel{\text{Area More}}} \times 100$$

$$\frac{(82.96)^2 - (66.368)^2}{(69.6)^2} \times 100$$

$$\frac{(\text{Solid})\text{Area} - \text{Holl. Area}}{\text{solid Area}}$$

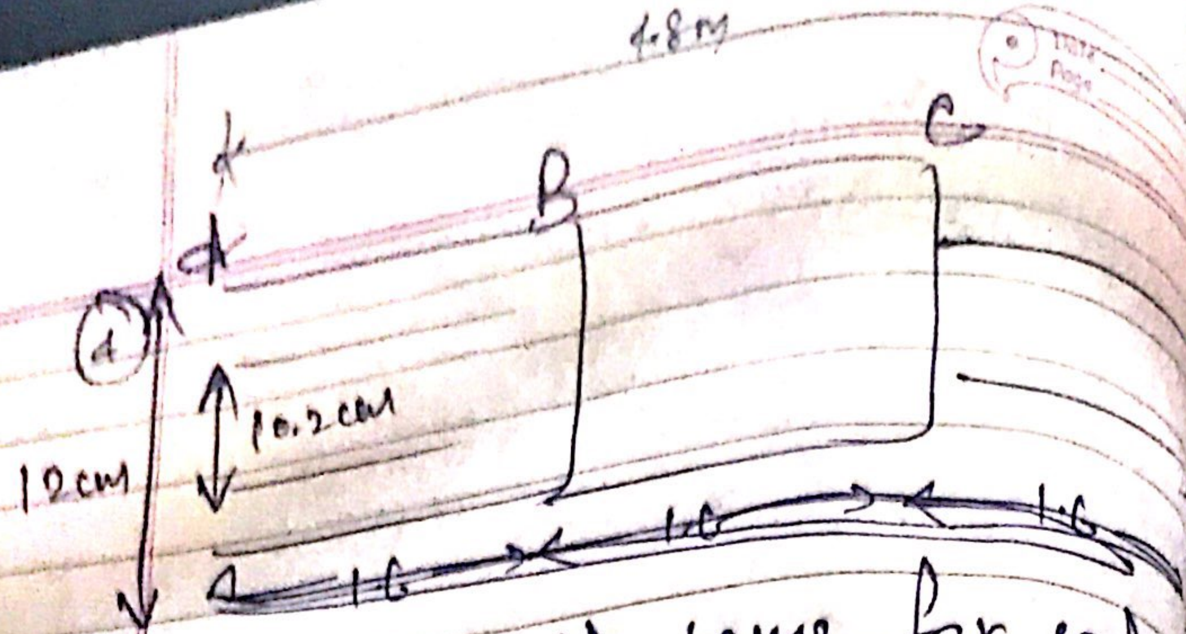
$$\%\ \text{Saved} = \frac{(69.6)^2 - \left[(82.96)^2 - (66.368)^2\right]}{(66.368)^2} \times 100$$

$$= 48.85\%$$

$d_o = 82.96$ mm

$d_i = 0.8 \times d_o = 66.368.$

$\Rightarrow c = 0.04148$ m



Angle of twist same for each

Length of each ?

Applied torque : ?

Total angle of twist

$$\frac{\tau_{max}}{c} = \frac{T}{I_p} = \frac{G\theta}{L}$$

$\tau_{max} = 50\ MPa$

$G = 84\ GPa$

$$\frac{\tau_{max}}{c} = \frac{T}{I_p}$$

$$\frac{50 \times 10^6}{0.12} = \frac{T_2}{\frac{\pi}{2}(0.12)^4}$$

$T_2 = 0.1357 \times 10^6\ Nm$

$$\frac{50 \times 10^6}{0.12} = \frac{T_1}{\frac{\pi}{2}\left((0.12)^4 - (0.102)^4\right)}$$

$T_1 = 0.06 \times 10^6\ Nm$

$$\frac{50 \times 10^6}{0.096} = \frac{T_3}{\frac{\pi}{2}(0.096)^4}$$

$T_3 = 7.539 \times 10^6\ Nm$

$$\tau_{max} = \frac{C\theta c}{L}$$

con

Since same material τ_{max} would be same for all 3.

Also given angle of twist same

$$\frac{c_1}{L_1} = \frac{c_2}{L_2} = \frac{c_3}{L_3}$$

$$\frac{12}{L_1} = \frac{12}{L_2} = \frac{9.6}{L_3} = k$$

$$L_1 = \frac{12}{k} \qquad L_2 = \frac{12}{k} \qquad L_3 = \frac{9.6}{k}$$

$$L_1 + L_2 + L_3 = 480 \text{ cm}$$

$$\frac{1}{k}(12+12+9.6) = 480$$

$$k = \frac{33.6}{480} = 0.07$$

$$L_1 = \frac{12}{0.07} = 171.42 \text{ cm}$$

$$= 1.71 \text{ m}$$

$$L_2 = 1.71 \text{ m}$$

$$L_3 = 1.37 \text{ m}$$

$$\frac{\tau_{max}}{c} = \frac{T}{I_p}$$

$$\frac{50\times10^6}{0.12} = \frac{T}{\frac{\pi}{2}\left((0.12)^4 - (0.102)^4\right)}$$

$$\Rightarrow T = 0.06\times10^6 \text{ Nm}$$

$$\frac{\tau_{max}}{c} = \frac{C\theta}{L}$$

$$\frac{50\times\cancel{10^6}}{0.12} = \frac{84\times\cancel{10^{9}}\,10^3\times\theta}{1.71} \Rightarrow \theta = 8.482\times10^{-3} \text{ rad}$$

$$= 0.48^\circ$$

$$3\theta = 1.45^\circ$$

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(5) $P = 100\ kW$

$40\ rpm$ $= \frac{4\pi}{3}\ rad/sec = \omega$

$= \frac{40 \times 2\pi}{3\ 60}\ rad/sec$

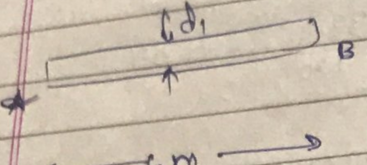
$P = T\omega$

$100 \times 10^3 = T\,\frac{4\pi}{3}$

$T = \frac{100 \times 10^3}{4\pi} \times 3$

$T_B = 2.387 \times 10^4\ Nm$

$\frac{32 \times 10^3 \times 3}{4\pi} = T_{AB}$



$T_{AB} = 7639.43\ Nm$

$T_{BC} = 16233.80\ Nm$

$\frac{\tau_{max}}{c} = \frac{T}{J} = \frac{G\theta}{L}$

$\tau_{max} = 50\ MPa$

$G = 80\ GPa$

$$\frac{50 \times 10^6}{d_1/2} = \frac{7639.43}{\frac{\pi}{2}\left(\frac{d_1}{2}\right)^4} = \frac{80 \times 10^9 \times \theta_1}{6}$$

$$\left(\frac{d_1}{2}\right)^3 = \frac{7639.43}{\pi/2 \times 50 \times 10^6}$$

$$\left(\frac{d_1}{2}\right)^3 = (9.726) \times 10^{-5}$$

$$\Rightarrow d_1 = 0.09198\ m$$

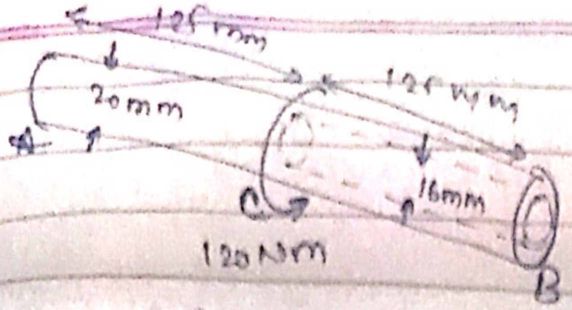
$$\frac{7639.43}{\frac{\pi}{2}\left(\frac{d_1}{2}\right)^4} = \frac{80 \times 10^9}{6} \times \theta_1$$

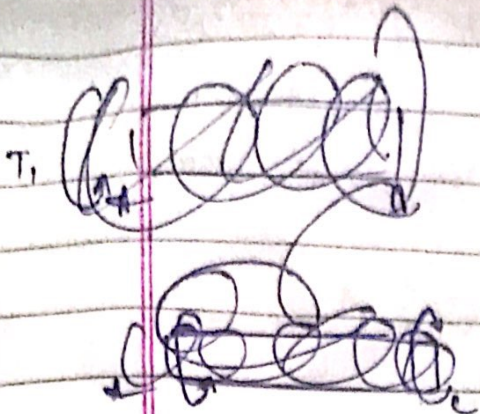
$$\theta_1 = 0.0815\ rad$$

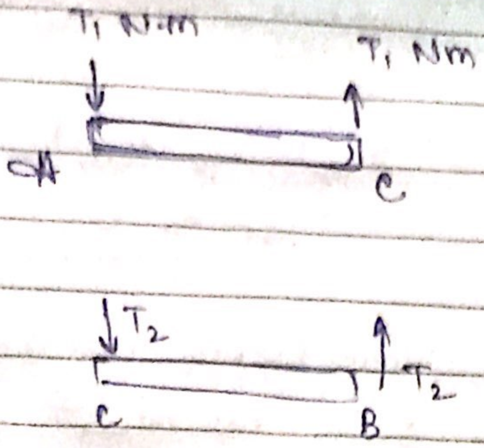
$$= 4.67^\circ$$

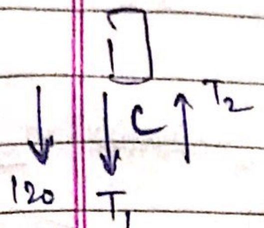
Similarly get d_2 & θ_2.

(6)









Since both sides attached to fixed support

$\theta_1 = \theta_2$.

$120 + T_1 = T_2$

$T_2 - T_1 = 120 \text{ Nm}$

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{TL}{J} = G\theta = k$$

$$\frac{T_1 L_1}{J_1} = \frac{T_2 L_2}{J_2} = C$$

T_1 & T_2 ?

$$T_1 = \frac{C J_1}{L_1} \qquad T_2 = \frac{C J_2}{L_2}$$

$$T_1 = C \frac{\frac{\pi}{2}(10 \times 10^{-3})^4}{125 \times 10^{-3}}$$

$$T_2 = C \frac{\frac{\pi}{2}\left((10 \times 10^{-3})^4 - (8 \times 10^{-3}\right.}{125 \times 10}$$

$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau_{max}}{c}$

(7) Steel Shaft & Aluminium Tube.

$T_{max} = ?$ (Max. Torque)

$\tau_{max}(\text{steel}) = 120\,MPa$; $G = 80\,GPa$

$\tau_{max}(Al) = 70\,MPa$; $G = 27\,GPa$

Angle of twist (θ) would be same for both.

Q How would the net torque be divided onto the shaft & alum. tube.

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{TL}{GJ} = \theta$$

$$\frac{T_1 \cancel{L_1}}{G_1 J_1} = \frac{T_2 \cancel{L_2}}{G_2 J_2}$$

$$\frac{T_1}{G_1 J_1} = \frac{T_2}{G_2 J_2}$$

$$\frac{T_1}{T_2} = \frac{G_1 J_1}{G_2 J_2}$$

$$\frac{T_1}{\frac{\pi}{2} \times \left(\frac{50\times10^{-3}}{2}\right)^{3}} = \frac{120}{\frac{50}{2}}$$

$$T_1 = \frac{\pi}{2} \times \left(\frac{50\times10^{-3}}{2}\right)^3 \times 120 \times$$

$$= \frac{\pi}{2} \times \frac{50^3}{8} \times 120 \times 10^{-3}$$

$$2945.24 = \frac{23561.94}{8}\ Nm$$

Corresponding T_2 :

$$T_2 = \frac{T_1 G_2 J_2}{G_1 J_1}$$

$$= \frac{T_1 \times 27}{80} \times \frac{\cancel{\frac{\pi}{2}}\left(\left(\frac{76\times10^{-3}}{2}\right)^4 - \left(\frac{60\times}{2}\right)\right)}{\cancel{\frac{\pi}{2}}\left(\frac{50\times10^{-3}}{2}\right)^4}$$

$(38)^4 - 30^4$

25

38

$= \frac{T}{38\times10^{-3}}$

$\frac{T}{\frac{\pi}{2}(38^4 - 30^4)\times10^{-12}}$

$\times 10^{-9}$

$= 61559983.1$

$\tau_2 = 61.55\ GPa$

$= 994.019 \times \left(\quad\right)$

Possible

$$T_2 = 3244.825\ Nm$$

$(38^4 - 30^4) \times 10^{-12}$ $\quad \dfrac{\ldots}{38 \times 10^{-3}}$

$$T_2 = 3689.69 \text{ Nm}$$

Corresponding T_1 :

$$T_1 = \frac{C_1 J_1}{C_2 J_2} \times T_2$$

$$= \frac{80}{27} \times \frac{(25)^4}{38^4 - 30^4} \times 3689.69$$

$$= 3349.04 \text{ Nm}$$

↳ Since greater than the limit got from τ_{max} of steel

∴ Not possible.

$$\Rightarrow T_1 = 2945.24 \text{ Nm}$$

$$T_2 = 3244.825 \text{ Nm}$$

$$T_{net} = T_1 + T_2$$

$$= 6190.065 \text{ Nm}$$

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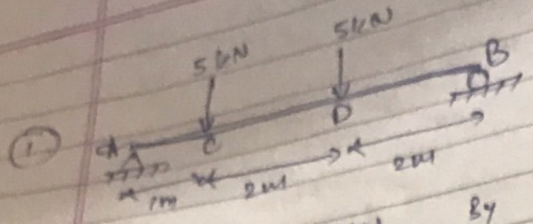
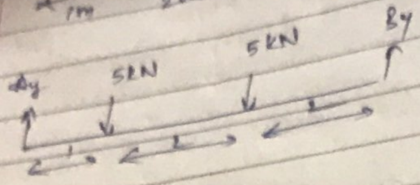
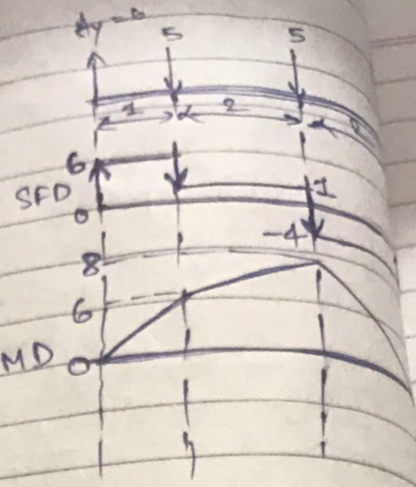
Tut-5

①





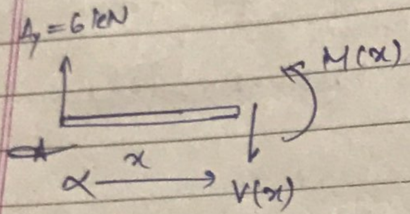


$A_y + B_y = 10$

$5(1) + 5(3) = 5(B_y)$

$B_y = 4$ kN

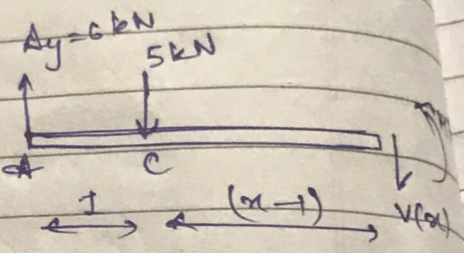
$\Rightarrow A_y = 6$ kN



$0 < x < 1$

$V(x) = 6$ kN

$6x = M(x)$



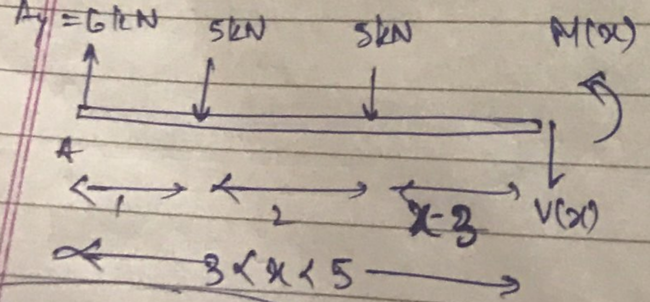
$1 < x < 3$

$6 = 5 + V(x)$

$V(x) = 1$ kN

$6(x) - 5(x-1) - M(x) = 0$

$M(x) = x + 5$



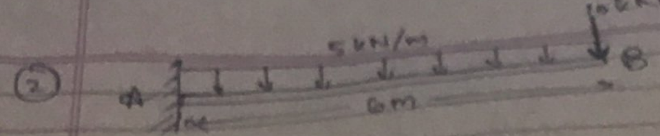
$3 < x < 5$

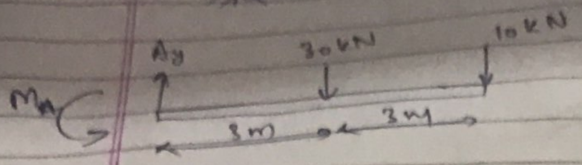
$V(x) = -4$ kN

$6(x) - 5(x-1) - 5(x-3) - M(x) = 0$

$M(x) = -4x + 20$

②

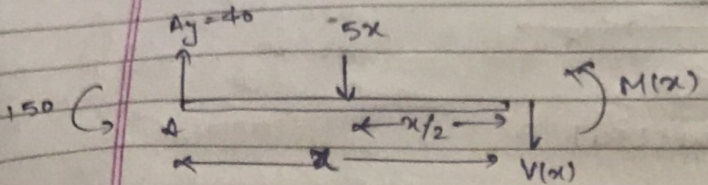




$A_y = 40\ kN$

$30(3) + 10(6) = M_A$

$M_A = 150\ kNm$

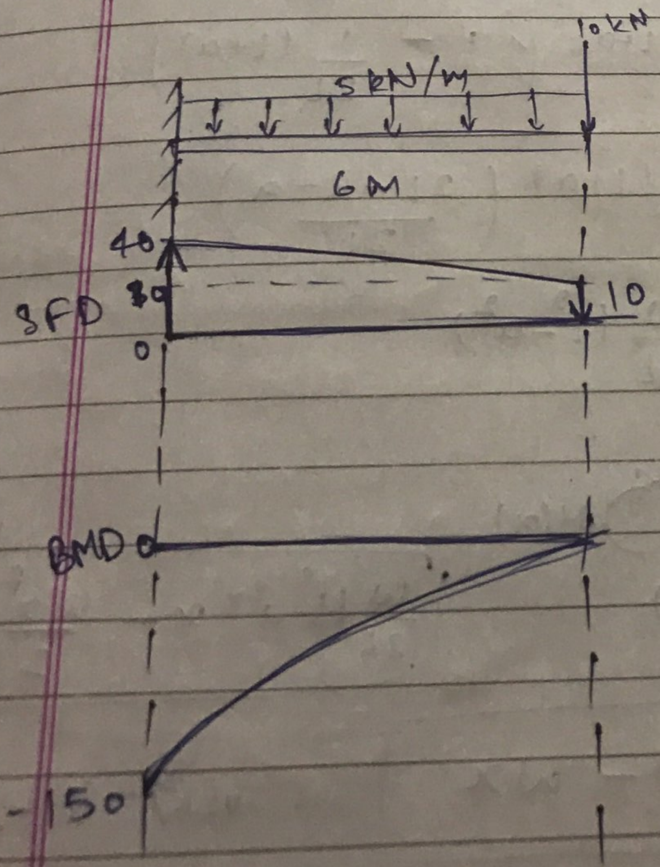


$0 < x < 6$

$40 - 5(x) = V(x)$

$-150 + 40(x) - 5(x)\left(\frac{x}{2}\right) - M(x) = 0$

$M(x) = -\frac{5x^2}{2} + 40x - 150$



$M(0) = -150$

$M(3) = -22.5 + 120 - 150$

$= -52.5$

$M(6) = -90 + 240 - 150$

$= 0$

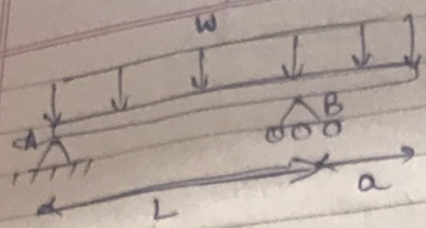
$M'(x) = -5x + 40$

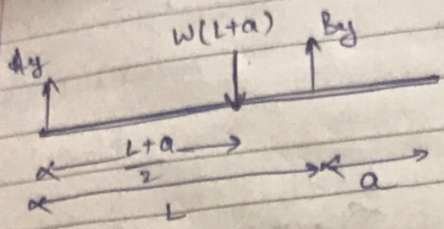
Max/Min at

$x = 8$

Date
Page

(3)





$A_y + B_y = w(L+a)$

$\frac{w(L+a)^2}{2} = B_y(L)$

$B_y = \frac{w}{2L}(L+a)^2$

$A_y = w(L+a) - \frac{w}{2L}(L+a)^2$

$= w(L+a)\left(1 - \frac{1}{2L}(L+a)\right)$

$= w(L+a)\left(\frac{2L-L-a}{2L}\right)$

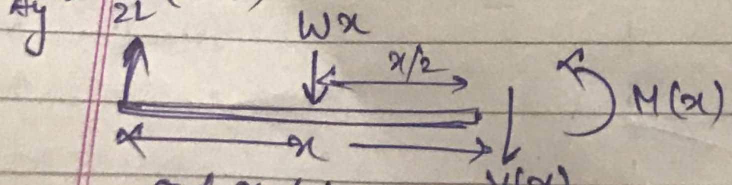
$A_y = \frac{w}{2L}(L^2-a^2)$

$\frac{w}{2}(L^2-a^2) - \frac{wL^2}{2}$

$-\frac{wa^2}{2}$

$x = \frac{(L^2-a^2)}{2L}$

$= L$



$0 < x < L$

$$\boxed{\frac{w}{2L}(L^2-a^2)x - \frac{wx^2}{2} = M(x)}$$

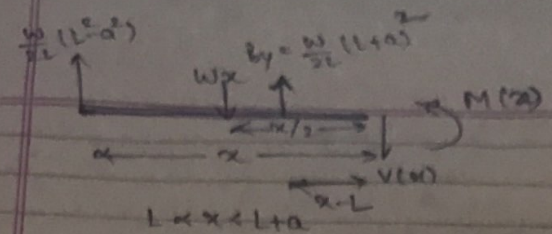
$$\boxed{V(x) = \frac{w}{2L}(L^2-a^2) - wx}$$

$V(L) = \frac{wL}{2} - \frac{wa^2}{2L} - w$

$= -\frac{wL}{} - \frac{wa^2}{}$

$\frac{w}{2L}(L^2-a^2)$

$\frac{w}{2L}$

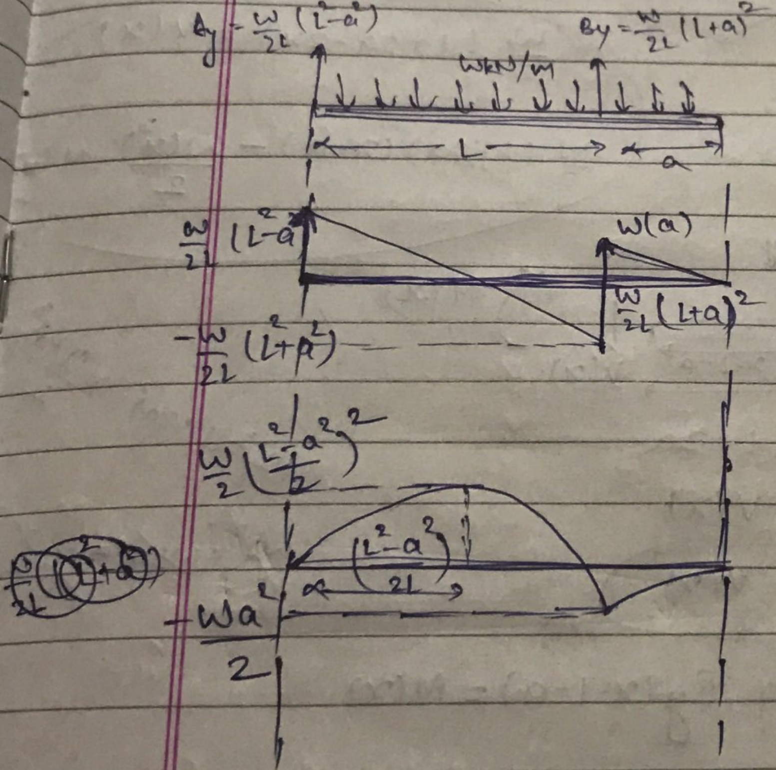


$$\frac{w}{2L}(L^2 - \cancel{a^2} + L^2 + \cancel{a^2} + 2La) - w(x) = V(x)$$

$$\frac{w}{\cancel{2L}}\cancel{2L}(L+a) - w(x) = V(x)$$

$$w(L+a-x) = V(x) \Rightarrow w(L+a) - w(x)$$

$$\frac{w}{2L}(L^2-a^2)(x) - wx\left(\frac{x}{2}\right) + \frac{w}{2L}(L+a)^2(x-L) = M(x)$$

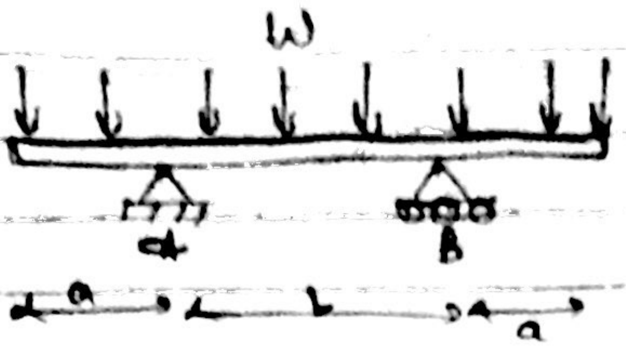


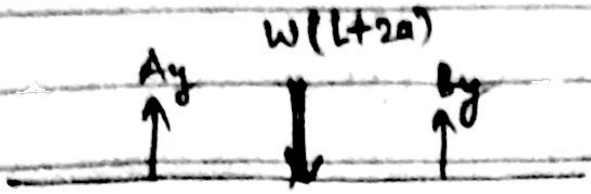
$$\frac{w}{2L}\frac{(L^2-a^2)^2}{(2L)} - \frac{w}{2}\left(\frac{L^2-a^2}{2L}\right)^2$$

$$\frac{w}{2}\left(\frac{L^2-a^2}{2L}\right)^2$$

$$\frac{w}{2L}(L-a)(L+a)^2 - \frac{w}{2}(L+a)^2 + \frac{w}{2L}(L+a)^2(a)$$

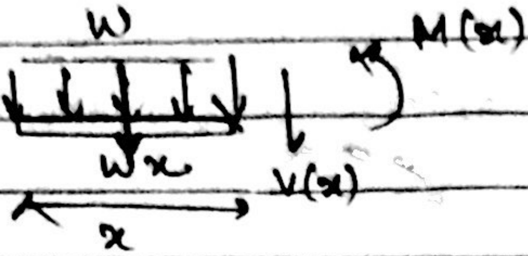
$$\frac{w(L+a)^2}{2}\left(\frac{L-a}{L} - 1 + \frac{a}{L}\right) = 0$$

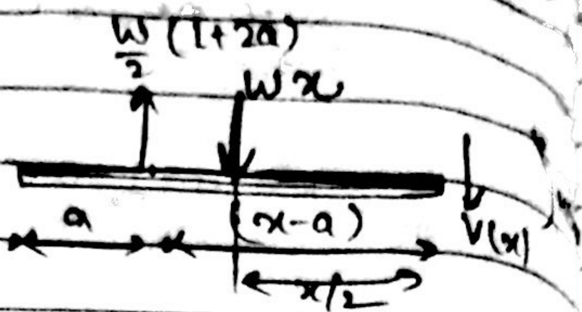




$A_y = B_y = \frac{W}{2}(L+2a)$

SFD





$0 < x < a$

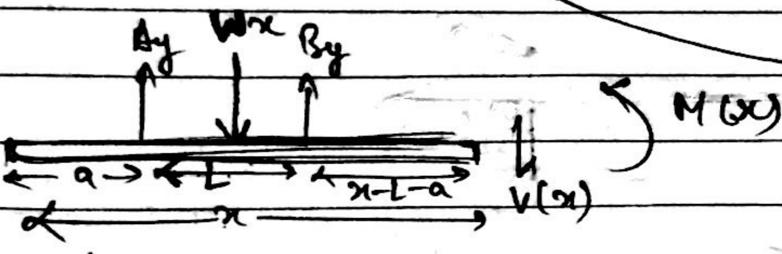
$V(x) = -W(x)$

$M(x) = -\frac{Wx^2}{2}$

$a < x < (L+a)$

$\frac{W}{2}(L+2a-2x) = V(x)$

$\frac{W}{2}(L+2a)(x-a) - \frac{W(x^2)}{2} = M$



$L+a < x < L+2a$

$A_y + B_y - Wx = V(x)$

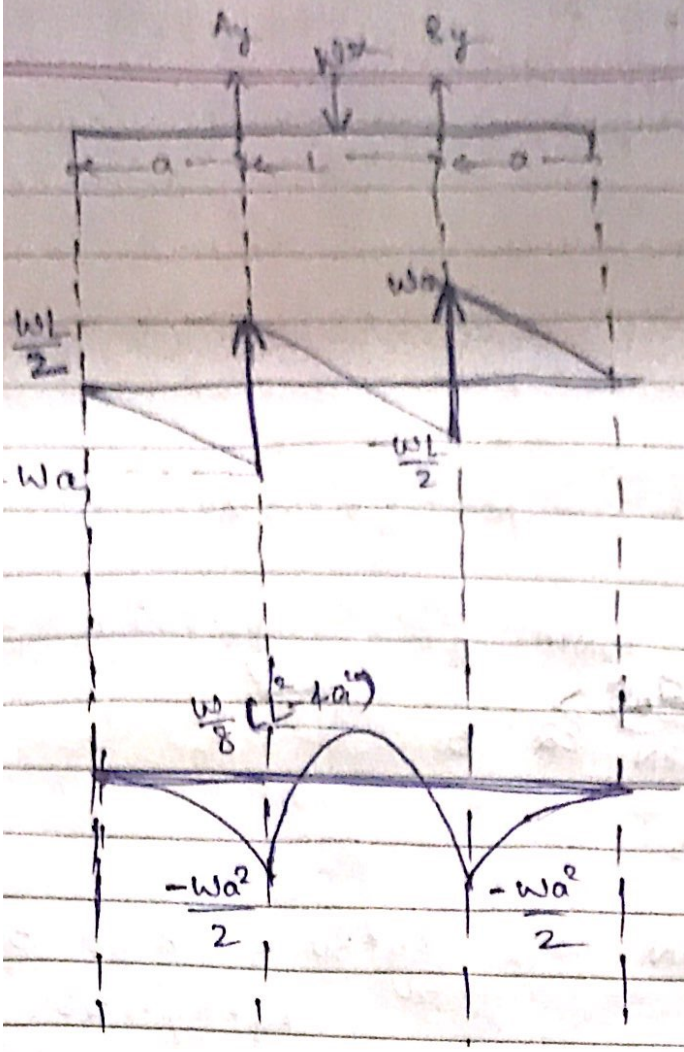
$W(L+2a-x) = V(x)$

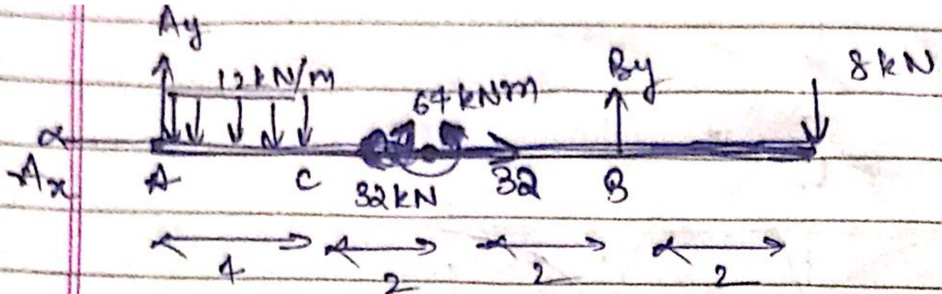
$A_y(x-a) - W\frac{x^2}{2} + B_y(x-L-a) = M(x)$

$\frac{WL}{2}(L+2a) - \frac{W(L+a)^2}{2}$

$\cancel{\frac{WL^2}{2}} + \cancel{WLa} - \cancel{\frac{WL^2}{2}} - \frac{Wa^2}{2} - \cancel{WLa}$

$\frac{w}{2}$ (1.2a - 2L - 2a)





$A_x = 32$ kN

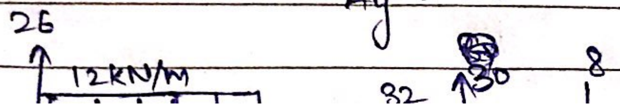
$A_y + B_y = 48 + 8 = 56$

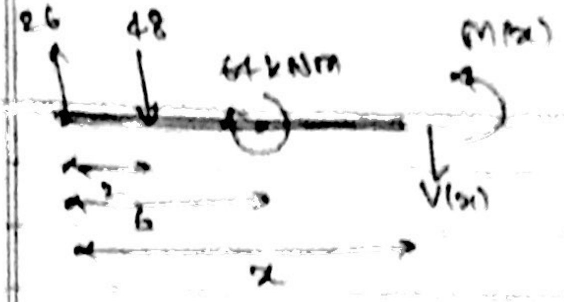
$A_y + B_y = 56$

$\overset{6}{\cancel{48}}(2) + \overset{8}{\cancel{64}} + \cancel{8}(10) = B_y \cancel{(8)}$

$\Rightarrow B_y = 30$

$A_y = 26$



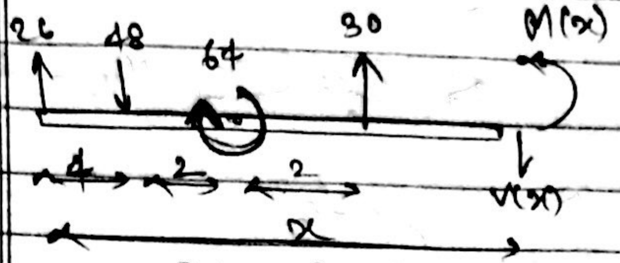


$6 < x < 8$

$V(x) = -22$

$26(x) - 48(x-2) + 64 = M(x)$

$M(x) = -22x + 160$



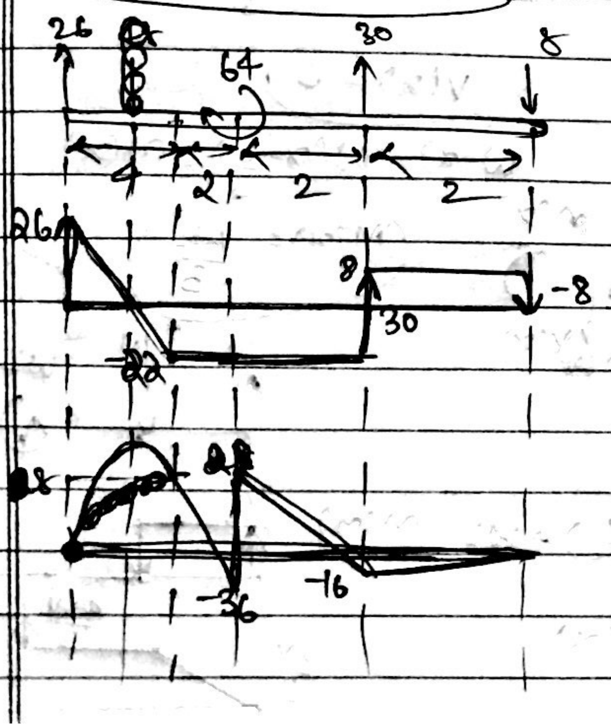
$8 < x < 10$

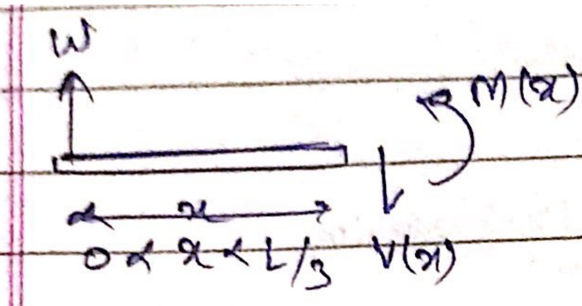
$V(x) = 26 + 30 - 48 = 8$

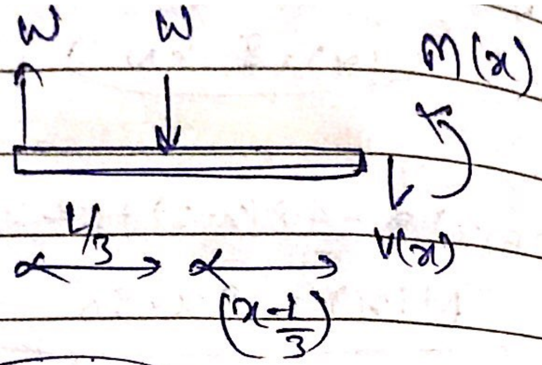
$V(x) = 8$ kN

$26(x) - 48(x-2) + 64 + 30(x-8) = M(x)$

$M(x) = 8x - 80$





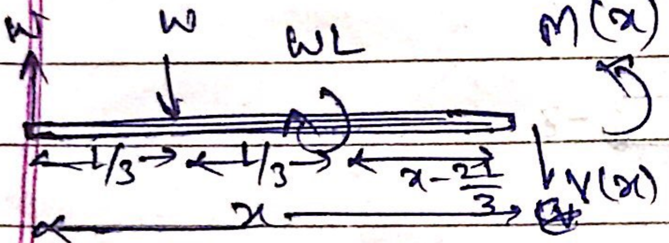


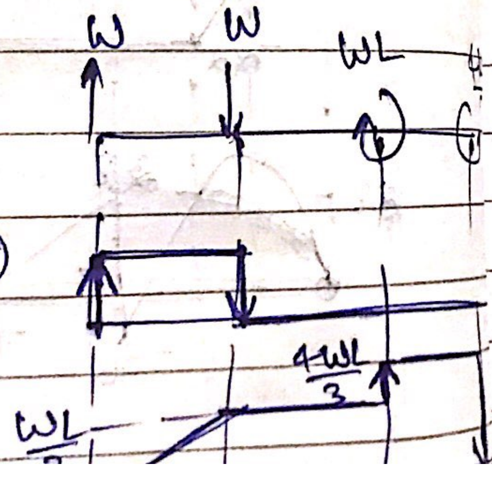
$$V(x) = W$$
$$M(x) = Wx$$

$$V(x) = 0$$

$$W(x) - W\left(x - \frac{L}{3}\right) = M(x)$$

$$M(x) = \frac{WL}{3}$$



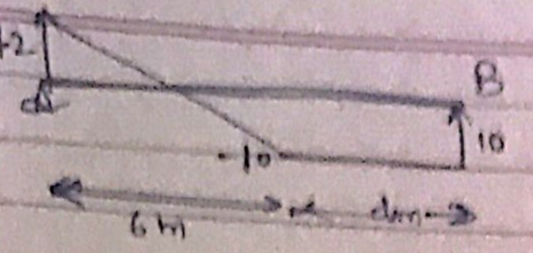


$$V(x) = 0$$

$$W(x) - W\left(x - \frac{L}{3}\right) + WL = M(x)$$

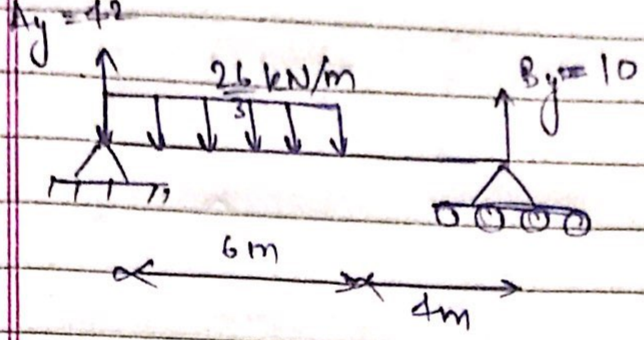
$$\frac{4WL}{3} = M(x)$$

7.

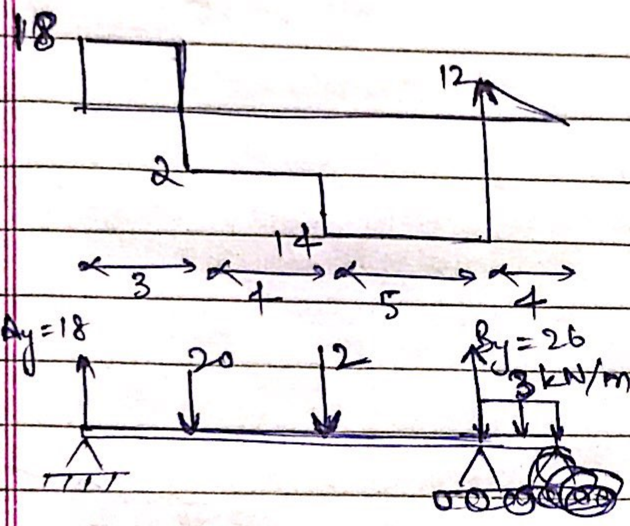


$-10 = m(6) + 42$

$\frac{-52}{6} = m = -\frac{26}{3}$



8.



$0 = m(4) + 12$

$m = -3$

$y = -3x + 12$