LMAPR2020: Materials Selection

Second homework on influence of shape in material selection

Work performed by Group L:

Gad Omar (15712400)

Vansnick François (74871900)

April 19, 2025

Question 1:

a.

For the hollow rectangle we get:

$$I_{min} = \int_{-4t}^{4t} y^2 b(y) dy = \left[\frac{y^3}{3}\right]_{-4t}^{4t} 8t = (\frac{64t^3}{3} + \frac{64t^3}{3})8t = \frac{1024t^4}{3}$$

$$I_{max} = \int_{-5t}^{5t} y^2 b(y) dy = \left[\frac{y^3}{3}\right]_{-5t}^{5t} 10t = (\frac{125t^3}{3} + \frac{125t^3}{3})10t = \frac{2500t^4}{3}$$

$$I_{tot} = \frac{2500t^4}{3} - \frac{1024t^4}{3} = 492t^4$$

$$and$$

$$I_{top} = I_{bottom} = \frac{10t^4}{12} + 10t^2\left(\frac{9t}{2}\right)^2 = \frac{2440t^4}{12}$$

$$I_{left} = I_{right} = \frac{512t^4}{12}$$

$$I_{total} = I_{top} + I_{bottom} + I_{left} + I_{right} = \frac{4880t^4 + 1024t^4}{12} = 492t^4$$

For the I shape we get:

$$I_{top} = I' + Ad^2 = \frac{5t^4}{12} + 5t^2\left(\frac{9t}{2}\right)^2 = \frac{1220t^4}{12}$$

$$I_{center} = I' + Ad^2 = \frac{(8t)^3 t}{12} = \frac{524t^4}{12}$$

$$I_{bottom} = I' + Ad^2 = \frac{7t^4}{12} + 7t^2\left(\frac{9t}{2}\right)^2 = \frac{1708t^4}{12}$$

$$I_{total} = \frac{1220t^4}{12} + \frac{524t^4}{12} + \frac{1708t^4}{12} = \frac{863t^4}{3}$$

For the hollow circle we get:

$$I_{min} = \int_0^{2\pi}\int_0^r (rsin\theta)^2 rdrd\theta = \int_0^{2\pi} \frac{r^4}{4} sin^2\theta d\theta = \frac{r^4}{4}\left[\frac{1}{2}(\theta - sin\theta cos\theta)\right]_0^{2\pi} = \frac{\pi}{4}r^4$$

$$I_{max} = \int_0^{2\pi}\int_0^R (rsin\theta)^2 rdrd\theta = \int_0^{2\pi} \frac{r^4}{4} sin^2\theta d\theta = \frac{R^4}{4}\left[\frac{1}{2}(\theta - sin\theta cos\theta)\right]_0^{2\pi} = \frac{\pi}{4}R^4$$

$$I_{tot} = \frac{\pi}{4}R^4 - \frac{\pi}{4}r^4 = \frac{\pi}{4}((7t)^4 - (6t)^4) = \frac{\pi}{4}(2401t^4 - 1296t^4) = \frac{\pi}{4}1105t^4$$

b.

In order to compute the shape factor, we compute the second moment of area for a square of the same cross section (same mass per unit of length) as the modified shape. (Note that we take the same material each time so the shape factor depends only on I and not E)

$$A_{hollowsquare} = 2(10t * t) + 2(8t * t) = 36t^2 \text{ and } I_{ref} = \frac{A^2}{12} = 108t^4$$

$$\text{For a hollow square shape : } \phi_B^e = \frac{I}{I_{ref}} = \frac{492}{108} = 4.56$$

$$A_{Ishape} = (5t * t) + (8t * t) + (7t * t) = 20t^2 \text{ and } I_{ref} = \frac{A^2}{12} = \frac{400t^4}{12}$$

$$\text{For a I shape : } \phi_B^e = \frac{I}{I_{ref}} = \frac{863/3}{400/12} = 8.63$$

$$A_{hollowcircle} = \pi(7^2 - 6^2)t^2 = 13\pi t^2 \text{ and } I_{ref} = \frac{A^2}{12} = \frac{169\pi^2 t^4}{12}$$

$$\text{For a hollow circle shape : } \phi_B^e = \frac{I}{I_{ref}} = \frac{1105\pi/4}{169\pi^2/12} = 6.24$$

c.

As explained in Ashby's book, the shape factor is dimensionless. This means that it depends only on the shape and not on the scale. So even if the cross-section area is different for the three models, it does not change the result. Therefore, we can confirm that the I shape will always be more efficient in stiffness than a hollow circle and even more than a hollow square.

Question 2:

a.

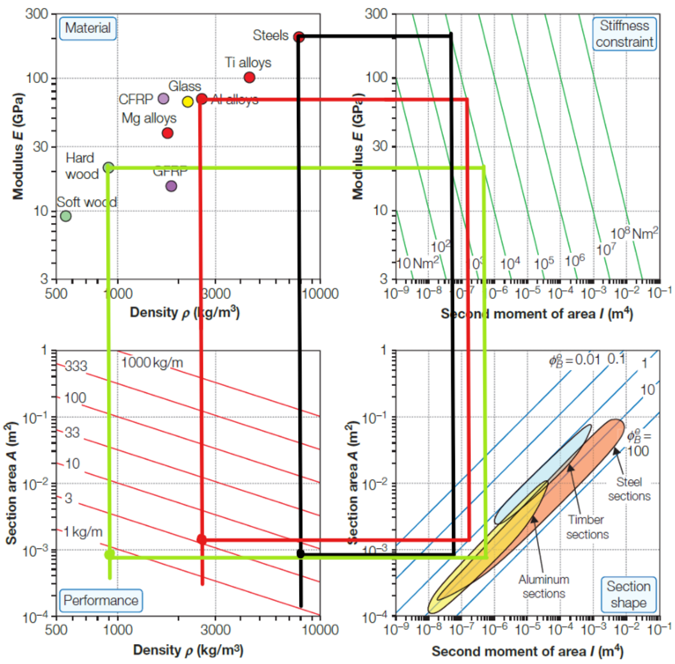

Figure 1: A comparison of steel, aluminum, and hard wood for a stiffness-limited design with $EI = 10^4 Nm2$.

We plot the various lines based on the data provided and the method used in Ashby's book. As we can see, steel weighs approximately 7 kg/m, aluminum 3.5 kg/m, and hardwood 0.8 kg/m.

b.

To compute the linear density $\frac{m}{L}$, we take the relation for stiffness-limited beam design:

$$EI = 10^4\,\mathrm{Nm}^2$$

$$I = \frac{EI}{E} \qquad A^2 = \frac{12I}{\phi_B^e} = \frac{12EI}{E \cdot \phi_B^e} \qquad \Rightarrow \qquad \frac{m}{L} = \rho A = \rho\sqrt{\frac{EI}{E \cdot \phi_B^e}}$$

Finally we get:

$$\frac{m}{L} = \rho\sqrt{\frac{12 \cdot 10^4}{E \cdot \phi_B^e}}$$

For the steel:

$$\frac{m}{L} = \rho\sqrt{\frac{12 \cdot 10^4}{E \cdot \phi_B^e}} = 7900\sqrt{\frac{12 \cdot 10^4}{200 \times 10^9 \cdot 1}} = 6.12 \quad [kg/m]$$

For the aluminum:

$$\frac{m}{L} = \rho\sqrt{\frac{12 \cdot 10^4}{E \cdot \phi_B^e}} = 2700\sqrt{\frac{12 \cdot 10^4}{70 \times 10^9 \cdot 1}} = 3.53 \quad [kg/m]$$

For the hard wood:

$$\frac{m}{L} = \rho\sqrt{\frac{12 \cdot 10^4}{E \cdot \phi_B^e}} = 900\sqrt{\frac{12 \cdot 10^4}{23 \times 10^9 \cdot 10}} = 0.65 \quad [kg/m]$$

We note that we mathematically obtain values approximately similar to those obtained via the 4-quadrant chart.

Question 3:

a.

Specifications:

- Function: Designing a stiff and light beam
- Constraint: Stiff beam that meets the required bending stiffness S^*
- Objective: minimize weight
- Free Variable: shape of the beam and material choice

b.

$$m = \rho.A.L \quad and \quad S = \frac{C_1 EI}{L^3}$$

$$\phi_B^e = \frac{12I}{A^2} \quad => \quad S = \frac{C_1}{12}\frac{E}{L^3}\phi_B^e A^2$$

$$m = \left(\frac{12S^*}{C_1}\right)^{1/2} L^{5/2}\left[\frac{\rho}{(\phi_B^e E)^{1/2}}\right]$$

So the material index is : $M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho}$ and if the shape is the same $M_1 = \frac{E^{1/2}}{\rho}$

c. Regardless of shape, we have three different materials that maximize our material index: balsa wood, femur trabecular bone, and radius trabecular bone. Since two of these are bones, they are not relevant for our application. Therefore, the best choice seems to be balsa wood (ochroma spp.)(0.09-0.11)(L).

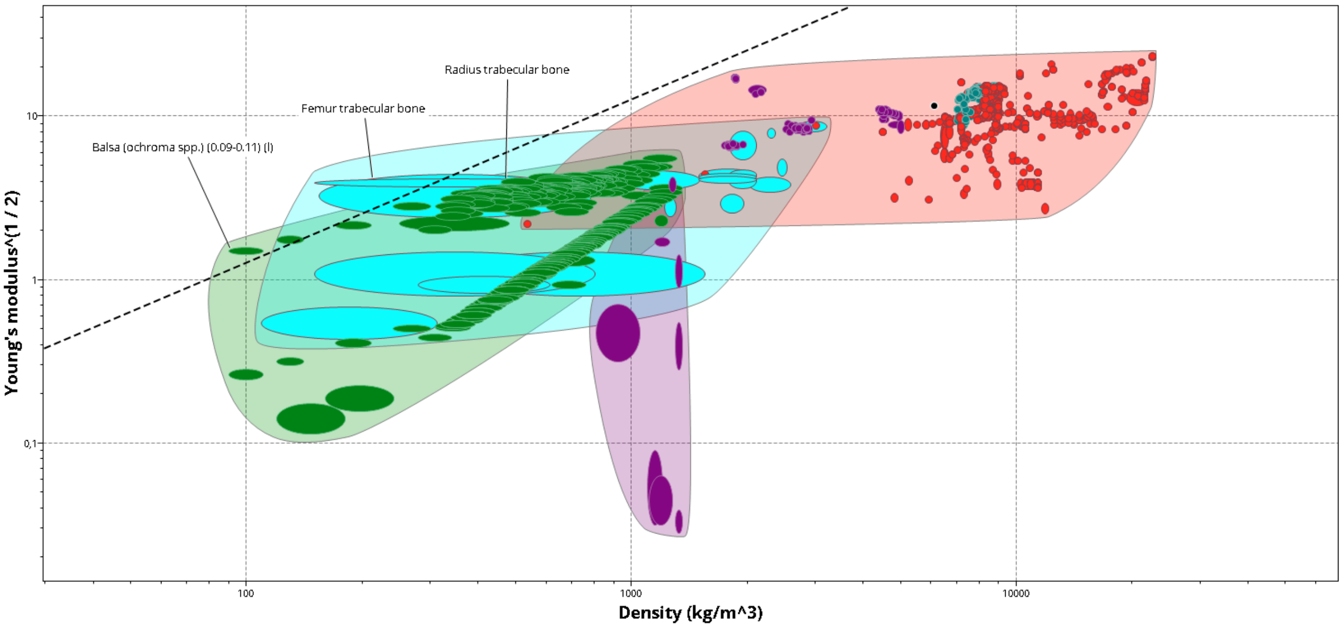

Figure 2: Graph of the Young's Modulus as a function of the density

d. In order to create the pseudo materials, we need to redefine the material index to be able to display them on Granta. We therefore have:

$$M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho} = \frac{(\phi_B^e E)^{1/2}/\phi_B^e}{\rho/\phi_B^e} = \frac{(E/\phi_B^e)^{1/2}}{\rho/\phi_B^e} = \frac{(E^*)^{1/2}}{\rho^*}$$

	Density (kg/m^3)	Young's Modulus (GPa)	$\phi_B^e, 1$	$\frac{E^{1/2}}{\rho}$	$\frac{(E^*)^{1/2}}{\rho^*}$	$\phi_B^e, 2$	$\frac{(E^*)^{1/2}}{\rho^*}$
Balsa Wood	90 - 110	2,1 - 2,5	10	16,85	53,28	1	16,85
Al_8090_T851	$2,52 \times 10^3$ - $2,57 \times 10^3$	80 - 84	10	3,59	11,36	15	13,91
Berylium	$1,85 \times 10^3$ - $1,86 \times 10^3$	290 - 305	10	9,29	29,4	10	29,4
1020 Steel	$7,87 \times 10^3$ - $8,07 \times 10^3$	195 - 197	10	1,76	5,55	20	7,85
Oak	690 - 840	12,1 - 14,8	10	4,77	0,015	2	6,77

Table 1: Table with shape improvements for different materials

When a shape factor of 10 is applied to all materials, balsa wood still maximize the material index, thanks to its extremely low density. However, this assumption does not reflect real-world design constraints. In practice, not all materials can be shaped with the same freedom. For example, balsa wood is very light but

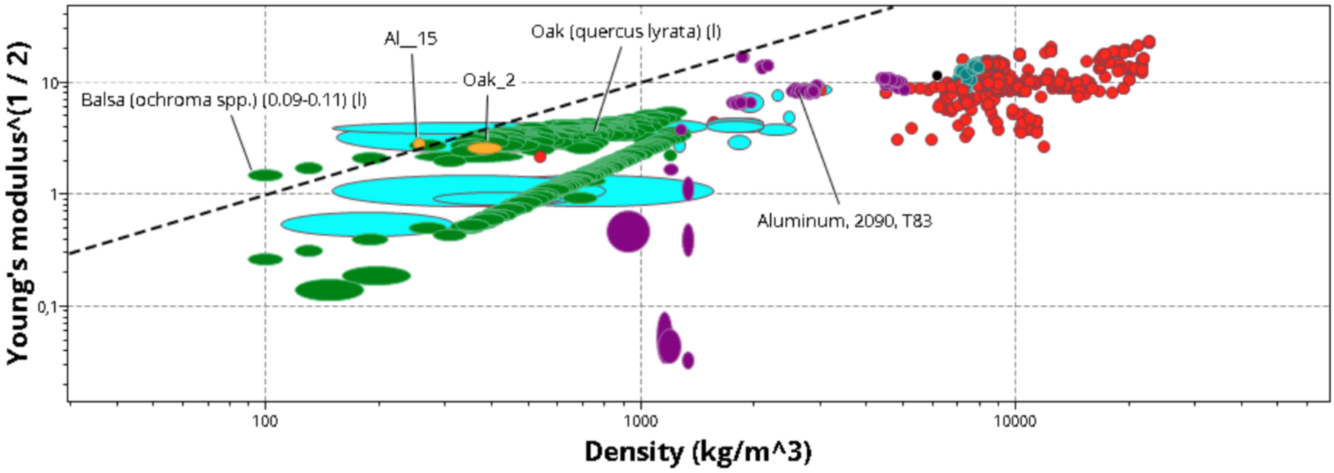

Figure 3: Graph of the Young's Modulus as a function of the density with pseudo material

also anisotropic, brittle, and difficult to machine. These properties limit its ability to be transformed into optimized structures such as I-beams. For this reason, in Ashby's book they take a maximum shape factor of 2 for woods, which significantly reduces their effective performance in structural applications. Conversely, metallic materials such as steel or aluminum have an excellent suitability for manufacturing optimized shapes. Their typical shape factors range from 15 to 20, allowing them to take full advantage of their mechanical properties. As you can see Beryllium seems to be the best material but its cost and toxicity made of him a bad candidate. Finally, the best material for industrial application seems to be Al_8090_T851.

e.

f.

References