LMAPR2020: Materials Selection

Second homework on multiple constraints and conficting objectives

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March 31, 2025

Part 1.

Question 1:

For the first we have to select 2 different materials so we can divide the problem as:

- **The Structural Material:**
  - Function : Support the tank structure under the weight of the payload
  - Constraint : Must resist buckling under a payload of 5000 kg and can't exceed 1000 euros/kg
  - Objective : Minimize the mass
  - Free Variable : Material and thickness
- **Insulation Material:**
  - Function : Insulate the tank to prevent fuel evaporation
  - Constraint : Can't exceed 1000 euros/kg
  - Objective : Minimize the mass and thermal conductivity λ
  - Free Variable : Material and thickness

So we took the formula on appendix B from Ashby's book with a fixed embedding in the ground and free at the top. Therefore $n = 1/2$ and if we replace by the inertia of a hollow cylinder via the book approximation we find:

$$F_{crit} = \frac{\pi^2 EI}{4L^2} \qquad => \qquad F_{crit} = \frac{\pi^3 Er^3 t}{4L^2}$$

The objective is to minimize the mass so we use the formula:

$$m = \rho AL = \rho 2\pi rtL$$

By replacing t (the thickness of the wall) into the buckling equation we found:

$$F_{crit} = \frac{\pi^2 Er^2 m}{8L^2 \rho} \geq 49050[N]$$

So we get:

$$m \geq 49050 \frac{\rho L^2 8}{\pi^2 Er^2} = 15903372.98 \times (\frac{\rho}{E})$$

As observed in the graph 1, to maximize the material index $M_1 = \frac{E}{\rho}$, the optimal material should be positioned above the selection line. Based on this criterion, two potential choices emerge: carbon fiber and boron carbide. However, boron carbide is a ceramic material, so it's relatively brittle instead of carbon fiber who offers a significantly higher toughness. Therefore, **carbon fiber reinforced composites** is selected as the preferred material. As explained in the video, carbon fiber reinforced composites is one of the materials

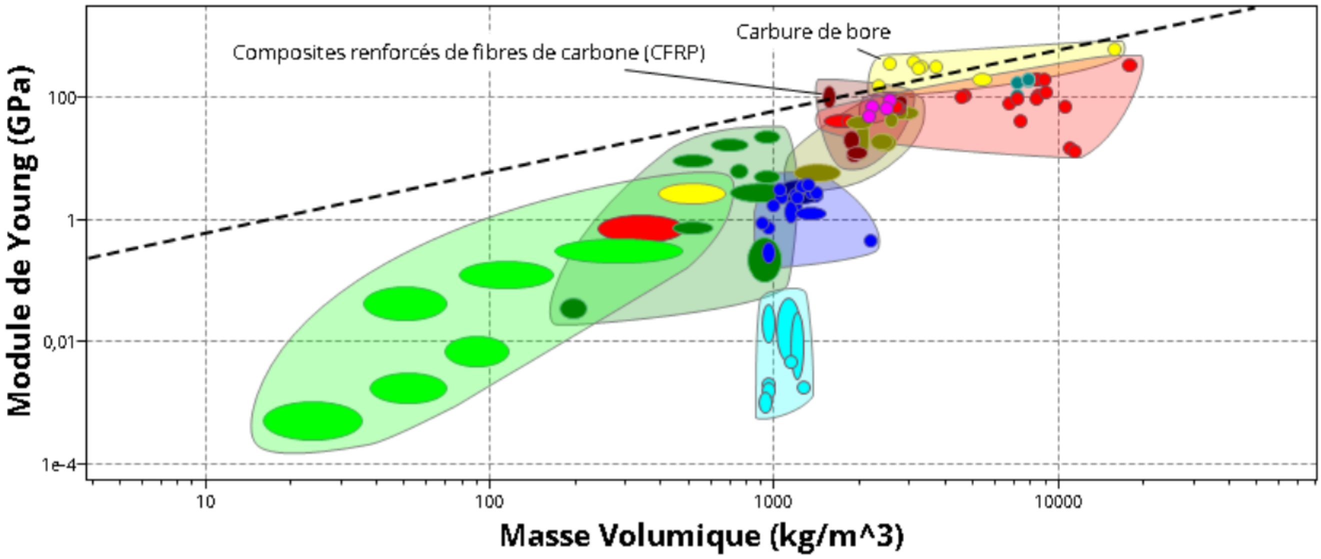

Figure 1: Young Modulus as a function of the density

commonly used for this type of application, along with aluminum alloys. This confirms our initial choice. For the insulation material, we want to prevent fuel evaporation due to conduction. This means we need to minimize the thermal conductivity, as described by Fourier's equation: $Q = -\lambda \nabla T$, where Q is the heat flux, λ is the thermal conductivity, and ∇T represents the temperature gradient. Moreover, density is also an important factor to consider. We plotted the graph below and selected the material in the bottom left corner. Finally, **Flexible Polymer Foam** appears to be the most suitable material for our application.

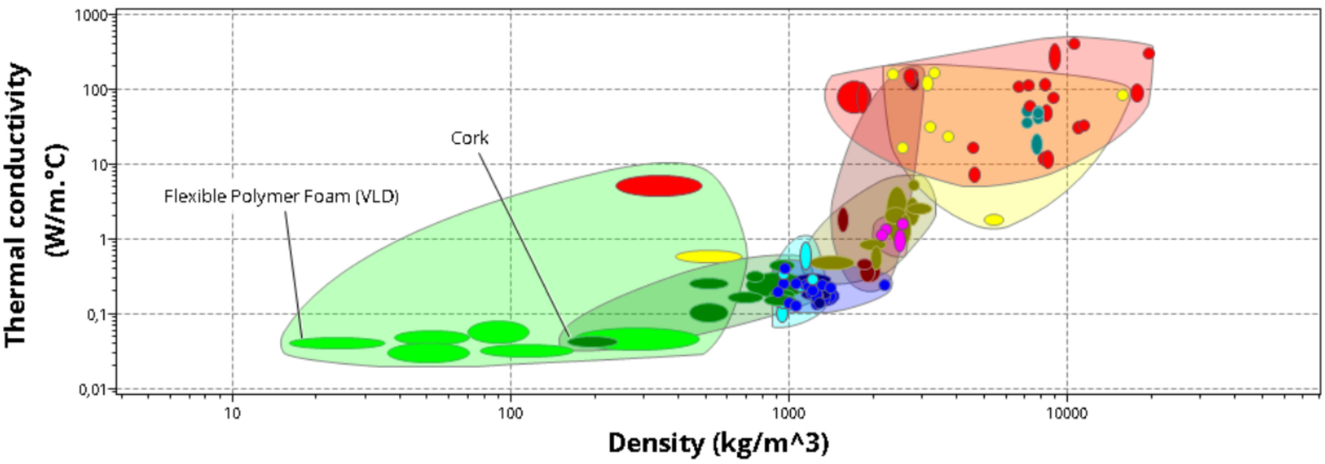

Figure 2: Thermal conductivity as a function of the density

After some research, we found that NASA generally uses composite materials for insulation. One article talks about a composite made from cork and foam, which is very effective for insulating against extreme temperatures. This material helps protect the spacecraft from both heat and cold.

Question 2:

So now we need to deal with a material that allow us to use only one instead of two. Basically, we want to find a material with a good insulation and a minimum mass which resist to buckling.

- **Structural Material:**
  - Function : Support the tank structure under the weight of the payload and Insulate the tank to prevent fuel evaporation
  - Constraint : Must resist buckling under a payload of 5000 kg and can't exceed 1000 euros/kg
  - Objective : Minimize the mass and Minimize thermal conduction
  - Free Variable : Material and thickness

So we can't use the same methodology as in the first exercise. Here we have two objectives, maximize the insulation that's mean minimize the heat flux loss by conduction and minimize the mass::

$$Q = -\lambda \frac{dT}{dx} = -\lambda \frac{(T_2 - T_1)}{t} \qquad and \qquad m = \rho 2\pi r t L$$

The only constraint is the same as before:

$$F_{crit} = \frac{\pi^3 E r^3 t}{4L^2}$$

So we have two material indices to minimize: $M_1 = (\frac{\rho}{E})$ and the second one is, by replacing $t = \frac{\lambda \Delta T}{Q}$ in the F_{crit} equation:

$$Q = \frac{1}{49050} \frac{\pi^3 E r^3 \lambda \Delta T}{4L^2} \qquad => \qquad M_2 = (E\lambda)$$

Following Ashby's method, we introduce a penalty function to balance competing objectives:

$$Z = \alpha_m (\frac{m_a}{m_{a,0}}) + \alpha_q (\frac{q_a}{q_{a,0}})$$

With α_m and α_q are weighting factors reflecting the importance of mass reduction and heat insulation. $\frac{m}{m_{a,0}}$ and $\frac{q_a}{q_{a,0}}$ are relative mass and heat transfer indices, normalized to a reference material and they are defined as:

$$\frac{m}{m_{a,0}} = \frac{\rho}{E} \times \frac{E_0}{\rho_0} \qquad and \qquad \frac{q}{q_{a,0}} = \frac{E\lambda}{1} \times \frac{1}{E_0 \lambda_0}$$

Depending on the choice of α_m/α_q, different materials become optimal.

As can be seen in graph 3, if we prioritize mass minimization with a ratio of $\frac{\alpha_m}{\alpha_q} = 10$, CFRP (Carbon Fiber Reinforced Polymer) appears to be the best material, it is also much better than stainless steel in many ways. Indeed, according to [2], "Carbon fiber has impressive strength, stiffness, and lightness. Replacing traditional materials with carbon fiber composites would greatly reduce the rocket mass, greatly increasing the rocket's payload capacity." However, Space X ultimately abandoned the use of CFRP for two major reasons. First, stainless steel is much more resistant to extreme temperatures, which is an essential criterion for a space launch vehicle. Although carbon fiber withstands temperature variations on Earth well, it cannot withstand atmospheric reentry conditions, where temperatures become extreme [3]. Then, cost is a determining factor. Carbon fiber costs around 130,000$ per ton, compared to 2,500$ for stainless steel, making the latter much more attractive for large-scale production. However, if cost were not a constraint, CFRP would remain a very attractive option due to its exceptional strength-to-weight ratio.

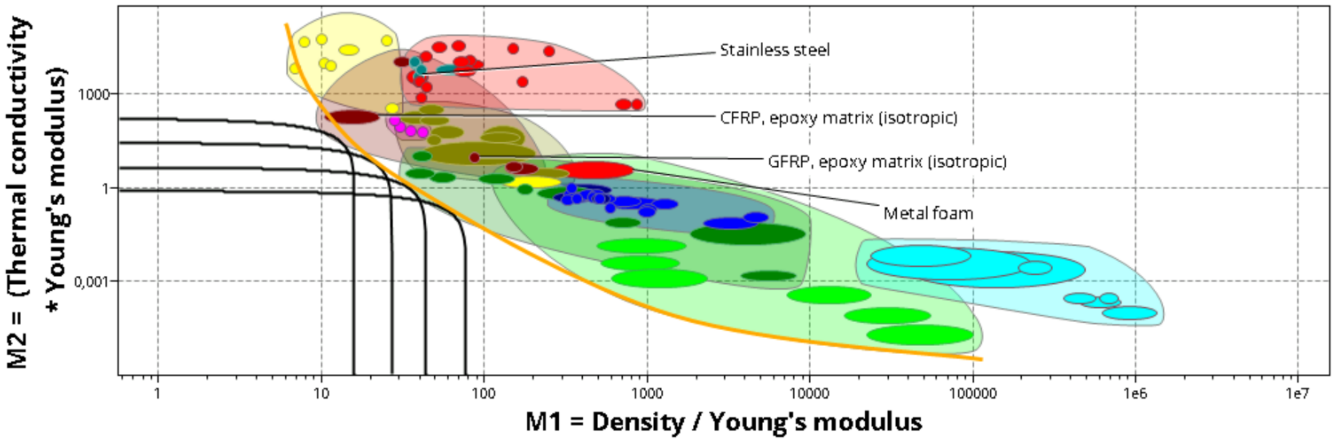

Figure 3: M1 vs M2

Part 2.

Question 1:

- **First Example: A thin lightweight table column**
  - Function : A column that support vertical loads while being as light and thin as possible.
  - Constraint : Must resist buckling under payload
  - Conflicting objectives : Minimize the thickness and mass
  - Free Variables : Material section

Major performance categories:

- Density (ρ) as it affects weight and Stiffness (E) as it affects the bending and buckling

Material indices and value function:

$$M_1 = \frac{\rho}{E^{1/2}} \qquad and \qquad M_2 = \frac{1}{E} \qquad => \qquad Z = \alpha_1 \cdot \frac{\rho}{E^{1/2}} + \alpha_2 \cdot \frac{1}{E}$$

- **Second example: Thin Phone case**
  - Function : A cheap thin phone case that is stiff
  - Constraint : Must resist deformation under load
  - Conflicting objectives : Minimizing mass and thickness
  - Free Variable : Material selection

Major performance categories:

- mass m as it affect the price and Stiffness (E) as it affects the bending and buckling

Material indices:

$$M_1 = \frac{\rho}{E^{1/2}} \qquad \textit{and} \qquad M_2 = \frac{1}{E} \qquad \textit{and} \qquad Z = \alpha_1 \cdot \frac{\rho}{E^{1/3}} + \alpha_2 \cdot \frac{1}{E}$$

Question 2:

Function: Material for fridge insulation

Objectives:

- Minimize cost of the insulation material (C_m) and Minimize thermal conductivity (λ)

Constraints: /

Free variables: Material and Thickness

So we have $C = 2t(h+b) \times \rho \times C_m$ and $Q = \lambda \frac{\Delta T}{t}$ so the penalty function can be written as follow:

$$Z = \alpha_m \cdot \frac{C_m \rho}{C_{m0} \rho_0} + \alpha_2 \cdot \frac{\lambda}{\lambda_0}$$

Flexible polymeric foam appears to be a good candidate because it minimizes thermal conductivity while

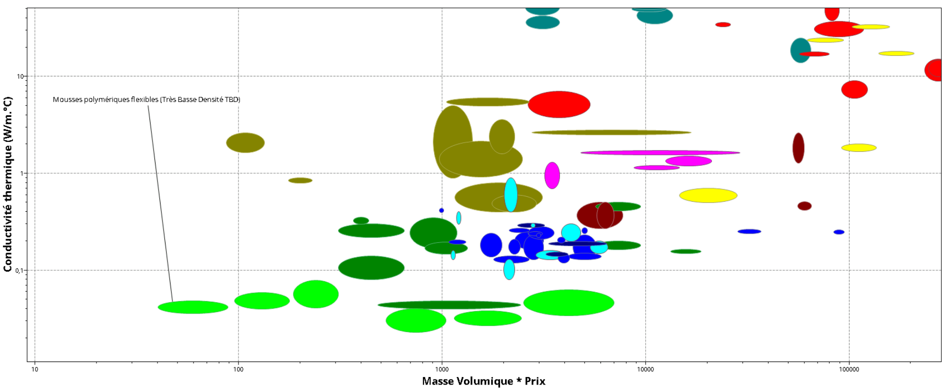

Figure 4: Thermal conductivity as a function of the material cost

being the least expensive. After some research, Polyurethane rigid foam is the insulating material which is most widely used throughout the world for refrigerators and freezers.[4]

1. **Cold storage truck** for a cold storage truck it has high exposure frequently opened it should be able operate in hot ambient conditions while maintaining its temp so as efficiency is critical we need a lower λ

2. **Domestic freezer** for a domestic freezer it has less exposure and we can be flexible with the α to reduce cost and make it convenient for domestic use

Conclusion The value of α tells us how much we are willing to pay in order to reduce thermal conductivity. It represents the trade-off between upfront material cost and operational energy savings.

References

[1] NASA, "Foam and Cork Insulation Protects Deep-Space Rocket from Fire and Ice," *NASA*, 2025, https://www.nasa.gov/missions/artemis/orion/foam-and-cork-insulation-protects-deep-space-rocket-from-fire-and-ice/?utm_source=chatgpt.com.

[2] Jason Zhao, "Carbon Fiber Applications in Modern Rockets," *ResearchGate*, 2023. Available: https://www.researchgate.net/publication/371667232_Carbon_Fiber_Applications_in_Modern_Rockets#:~:text=Carbon%20fiber%20has%20impressive%20strength,increasing%20the%20rocket%27s%20payload%20capacity. [Accessed: 29-Mar-2025]

[3] Rock West Composites, "We Now Know Why SpaceX Abandoned Carbon Fiber," *Rock West Composites*, 2023. Available: https://www.rockwestcomposites.com/we-now-know-why-spacex-abandoned-carbon-fiber.html#:~:text=Because%20stainless%20steel%20is%20much,per%20ton%20for%20stainless%20steel. [Accessed: 29-Mar-2025]

[4] Polyurethanes.org, "Refrigeration and Freezers: How Polyurethane is Used". Available: https://www.polyurethanes.org/where-is-it/refrigeration-and-freezers/. [Accessed: 30-Mar-2025]