LMAPR2020: Materials Selection

First homework on influence of shape in material selection

Work performed by Group L:

Gad Omar (15712400)

Vansnick François (74871900)

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Questions

1. In axial tension, the cross-sectional shape has no impact on the stress, which is defined by: $\sigma = \frac{F}{A}$. Thus, only the cross-sectional **area** influences the material's strength and not the shape. However, for bending the stress is $\sigma = \frac{My}{I}$, torsion is $\tau = \frac{Tr}{J}$, and compression is $F_{cr} = \frac{n^2\pi^2 EI}{L^2}$, the stress depends on the moment of inertia of the cross section or polar moment of inertia, **which vary significantly based on the cross-sectional shape**. For the bending, I is in the denominator so maximizing I reduces stress and increases resistance. For the torsion, the polar moment of inertia J depends on I, so shapes with a high J are much more efficient in resisting torsion. Finally, for the compression, the smallest moment of inertia I_{min} determines buckling resistance, so maximizing it is essential to improve resistance to buckling.

2. As explained in Ashby's book, the shape factor for elastic torsion is defined as the ratio of the torsional stiffness of a given shaped section S_T to that of a solid square shaft S_{T0}. This gives: $\phi_T^e = \frac{K}{K_0}$ where K is the torsional stiffness of the selected shape and K_0 of the reference. So we need to maximize K. For a given mass, the circular hollow section is the most efficient shape because it maximizes the polar moment of inertia, which directly influences the torsional stiffness. This means that, for the same material and mass, a hollow circular section provides the best performance in resisting torsion. For bending, the shape factor for elastic bending is given by $\phi_B^e = \frac{S}{S_0} = \frac{I}{I_0}$ where I is the second moment of area (moment of inertia). To maximize bending stiffness, we must maximize I. The optimal shape for bending stiffness is the I-beam, as it efficiently distributes material in the flanges, where bending stresses are highest. Finally, these results are in agreement with those found in the documentation.

3. To compare the buckling efficiency of a hollow column versus a solid one for the same mass, we start with the critical buckling load: $F_c = \frac{n^2\pi^2 EI_{\text{min}}}{L^2}$. Since mass is given by $m = \rho AL$, we ensure that both columns have the same cross-sectional area A. For a solid circular column of radius R, the moment of inertia is: $I_{\text{solid}} = \frac{\pi R^4}{4}$ For a hollow circular column with external radius R and internal radius r, ensuring the same area: $A_{\text{hollow}} = \pi(R^2 - r^2) = A_{\text{solid}} = \pi R^2$. Solving for r, we get $r = R/\sqrt{2}$, leading to: $I_{\text{hollow}} = \frac{\pi}{4}(R^4 - r^4) = \frac{\pi}{4}\left(R^4 - \frac{R^4}{4}\right) = \frac{3\pi R^4}{16}$. Thus, the efficiency gain is: $\phi = \frac{F_{c,hollow}}{F_{c,solid}} = \frac{I_{\text{hollow}}}{I_{\text{solid}}} = \frac{\frac{3\pi R^4}{16}}{\frac{\pi R^4}{4}} = \frac{3}{4} \times 2 = \frac{3}{2}$.

4. Material indices can be extended to include shape by incorporating shape factors (ϕ) into performance equations. This is essential in cases where structural efficiency depends on geometry, such as bending, torsion, and buckling. For bending stiffness, the performance index becomes: $M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho}$ where ϕ_B^e is the bending shape factor. For torsional stiffness: $M_2 = \frac{(\phi_T^e E)^{1/2}}{\rho}$ where ϕ_T^e is the torsional shape factor. Similarly, for failure-limited design: $M_3 = \frac{(\phi_B^f \sigma_f)^{2/3}}{\rho}$, $M_4 = \frac{(\phi_T^f \sigma_f)^{2/3}}{\rho}$ where ϕ_B^f and ϕ_T^f are failure-related shape factors. These indices allow for the selection of optimal material-shape combinations for minimum weight designs in stiffness- and strength-limited applications.

Exercices

5. In our case, the supports will essentially have to have a high buckling resistance since they are held between the ground and the roof. These supports are generally of hollow circular section, as can be seen in the photo below (my dad is a carpenter and has a lot of support of this type, we call it "étançon" in French).

Figure 1: Supports frequently used on construction sites

But this choice can be justified mathematically by first comparing the buckling resistance of the two shapes. If we take the equation for the critical buckling load for a beam of length L subjected to axial compression, we have:

$$F_c = \frac{n^2\pi^2 EI_{min}}{L^2}$$

Calculating the minimum moment of inertia for each shape, we have: For a hollow circular tube with a length of $r_{in} = 48mm$ and a thickness of $t = 9mm$, with $r_{out} = 57mm$. The moment of inertia is given by:

$$I_{cyl} = \frac{\pi}{4}(r_{out}^4 - r_{in}^4) \approx 4.12 \times 10^{-6}[m^4]$$

For the moment of inertia **minimum** of the hollow square of $b = h = 91.5mm$ and thickness $t = 9mm$. It is given by:

$$I_{\text{squ}} = \frac{1}{12}(B^4 - b^4)$$

With $b = B - 2t = 91.5 - 2 * 9 = 73.5mm$ (inner side).

$$I_{squ} = \frac{1}{12}((91.5 \times 10^{-3})^4 - (73.5 \times 10^{-3})^4) = 3.41 \times 10^{-6}[m^4]$$

We can see that our hypothesis is confirmed since the minimum inertia on which the critical buckling force depends is maximized with a circular hollow force. Let us then calculate the bending fracture and bending stiffness for each case. For the circular shape, we have:

$$\phi_B^e = \frac{3}{\pi}(\frac{r}{t}) = \frac{3}{\pi}(\frac{48 \times 10^{-3}}{9 \times 10^{-3}}) = 5.09$$

$$\phi_B^f = \frac{3}{\sqrt{2\pi}}(\sqrt{\frac{r}{t}}) = 2.764$$

For the square shape:

$$\phi_B^e = \frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2} = 5.083$$

$$\phi_B^f = \frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{(1+3b/h)}{(1+b/h)^{3/2}} = 3.189$$

Finally, we can also calculate the mass which depends on the area of the section:

$$A_{cyl} = \pi((57 \times 10^{-3})^2 - (48 \times 10^{-3})^2) = 2.969 \times 10^{-3}[m^2]$$

$$A_{squ} = (91.5 \times 10^{-3})^2 - (91.5 \times 10^{-3} - 9 \times 10^{-3})^2 = 1.566 \times 10^{-3}[m^2]$$

To conclude, the critical buckling load is proportional to $I_{\min}$, the hollow circular section offers better buckling resistance than the hollow square section. Although the bending stiffness is similar, the ultimate bending strength is higher. However, in our case, where buckling is the dominant criterion, the circular section is superior. This explains why the props used in carpentry are generally circular. Based on the literature, other factors may come into play to justify this choice:

- **Uniform stress distribution:** The circular section distributes loads better and avoids stress concentration points.
- **Ease of adjustment:** Circular tubes can be easily adjusted with telescopic systems, which is common for construction props. Square props would be more complicated to slide together.
- **Manufacturing:** More tolerant of manufacturing and installation imperfections, alignment errors have less impact on a tube than a square. Therefore, easier manufacturing and assembly, connections, fasteners, and adjustments are simpler with a tube.
- **Weight:** Although circular tubes appear stronger, the disadvantage is that they weigh more than square-section supports.

6.

1. My car simulator support is made of hollow square sections.

This choice is motivated by several factors:

- **Weight Reduction**: A hollow section saves material while maintaining good rigidity.

- **Good Bending and Torsional Strength**: The square section effectively resists lateral loads and torsional forces.
- **Ease of Assembly**: Unlike round tubes, square tubes are easier to weld and secure with plates or bolts.

Conclusion: Choosing a hollow square section optimizes the *mass/rigidity* ratio while remaining practical to assemble compare to a solid section would increase the mass considerably or a circular section would be **less practical** to assemble with flat fasteners.

2. My arbor posts are made of hollow circular tubes, a choice structured by strength and environmental constraints:

- **Excellent resistance to vertical loads**, limiting the risk of buckling.
- **Reduced Wind Resistance**: A circular shape reduces aerodynamic drag compared to a square section.
- **Durability and Ease of Assembly**: The tubes are easy to attach with suitable sleeves and fittings.

Conclusion: The hollow circular shape optimizes mechanical resistance and adaptability to external conditions compare to a square section would have a larger surface area exposed to the wind and it would be less effective against buckling in certain directions.

3. My shovel handle is designed with a full circular cross-section, a logical choice for several reasons:

- **Better grip**: A cylinder is ergonomic and avoids uncomfortable sharp edges.
- **Even stress distribution**: The cylindrical shape limits potential breaking points.
- **Ease of handling**: It's easy to rotate the handle in your hand to adjust your grip.

Conclusion: The full circular shape offers good ergonomics and effective mechanical resistance compare to a square shape which would be difficult to grip and would cause discomfort during prolonged use.

7.

In mechanics and design, we use a Factor of Safety to ensure structural reliability, choosing values based on application-specific risks.

From a **material selection perspective**, does the Factor of Safety influence the material choice, or is it purely a design constraint ?

Specifically:

When selecting materials, multiple candidates may satisfy performance requirements. However, if a specific Factor of Safety is required for the application, will this affect the selection process?

(a) shovel

(b) Racing simulator support

Figure 3: Arbor bars