Fluid Mechanics II (LMECA2322)

Compressible flows

Homework Report

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1 Methodology and results

To find the geometry of the pipe, we determined $r_2 = 0.2923\,\text{m}$ by solving the system:

$$r_2 \sin(\alpha) = 0.04, \quad r_2 + 0.00975 - 0.0125 = r_2 \sin(\alpha),$$

Which gives us,

$$r_2^2(\sin^2(\alpha) + \cos^2(\alpha)) = 0.04^2 + (r_2 + 0.00975 - 0.0125)^2.$$

Next, we calculated $r_1 = 0.04319\,\text{m}$ from:

$$r_1^2 = (-0.1 - x_{C1})^2 + (0.025 - y_{C1})^2, \quad (r_1 + r_2)^2 = (x_{C2} - x_{C1})^2 + (y_{C2} - y_{C1})^2,$$

$$y_{C1} + r_1 = 0.025.$$

With the circle equations, we found the tangent point $(x_{tg}, y_{tg}) = (-0.08713, 0.02304)$ and computed the pipe profile.

The process to calculate the pressure $p_{0,\text{min}}$ to achieve choked conditions is as follows. First, it is known that for that case, the Mach number is equal to 1 exactly at the throat. It thus means that $A_t = A^*$. Using equation (1.52) in the formula table linking A^*/A_i and M_i, it is possible to find M_e after a couple of iterations. This M_e is subsonic. Assuming that $p_e = p_{\text{atm}}$ and using the isentropic relationship, $\boldsymbol{p_{0,\text{min}}}$ **can be found, and is equal to 112186.341 Pa.** With equation (1.53), it is found that $\boldsymbol{Q_{\text{max},p_{0,\text{min}}}} = \mathbf{0.07817}\,\text{kg/s}$. This methodology is used to plot the evolution of the Mach number $M(x)$ and the static pressure $p(x)$, but this time the calculation is done for each area slice of the nozzle.

The process to calculate the pressure $p_{0,max}$ is a bit different, as there's a shock located right at the exit. The flow has to be supersonic before a shock, which means that after achieving sonic condition at the throat, it is supersonic up to the end of the nozzle. Equation (1.52) is used again for iterating, but this time to compute $M_{sh,1}$, the Mach right before the shock. Using equation (1.56), $M_{sh,2}$, the Mach after the shock, is computed. The entropic relation can once more be used to find this time $p_{0,2}$, which is then linked to find $\boldsymbol{p_{0,1} = p_{0,max} = 173357.445 Pa}$ with the equation of normal shock (1.60). The corresponding mass flow-rate is $\boldsymbol{Q_{max,p_{0,max}}} = \mathbf{0.12079}$ kg/s. For plotting the evolutions, (1.52) is used only for the supersonic flow in the diverging part of the nozzle, as the allure of the subsonic part before the throat is exactly the same as for the previous case.

Lastly, to compute the shock positions, the following method has been used. Knowing that the throat is sonic, an area is choosen in the diverging part. Using (1.52), the supersonic $M_{sh,1}$ is found, as well as $M_{sh,2}$ with equation (1.56). Along with the area of the nozzle at the shock location, the area of the sonic section downstream of the shock A_2^* is computed, which then leads to M_e using (1.52). The exit pressure matches the atmospheric pressure, as the flow is subsonic downstream of the shock. It is then possible to compute $p_{0,e}$=$p_{0,2}$ with the isentropic relation, and link it with $p_{0,1}$ with the normal shock relation. This process is repeated as much as needed, to find the correct matching of each p_0 and A_sh. On figure (1) and (2), 9 shocks have been represented, as well as the locus of the jumps with the dotted line. The evolutions on figures (3) and (4) have been represented for $\boldsymbol{p_0 = 132403.379 Pa}$, and the table (1) summarizes the important data of this specific case.

	A_{sh} [cm^2]	x_{sh} [m]	M_{sh1}	M_{sh2}	p_{sh1} [Pa]	p_{sh2} [Pa]
$p_0 = 132403.379$ Pa	3.8335	0.0275	1.6401	0.6567	29347.612	87207.704

	Total pressure drop [Pa]	Change in entropy [J/kgK]
$p_0 = 132403.379$ Pa	15903.568	36.738

Table 1: Data for one intermediate total pressure

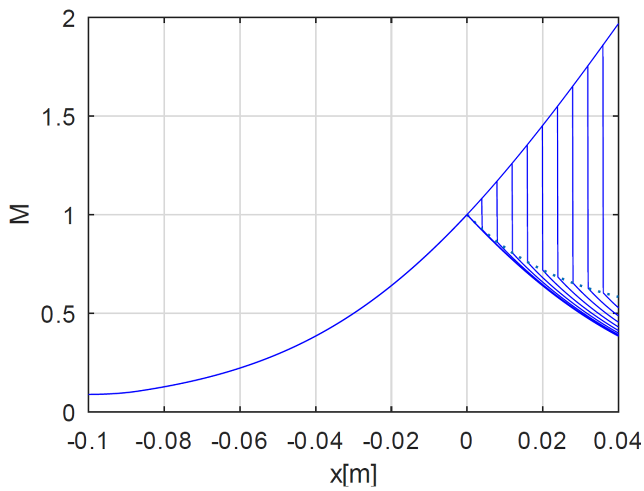

Figure 1: Evolution of the Mach number for $p_{0,min}$, $p_{0,max}$ and some total pressures in between

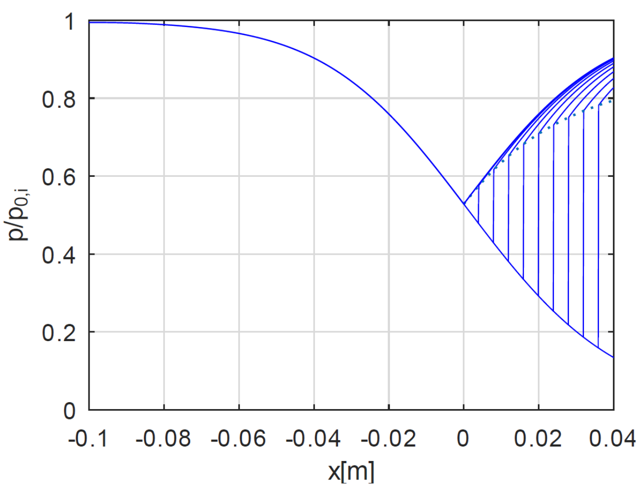

Figure 2: Evolution of the normalized static pressure for $p_{0,min}$, $p_{0,max}$ and some total pressures in between

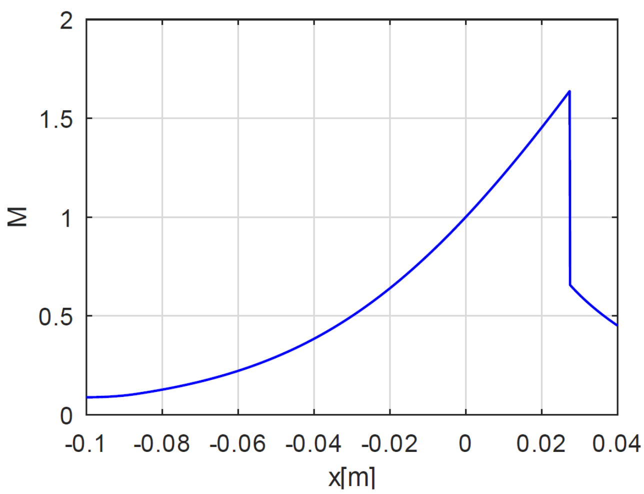

Figure 3: Evolution of the Mach number for $p_0 = 132403.379 Pa$

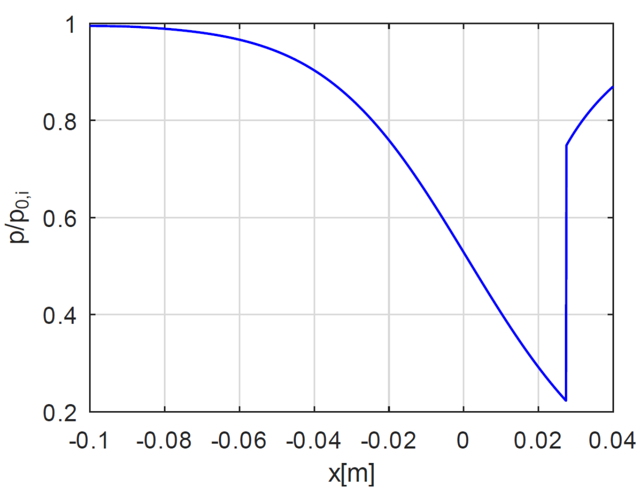

Figure 4: Evolution of the normalized static pressure for $p_0 = 132403.379 Pa$