LMECA2854 - Heat & Mass Transfert II

Heat Exchanger Laboratory Preparation

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Group 4

1 Introduction

The laboratory aims to compare the theoretical performance of a simplified water-water heat exchanger with experimental results. The setup consists of two separate circuits: a hot circuit and a cold one. Heat exchange occurs in coaxial tube assemblies, where hot fluid flows inside the inner tube and cold fluid flows in the external annulus.

Installation allows for different flow patterns using different valve settings and connection points. Three basic configurations are possible: using one coaxial assembly, using both in parallel, or using both in series. Temperature is measured using thermocouples at the extremities of the coaxial assemblies. Mass flow rates are measured using rotameters with a maximum capacity of 40 kg/h. An electrical resistance is used to heat the hot fluid, dissipating 972 W, and should only be activated with a sufficient flow rate (15-20 kg/h).

2 Preparation

2.1 Show that both flows in one coaxial assembly (annular and interior) are laminar. Are they fully developed or should you consider the effect of the entry ?

The Reynolds number, Re, is used to determine whether the fluid flow is laminar or turbulent. It can be computated such as :

$$Re_D = \frac{U_m D}{\nu} \qquad With : \nu = \frac{\mu}{\rho}$$

Knowing that :

$$\dot{m} = A U_m \rho$$

Leads to :

$$Re_D = \frac{\dot{m} D}{A \mu} \tag{1}$$

A flow can be considered laminar if $Re_D < 2300$.

For the hot fluid, the characteristic length is the inside diameter of the inside tube = 0,0104 m, and its area $= 8.495 \times 10^{-5} m^2$. For the cold fluid, the characteristic length is the distance between the inside of the outer tube and the outside of the inner tube = 0,0038 m, and the area of the outer annulus $= 1.886 \times 10^{-4} m^2$. The installation is connected to the belgian water supply network, that is why an average value of 15 °C = 288,15 K will be used as the entrance temperature in the device. The maximum temperature that the water can reach after being heated by the power provided by the electrical resistance can be calculated thanks to :

$$Q = \dot{m}c\Delta T \quad \Rightarrow \quad P = \dot{m}c\Delta T \quad \Rightarrow \quad \Delta T = \frac{P}{\dot{m}c} \tag{2}$$

An attention must be paid to the mass flow rate $\dot{m}$ of the hot fluid, as it can massively change the temperature reached after the resistance. Two cases will be studied here, one corresponding to the lower limit (15 kg/h) and the other one corresponding to the upper limit (40 kg/h). The corresponding dynamic viscosites will be each time taken from the tables at the end of the formula sheet, as they vary with the temperature.

2.1.1 Lower limit of $\dot{m}_h$ = 15 kg/h

Using equation (2), it is found that :

$$\Delta T = \frac{P}{\dot{m}c} = \frac{972}{\frac{15}{3600}4180} = 55.8\,°\mathrm{C}$$

The maximum temperature reached by the hot fluid after the resistance will then be 70.8 °C = 343,95 K. The mass flow rate $\dot{m}_c$ used for computation will be 40 kg/h, the one that could have the higher influence on the Reynolds number of the cold fluid.

	Temperature $[K]$	Dynamic viscosity μ [Pa.s]	Re
Cold fluid	290	0,001080	207,27
	300	0,000855	261,81
	310	0,000695	322,08
	320	0,000577	387,95
Hot fluid	320	0,000577	884,08
	330	0,000489	1043,2
	340	0,000420	1214,6
	345	0,000389	1311,3

Table 1: Values of the Reynolds number for $\dot{m}_h = 15 kg/h$

The Reynolds number, Re, is always lower than 2300 : the flows can then always be considered laminar. To know if those flows are fully developped, $x_{dev,h}$ and $x_{dev,t}$ can be computated, using each time the highest Re in the tabular (1) :

- Cold fluid

$$x_{dev,h,c} \approx 0,05Re_D D = 0,0737m \tag{3}$$

$$x_{dev,t,c} \approx 0,05Re_D Pr D = 0,2867m \tag{4}$$

This flow can't be considered as a fully developed but as a thermal entry flow.

- Hot fluid

$$x_{dev,h,h} \approx 0,05Re_D D = 0,6819m \tag{5}$$

$$x_{dev,t,h} \approx 0,05Re_D Pr D = 1,7883m \tag{6}$$

This flow can't be considered as a fully developed but as a combined entry flow.

2.1.2 Upper limit of $\dot{m}_h$ = 40 kg/h

Once again, using equation (2), it is found that :

$$\Delta T = \frac{P}{\dot{m}c} = \frac{972}{\frac{40}{3600}4180} = 20.9C$$

The maximum temperature reached by the hot fluid after the resistance will then be 35.9 °C = 309,05 K. The mass flow rate $\dot{m}_c$ used for computation will once again be 40 kg/h.

	Temperature $[K]$	Dynamic viscosity μ [Pa.s]	Re
Cold fluid	290	0,001080	207,27
	295	0,000959	233,42
	300	0,000855	261,81
Hot fluid	300	0,000855	1591
	305	0,000769	1768,9
	310	0,000695	1957,3

Table 2: Values of the Reynolds number for $\dot{m}_h = 40kg/h$

Once again, the Reynolds number, Re, is always lower than 2300 : the flows can then always be considered laminar, too. To know if those flows are also fully developped, $x_{dev,h}$ and $x_{dev,t}$ can be computated, using each time the highest Re in the tabular (2) :

- Cold fluid

$$x_{dev,h,c} \approx 0,05 Re_D D = 0,0497m \tag{7}$$

$$x_{dev,t,c} \approx 0,05 Re_D Pr D = 0,2867m \tag{8}$$

  This flow can be considered as a thermal entry flow.

- Hot fluid

$$x_{dev,h,h} \approx 0,05 Re_D D = 1,0178m \tag{9}$$

$$x_{dev,t,h} \approx 0,05 Re_D Pr D = 4,769m \tag{10}$$

  This flow can't be considered as a combined entry flow.

2.2 Compute the global heat transfer coefficient $U = 1/(R_{tot}A)$ between the hot and cold fluids.

Three phenomenons must be taken into account to find R_{tot} : forced convection in the inner pipe, conduction in the pipe and forced convection in the outer annulus. Each one needs to be computated differently.

- $R_{conv,h}$

  For this flow, the methodology for combined entry with uniform T_w in the formula sheet page 4 will be followed.

  The Nusselt number $\overline{Nu(L)}$ is given by :

$$\overline{Nu(L)} = \frac{A+B}{C} = \frac{hD}{k} \tag{11}$$

  With :

$$A = \frac{3,66}{tanh(2,264 G_{z_D}^{\frac{-1}{3}}(L) + 1,7 G_{z_D}^{\frac{-2}{3}}(L))} \quad With \quad G_z(L) = \frac{RePr}{\frac{L}{D}} \tag{12}$$

$$B = 0,0499 G_{zD}(L) tanh(G_{z_D}^{-1}(L)) \tag{13}$$

$$C = tanh(2,432 Pr^{\frac{1}{6}} G_{z_D}^{\frac{-1}{6}}(L)) \tag{14}$$

R_{conv},h can then be found using :

$$R_{conv,h} = \frac{1}{h_h A} \tag{15}$$

$\dot{m}_h[kg/h]$	Highest Re	Pr	G$_z(L)$	Nu(L)	h $[W/m^2K]$	$R_{conv,h}[K/W]$
15	1311,3	2,6226	17,0317	4,8547	289,42	0.0504
40	1957,3	4,6856	45,4189	6.1991	369.56	0.0394

Table 3: Values of Nu for different $\dot{m}_h$

- R_{cond}

  Due to the configuration, it can be found using :

$$R_{cond} = \frac{\log \frac{r_o}{r_i}}{2\pi L K_{Wall}} = 0.00107K/W$$

- $R_{conv,c}$

  For this flow, the methodology for thermal entry with uniform T_w in the formula sheet page 4 will be followed.

  The average Nusselt number $\overline{Nu(L)}$ is given by the Haussen correlation :

$$\overline{Nu(L)} = \frac{1}{L}\int_0^L Nu(x)\, dx = \frac{\overline{h}D}{k} \tag{16}$$

In our case, it can be approximated by:

$$\overline{Nu(L)} = 3,66 + \frac{0,0668 G_z(L)}{1 + 0,04 G_z(L)^{\frac{2}{3}}} \quad With \quad G_z(L) = \frac{RePr}{\frac{L}{D}} \tag{17}$$

For this case, the hydraulic diameter will be used for D, which can be computated with :

$$D_{hydraulic} = \frac{4A_{wet}}{P_{wet}} = 0.0092m \tag{18}$$

$R_{conv,c}$ can then be calculated using :

$$R_{conv,c} = \frac{1}{h_c A} \tag{19}$$

$\dot{m}_h[kg/h]$	$\dot{m}_c[kg/h]$	Highest Re	Pr	$G_z(L)$	Nu(L)	h $[W/m^2K]$	$R_{conv,c}[K/W]$
15	40	387,95	3,89	6,6114	4,0471	272,74	0,0463
40	40	261,81	5,76	6,6066	4,0468	272,72	0,0463

Table 4: Values of Nu

U can then be computated using :

$$U = \frac{1}{(R_{conv,h} + R_{cond} + R_{conv,c})A} \tag{20}$$

$\dot{m}_h[kg/h]$	$\dot{m}_c[kg/h]$	$R_{conv,h}$ [K/W]	R_{cond} [K/W]	$R_{conv,c}$ [K/W]	U [W/Km^2]
15	40	0,0504	0,00107	0,0463	129,19
40	40	0,0394	0,00107	0,0463	145,57

Table 5: Values of U

2.3 List of Cases to Investigate During the Lab to Best Satisfy Objectives

Assuming that each case must be treated we will have to analyze 8 different configurations. That is to say the four presented in the report in co-flow and four in counter-flow.

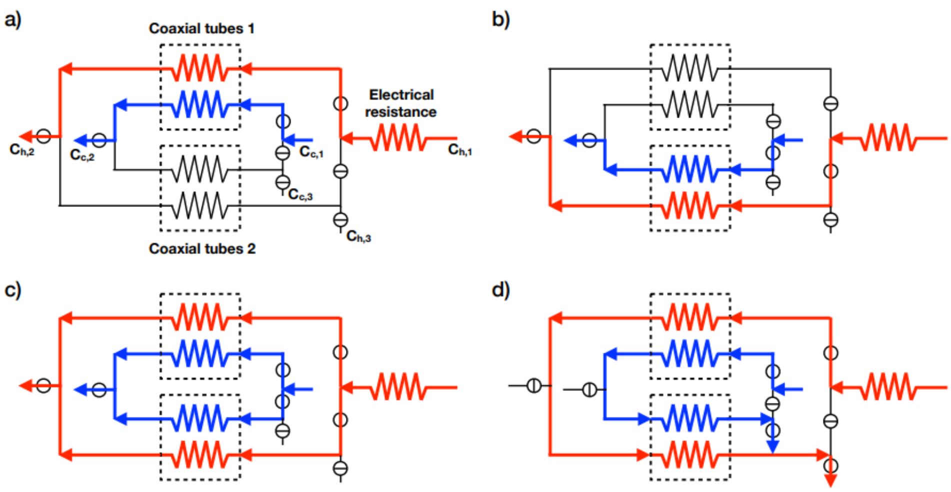

Figure 1: Flow patterns which can be obtained in the installation

Firstly, as we only have one resistance, the hot water input will be the same in each case.In the table below, we have included all the valves which are open in each case. By default, if a valve is not included in the table, it is in a closed state. We will also assume that the inlet of water means that the inlet is connected to a water inlet while the outlet will be connected to the drain.We also made sure that water always passed through the rotameter in each circuit to have flow data.

		Co-Flow	Conter-Flow			Co-Flow	Conter-Flow
a	Hot Fluid	VA201 (inlet) VA202 VA206	VA201 (inlet) VA202 VA206	b	Hot Fluid	VA201 (inlet) VA203 VA206 (outlet)	VA201 (inlet) VA203 VA206 (outlet)
	Cold Fluid	VA101 (inlet) VA102 VA104 (outlet)	VA104 (inlet) VA102 VA101 (outlet)		Cold Fluid	VA101 (inlet) VA103 VA104 (inlet)	VA104 (inlet) VA103 VA101 outlet
c	Hot Fluid	VA201 (inlet) VA202 VA203 VA206 (outlet)	VA201 (inlet) VA202 VA203 VA206 (outlet)	d	Hot Fluid	VA201 (inlet) VA202 VA204 (outlet)	VA201 (inlet) VA202 VA204 (outlet)
	Cold Fluid	VA101 (inlet) VA102 VA103 VA104 (outlet)	VA104 (inlet) VA102 VA103 VA101 (outlet)		Cold Fluid	VA101 (inlet) VA102 VA105 (outlet)	VA105 (inlet) VA102 VA101 (outlet)

The next step is to find the most appropriate mass flow rate for each configuration. To do this, it is necessary to find the flow rate which maximizes the heat exchange between the two fluids. We can already estimate that the greater the flow rate, the less resistance will be able to heat the fluid; on the contrary, if the Reynolds number of the fluid is greater, the heat exchange will take place more easily.

The main equations for a heat exchanger are:

Energy Balance:

$$\dot{m}_c c_{p,c} dT_c = C_c dT_c = dQ$$

$$\dot{m}_h c_{p,h} dT_h = C_h dT_h = dQ$$

Heat Flux:

$$dQ = UdA(T_h - T_c)$$

By developing these equations as we did in class we obtain a parameter ϵ, which represents the efficiency of the heat exchanger. If it is close to 1, it means that the heat exchanger has successfully transferred all the heat possible between the two fluids. So we need to find the mass flow rate that maximizes this parameter. (Nb: this is involved in the calculation of C_c and C_h)

For a parallel flow configuration:

$$\epsilon = \frac{1-exp[-NTU(1+C_r)]}{1+C_r}$$

For a counterflow configuration:

$$\epsilon = \frac{1-exp[-NTU(1-C_r]}{1-C_r exp[-NTU(1-C_r)]}$$

With:

$$C_r = \frac{C_{min}}{C_{max}} \qquad NTU = \frac{UA}{C_{min}} \qquad C_{min} = min(C_h, C_c)$$

We can conclude that if the mass flow rate is minimal for both, the heat exchange is greater, which is logical since if the cold water has a flow rate close to zero, it will quickly reach the temperature of T_h. On the contrary, if the two fluids have a very high flow rate, the heat exchange does not have time to take place. But, as the Reynolds varies according to the speed of the fluid and the heat transfer coefficient U is itself a function of h and therefore of the Reynolds. We must also take into account that heat transfer improves with speed.

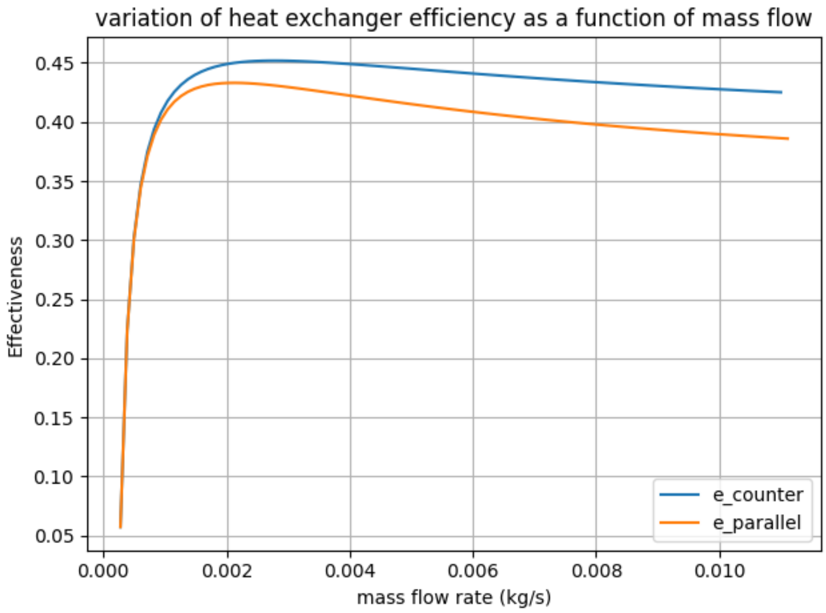

Figure 2: Variation of the heat exchanger effectiveness as a function of mass flow rate

By taking into account the variation of U and using the formulas described above. We note that the optimal value for cold water is close to 10kg/h and that for hot water is close to 15kg/h.

For the last part of the lab preparation we created a python script which allows, once the laboratory data has been acquired, to obtain the different results and interpret them.

```
import numpy as np
import math
import matplotlib.pyplot as plt

# Param tres

Cp = 4180  # C_p de l'eau
Ah = 2 * math.pi * 0.0052 * 2.1
Ac = 2 * math.pi * 0.006 * 2.1
P = 972
Tc = 287.15  # [ K ]

R_cond = 0.00107
R_convH = 1  #    modifier
```

```
R_convC = 1  #    modifier
U = (1/R_cond + 1/R_convH + 1/R_convC)**-1


# Calcul des valeurs de mDotC et mDot

mDotC = np.linspace(1, 40, 100) / 3600
mDoth = np.linspace(15, 40, 100) / 3600

# D finition des fonctions de Th, Cc et Ch
def Th():
    return Tc + (P / (mDoth * Cp))

def Cc():
    return mDotC * Cp

def Ch():
    return mDoth * Cp

#-----Estimation of the global heat transfert Coefficient -----#

Uh = mDoth * Cp *   math.log(Th() - Tc)/Ah
Uc = mDotC * Cp * - math.log(Th() - Tc)/Ah

file_path = "Estimation of the global heat transfert.txt"

with open(file_path, "w") as file:
    #  crire  les donn es ligne par ligne
    file.write("Heat transfert for heat flux:\n")
    file.write("U = " + str(Uh) + " \n")
    file.write("Heat transfert for cold flux:\n")
    file.write("U = " + str(Uc) + " \n")

#-----T   en fonction de la configuration (1 ou 2 tubes)-----#

# One coaxial assembly

T2c = 0 #    modifier
T2h = 0 #    modifier
```

```
dQc = mDotC * Cp * (T2c - Tc)
dQh = mDoth * Cp * (T2h - Th)

file_path = "Effect_of_configuration(1or2).txt"

with open(file_path, "w") as file:
    #  crire  les donn es ligne par ligne
    file.write("One coaxial assembly:\n")
    file.write("T2c = " + str(T2c) + "  dQc = " + str(dQc) + " \n")
    file.write("T2h = " + str(T2h) + "  dQh = " + str(dQh) + " \n")

# Two coaxial assembly in series

T2c = 0 #    modifier
T2h = 0 #    modifier
dQc = mDotC * Cp * (T2c - Tc)
dQh = mDoth * Cp * (T2h - Th)

with open(file_path, "w") as file:
    #  crire  les donn es ligne par ligne
    file.write("Two coaxial assembly in series:\n")
    file.write("T2c = " + str(T2c) + "  dQc = " + str(dQc) + " \n")
    file.write("T2h = " + str(T2h) + "  dQh = " + str(dQh) + " \n")

#----------------------------------------#

""""# D finition des fonctions DTcFinal et DThFinal
def DTcFinal():
    return (U * Ah * (Th() - Tc)) / (mDotC * Cp)

def DThFinal():
    return -(U * Ah * (Th() - Tc)) / (mDoth * Cp)"""

#-----T   en fonction de la configuration (co ou counter)-----#

# Co-Flow

T2c = 0 #    modifier
T2h = 0 #    modifier
```

```
dQc = mDotC * Cp * (T2c - Tc)
dQh = mDoth * Cp * (T2h - Th)

file_path = "Effect_of_configuration.txt"

with open(file_path, "w") as file:
    #  crire  les donn es ligne par ligne
    file.write("Co-Flow configuration:\n")
    file.write("T2c = " + str(T2c) + "  dQc = " + str(dQc) + " \n")
    file.write("T2h = " + str(T2h) + "  dQh = " + str(dQh) + " \n")

# Counter Flow

T2c = 0 #    modifier
T2h = 0 #    modifier
dQc = mDotC * Cp * (T2c - Tc)
dQh = mDoth * Cp * (T2h - Th)

with open(file_path, "w") as file:
    #  crire  les donn es ligne par ligne
    file.write("Counter Flow configuration:\n")
    file.write("T2c = " + str(T2c) + "  dQc = " + str(dQc) + " \n")
    file.write("T2h = " + str(T2h) + "  dQh = " + str(dQh) + " \n")

#------------------------------------------#

#-----Effectiveness of the heat exchanger-----#
# Calcul de Cmin et Cmax
Cmin = np.minimum(Cc(), Ch())
Cmax = np.maximum(Cc(), Ch())
CR = Cmin / Cmax

# D finition du NTU
def NTU():
    return (U * Ah) / Cmin

# D finition des fonctions d'effectiveness
def e_parallel():
    return (1 - np.exp(-NTU() * (1 + CR))) / (1 + CR)
```

```python

def e_counter():
    return (1 - np.exp(-NTU() * (1 - CR))) / (1 - CR * np.exp(-NTU() * (1 - CR)))

#--------------------------------------#

# Trace le graphique
plt.figure(figsize=(10, 6))

plt.subplot(2, 1, 1)
"""plt.plot(mDoth, DThFinal(), label='DThFinal')
plt.plot(mDotC, DTcFinal(), label='DTcFinal')
plt.xlabel('mass flow rate (kg/s)')
plt.ylabel('Delta T (K)')
plt.title('Variation of Delta T as a function of mass flow')
plt.grid(True)
plt.legend()"""

plt.subplot(2, 1, 2)
plt.plot(mDotC, e_counter(), label='e_counter')
plt.plot(mDotC, e_parallel(), label='e_parallel')
plt.xlabel('mass flow rate (kg/s)')
plt.ylabel('Effectiveness')
plt.title('variation of heat exchanger efficiency as a function of mass flow')
plt.grid(True)
plt.legend()

plt.tight_layout()
plt.show()
```

Listing 1: Script for the Lab