LMECA2410 – Mechanics of Materials

Fatigue Mechanics Project

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1 Fatigue failure of aircraft structures

1.1 Different Stages of Fatigue

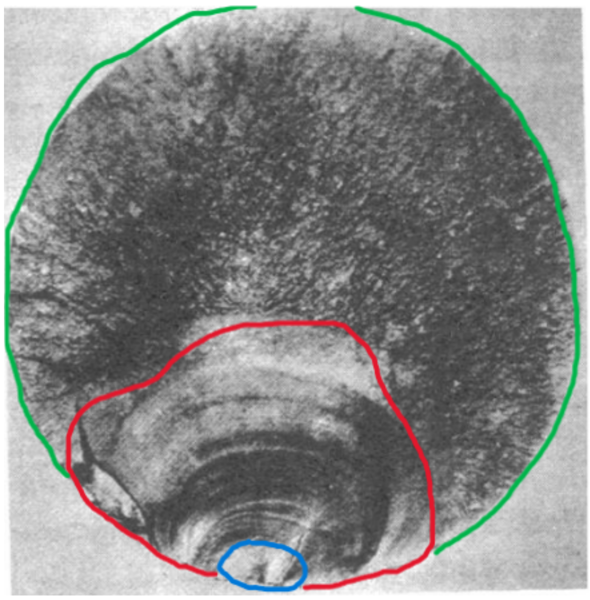

Figure 1: Image of a fracture surface after fatigue cracking

The Blue zone represent the **Crack initiation stage**. This can be identified by the pits at the bottom of the surface (the dark spot or irregularity).

- In the crack initiation stage at the surface, variations in grain size, distribution, and orientation can influence the nucleation and growth of microcracks which brings about the irregularities observed.

- Also, Residual stresses from manufacturing processes or previous loading cycles can act as local stress raisers, facilitating crack initiation

The Red zone represent the **Crack growth stage**. This can be identified by the presence of beachmarks (Striations).

- In the crack growth stage, striations are produced due to the loading/unloading of the material that each time enlarges the crack and propagates it by the blunting phenomenon. Higher loads can result in faster crack propagation, leading to longer and more pronounced striations.

- Also, higher ductility can result in more extensive plastic deformation around the crack tip, influencing the appearance of striations.

The green zone represent the **fast cracking fracture stage** (Ductile failure).

- This can be identified by the coarse surface appearance with dimples and ridges. In the fast cracking stage, the propagation of striations contributes to the weakening of the material and reduces its load-bearing capacity. As fatigue damage accumulates, the material becomes more susceptible to sudden failure.

- Environmental conditions such as temperature, humidity, and the presence of corrosive agents can affect material properties and accelerate fatigue crack growth. For example, corrosion can weaken the material and promote crack propagation, hastening the onset of fracture rupture.

1.2 Fatigue Crack Growth Analysis of Additive Manufactured and Wrought Al7075 Alloys

A high strength Al7075 aluminium alloy produced by laser powder bed fusion (L-PBF) [Material A] and a wrought Al7075 [Material B] have been tested for fatigue crack growth following the E-647 standard for Compact Tension (CT). The two samples undergone different heat treatment. The tested samples are under plain strain conditions, which means we can consider Kc = Kic.

1.2.1 Analysis of Crack Length Evolution Over Cycles and Comparative Evaluation

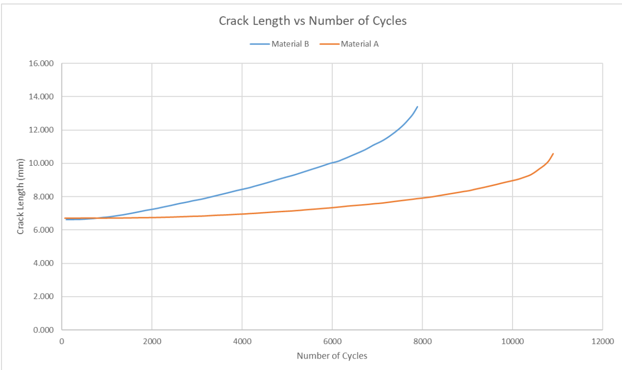

Figure 2: Crack length over number of cycles

From figure (2) we can see that Material A has much higher resistance to crack growth as it has a less steep evolution with the number of cycles and also does enfure more cycles overall. This initially seems to indicate that Material A is more resistant to fatigue than Material B.

When comparing the two materials crack growth rate is much more important to determine the fatigue resistance of a material. The rate of crack growth is da/dN and is the instantaneous slope of the crack growth curve. Therefore comparing the two curves doesn't necesarrily give us definitive information but it can be useful to get an idea of the material behaviour.

1.2.2 Crack growth rate plots

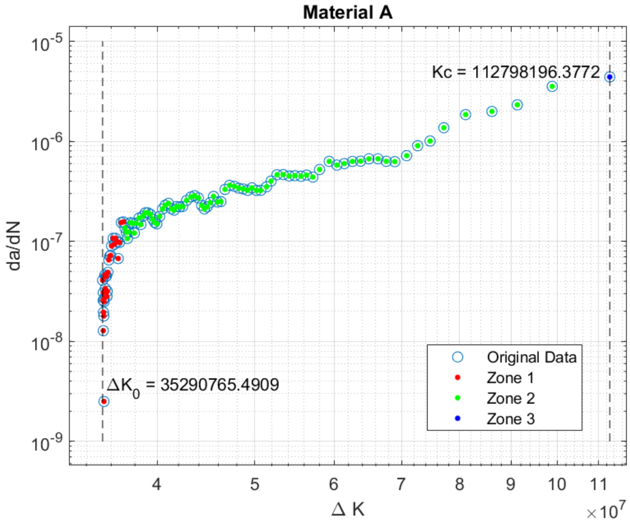

Figure 3: Crack growth rate of material A

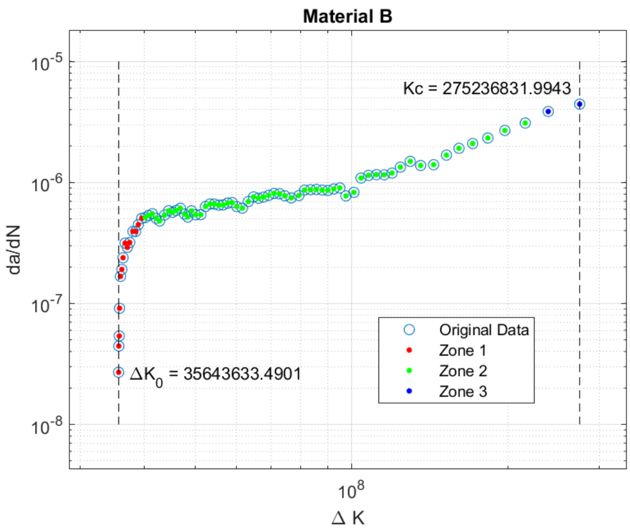

Figure 4: Crack growth rate of material B

Zone Indications

- Zone 1 indicates the low growth rate zone where the cracks propagate very slowly.
- Zone 2 indicates zone where the crack propagates faster under the paris law
- Zone 3 is where the crack suddenly increases up to failure

Definitions

1. **Stress Intensity Range at Threshold (ΔK_0):**

- The stress intensity range at threshold, denoted as ΔK_0, represents the minimum cyclic stress intensity factor range required for crack propagation to initiate or accelerate significantly. It indicates the onset of fatigue crack growth under cyclic loading conditions.
- Materials with lower ΔK_0 values exhibit better resistance to fatigue crack initiation and have higher fatigue thresholds.
- Experimentaly it corresponds to the ΔK at initial stages (i.e The lower bound) from the crack growth rate graph.

2. **Final Stress Intensity Factor at Failure (K_c):**

- The final stress intensity factor at failure, denoted as K_c, represents the stress intensity factor when the material ultimately fails under fatigue loading conditions.It characterizes the material's fracture toughness, indicating its ability to resist crack propagation and withstand catastrophic failure.
- Materials with higher K_c values exhibit greater resistance to crack growth and higher fracture toughness, resulting in better fatigue performance and improved structural integrity.
- Experimentaly it corresponds to the ΔK at fracture (i.e The upper bound) from the crack growth rate graph.

The calculated values for the parameters of the Materials are summarized in the table below.

	Material A	Material B
C	$7.5762\times10^{-30}\sqrt{m}$/cycle	$3.3252\times10^{-14}\sqrt{m}$/cycle
m	2.9431	0.93957
K$_c$	35 290 765.4909 Pa$\sqrt{m}$	275 236 831.9943 Pa$\sqrt{m}$
ΔK_0	112 798 196.3772 Pa$\sqrt{m}$	35 643 633.4901 Pa$\sqrt{m}$

Table 1: Parameters for material A and B

The C and m coefficient are calculated by fitting the Paris law to the middle section of the graph illustrated in the figures (5) and (6), which will be explained in the following sections.

1.2.3 Effect of changing the loading condition

Changing the loading value highlights the relationship between fatigue resistance and loading. For example, the rate of crack initiation and growth will be faster under higher loading. Additionally, increasing Pmax can also impact the material's behavior; for instance, if the load exceeds the material's plastic limit, it will deform along with the crack. Conversely, having a Pmax that is too low and below the material's elastic limit would imply that the material would not fail due to fatigue.

1.2.4 Military aircraft applications

The material constants C and m were determined from the graphs below, where the Paris law was applied to derive these parameters for Material A and Material B.

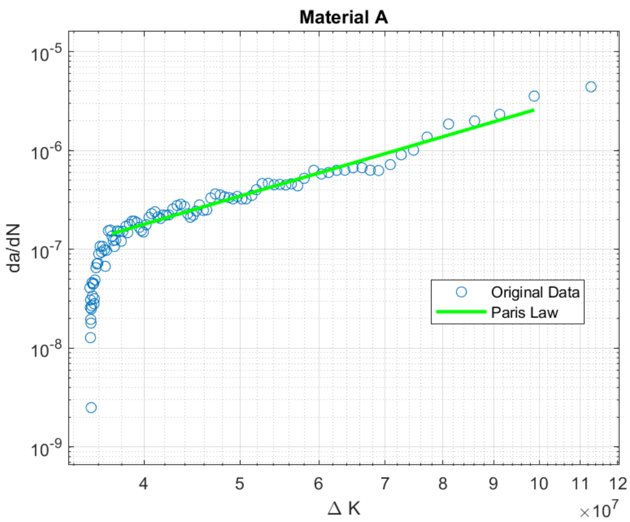

Figure 5: Paris Law Curve fitting for Material A

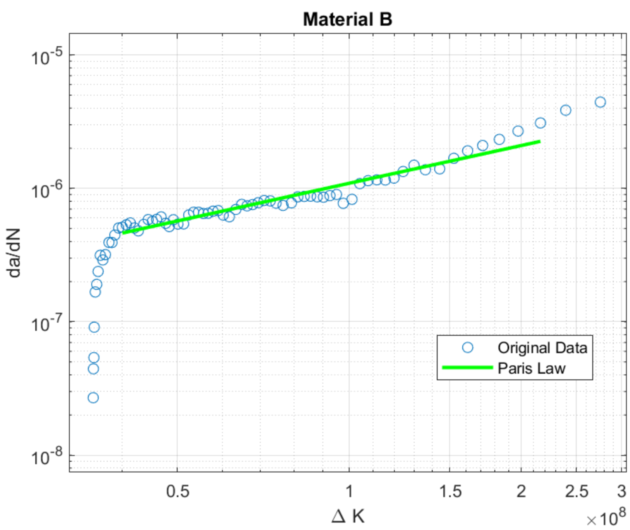

Figure 6: Paris Law Curve Fitting for Material B

C and m material constants for the Al7075, material B only. We know from Paris law that : $\frac{da}{dN} = C \cdot (\Delta K)^m$. To find the parameters C and m we know that $\frac{da}{dN}$ is the slope of the line in region II that we have drawn on figure 5 and 6. Indeed, by taking the logarithm we find : $\log(C) + m \cdot \log(\Delta K)$. The values for C and m are written in table 1.

The critical crack length in the tail section which induces its complete failure (for material B only) To find the critical crack length we use the following relation : $K_{IC} = \sigma_{max} \cdot \sqrt{\pi \cdot a_c}$. Since it is assumed $K_{IC} = K_c$, using the appliend maximum stress given in the questionaire we thus have :

$$a_c = \frac{1}{\pi} \cdot \left(\frac{K_{IC}}{\sigma_{max}}\right)^2 = 0.868m \tag{1}$$

The lifetime of the tail section for material B only. To find the lifetime we take again the Paris law and integrate it. For that we use :

$$\Delta K = K_{max} - K_{min} = (\sigma_{max} - \sigma_{min}) \cdot \sqrt{\pi \cdot a} \tag{2}$$

We arrange the equation and find the equation to solve :

$$\frac{da}{a^{m/2}} = C \cdot (\sigma_{max} - \sigma_{min})^m \cdot \pi^{m/2} \cdot dN \tag{3}$$

To find the number of cycles we integrate the equation using boundaries of crack length. Since the resolution is 0.01 mm, we will use it as our lower bound as we assume the smallest crack is of that order of magnitude. We integrate it the up to the critical crack length of 0.86 m to find the number of cycles

$$\int_{a_0}^{a_c} \frac{1}{a^m} \, da = \int_0^N \left(C \cdot \left((\sigma_{\max} - \sigma_{\min}) \cdot \sqrt{\pi}\right)^m\right) \, dN \tag{4}$$

$$N = 3948935.6547$$

Therefore Material B will endure close to 4 million cycles before Fracture.

1.2.5 The most Significant factor in safe design of the component

Plane strain fracture toughness (K_{IC}) is often considered the most important factor in safe design against catastrophic failure. This is because K_{IC} directly measures a material's resistance to crack propagation under static loading conditions. A high K_{IC} value indicates that the material can withstand the presence of cracks without rapid propagation, thus reducing the likelihood of sudden failure. Additionally, K_{IC} provides a critical parameter for assessing the structural integrity of components, making it a fundamental consideration in engineering design to ensure safety and reliability.

The other two factors are also useful to help us determine the material behaviour like the threshold stress intensity factor range (ΔK_{th}) which defines the stress level below which crack growth is negligible, helping to prevent sudden failure initiation. The rate of crack growth in the Paris regime is also significant as it indicates how quickly cracks propagate under cyclic loading, allowing engineers to predict component life and schedule maintenance.

In summary, all three factors contribute to safe design, with K_{IC} and ΔK_{th} playing key roles in preventing catastrophic failure initiation, while the crack growth rate helps in predicting long-term component reliability. However, the most important parameter is the Plane strain fracture toughness (K_{IC})

2 Case Study 2

2.1 Fatigue Life Testing for Evaluating Suitability of New Material

2.1.1 Wöhler curve

Thanks to the data recorded during the tests, we were able to recreate a graph allowing us to highlight the minimum stress as a function of the cycle for a fatigue breakage.

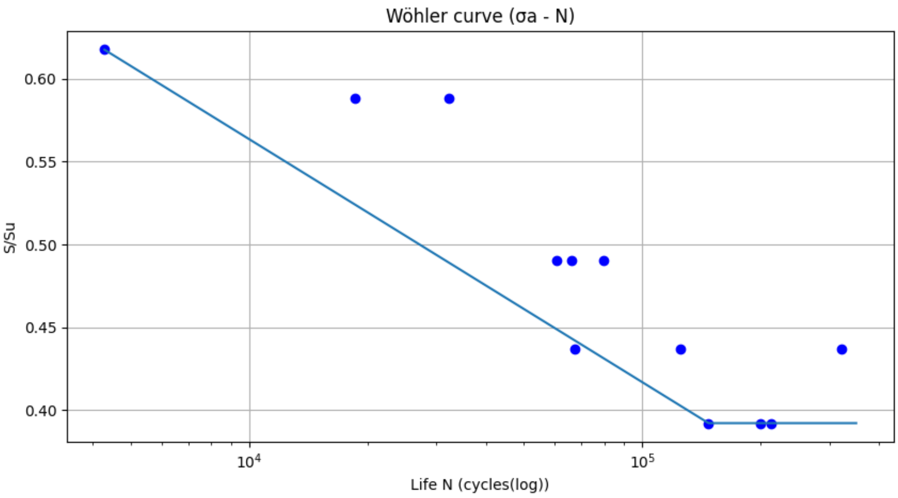

Figure 7: Wöhler curve ($\sigma a - N$)

2.1.2 Sine curves for R = -0.5 and R = 0

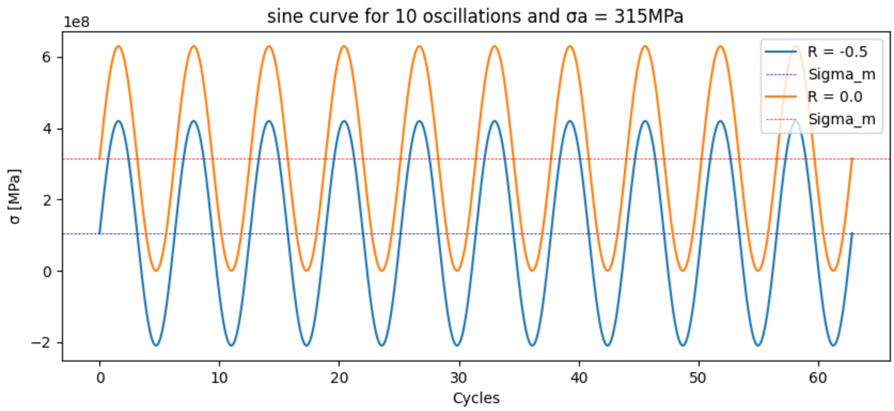

Figure 8: Sine curve for R = -0.5 and for R = 0

2.1.3 Effect of R on fatigue life

As we can see in the graph in the previous point, if R goes from -0.5 to 0 this means that the maximum load will be greater since $R = \frac{\sigma_{min}}{\sigma_{max}}$ and since the maximum load is greater, the life of the material will be shorter.

2.2 Understand Material Behavior at High Fatigue Cycles

2.2.1 Graph $\Delta\epsilon - N$

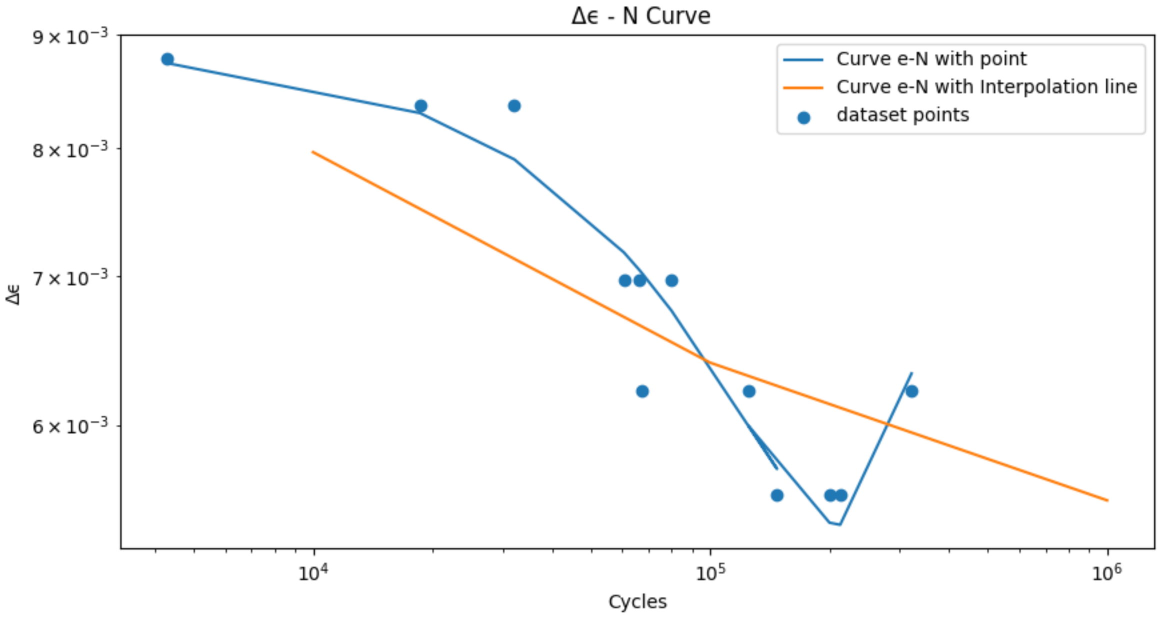

Figure 9: Delta Epsilon - N Curve

Thanks to the data provided and that retrieved at point one of the exercise, we were able to draw the $\Delta\epsilon - N$ graph. By comparing with the course graphs we notice that the curves are quite similar and have the same shape.

2.2.2 Numerical values of the coefficients of the law

We are here in the case of a high fatigue cycle. This means that we must use the Baskin law. We easily find the coefficients thanks to the previous graph :

$\Delta\epsilon = \frac{\Delta\sigma}{E} = \frac{C_1/E}{N_f^b}$ where $b \approx 0.5 - 0.13$ or $log(\Delta\epsilon) = -b.log(N_f) + log(C_1/E)$

We can also extrapolate the values to estimate the coefficients in LOw cycle fatigue, we must then use Coffin's law :

$\Delta\epsilon_{pl} = \frac{C_2}{N_f^C}$ where c » b, $c \approx 0.5$ or $log(\Delta\epsilon) = -c.log(N_f) + log(C_2)$

After some iterations we found the best parameter to minimize error, we took $b = 0.154$ (and $c \approx 0.5$). Finally, we got $C1 = 706744377$ (and $C2 = 0.02257$).

2.2.3 Amplitude stresses σ_a and mean stresses σ_m for different fatigue life

Thanks to the S-N graph we were able to find the values of σ_a for certain specific cycle values and by taking R = -0.5, find σ_m.

	10^4	10^5	10^6
$\frac{S}{S_u}$	0.56	0.45	0.39
σ_a [GPa]	285.6	229.5	198.9
σ_m [GPa]	190.4	153	132.6

Table 2: amplitude stresses σ_a and mean stresses σ_m

2.2.4 Haigh's endurance diagram

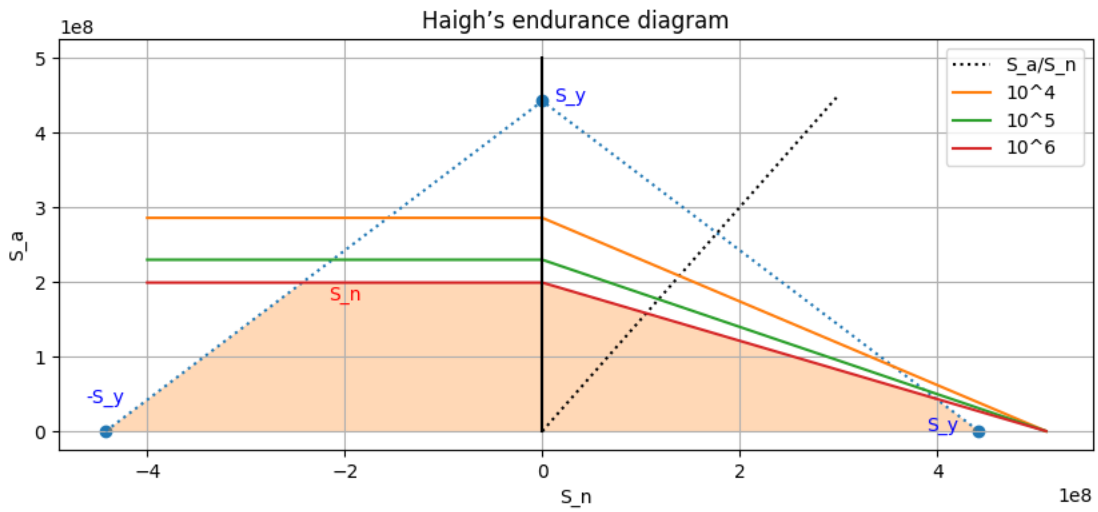

Figure 10: Haigh's endurance diagram

First, the orange area represents the safe area, free of failure and macroscopic plastic strains. It is formed from the minimum between the S-N curve and the yield curve.

Second, the black dotted line shows us that in case of higher load, the shaft will fail more easily due to fatigue before entering the deformation zone. To avoid this we could take a smaller R to reduce the slope of the line.