Copyright Notice

Derivatives of Activation Functions

Gradient Descent for Neural Networks

Backpropagation Intuition (Optional)

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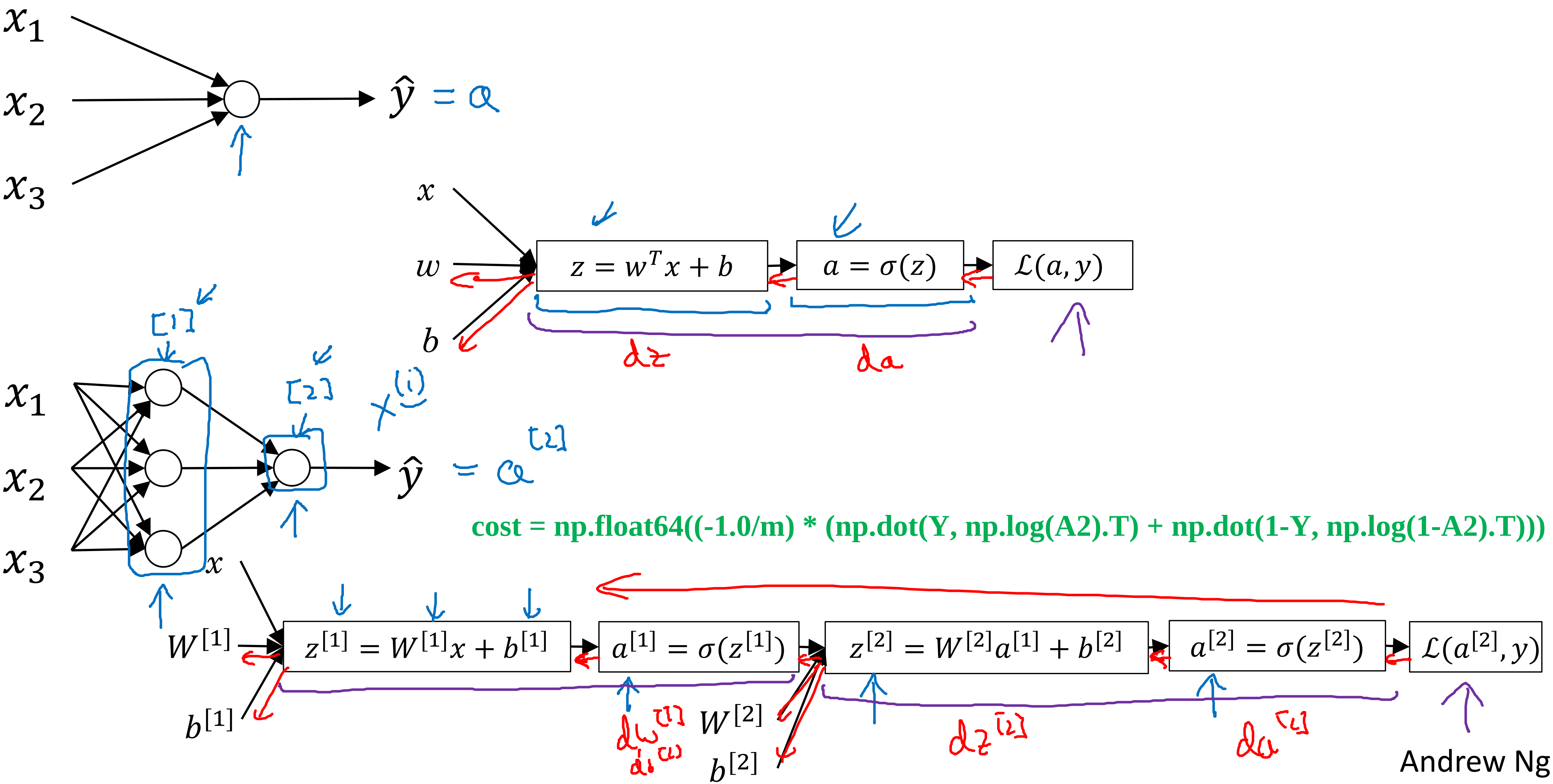
One hidden layer Neural Network

Neural Networks Overview

What is a Neural Network?



```
cost = np.float64((-1.0/m) * (np.dot(Y, np.log(A2).T) + np.dot(1-Y, np.log(1-A2).T)))
```

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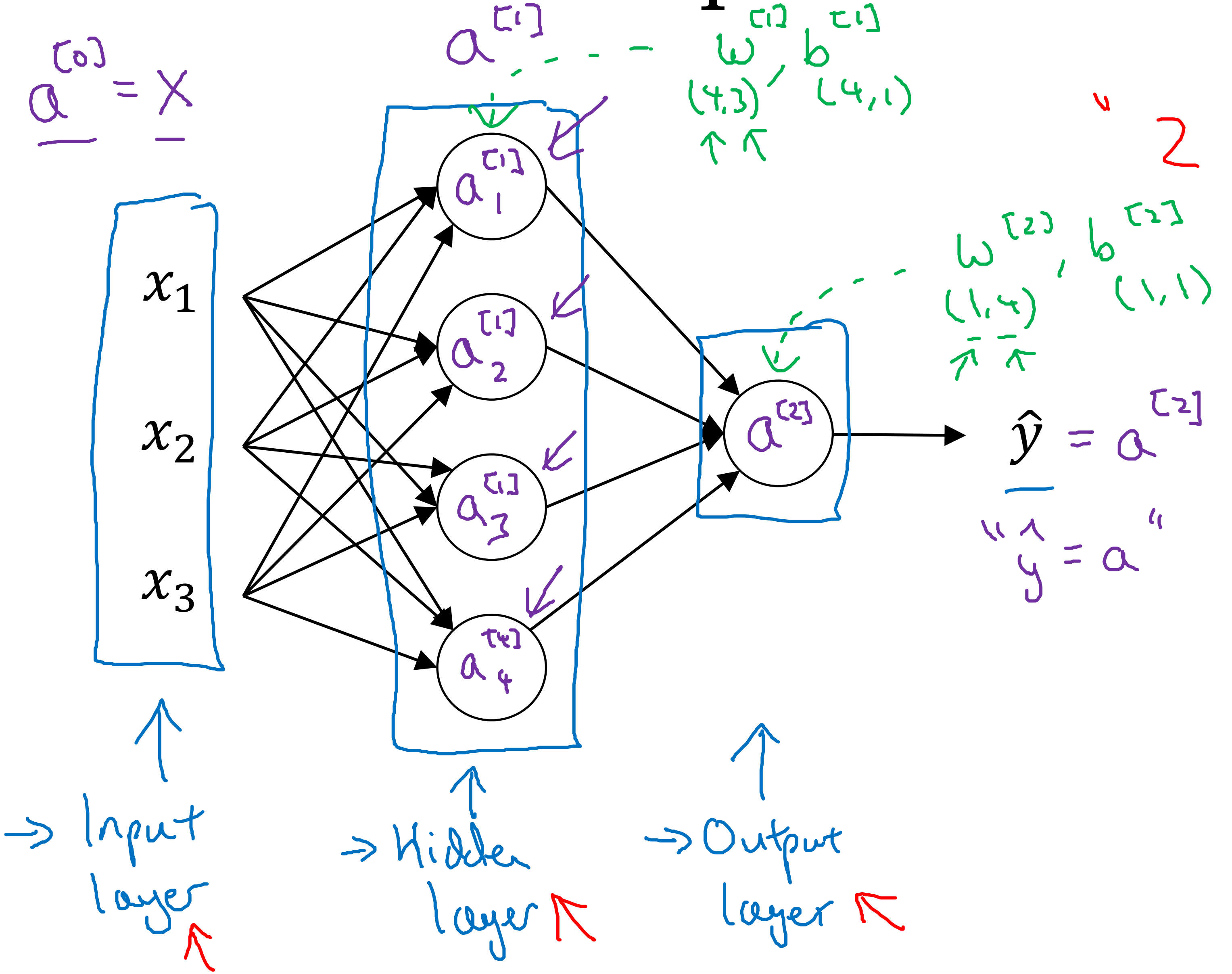
One hidden layer Neural Network

Neural Network Representation

Neural Network Representation



"2 layer NN"

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix}$$

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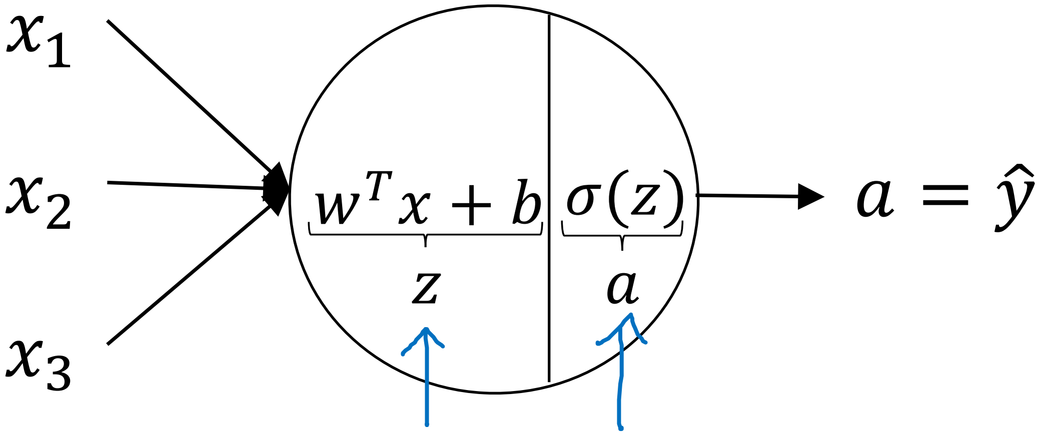
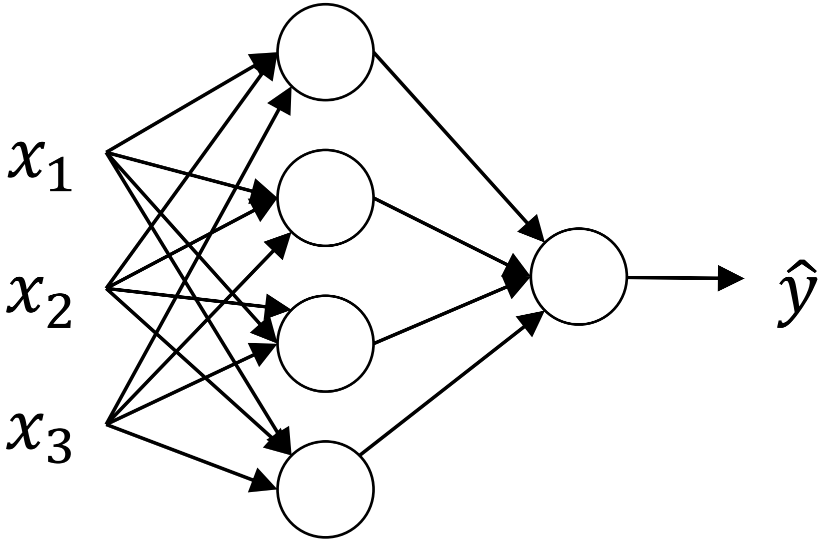
One hidden layer Neural Network

Computing a Neural Network's Output

Neural Network Representation

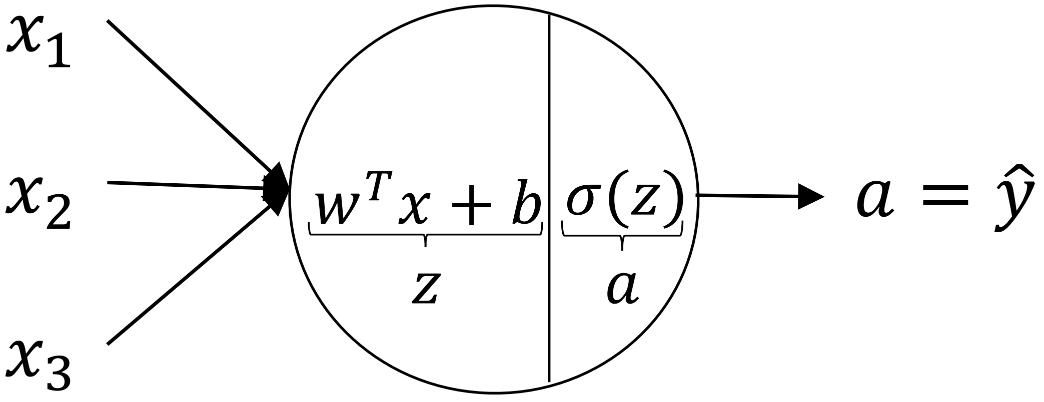




$$z = w^T x + b$$

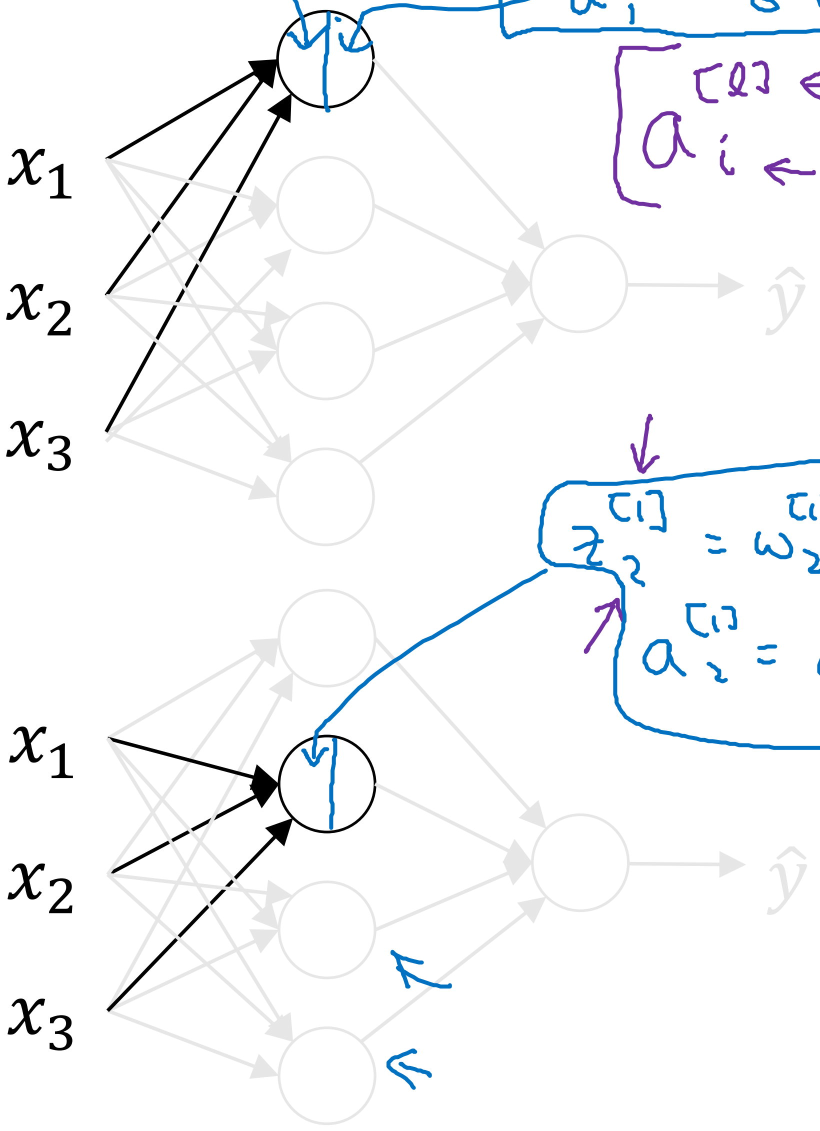
$$a = \sigma(z)$$

Neural Network Representation



$$z = w^Tx + b$$

$$a = \sigma(z)$$



$$z_1^{[1]} = w_1^{[1]T} x + b_1^{[1]}$$

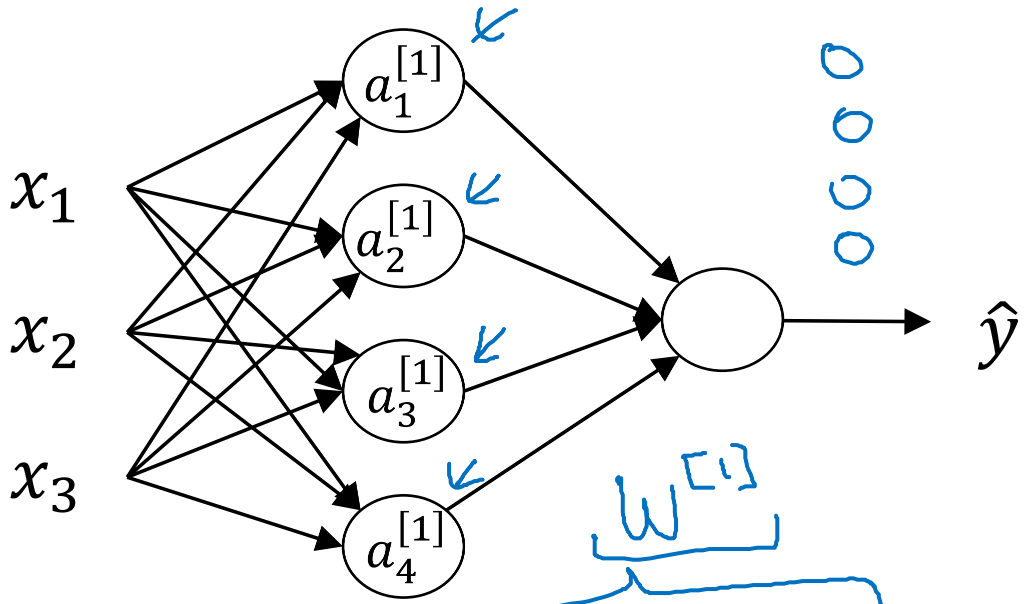
$$a_1^{[1]} = \sigma(z_1^{[1]})$$

$a_i^{[l]}$ ← layer
← node in layer.

$$z_2^{[1]} = w_2^{[1]T} x + b_2^{[1]}$$

$$a_2^{[1]} = \sigma(z_2^{[1]})$$

Neural Network Representation



$(w_1^{[1]})^T x$ $\qquad a^{[1]}$

$$z_1^{[1]} = w_1^{[1]T}x + b_1^{[1]}, \quad a_1^{[1]} = \sigma(z_1^{[1]})$$

$$z_2^{[1]} = w_2^{[1]T}x + b_2^{[1]}, \quad a_2^{[1]} = \sigma(z_2^{[1]})$$

$$z_3^{[1]} = w_3^{[1]T}x + b_3^{[1]}, \quad a_3^{[1]} = \sigma(z_3^{[1]})$$

$$z_4^{[1]} = w_4^{[1]T}x + b_4^{[1]}, \quad a_4^{[1]} = \sigma(z_4^{[1]})$$

$$z^{[1]} = \overbrace{\begin{bmatrix} — w_1^{[1]T} — \\ — w_2^{[1]T} — \\ — w_3^{[1]T} — \\ — w_4^{[1]T} — \end{bmatrix}}^{W^{[1]}}_{(4,3)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} b_1^{[1]} \\ b_2^{[1]} \\ b_3^{[1]} \\ b_4^{[1]} \end{bmatrix}}_{b^{[1]}} = \begin{bmatrix} w_1^{[1]T}x + b_1^{[1]} \\ w_2^{[1]T}x + b_2^{[1]} \\ w_3^{[1]T}x + b_3^{[1]} \\ w_4^{[1]T}x + b_4^{[1]} \end{bmatrix} = \begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \\ z_4^{[1]} \end{bmatrix}$$

$(4,1)$

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ \vdots \\ a_4^{[1]} \end{bmatrix} = \sigma(z^{[1]})$$

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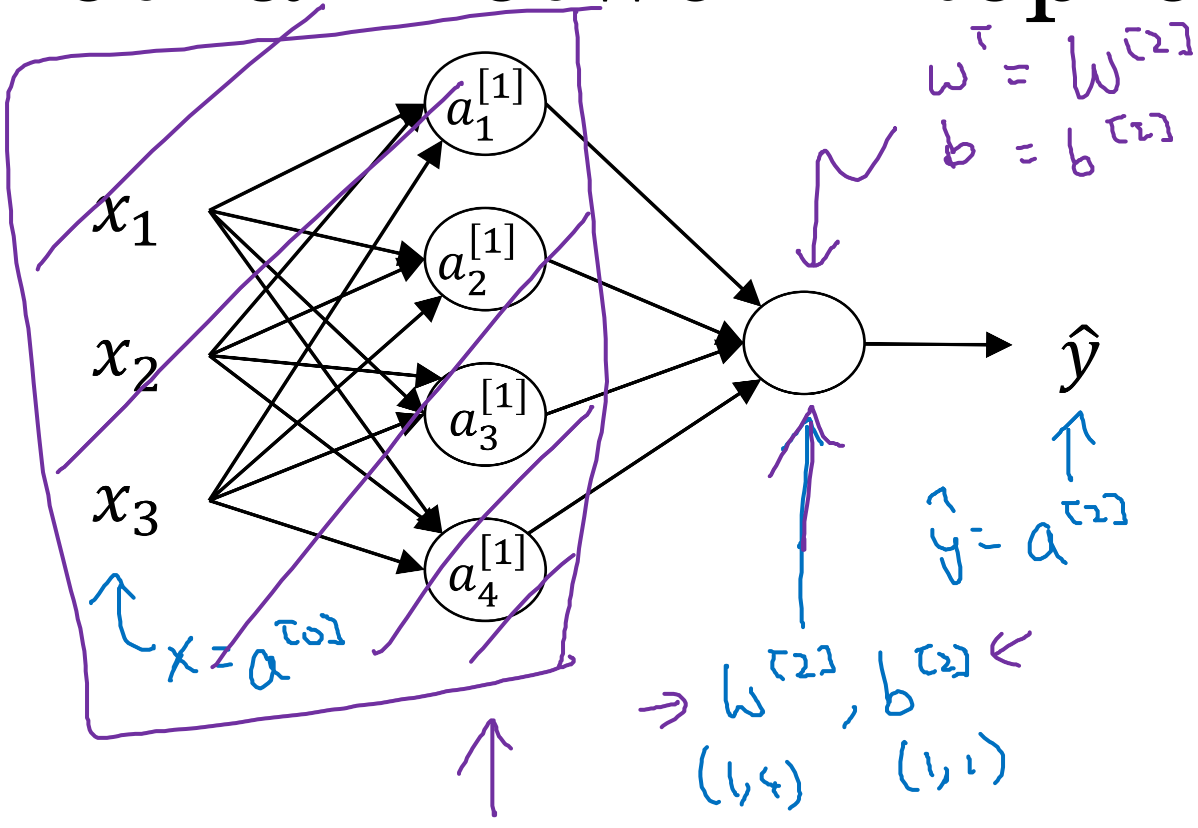
Neural Network Representation learning



$z = w^T x + b$

$\hat{y} = a = \sigma(z)$

Given input x:

$$\rightarrow \underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\overset{a^{[0]}}{x}} + \underset{(4,1)}{b^{[1]}}$$

$$\rightarrow \underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\rightarrow \underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\rightarrow \underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

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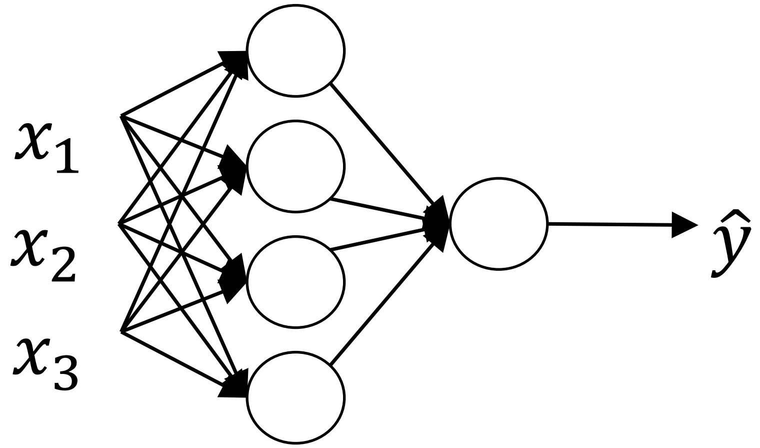
One hidden layer Neural Network

Vectorizing across multiple examples

Vectorizing across multiple examples



$$x \longrightarrow a^{[2]} = \hat{y}$$

$$x^{(1)} \longrightarrow a^{[2](1)} = \hat{y}^{(1)}$$

$$x^{(2)} \longrightarrow a^{[2](2)} \quad \hat{y}^{(2)}$$

$$\vdots$$

$$x^{(m)} \longrightarrow a^{[2](m)} \quad \hat{y}^{(m)}$$

$$a^{[2](i)}$$

← example i

layer 2

$$\left\{\begin{aligned} z^{[1]} &= W^{[1]}x + b^{[1]} \\ a^{[1]} &= \sigma(z^{[1]}) \\ z^{[2]} &= W^{[2]}a^{[1]} + b^{[2]} \\ a^{[2]} &= \sigma(z^{[2]}) \end{aligned}\right\} \leftarrow$$

→ for $i = 1$ to m,

$$z^{[1](i)} = W^{[1]} x^{(i)} + b^{[1]}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = W^{[2]} a^{[1](i)} + b^{[2]}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

Vectorizing across multiple examples

```
for i = 1 to m:
    $\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$
    $\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$
    $\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$
```

layer **1**

layer **2**

layer **1**

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

layer **2**

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

(n_x, m)

training examples

hidden units.

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

hidden units

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One hidden layer Neural Network

Explanation for vectorized implementation

Justification for vectorized implementation

$$z^{[1](1)} = W^{[1]}x^{(1)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](2)} = W^{[1]}x^{(2)} + \cancel{b^{[1]}} \to 0, \quad z^{[1](3)} = W^{[1]}x^{(3)} + \cancel{b^{[1]}} \to 0$$

$$W^{[1]} = \begin{bmatrix} \text{———} \\ \text{———} \\ \text{———} \\ \text{———} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix}$$

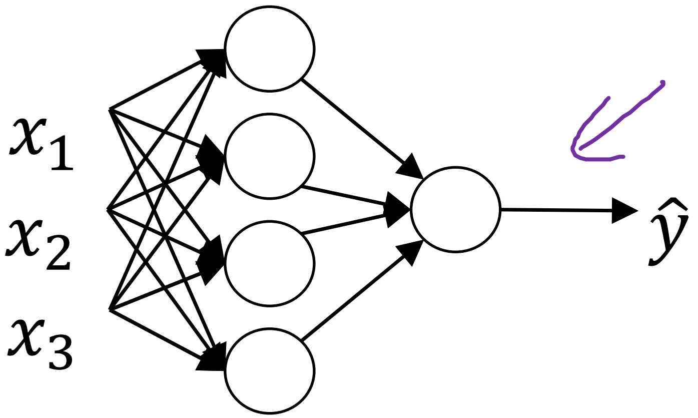
layer 1

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \\ \uparrow & \uparrow & \uparrow & \\ +b^{[1]} & +b^{[1]} & +b^{[1]} & \end{bmatrix} = Z^{[1]}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

Recap of vectorizing across multiple examples



$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$\underline{A^{[1]}} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

```
for i = 1 to m
    $\boxed{z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}}$
    $a^{[1](i)} = \sigma(z^{[1](i)})$
    $z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$
    $a^{[2](i)} = \sigma(z^{[2](i)})$
```

$A^{[0]}$

$X = a^{[0]}$

$x^{(i)} = a^{[0](i)}$

$$Z^{[1]} = W^{[1]}\boxed{X} + b^{[1]} \leftarrow W^{[1]}A^{[0]} + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

One hidden layer Neural Network

Activation functions

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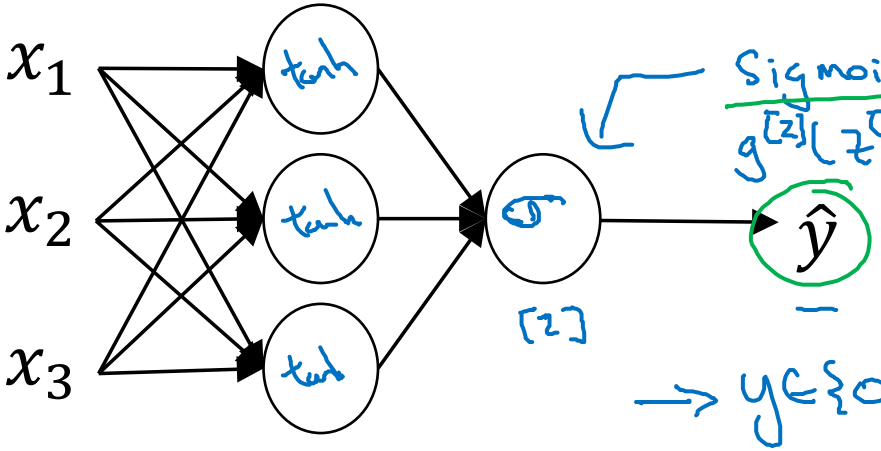
Activation functions

$g^{[1]}(z^{[1]}) = \tanh(z^{[1]})$



Sigmoid

$g^{[2]}(z^{[2]}) = \sigma(z^{[2]})$

[1]

→ $y \in \{0,1\}$

$0 \le \hat{y} \le 1$

-1 1

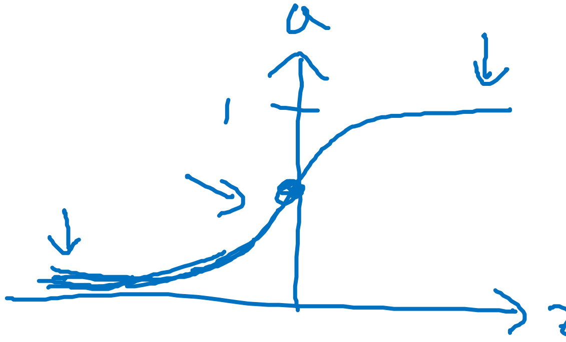
Given x:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

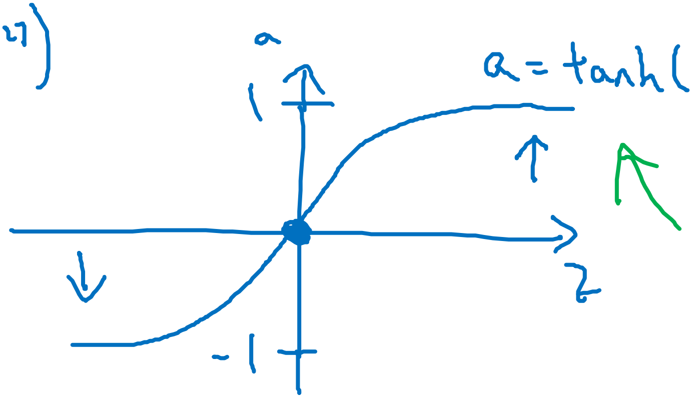
$$\rightarrow \quad a^{[1]} = \cancel{\sigma(z^{[1]})} \; g^{[1]}(z^{[1]})$$

$$z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

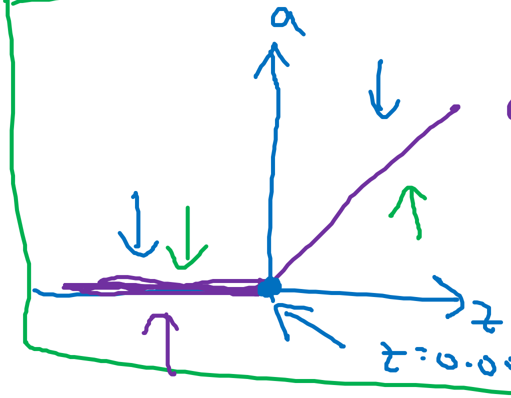
$$\rightarrow \quad a^{[2]} = \cancel{\sigma(z^{[2]})} \; g^{[2]}(z^{[2]})$$



$a = \frac{1}{1+e^{-z}}$



$a = \tanh(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$



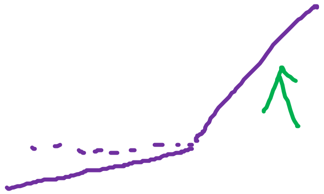
$z > 0$

$a = \max(0, z)$

ReLU

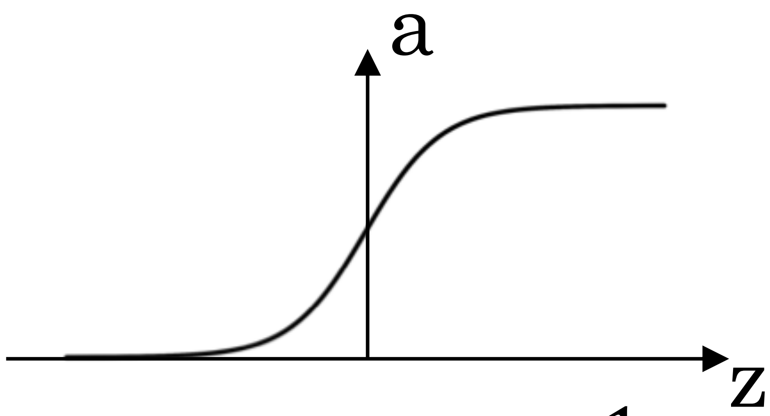
Rectified Linear Unit

$z = 0.0000000000000$

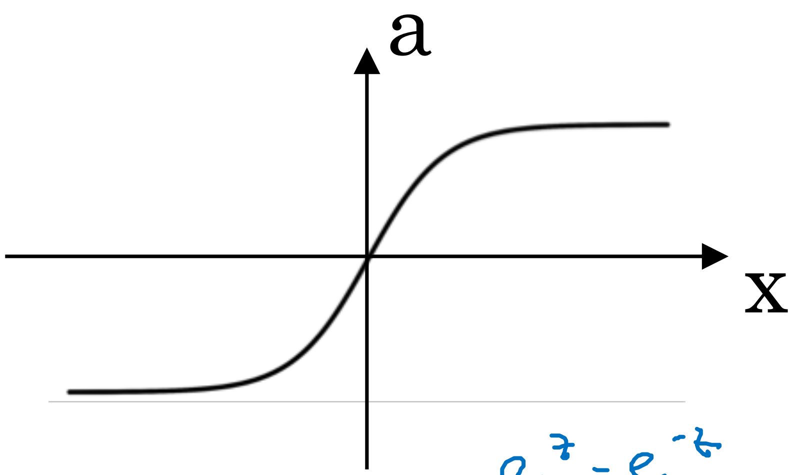


Leaky ReLU

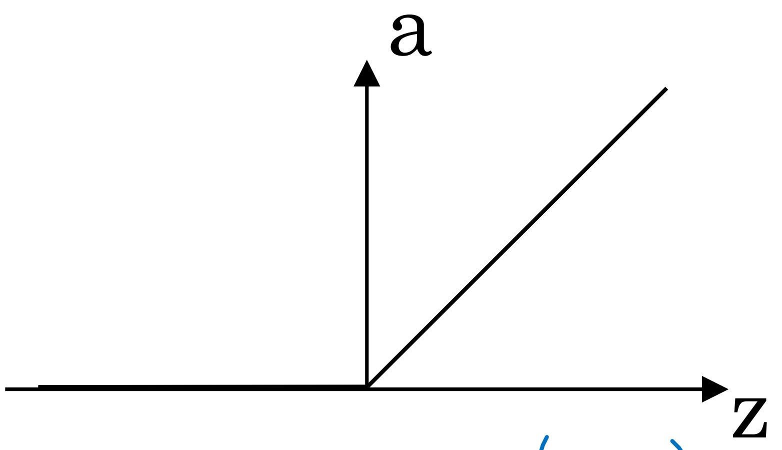
Pros and cons of activation functions



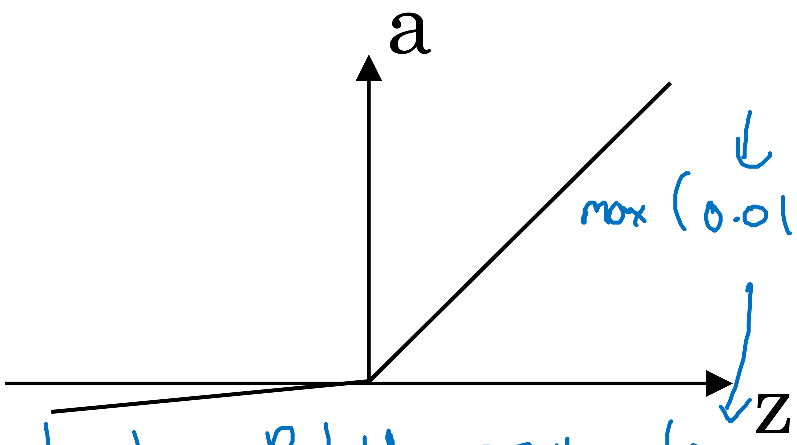
sigmoid: $a = \frac{1}{1+e^{-z}}$



tanh: $a = \frac{e^z - e^{-z}}{e^z + e^z}$



ReLU $a = \max(0, z)$



$\max(0.01z, z)$

Leaky ReLU $a = \max(0.01z, z)$

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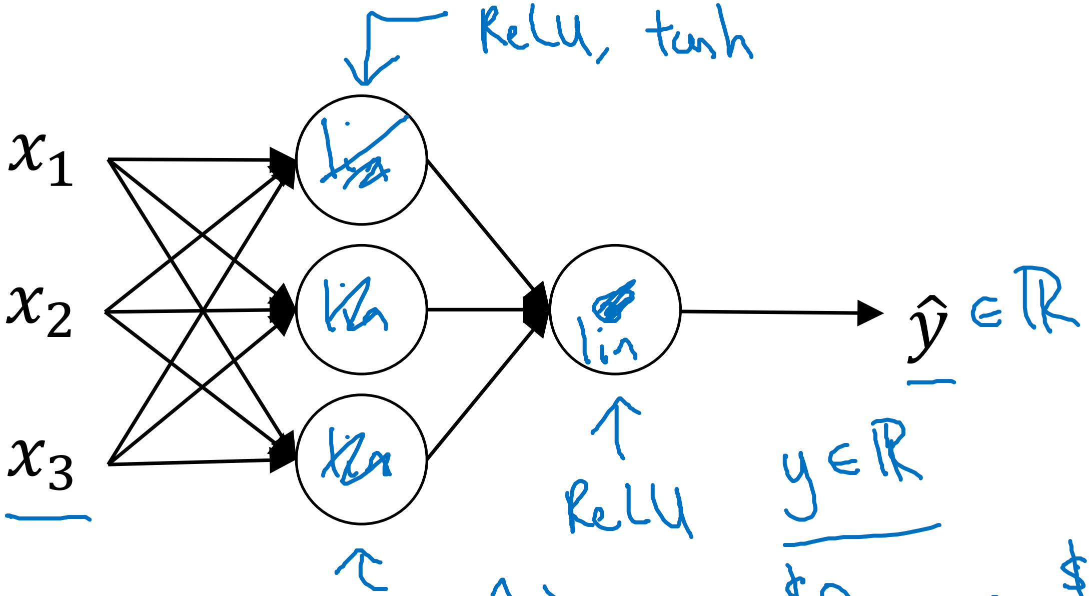
One hidden layer Neural Network

Why do you need non-linear activation functions?

Activation function



$y \in \mathbb{R}$

\$0 ..... \$1,000,000\$

$\hat{y} \geq 0$.

Given x:

→ $z^{[1]} = W^{[1]}x + b^{[1]}$

→ $a^{[1]} = \cancel{g^{[1]}(z^{[1]})}$ $z^{[1]}$

→ $z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$

→ $a^{[2]} = \cancel{g^{[2]}(z^{[2]})}$ $z^{[2]}$

$g(z) = z$

"linear activation function"

$$a^{[1]} = z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[2]} = z^{[2]} = W^{[2]}a^{[1]} + b^{[2]}$$

$$a^{[2]} = W^{[2]}(\underbrace{W^{[1]}x + b^{[1]}}_{a^{[1]}}) + b^{[2]}$$

$$= \underbrace{(W^{[2]}W^{[1]})}_{W'}x + \underbrace{(W^{[2]}b^{[1]} + b^{[2]})}_{b'}$$

$$= W'x + b'$$

$g(z) = z$

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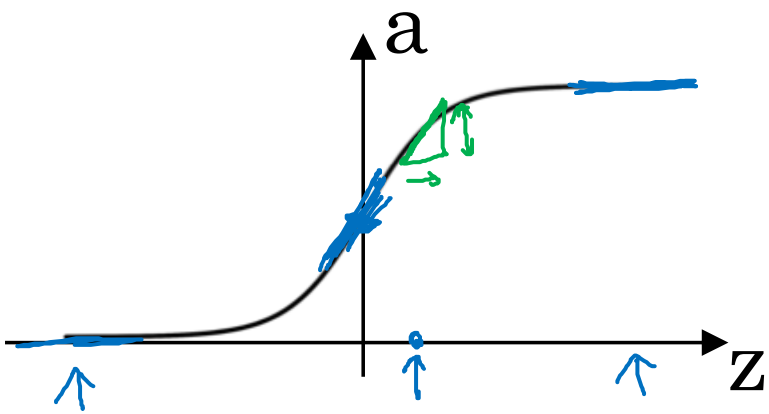
One hidden layer Neural Network

Derivatives of activation functions

Sigmoid activation function



$$\underline{g(z)} = \frac{1}{\underline{1+e^{-z}}}$$

$$a = g(z) = \frac{1}{1+e^{-z}}$$ **in Neural Network**

$$\boxed{g'(z)} = \boxed{\frac{d}{dz} g(z)} = \text{slope of } g(x) \text{ at } z$$

slope

$$= \frac{1}{1+e^{-z}}\left(1-\frac{1}{1+e^{-z}}\right)$$

$$= \underbrace{g(z)\left(1-g(z)\right)} \leftarrow$$

$$= \boxed{a(1-a)}$$

$$g'(z) = a(1-a)$$
$$\uparrow$$

in Neural Network

$z = 10. \quad g(z) \approx 1$

$\frac{d}{dz} g(z) \approx 1(1-1) \approx 0$

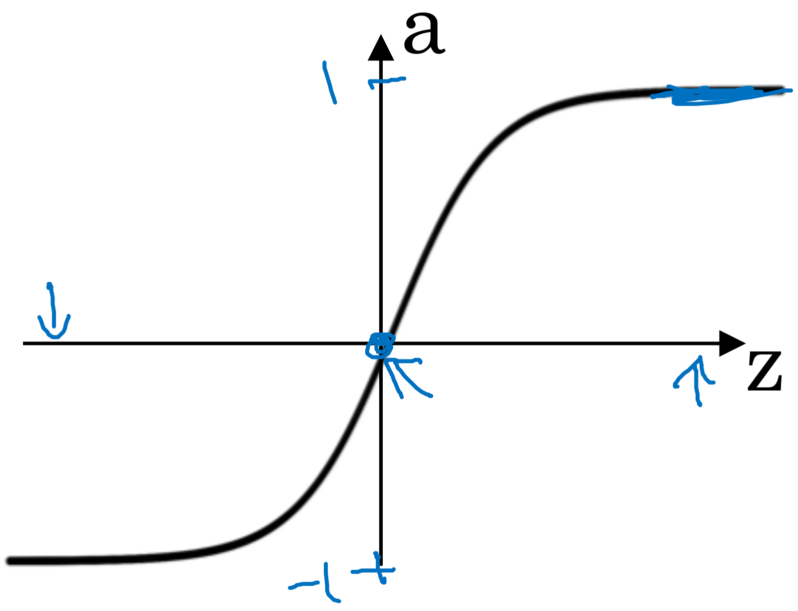
$z = -10 \quad g(z) \approx 0$

$\frac{d}{dz} g(z) \approx 0 \cdot (1-0) \approx 0$

$z = 0 \quad g(z) = \frac{1}{2}$

$\frac{d}{dz} g(z) = \frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$

Tanh activation function



Z1 = np.dot(W1, X) + b1
A1 = np.tanh(Z1)

$$g(z) = \tanh(z)$$

$$= \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$z = 10$ $\tanh(z) \approx 1$

$g'(z) \approx 0$

$z = -10$ $\tanh(z) \approx -1$

$g'(z) \approx 0$

$z = 0$ $\tanh(z) = 0$

$g'(z) = 1$

$$g'(z) = \frac{d}{dz} g(z) = \text{slope of } g(z) \text{ at } z$$

$$= \underline{1 - (\underline{\tanh}(z))^2} \leftarrow$$

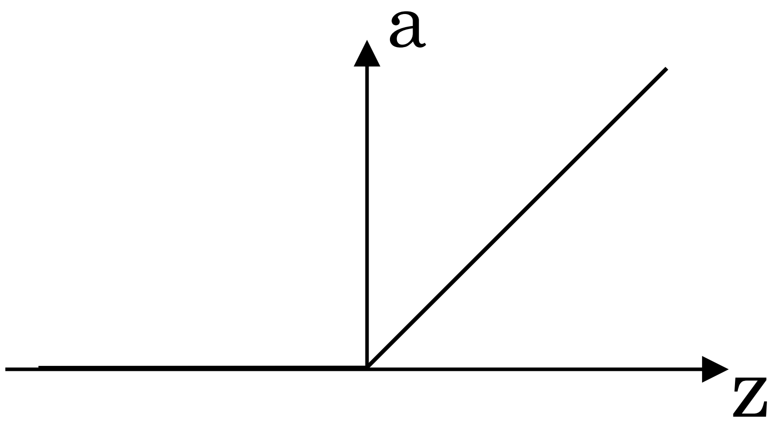
in Neural Network

$a = g(z)$,

slope

$$\underset{\uparrow}{g}'(z) = 1 - \underset{\uparrow}{a}^2$$

ReLU and Leaky ReLU

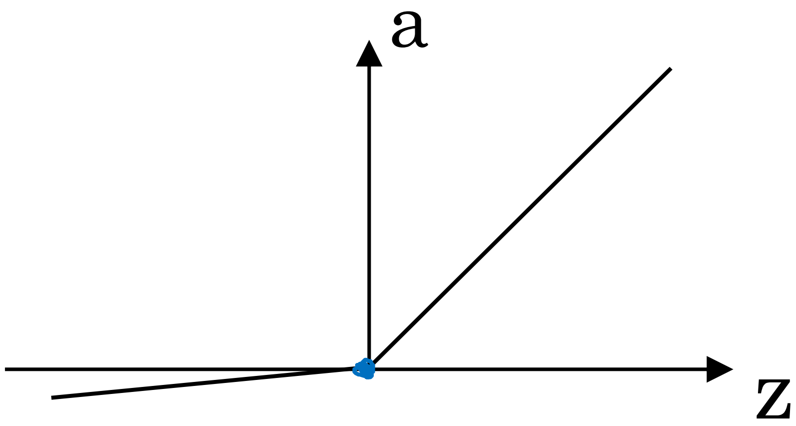


ReLU

$$g(z) = \max(0, z)$$

$$\rightarrow g'(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \\ \cancel{\text{undef}} & \cancel{\text{if } z = 0} \end{cases}$$

$$z = 0.00000\cdots 0$$



Leaky ReLU

$$g(z) = \max(0.01z, z)$$

$$g'(z) = \begin{cases} 0.01 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$$

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One hidden layer Neural Network

Gradient descent for neural networks

Gradient descent for neural networks

Parameters: $\underset{(n^{[1]}, n^{[0]})}{W^{[1]}}, \underset{(n^{[1]}, 1)}{b^{[1]}}, \underset{(n^{[2]}, n^{[1]})}{W^{[2]}}, \underset{(n^{[2]}, 1)}{b^{[2]}}$

$n_x = n^{[0]}, n^{[1]}, \underline{n^{[2]} = 1}$

Cost function: $J(W^{[1]}, b^{[1]}, \underset{\uparrow}{\underline{W^{[2]}}}, \underset{\uparrow}{\underline{b^{[2]}}}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\underset{\nwarrow a^{[2]}}{\hat{y}}, y)$

Gradient descent:

→ Repeat {

→ Compute predicts $(\hat{y}^{(i)}, i = 1 \dots, m)$

$\underline{dW^{[1]}} = \frac{\partial J}{\partial W^{[1]}}, \quad \underline{db^{[1]}} = \frac{\partial J}{\partial b^{[1]}}, \dots$

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

$W^{[2]} := \dots \qquad b^{[2]} := \dots$

}

Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]}X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]})$ ←

$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y$ ←

$Y = [y^{(1)}\ y^{(2)} \dots, y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$

$(n^{[2]},)$ ←

$(n^{[2]}, 1)$ ←

$db^{[2]} = \frac{1}{m}$ np.sum($dZ^{[2]}$, axis=1, keepdims=True)

$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]},m)}$

↑ element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$db^{[1]} = \frac{1}{m}$ np.sum($dZ^{[1]}$, axis=1, keepdims=True)

$(n^{[1]}, 1)$ $(n^{[1]},)$ ↑ reshape

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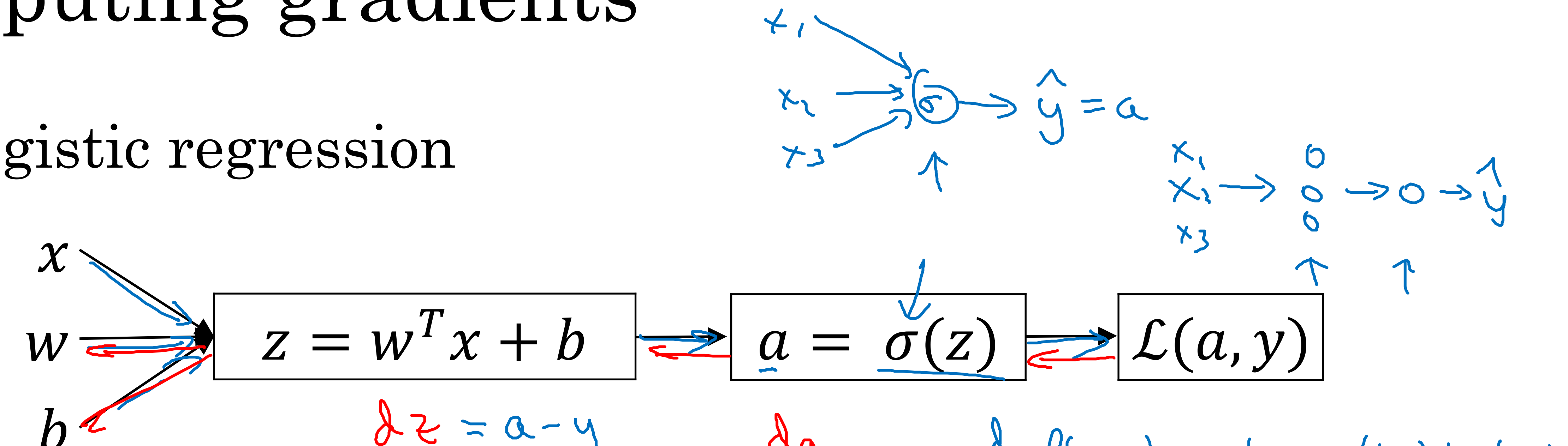
One hidden layer Neural Network

Backpropagation intuition (Optional)

Computing gradients

Logistic regression



$dz = a - y$

$da = \frac{d}{da}\mathcal{L}(a,y) = -y\log a - (1-y)\log(1-a)$

$= -\frac{y}{a} + \frac{1-y}{1-a}$

$dw = dz \cdot x$

$db = dz$

$\boxed{dz = da \cdot g'(z)}$

$g(z) = \sigma(z)$

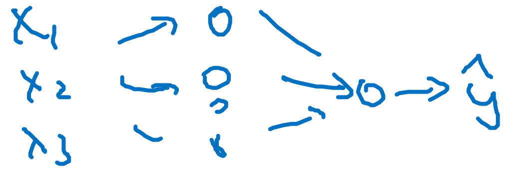
$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$

"dz" = "da"

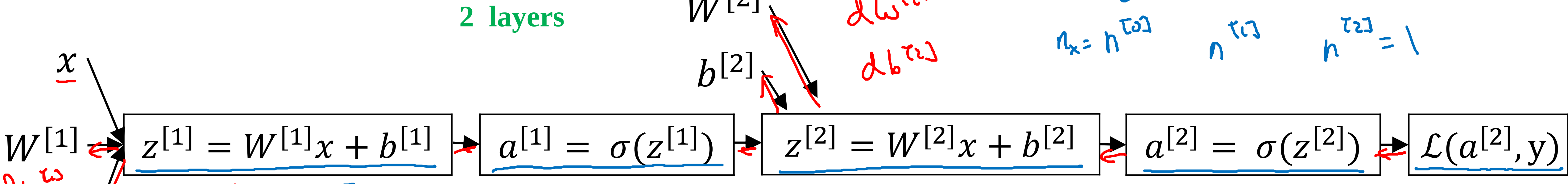
$\frac{d}{dz} g(z) = g'(z)$

Neural network gradients

2 layers



$n_x = n^{[0]}$ $n^{[1]}$ $n^{[2]} = 1$



x

$W^{[1]}$

$b^{[1]}$

$W^{[2]}$

$b^{[2]}$

$z^{[1]} = W^{[1]}x + b^{[1]}$

$a^{[1]} = \sigma(z^{[1]})$

$z^{[2]} = W^{[2]}x + b^{[2]}$

$a^{[2]} = \sigma(z^{[2]})$

$\mathcal{L}(a^{[2]}, y)$

$dW^{[1]}$ $db^{[1]}$ $dW^{[2]}$ $db^{[2]}$

$dz^{[1]} = W^{[2]T} dz^{[2]}$

$* g^{[1]\prime}(z^{[1]})$

← element wise product

$da^{[1]} = \dots$

$dz^{[2]} = a^{[2]} - y$

$da^{[2]}$

$dW^{[2]} = dz^{[2]} a^{[1]T}$

$db^{[2]} = dz^{[2]}$

"$dw = dz \cdot x$"

$W^{[1]}$: [——]

$W^{[2]}$ $(n^{[2]}, n^{[1]})$

$z^{[2]}, dz^{[2]}$ $(n^{[2]}, 1)$ $-(1,1)$

$z^{[1]}, dz^{[1]}$ $(n^{[1]}, 1)$

$dz^{[1]} = \boxed{W^{[2]T} dz^{[2]}} * \boxed{g^{[1]\prime}(z^{[1]})}$

$(n^{[1]}, 1) = (n^{[1]}, n^{[2]}) (n^{[2]}, 1) * (n^{[1]}, 1)$

foo

dfoo

$W^{[1]}$

$dW^{[2]}$

$dW^{[1]} = dz^{[1]} \cdot x^T$

$db^{[1]} = dz^{[1]}$

$a^{[0]T}$

Summary of gradient descent

$$dz^{[2]} = a^{[2]} - y$$

$$dW^{[2]} = dz^{[2]}a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$dz^{[1]} = W^{[2]T}dz^{[2]} * g^{[1]\prime}(\mathrm{z}^{[1]})$$

$$dW^{[1]} = dz^{[1]}x^T$$

$$db^{[1]} = dz^{[1]}$$

Vectorized Implementation:

$$z^{[1]} = W^{[1]}x + b^{[1]}$$

$$a^{[1]} = g^{[1]}(z^{[1]})$$

$$Z^{[1]} = \begin{bmatrix} \vert & \vert & & \vert \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ \vert & \vert & & \vert \end{bmatrix}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$A^{[1]} = g^{[1]}(Z^{[1]}).$$

Summary of gradient descent (backward_propagation)

in NumPy

với **Sigmoid** function

$$\underline{dz^{[2]}} = \underline{a^{[2]}} - \underline{y}$$

$$dW^{[2]} = dz^{[2]} a^{[1]^T}$$

$$db^{[2]} = dz^{[2]}$$

$$\underset{(n^{[1]},1)}{dz^{[1]}} = W^{[2]T} dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dW^{[1]} = dz^{[1]} x^T$$

$$db^{[1]} = dz^{[1]}$$

$$\underline{dZ^{[2]} = A^{[2]} - Y}$$

```
dZ2 = A2 - Y
```

$$J(\ldots) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$$

$$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]^T}$$

```
dW2 = (1/m) * np.dot(dZ2,A1.T)
```

```
db2 = (1/m) * np.sum(dZ2, axis=1, keepdims=True)
```

$$db^{[2]} = \frac{1}{m} np.sum(dZ^{[2]}, axis = 1, keepdims = True)$$

elementwise product

$$\underset{(n^{[1]},m)}{dZ^{[1]}} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]},m)} * \underbrace{g^{[1]\prime}(Z^{[1]})}_{(n^{[1]},m)}$$

```
dZ1 = np.dot(W2.T,dZ2) * (1 - np.power(A1,2))
```

$$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$$

```
dW1 = (1/m) * np.dot(dZ1,X.T)
```

$$db^{[1]} = \frac{1}{m} np.sum(dZ^{[1]}, axis = 1, keepdims = True)$$

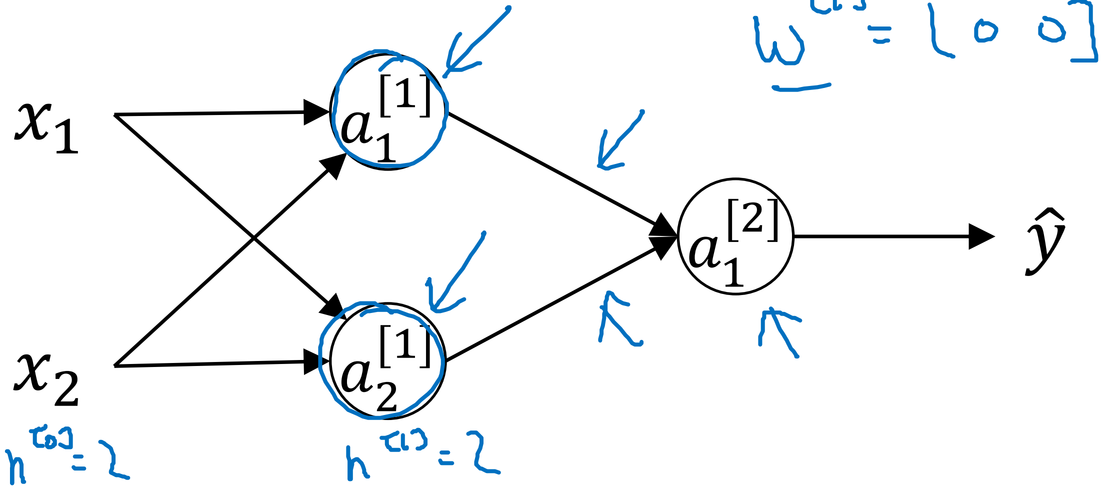
```
db1 = (1/m) * np.sum(dZ1, axis=1, keepdims=True)
```

One hidden layer Neural Network

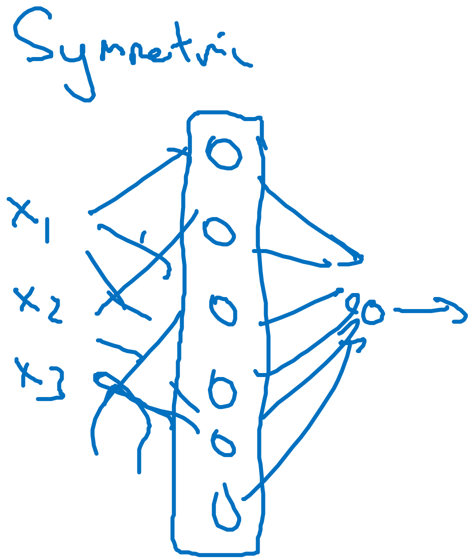
Random Initialization

deeplearning.ai

What happens if you initialize weights to zero?



$$W^{[2]} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$



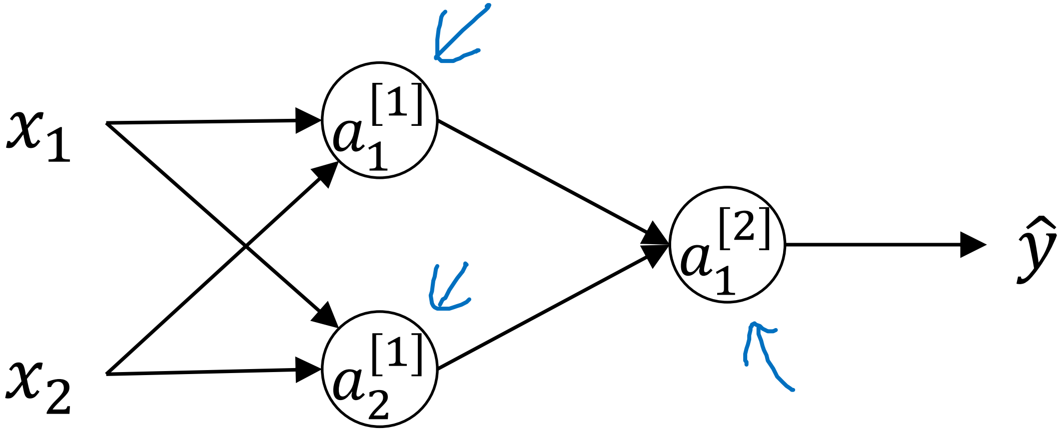
$$W^{[1]} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad b^{[1]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$W^{[1]} = \begin{bmatrix} \cdots\cdots \\ \cdots\cdots \end{bmatrix}$$

$$a_1^{[1]} = a_2^{[1]} \qquad dz_1^{[1]} = dz_2^{[1]}$$

$$dw = \begin{bmatrix} u & v \\ u & v \end{bmatrix} \qquad W^{[1]} = W^{[1]} - \alpha dW$$

Random initialization



W1 = np.random.randn(n_h, n_x)* 0.01
b1 = np.zeros((n_h, 1))

→ $w^{[1]} =$ np.random.randn((2,2)) $* \underline{0.01}$

100 ?

$b^{[1]} =$ np.zeros((2,1))

$w^{[2]} =$ np.random.randn((1,2)) $* 0.01$

$b^{[2]} = 0$

→ $z^{[1]} = w^{[1]} x + b^{[1]}$

$a^{[1]} = g^{[1]}(z^{[1]})$