PCA Computation - Tuan Tran

Covariate Matrix $C = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

1. Find eigen values by solving : $\det(C - \lambda I) = 0$

$\Leftrightarrow \det \begin{pmatrix} 1-\lambda & -2 & 0 \\ -2 & 5-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{pmatrix} = 0$

$\Leftrightarrow (1-\lambda)(5-\lambda)(2-\lambda) + (-2).0.0 + 0(-2).0$
$- (0).(5-\lambda).0 - 0.0.(1-\lambda) - (2-\lambda)(-2)(-2) = 0$

$\Leftrightarrow (2-\lambda)(\lambda^2 - 6\lambda + 1) = 0$

$\Leftrightarrow \left[\begin{array}{l} \lambda = 2 \\ \lambda = 3 \pm 2\sqrt{2} \end{array}\right.$

2. Find i^{th} eigenvector by solving: $Ce_i = \lambda_i e_i$

• $\lambda = 2$: $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} e_{1,1} \\ e_{1,2} \\ e_{1,3} \end{bmatrix} = 2 \begin{bmatrix} e_{1,1} \\ e_{1,2} \\ e_{1,3} \end{bmatrix} \rightarrow \begin{cases} e_{1,1} - 2e_{1,2} = 2e_{1,1} \\ -2e_{1,1} + 5e_{1,2} = 2e_{1,2} \\ 2e_{1,3} = 2e_{1,3} \end{cases}$

(Choose the eigen vector with unit length)

$\longrightarrow e_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

• $\lambda = 3 + 2\sqrt{2}$

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- $\lambda = 3 + 2\sqrt{2}$

$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} e_{2,1} \\ e_{2,2} \\ e_{2,3} \end{bmatrix} = (3+2\sqrt{2}) \begin{bmatrix} e_{2,1} \\ e_{2,2} \\ e_{2,3} \end{bmatrix}$$

$$\Rightarrow \begin{cases} e_{2,1} - 2e_{2,2} = (3+2\sqrt{2})\, e_{2,1} \\ -2e_{2,1} + 5e_{2,2} = (3+2\sqrt{2})\, e_{2,2} \\ 2e_{2,3} = (3+2\sqrt{2})\, e_{2,3} \end{cases} \rightarrow \begin{cases} e_{2,1} = 1 \\ e_{2,2} = (-1+\sqrt{2}) \\ e_{2,3} = 0 \end{cases}$$

Choose the eigen vector with unit length

$$\rightarrow e_2 = \begin{bmatrix} 0{,}9239 \\ 0{,}3827 \\ 0 \end{bmatrix}$$

- $\lambda = 3 - 2\sqrt{2}$

$$\rightarrow \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} e_{3,1} \\ e_{3,2} \\ e_{3,3} \end{bmatrix} = (3-2\sqrt{2}) \begin{bmatrix} e_{3,1} \\ e_{3,2} \\ e_{3,3} \end{bmatrix}$$

Choose the eigen vector with unit length

$$\rightarrow e_3 = \begin{bmatrix} 0{,}9239 \\ 0{,}3827 \\ 0 \end{bmatrix} \rightarrow \text{Same as } e_2$$