INTRODUCTION TO

# FUNCTIONAL PROGRAMMING

# OUTLINE

- Numerical Type Classes
- Semigroup
- Newtype
- Record
- Monoid
- Functor

# NUMERICAL DATA TYPES

| Type | Description |
|---|---|
| Double | Double-precision floating point. A common choice for floating-point data. |
| Float | Single-precision floating point. Often used when interfacing with C. |
| Int | Fixed-precision signed integer; minimum range [-2^29..2^29-1]. Commonly used. |
| Int8 | 8-bit signed integer |
| Int16 | 16-bit signed integer |
| Int32 | 32-bit signed integer |
| Int64 | 64-bit signed integer |
| Integer | Arbitrary-precision signed integer; range limited only by machine resources. Commonly used. |
| Rational | Arbitrary-precision rational numbers. Stored as a ratio of two Integers. |
| Word | Fixed-precision unsigned integer; storage size same as Int |
| Word8 | 8-bit unsigned integer |
| Word16 | 16-bit unsigned integer |
| Word32 | 32-bit unsigned integer |
| Word64 | 64-bit unsigned integer |

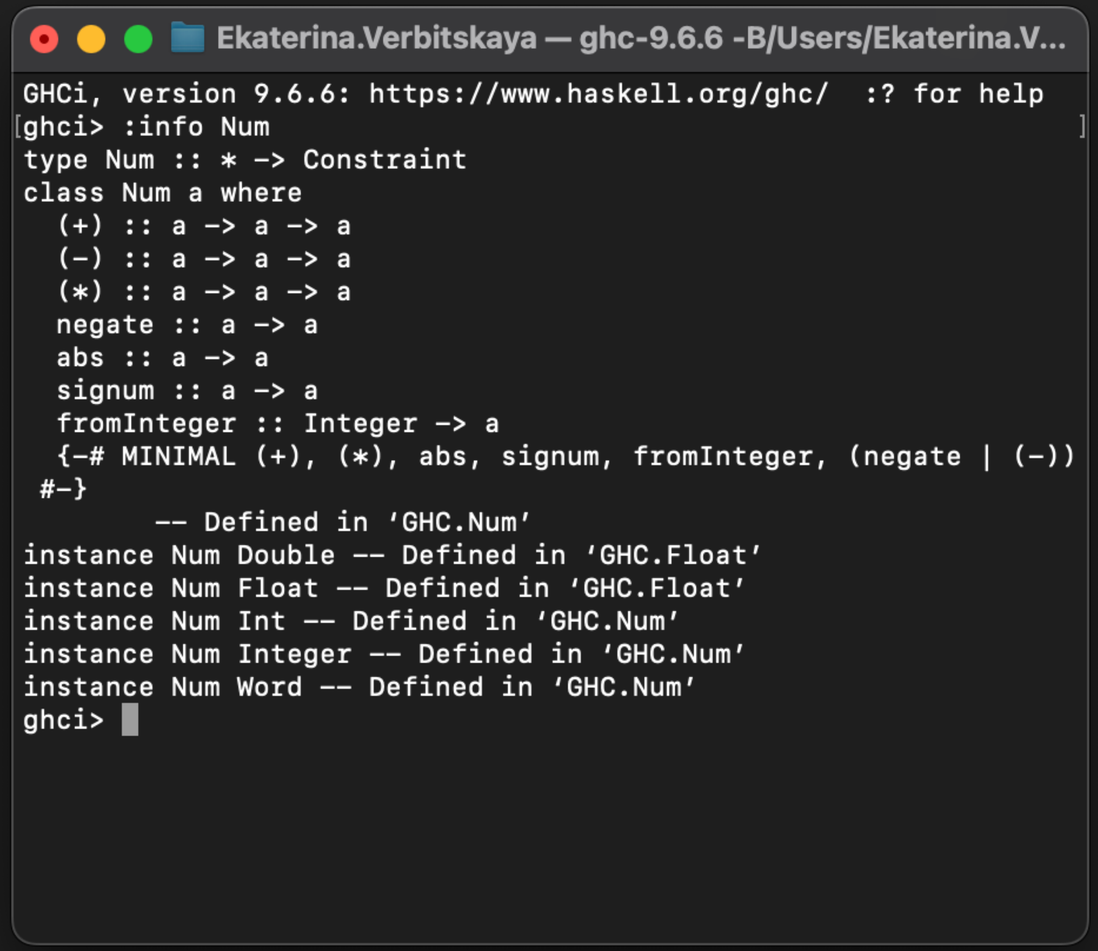
# SOME NUMERICAL OPERATIONS

| Function | Type | Description |
|---|---|---|
| (+), (-), (*) | Num a => a -> a -> a | Addition, subtraction, multiplication |
| (/) | Fractional a => a -> a -> a | Fractional divison |
| (^) | (Num a, Integral b) => a -> b -> a | Raise a number to a non-negative, integral power |
| (^^) | (Fractional a, Integral b) => a -> b -> a | Raise a fractional number to any integral power |
| (**), logBase | Floating a => a -> a -> a | Raise to the power of, log with explicit base |
| abs | Num a => a -> a | Absolute value |
| maxBound, minBound | Bounded a => a | The maximum/minimum value of a bounded type |
| pi | Floating a => a | Constant pi |
| sin, cos, tan, log, sqrt | Floating a => a -> a | Sine, cosine, tangent, natural logarithm, square root |
| div, mod, quot, rem | Integral a => a -> a -> a | Integer division, modulus, remainder |
| fromInteger | Num a => Integer -> a | Conversion from Integer into a numeric value |
| toInteger | Integral a => a -> Integer | Conversation of any Integral to Integer |
| fromRational | Fractional a => Rational -> a | Possibly lossy conversion from a Rational |
| toRational | Real a => a -> Rational | Convert losslessly to Rational |
| approxRational | RealFrac a => a -> a -> Rational | Ratio composition |
| fromIntegral | (Integral a, Num b) => a -> b | Convert from any Integral to any numeric type |
| round, truncate | (RealFrac a, Integral b) => a -> b | Rounds to nearest integer; truncates towards zero |

# TYPE CLASS INSTANCES FOR NUMERIC TYPES

| Type | Bits | Bounded | Floating | Fractional | Integral | Num | Real | RealFrac |
|---|---|---|---|---|---|---|---|---|
| Double | | | X | X | | X | X | X |
| Float | | | X | X | | X | X | X |
| Int | X | X | | | X | X | X | |
| Int16 | X | X | | | X | X | X | |
| Int32 | X | X | | | X | X | X | |
| Int64 | X | X | | | X | X | X | |
| Integer | X | | | | X | X | X | |
| Rational or any Ratio | | | | X | | X | X | X |
| Word | X | X | | | X | X | X | |
| Word16 | X | X | | | X | X | X | |
| Word32 | X | X | | | X | X | X | |
| Word64 | X | X | | | X | X | X | |

# NUM-BERS




`class Num a where` # Source

Basic numeric class.

The Haskell Report defines no laws for Num. However, `(+)` and `(*)` are customarily expected to define a ring and have the following properties:

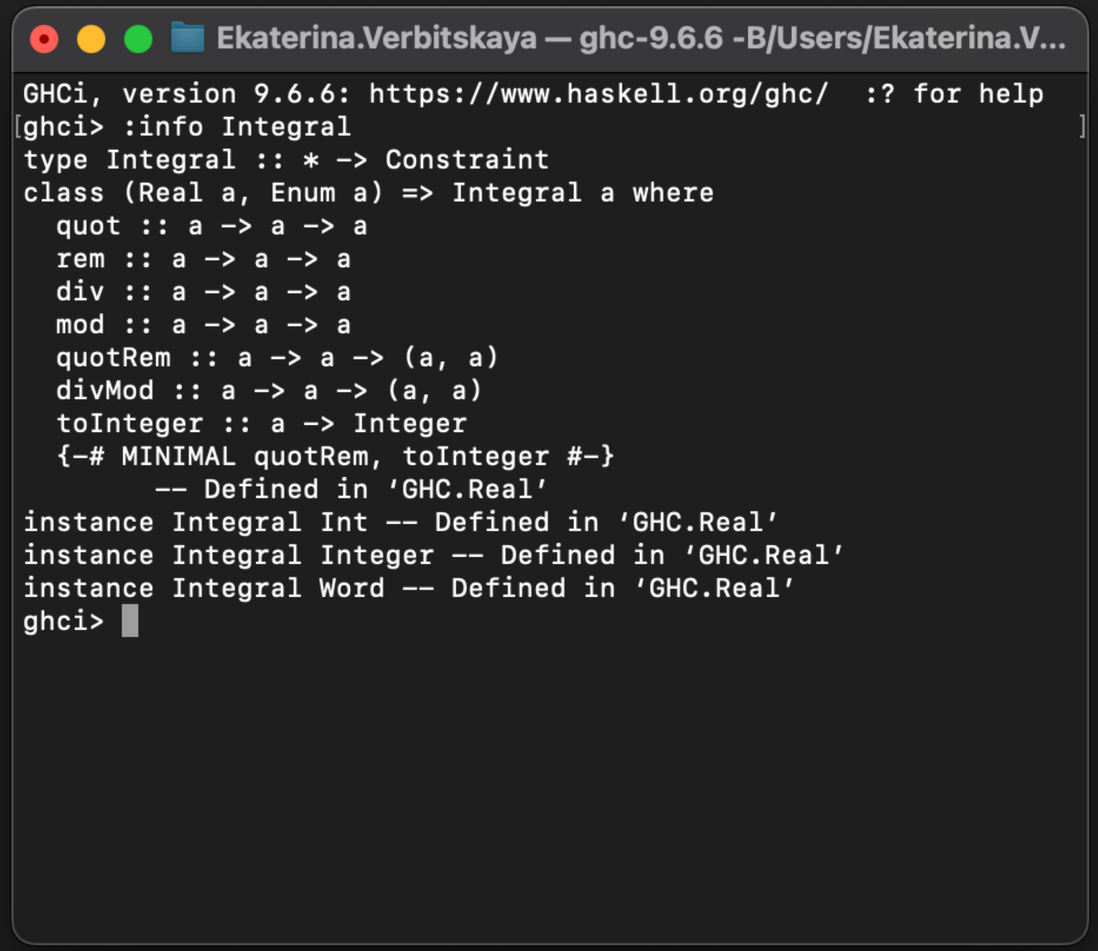
**Associativity of `(+)`**
`(x + y) + z = x + (y + z)`
**Commutativity of `(+)`**
`x + y = y + x`
**`fromInteger 0` is the additive identity**
`x + fromInteger 0 = x`
**`negate` gives the additive inverse**
`x + negate x = fromInteger 0`
**Associativity of `(*)`**
`(x * y) * z = x * (y * z)`
**`fromInteger 1` is the multiplicative identity**
`x * fromInteger 1 = x` and `fromInteger 1 * x = x`
**Distributivity of `(*)` with respect to `(+)`**
`a * (b + c) = (a * b) + (a * c)` and `(b + c) * a = (b * a) + (c * a)`
**Coherence with `toInteger`**
if the type also implements `Integral`, then `fromInteger` is a left inverse for `toInteger`, i.e. `fromInteger (toInteger i) == i`

Note that it *isn't* customarily expected that a type instance of both Num and Ord implement an ordered ring. Indeed, in `base` only `Integer` and `Rational` do.

**Minimal complete definition**

`(+), (*), abs, signum, fromInteger, (negate | (-))`

# INTEGRAL




`class (Real a, Enum a) => Integral a where` # Source

Integral numbers, supporting integer division.

The Haskell Report defines no laws for `Integral`. However, `Integral` instances are customarily expected to define a Euclidean domain and have the following properties for the `div`/`mod` and `quot`/`rem` pairs, given suitable Euclidean functions `f` and `g`:

- `x = y * quot x y + rem x y` with `rem x y = fromInteger 0` or `g (rem x y) < g y`
- `x = y * div x y + mod x y` with `mod x y = fromInteger 0` or `f (mod x y) < f y`

An example of a suitable Euclidean function, for `Integer`'s instance, is `abs`.

In addition, `toInteger` should be total, and `fromInteger` should be a left inverse for it, i.e. `fromInteger (toInteger i) = i`.

**Minimal complete definition**

`quotRem, toInteger`

# FRACTIONAL

Ekaterina.Verbitskaya — ghc-9.6.6 -B/Users/Ekaterina.V...

```
GHCi, version 9.6.6: https://www.haskell.org/ghc/  :? for help
ghci> :info Fractional
type Fractional :: * -> Constraint
class Num a => Fractional a where
  (/) :: a -> a -> a
  recip :: a -> a
  fromRational :: Rational -> a
  {-# MINIMAL fromRational, (recip | (/)) #-}
        -- Defined in 'GHC.Real'
instance Fractional Double -- Defined in 'GHC.Float'
instance Fractional Float -- Defined in 'GHC.Float'
ghci>
```

```
class Num a => Fractional a where                                # Source
```

Fractional numbers, supporting real division.

The Haskell Report defines no laws for `Fractional`. However, `(+)` and `(*)` are customarily expected to define a division ring and have the following properties:

**`recip` gives the multiplicative inverse**
`x * recip x = recip x * x = fromInteger 1`

**Totality of `toRational`**
`toRational` is total

**Coherence with `toRational`**
if the type also implements `Real`, then `fromRational` is a left inverse for `toRational`, i.e. `fromRational (toRational i) = i`

Note that it *isn't* customarily expected that a type instance of `Fractional` implement a field. However, all instances in `base` do.

**Minimal complete definition**

`fromRational`, (`recip` | (`/`))

# FLOATING

Ekaterina.Verbitskaya — ghc-9.6.6 -B/Users/Ekaterina.V...

```
ghci> :info Floating
type Floating :: * -> Constraint
class Fractional a => Floating a where
  pi :: a
  exp :: a -> a
  log :: a -> a
  sqrt :: a -> a
  (**) :: a -> a -> a
  logBase :: a -> a -> a
  sin :: a -> a
  cos :: a -> a
  tan :: a -> a
  asin :: a -> a
  acos :: a -> a
  atan :: a -> a
  sinh :: a -> a
  cosh :: a -> a
  tanh :: a -> a
  asinh :: a -> a
  acosh :: a -> a
  atanh :: a -> a
  GHC.Float.log1p :: a -> a
  GHC.Float.expm1 :: a -> a
  GHC.Float.log1pexp :: a -> a
  GHC.Float.log1mexp :: a -> a
  {-# MINIMAL pi, exp, log, sin, cos, asin, acos, atan, sinh, co
sh,
              asinh, acosh, atanh #-}
        -- Defined in ‘GHC.Float’
instance Floating Double -- Defined in ‘GHC.Float’
instance Floating Float -- Defined in ‘GHC.Float’
```

`class Fractional a => Floating a where` # Source

Trigonometric and hyperbolic functions and related functions.

The Haskell Report defines no laws for `Floating`. However, `(+)`, `(*)` and `exp` are customarily expected to define an exponential field and have the following properties:

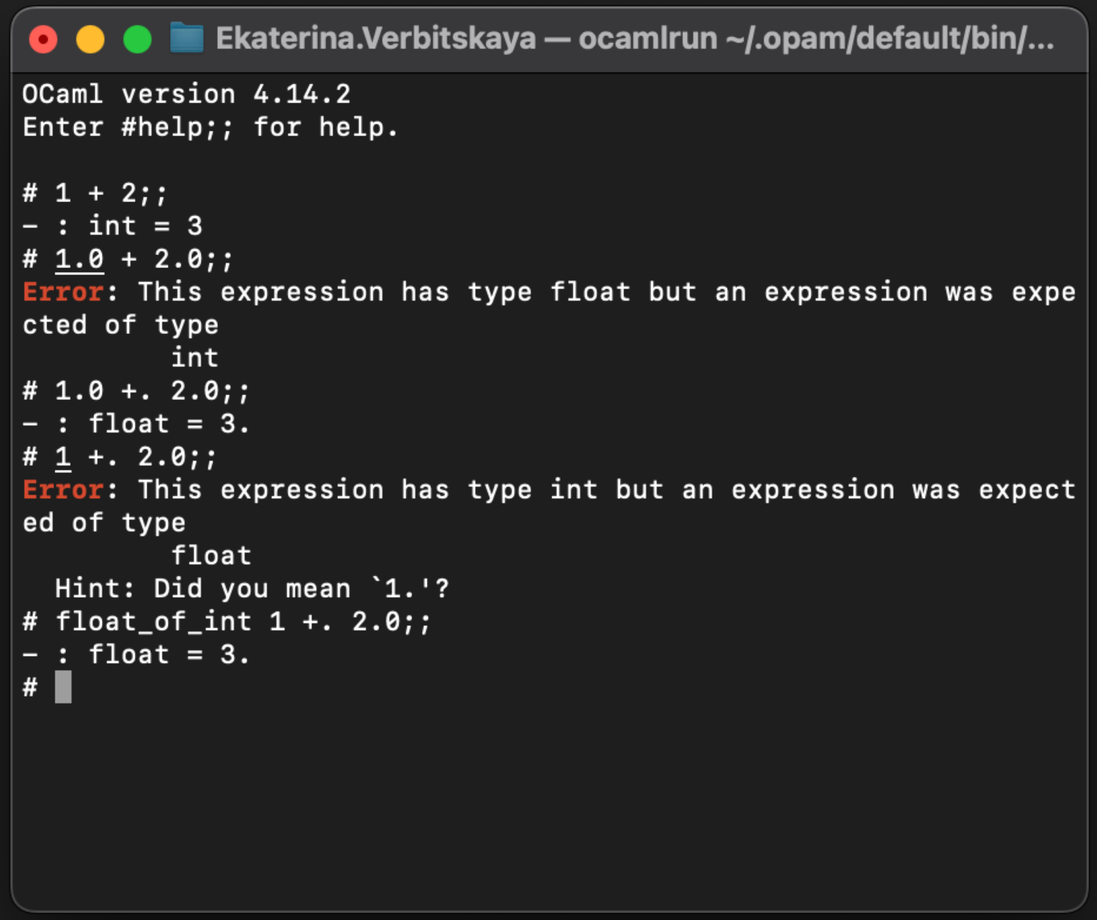
- `exp (a + b) = exp a * exp b`
- `exp (fromInteger 0) = fromInteger 1`

**Minimal complete definition**

`pi`, `exp`, `log`, `sin`, `cos`, `asin`, `acos`, `atan`, `sinh`, `cosh`, `asinh`, `acosh`, `atanh`

# SEEMS STUPID? IT COULD HAVE BEEN WORSE!

- OCaml – another statically typed functional programming language
- It doesn't have type classes
- `1.0` and `1` are not the same
- Functions over numbers don't convert any type on their own
- Specific operations for specific types



# EXERCISE

- Make `Expr` an instance of `Num`
  - Ignore those functions that aren't described by the `Expr` data type

# OUTLINE

- Numerical Type Classes
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- Newtype
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# SEMIGROUP

- Abstracts a type with an associative binary operation
  - doc
- Which operations are semigroups?
  - Integer addition
  - Integer subtraction
  - List concatenation

```
class Semigroup a where                                    # Source
```

The class of semigroups (types with an associative binary operation).

Instances should satisfy the following:

**Associativity**

```
x <> (y <> z) = (x <> y) <> z
```

You can alternatively define `sconcat` instead of (<>), in which case the laws are:

**Unit**

```
sconcat (pure x) = x
```

**Multiplication**

```
sconcat (join xss) = sconcat (fmap sconcat xss)
```

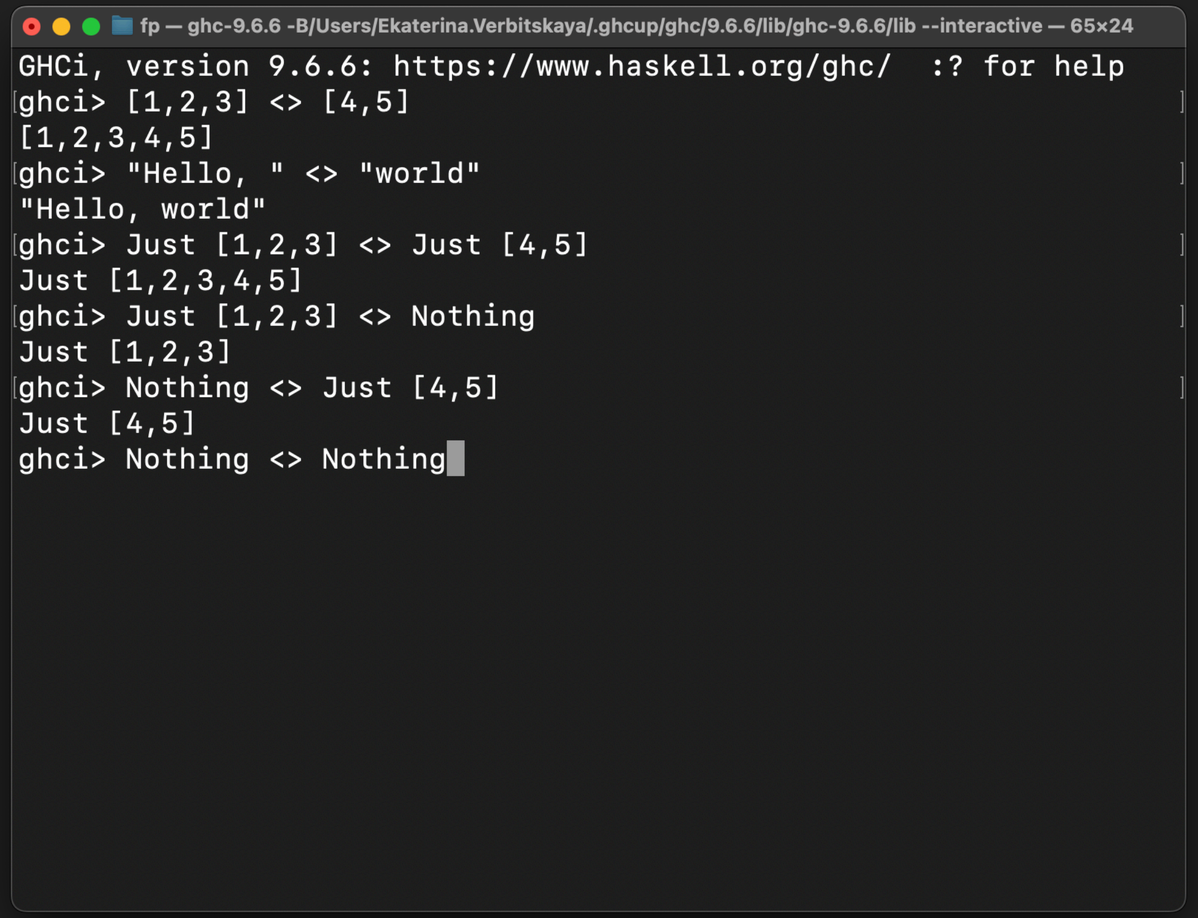
*Since: base-4.9.0.0*

**Minimal complete definition**

```
(<>) | sconcat
```

# EXAMPLES OF SEMIGROUP

- List concatenation
- String concatenation
- Merging two `Maybe` values
  - What's the result of `Nothing <> Nothing`?



# EXAMPLES OF SEMIGROUP

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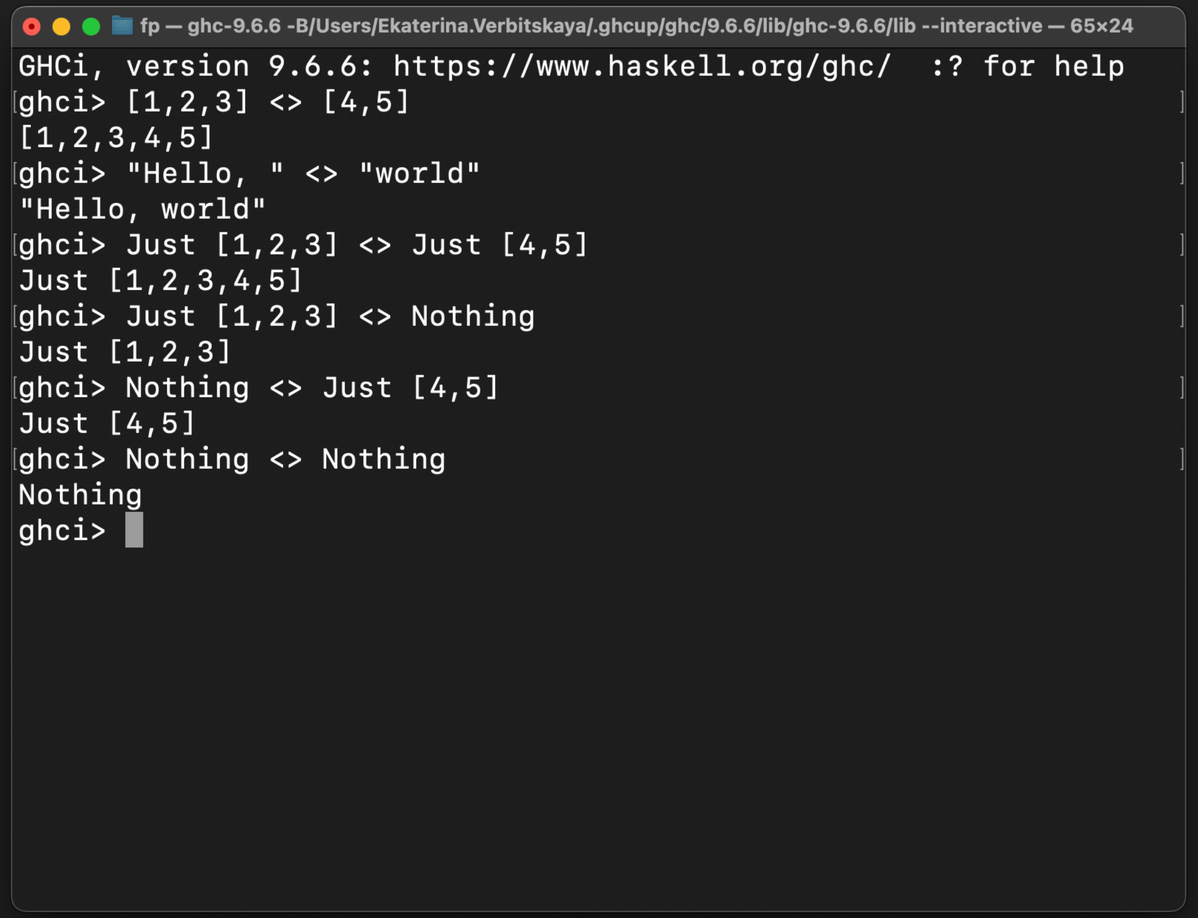

# IMPLEMENTATIONS OF SEMIGROUPS

- For `List`, we just concatenate
- For `Maybe a`, we concatenate the underlying values of type `a`, if there are any
- For `Int`?

```
instance Semigroup [a] where
  (<>) = (++)

instance Semigroup a ⇒ Semigroup (Maybe a) where

  Just x <> Just y = Just (x <> y)
  x <> Nothing = x
  Nothing <> y = y
```

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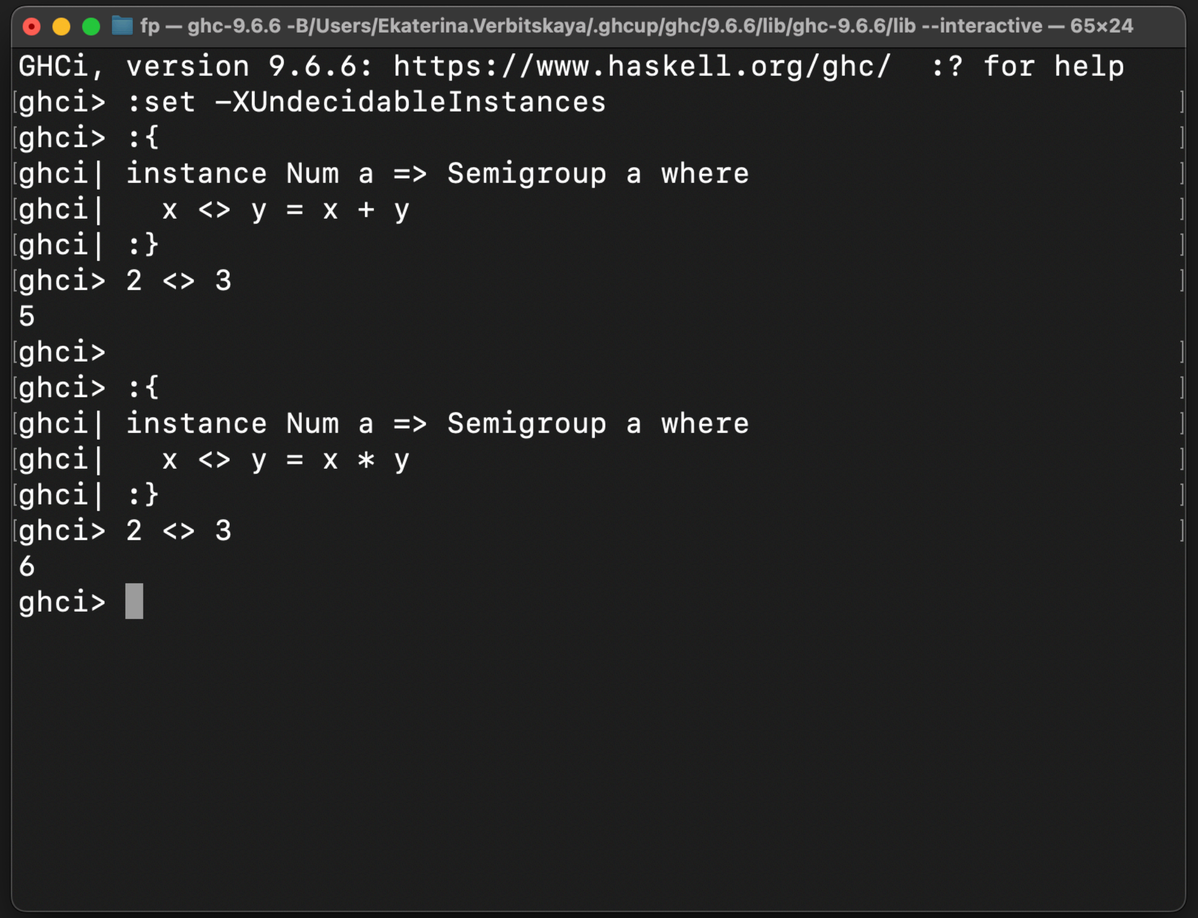
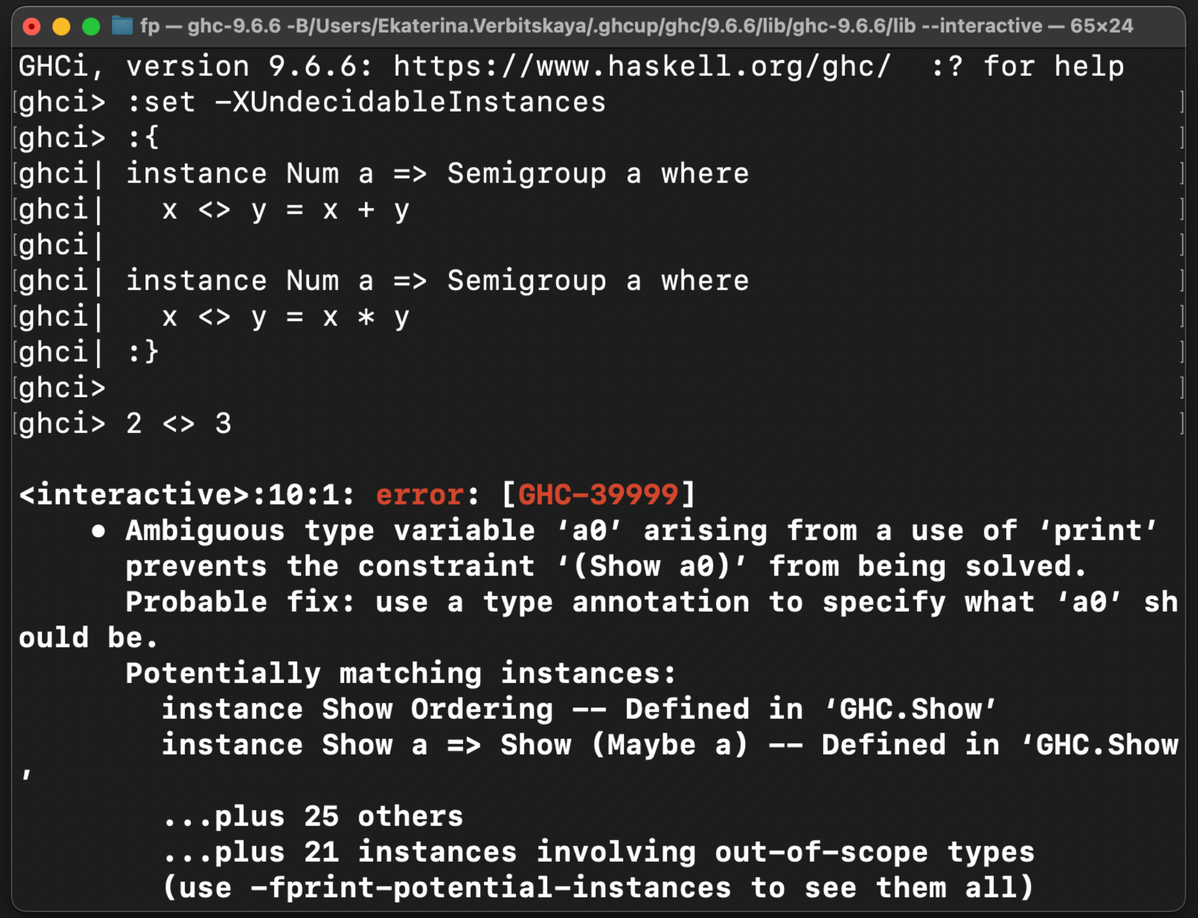
# NUMBERS: WHICH SEMIGROUP TO CHOOSE?

- There are two associative operations over `Num`
  - `(+)`
  - `(*)`



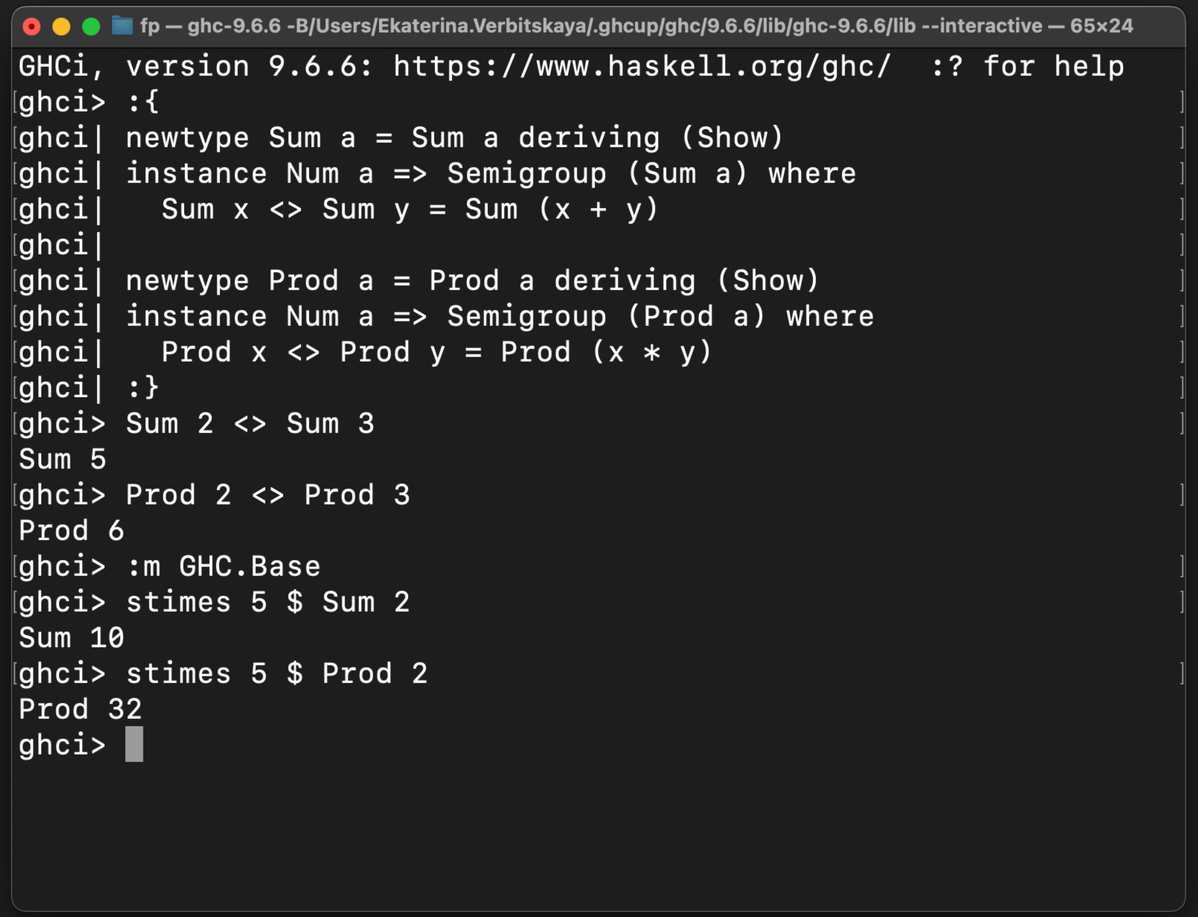
# NUMBERS: WHICH SEMIGROUP TO CHOOSE?

- There are two associative operations over `Num`
  - `(+)`
  - `(*)`
- Two instances cannot coexist easily
- Solution: `newtype`



# NEWTYPE

- Zero-weight wrapper
- Allows to define multiple behaviours for the same underlying data type
- Similar to `data`, but
  - Only one constructor
  - Only one field



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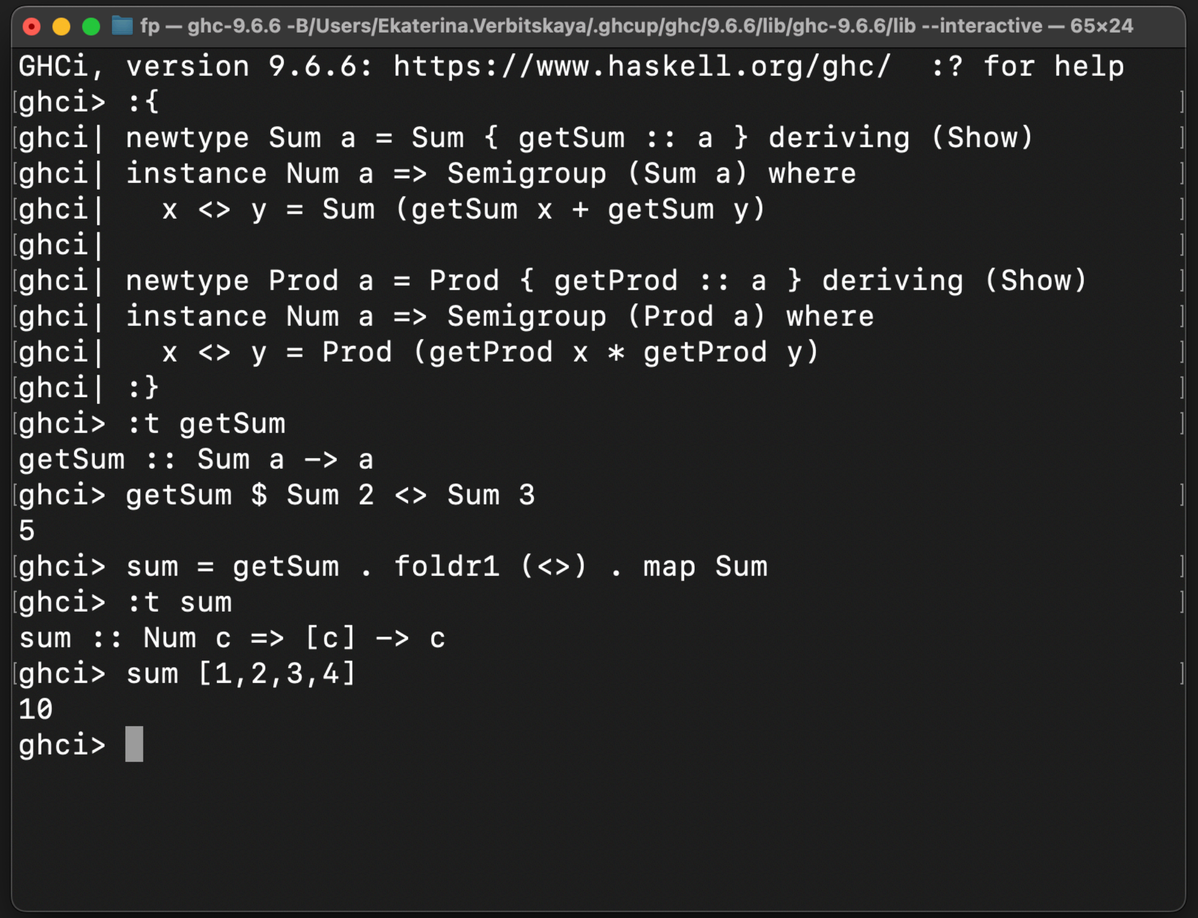
# RECORD SYNTAX IN NEWTYPES

- You can name fields of a constructor
- Most useful when you only have one constructor
- Be careful, names are global



# RECORD SYNTAX IN DATA

- When you have multiple fields in a constructor all of which have the same non-descriptive type
- Each field name is a function

```haskell
module Record where

data Person = Person String String Int String
            deriving (Show)

getAge :: Person → Int
getAge (Person _ _ age _) = age

me = Person "Ekaterina" "Verbitskaia" 31 "Netherlands"

data Person' = Person' { firstName :: String
                       , lastName :: String
                       , age :: Int
                       , residence :: String
                       }
               deriving (Show)

me' = Person "Ekaterina" "Verbitskaia" 31 "Netherlands"

getAge' = age
```

# PATTERN MATCHING AND VALUE CREATION

- No need to use wildcards and remember what the order of the field is
- You can bind variables in a pattern match
- Order of the fields is not important when they are referenced by name

```
module Record where

data Person = Person String String Int String
            deriving (Show)

me = Person "Ekaterina" "Verbitskaia" 31 "Netherlands"
notMe = Person "Verbitskaia" "Ekaterina" 31 "Netherlands"

data Person' = Person' { firstName :: String
                       , lastName :: String
                       , age :: Int
                       , residence :: String
                       }
                deriving (Show)

me' = Person' "Ekaterina" "Verbitskaia" 31 "Netherlands"

me'' = Person' { lastName = "Verbitskaia"
               , firstName = "Ekaterina"
               , residence = "Netherlands"
               , age = 30
               }

fullName (Person' { firstName = fn, lastName }) =
  fn ++ ' ' : lastName
```

# EASY UPDATES

- No mutation happens!
- A new value is created, with all old fields the same, and only the modified one changed

```haskell
module Record where

data Person = Person String String Int String
            deriving (Show)

me = Person "Ekaterina" "Verbitskaia" 31 "Netherlands"

ageUp :: Person → Person
ageUp (Person fN lN age r) = Person fN lN (age+1) r

data Person' = Person' { firstName :: String
                       , lastName :: String
                       , age :: Int
                       , residence :: String
                       }
               deriving (Show)

me' = Person' "Ekaterina" "Verbitskaia" 31 "Netherlands"

ageUp' :: Person' → Person'
ageUp' person = person { age = age person + 1 }
```

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# MONOID

- When your binary associative operation has a neutral element
  - `mempty <> x = x <> mempty = x`
- What are mempty of the types?
  - `[a]`
  - `Maybe a`
  - `Int`
  - `Bool`

```
class Semigroup a => Monoid a where                                  # Source
```

The class of monoids (types with an associative binary operation that has an identity). Instances should satisfy the following:

**Right identity**
```
x <> mempty = x
```
**Left identity**
```
mempty <> x = x
```
**Associativity**
```
x <> (y <> z) = (x <> y) <> z (Semigroup law)
```
**Concatenation**
```
mconcat = foldr (<>) mempty
```

You can alternatively define `mconcat` instead of `mempty`, in which case the laws are:

**Unit**
```
mconcat (pure x) = x
```
**Multiplication**
```
mconcat (join xss) = mconcat (fmap mconcat xss)
```
**Subclass**
```
mconcat (toList xs) = sconcat xs
```

The method names refer to the monoid of lists under concatenation, but there are many other instances.

Some types can be viewed as a monoid in more than one way, e.g. both addition and multiplication on numbers. In such cases we often define `newtypes` and make those instances of `Monoid`, e.g. `Sum` and `Product`.

**NOTE**: `Semigroup` is a superclass of `Monoid` since *base-4.11.0.0*.

**Minimal complete definition**

```
mempty | mconcat
```

# EXAMPLES

- `or [True, False, True] == True`
- `and [True, False, True] == False`
- `all even [1,2,3] == False`
- `any even [1,2,3] == True`
- `(Nothing, [1,2,3]) <> (Just “hi”, [4,5]) == (Just “hi”, [1,2,3,4,5])`

```
Any x <> Any y = Any (x || y)
All x <> All y = All (x && y)

Sum x <> Sum y = Sum (x + y)
Product x <> Product y = Product (x * y)

Endo f <> Endo g = Endo (f . g)

(x, y) <> (z, t) = (x <> z, y <> t)
```

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# FUNCTOR

- Generalisation of list `map`
- Transforms content, doesn't change shape

```
class Functor (f :: Type -> Type) where                    # Source
```

A type `f` is a Functor if it provides a function `fmap` which, given any types `a` and `b` lets you apply any function from `(a -> b)` to turn an `f a` into an `f b`, preserving the structure of `f`. Furthermore `f` needs to adhere to the following:

**Identity**

```
fmap id == id
```

**Composition**

```
fmap (f . g) == fmap f . fmap g
```

Note, that the second law follows from the free theorem of the type `fmap` and the first law, so you need only check that the former condition holds. See these articles by School of Haskell or David Luposchainsky for an explanation.

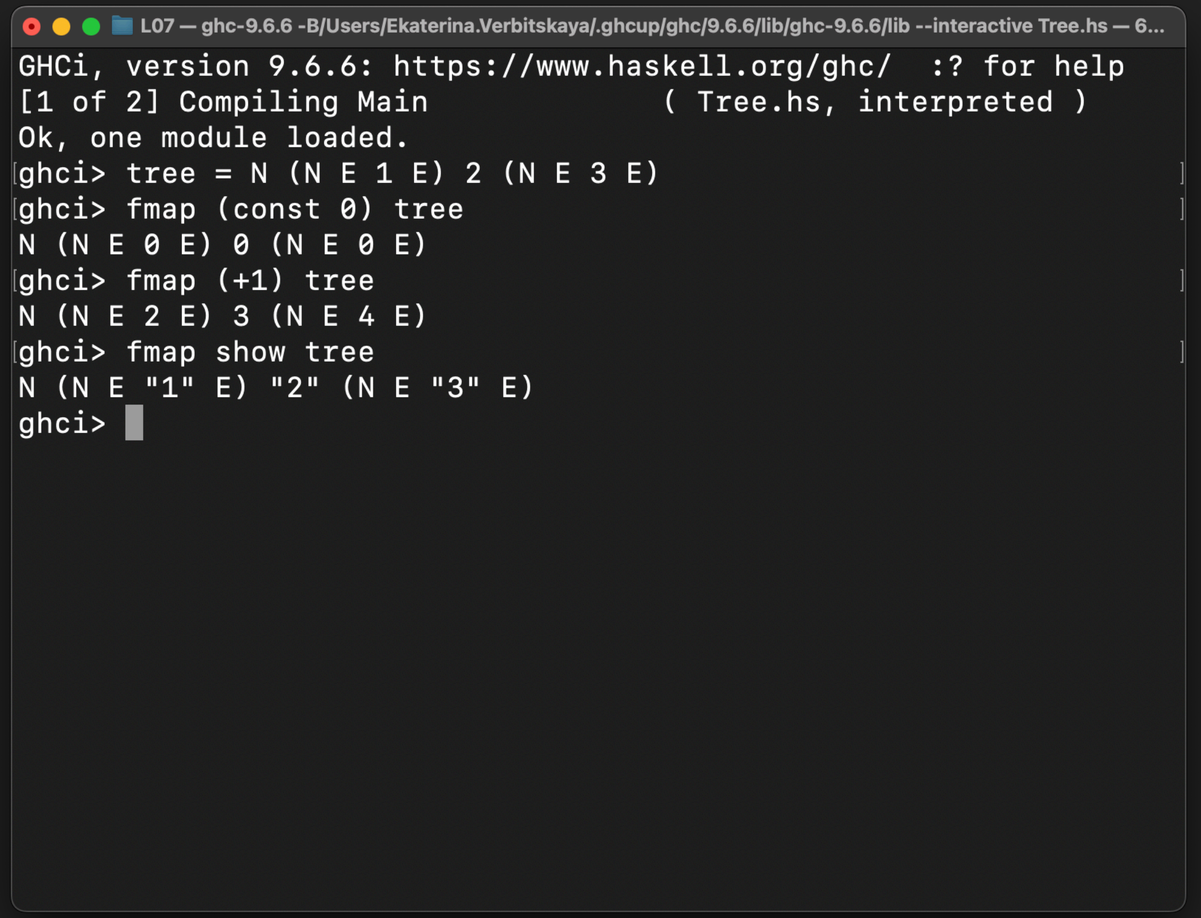
**Minimal complete definition**

```
fmap
```

# EXAMPLE

```haskell
data Tree a
  = E
  | N (Tree a) a (Tree a)
  deriving (Show, Eq, Read)

instance Functor Tree where
  fmap :: (a → b) → Tree a → Tree b
  fmap f E = E
  fmap f (N l x r) = N (fmap f l) (f x) (fmap f r)
```



# EXAMPLE

```haskell
data Tree a
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instance Functor Tree where
  fmap :: (a → b) → Tree a → Tree b
  fmap f E = E
  fmap f (N l x r) = N (fmap f l) (f x) (fmap f r)
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