2021 Spring VLSI DSP Homework Assignment #2

Due date: 2021/4/13

For a discrete wavelet transform (DWT) adopting (9/7) filters, i.e. the low pass filter $h(i)$ is 9-taped and the high pass filter $g(i)$ is 7-taped. Both filters are liner phased and have symmetric coefficients. The filter coefficients are given in Table 1. For a corresponding inverse discrete wavelet transform, the low pass filter $q(i)$ is 7-taped and the high pass filter $p(i)$ is 9-taped. The filter coefficients are given in Table 2.

Table 1. Analysis filter coefficients for the floating point 9/7 filter

Analysis Filter Coefficients		
i	Lowpass Filter h_i	Highpass Filter g_i
0	0.852698679009	-0.788485616406
±1	0.377402855613	0.418092273222
±2	-0.110624404418	0.040689417609
±3	-0.023849465020	-0.064538882629
±4	0.037828455507	

Note: the high pass and low pass filter notations here are opposite to those in the lecture note

Table 2. Synthesis filter coefficients for the floating point 9/7 filter

Synthesis Filter Coefficients		
i	Lowpass Filter q_i	Highpass Filter p_i
0	0.788485616406	-0.852698679009
±1	0.418092273222	0.377402855613
±2	-0.040689417609	0.110624404418
±3	-0.064538882629	-0.023849465020
±4		-0.037828455507

a) For a 256×256 gray scale image (will be provided from the course web site), please conduct a 2-D 3-level DWT transform (as shown in Figure 1) and show the transformed result. Then conduct a 2-D 3-level IDWT to convert it back. Please compare if the reconstructed image (after IDWT) is same as the original image

b) By setting all three level 1 sub-bands HL1, LH1 and HH1 coefficients to zeros and perform IDWT. See how the reconstructed image different from the original one

Note 1: the filter orders for analysis (DWT) is (low pass 9/ high pass 7) and the filter orders for synthesis is (low pass 7/ high pass 9). And the filter coefficients have the following relations $h(i) = (-1)^{i+1} p(i), \quad g(i) = (-1)^{i} q(i)$

Note 1: when performing down sampling in each octave, low pass filters always keep the **odd** numbered output data while high pass filters always keep the **even** numbered output data. In up-sampling process of IDWT, the discarded data are replaced with zeros and inserted to the output data stream.

Note 2: at the boundary of the image, you need to perform a symmetric extension to obtain the pixel values for $h(i)$, $g(i)$, $i<0$. The extension is illustrated in Figure 3.

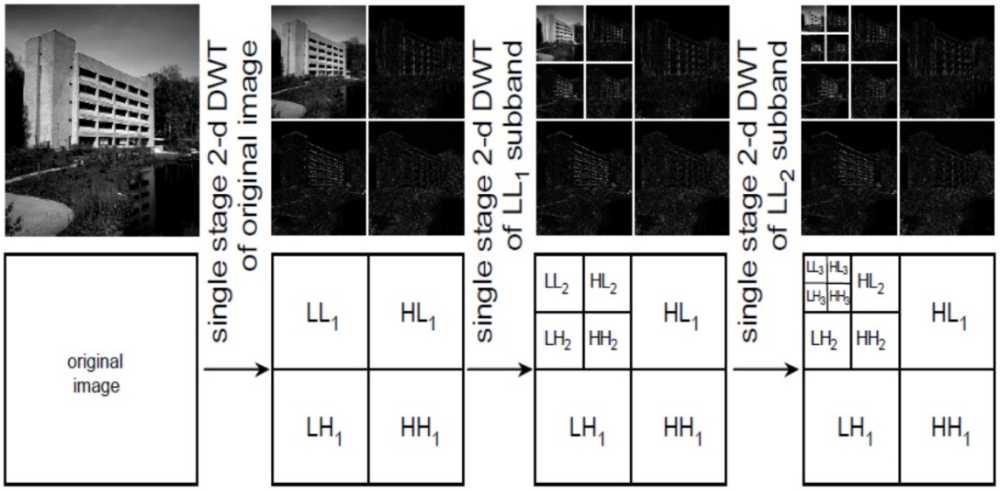
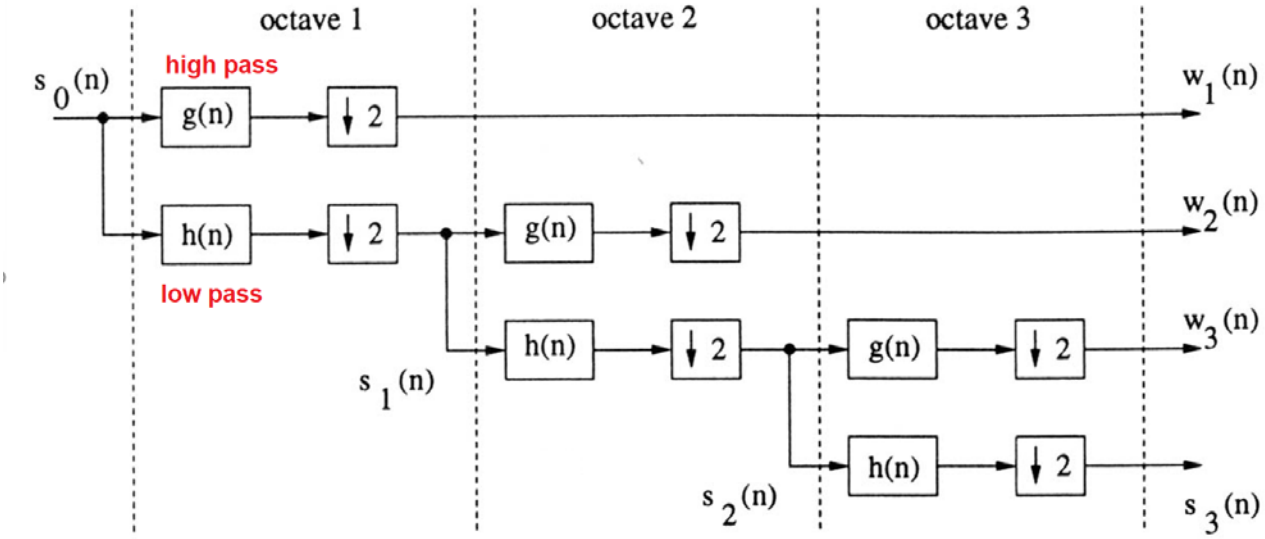
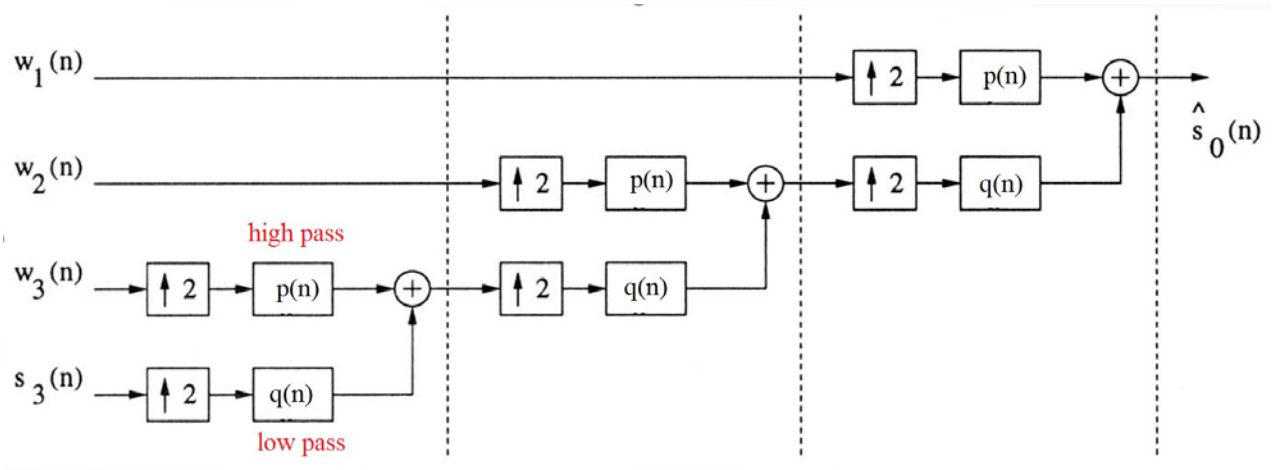


Figure 1. a 3-level DWT example



(a) One-dimensional 3-level DWT transform



(b) One-dimensional 3-level IDWT transform

Figure 2. 1-D 3-level DWT versus IDWT

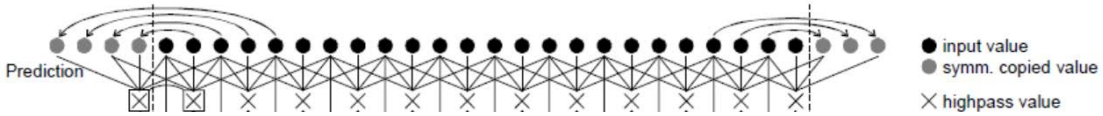


Figure 3. Symmetric extension scheme for boundary pixels

c) please conduct fixed point simulations to determine the DWT word length for the following items. Assume a floating point version IDWT is used, the PSNR (peak signal to noise ratio) of the reconstructed image should be no less than 40dB. Please use as small word length as possible to achieve this goal.

- The word length of the filter coefficients (Table 1), all coefficients should have the same word length
- The word length of the filter outputs at each level, i.e., level 1, 2 and 3 of DWT

Note 4: The PSNR is calculated as

$$\mathrm{MSE} = \frac{1}{\mathrm{M}\cdot\mathrm{N}}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left(I(i,j)-\hat{I}(i,j)\right)^2$$

$$PSNR = 10\log_{10}\left(\frac{MAXI^2}{\mathrm{MSE}}\right)$$

Where $I(i,j)$ is the original image and $\hat{I}(i,j)$ is the reconstructed image after performing DWT and IDWT. MAXI is the maximum possible value of a pixel. If it's an 8-bit pixel, MAXI is 255.