Table of Contents

Introduction

Armenia is experiencing rapid growth in distributed solar generation. Currently, thousands of commercial and residential buildings in Armenia have rooftop photovoltaic systems installed.

At the same time, time-of-use tariffs for commercial consumers have introduced a significant price difference between night and day, while export compensation for surplus solar energy remains low.

Adding a lithium-ion battery to the system allows the building owner to store excess midday solar production and either use it during expensive peak hours or sell it back to the grid.

Achieving the maximum possible daily profit requires making the correct decision every single hour of the day: charge, discharge, buy from the grid, or sell to the grid.

This decision problem is complex because:

- The battery cannot charge and discharge at the same time,
- Real inverters and batteries have losses that increase quadratically with power.

The goal of this diploma thesis is to develop mathematical optimisation models that find the absolute best 24-hour schedule for a typical commercial building in Yerevan under 2025 tariffs, and to quantify how much money can be saved when realistic nonlinear losses are taken into account compared to the simplified linear models commonly used in practice.

All calculations are performed using only open-source Python tools: PuLP + CBC for the linear case and GEKKO (APOPT + IPOPT) for the non-linear case.

Chapter 1: Analytical Review of Literature

1.1. Study of Existing Such Models

Optimal daily energy management of PV-battery systems under time-of-use tariffs has been a widely studied problem. Most works use mixed-integer linear programming (MILP) with constant efficiency assumption and achieve cost reduction compared to grid-only operation.

Recent studies show that real inverter and battery losses are quadratic in power and ignoring them overestimates savings. Such problems are formulated as convex mixed-integer nonlinear programs (MINLP) and solved to proven global optimality using the Outer Approximation algorithm implemented in tools like GEKKO/APOPT.

This thesis fills the identified research gap by developing and comparing linear and realistic nonlinear models using only open-source tools, and quantifying the exact impact of quadratic losses for a commercial building in Yerevan under current and proposed export compensation scenarios.

1.2 Problem Statement

Modern commercial and residential buildings increasingly combine rooftop photovoltaic systems, battery energy storage, and grid connection under time-of-use tariffs and net-billing regulations. In the Armenian context of 2025, such buildings face the challenge of optimally managing energy flows on an hourly basis to minimize electricity costs (or maximize financial benefit).

The core decision problem is to determine, for each hour of the day, the most economically advantageous operating strategy given:

- Time-varying electricity demand of the building
- Deterministic hourly solar generation from the rooftop PV system
- Time-of-use grid purchase prices and regulated export prices
- A lithium-ion battery with limited capacity, charge/discharge power, and round-trip efficiency

The system must decide:

- Quantity of electricity to purchase from or sell to the grid
- Battery charge and discharge rates

while respecting:

- Mutual exclusivity of simultaneous battery charging/discharging
- State-of-charge dynamics and power limits

Objective: maximize daily electricity cost savings compared to a baseline without local

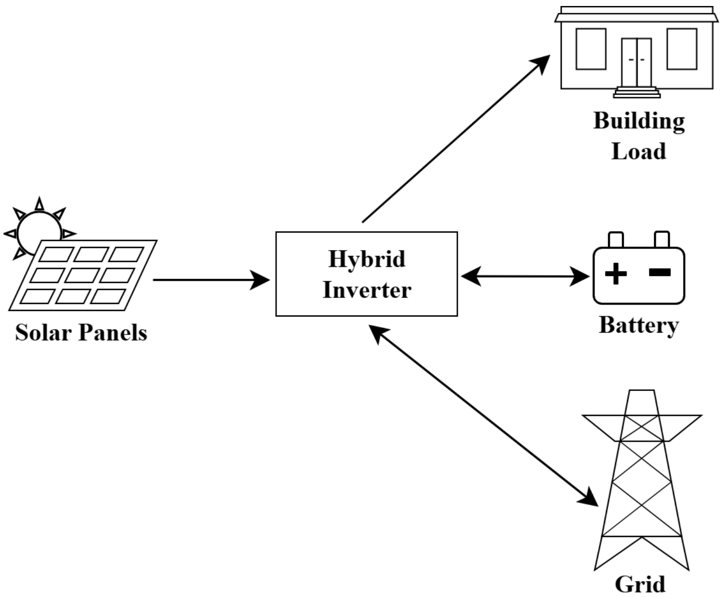


Figure 1. Schematic diagram of a grid-connected photovoltaic system with battery energy storage and hybrid inverter

generation/storage.

The problem is solved in two variants:

1. Mixed-integer linear program (MILP) using PuLP/CBC
2. Convex mixed-integer nonlinear program (MINLP) using GEKKO with Outer Approximation (APOPT, IPOPT).

The studied system consists of four main components connected through a single hybrid inverter:

- Rooftop photovoltaic panels (DC output)
- Lithium-ion battery storage (bidirectional DC)
- Building electrical load (AC)
- Utility grid connection operating under net-billing rules (bidirectional AC)

All energy flows are managed centrally by the hybrid inverter, which enables simultaneous solar generation, battery charging or discharging, and grid import or export while respecting the mutual-exclusivity constraints described below.

Chapter 2: Development and Solution of Mathematical Models

Two modelling approaches exist in the literature: simplified linear models assuming constant efficiency and more accurate nonlinear models with quadratic loss terms. Although the nonlinear formulation better reflects physical reality, the linear case serves as an important benchmark and is significantly easier to solve and analyse.

Therefore, to fully understand the more complex non-linear case and to clearly quantify the impact of realistic losses, we first study the linear formulation before extending it to the convex nonlinear case.

2.1 Linear Case

2.1.1 Mathematical Model

Indices

$t = 0, 2, 3, \dots, T$	Hours in the optimization period

Parameters

C_t^{buy}	Electricity purchase price at hour t (AMD/kWh)
C_t^{sell}	Electricity selling price at hour t (AMD/kWh)
E_t^{solar}	Solar generation at hour t (kWh)
E_t^{demand}	Load demand at hour t (kWh)
E^{cap}	Battery capacity (kWh)
$P_{charge}^{max}, P_{discharge}^{max}$	Max battery charge/discharge power (kWh)
$P_{buy}^{max}, P_{sell}^{max}$	Max grid buy/sell power (kWh)
$\eta_{charge}, \eta_{discharge}$	Battery efficiency
s_0	Initial battery state of charge

Variables

x_t^{buy}	Energy bought from the grid	(Continuous)

x_t^{sell}	Energy sold to the grid	(Continuous)
x_t^{charge}	Energy charged into battery	(Continuous)
$x_t^{discharge}$	Energy discharged from battery	(Continuous)
s_t	Battery state of charge	(Continuous)
$y_t^{charge}, y_t^{discharge} \in \{0,\ 1\}$	Whether charging or discharging battery	(Binary)

Objective Function

$$\sum_{t=0}^{T}\left(C_t^{sell}x_t^{sell} - C_t^{buy}x_t^{buy}\right) \rightarrow max$$

Constraints

For each hour $t\ =\ 0,\ 2,\ 3,\ \dots\ ,\ T$

- Power balance:

$$E_t^{demand} = E_t^{solar} + x_t^{buy} + x_t^{discharge} - x_t^{charge} - x_t^{sell}$$

- Battery state of charge:

$$s_t = s_0 + \eta_{charge}x_t^{charge} - \frac{x_t^{discharge}}{\eta_{discharge}}, \quad t = 0$$

$$s_t = s_{t-1} + \eta_{charge}x_t^{charge} - \frac{x_t^{discharge}}{\eta_{discharge}}, \quad t \geq 1$$

$$s_t \leq E^{cap} \quad , s_0\ given$$

- Battery charge/discharge limits:

$$x_t^{charge} \leq P_{max}^{charge}y_t^{charge}$$

$$x_t^{discharge} \leq P_{discharge}^{max}y_t^{discharge}$$

- No simultaneous battery charging and discharging:

$$y_t^{charge} + y_t^{discharge} \leq 1$$

$$y_t^{charge}, y_t^{discharge} \in \{0, 1\}$$

- <u>Grid buy/sell limits:</u>

$$x_t^{buy} \leq P_{buy}^{max}$$

$$x_t^{sell} \leq P_{sell}^{max}$$

- <u>Non-negativity:</u>

$$x_t^{buy}, x_t^{sell}, x_t^{charge}, x_t^{discharge}, s_t, y_t^{charge}, y_t^{discharge} \geq 0$$

2.1.2 Solution Methodology:

For each hour t, the formulated MILP contains 5 continuous variables and 2 binary variables. Therefore, for a 24 hour period, the formulated MILP contains 120 continuous variables and 48 binary variables.

This MILP formulation is necessary because the system contains both continuous variables (battery SOC and energy bought, sold, charged, or discharged) and binary variables (whether the battery is charging or discharging).

The objective of the model is to maximize profit (or equivalently, minimize the electricity cost) while satisfying all operational constraints

Solver and Algorithm:

The model is implemented in Python using the PuLP library, which interfaces with the CBC solver. The CBC solver then finds the optimal solution using the following steps:

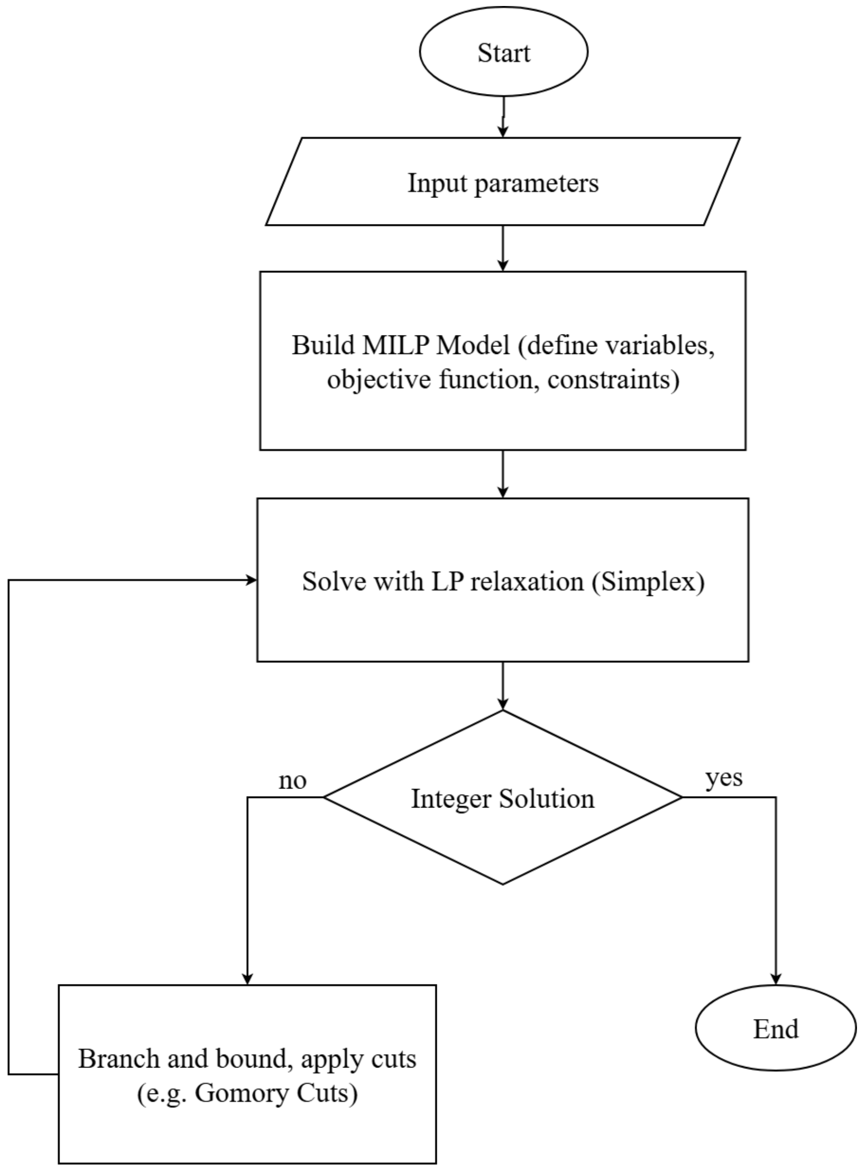


Figure 2. The solution algorithm for the linear case

LP Relaxation: All binary variables are temporarily relaxed to continuous values. The relaxed linear program is solved using the Simplex method.

Branch-and-Bound: If any binary variable in the LP solution is fractional, the solver creates two branches: one where the variable is forced to 0, and another where it is forced to 1. This process continues recursively, removing branches that cannot improve the current best solution.

Gomory Cuts: CBC automatically adds Gomory cuts, which are linear constraints that eliminate fractional solutions. These cuts work by adding a new linear constraint that cuts off the fractional solution but doesn't exclude any feasible integer solutions.

2.3. Non-Linear Case:

2.3.1 Mathematical Model:

For the non-linear case, we keep the variables and objective function and we introduce a new parameter:

k	Loss coefficient

We also adjust the battery SOC constraint to be convex and non-linear:

- Battery state of charge:

$$E_t^{\text{loss}} = k \cdot \left(\frac{(x_t^{charge})^2}{P_{charge}} + \frac{(x_t^{\text{discharge}})^2}{P_{\text{discharge}}} \right)$$

$$\mathrm{s}_t = \begin{cases} \mathrm{s}_0 + \eta_{\text{charge}} x_t^{charge} - \frac{x_t^{discharge}}{\eta_{\text{discharge}}} - E_t^{\text{loss}} & t = 0 \\ \mathrm{s}_{t-1} + \eta_{\text{charge}} x_t^{charge} - \frac{x_t^{discharge}}{\eta_{\text{discharge}}} - E_t^{\text{loss}} & t \geq 1 \end{cases}$$

$$s_t \leq E^{cap} \quad , s_0 \ given$$

All other constraints remain the same

2.3.2 Solution Methodology:

Adding the non-linear constraint turns the original MILP into a convex Mixed-Integer Nonlinear Program (MINLP) with 120 continuous variables, 48 binary variables (charge/discharge mode per hour) and 24 convex nonlinear constraints for the 24 hour case

Solver and Algorithm:

The problem is solved using the GEKKO library in python. GEKKO solves it to proven global optimality using the Outer Approximation strategy and IPOPT as the continuous NLP solver.. This combination is the standard for convex MINLPs of this size.

The algorithm executes the following steps:

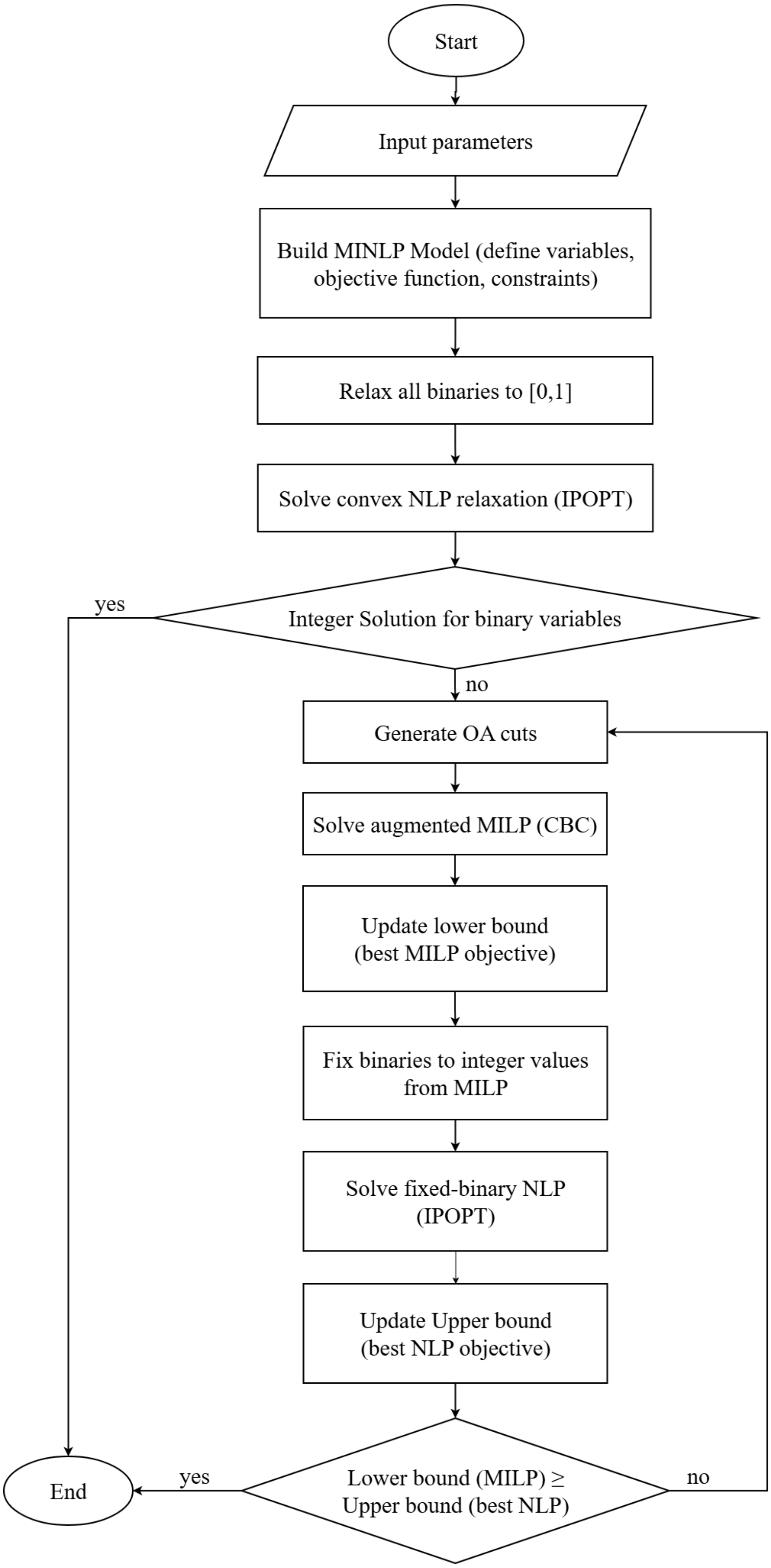


Figure 3. The solution algorithm for the non-linear case

1. Initialization: All binary variables $y_t^{charge}, y_t^{discharge}$ are relaxed to continuous variables in [0,1].

2. Solve NLP relaxation: The full problem with relaxed binaries is solved as a convex nonlinear program using IPOPT (primal-dual interior-point method with exact Hessian of the quadratic loss term).

3. Generate Outer Approximation (OA) cut: Using the fractional solution from step 2, a linear supporting hyperplane of the quadratic loss function is computed and added to the master problem.

4. Solve augmented MILP problem: The augmented linear problem (original linear constraints with all accumulated OA cuts and integer cuts added to it) is solved by CBC branch-and-cut. This provides a lower bound and a new candidate integer solution.

5. Fix binaries and solve NLP sub-problem: The binary variables are fixed to the integer values from step 4. The resulting convex NLP is solved again with IPOPT, resulting in a feasible solution and an upper bound.

6. Convergence check: If the lower bound (MILP) is greater than or equal to the upper bound (best NLP) then global optimum is found. Otherwise, the solver returns to step 3 and repeats.

2.4. Case Study and Numerical Results:

2.4.1 System parameters – Commercial building in Yerevan, 2025

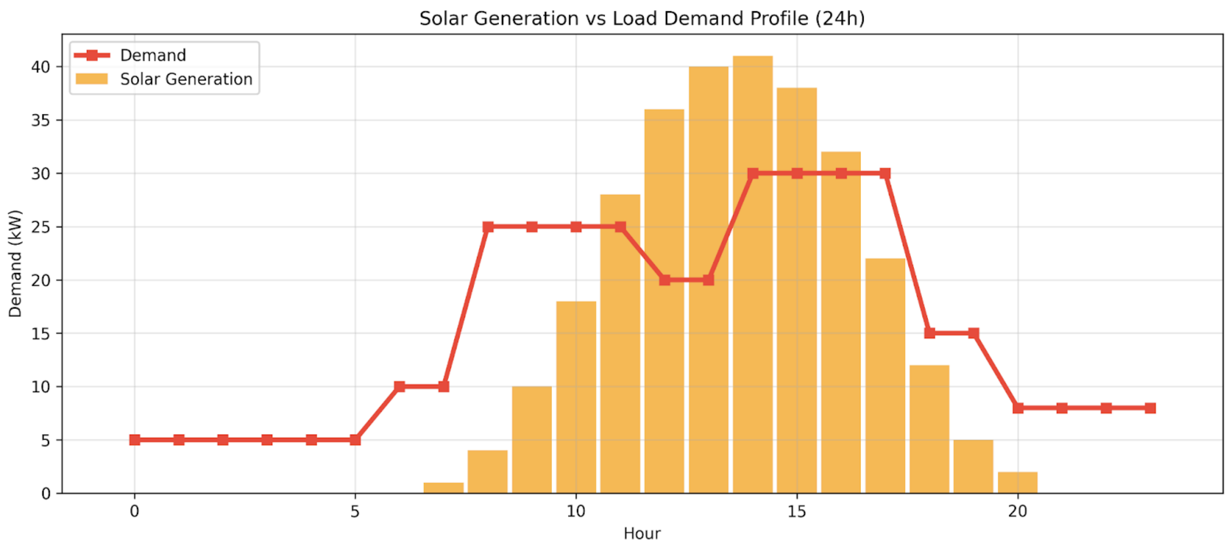
We consider the following system:

- 30 kWh lithium battery, 12 kW charge/discharge power, η = 95 %
- Grid tariffs: buy 38 AMD (00–07) / 52 AMD (rest) and 2 export price scenarios:
  - Scenario 1: 22 AMD
  - Scenario 2: 35 AMD (00–07) / 48 AMD (rest)
- Total daily load of 372 kWh
- Total rooftop solar production of 289 kWh
- Max grid buy power in one hour: 60kWh, sell power: 30kWh
- Initial battery state of charge: 15 kWh

The given parameters are:

E^{cap}	$30\ kWh$	P^{max}_{buy}	$60\ kWh$
P^{max}_{charge}, $P^{max}_{discharge}$	$12\ kWh$	η_{charge}, $\eta_{discharge}$	0.95
P^{max}_{sell}	$30\ kWh$	s_0	15 kWh

t	E^{solar}_t	E^{demand}_t	P^{buy}_t	P^{sell}_t (Scenario 1)	P^{sell}_t (Scenario 2)
0	0	5	38	22	35
1	0	5	38	22	35
2	0	5	38	22	35
3	0	5	38	22	35
4	0	5	38	22	35
5	0	5	38	22	35
6	0	10	38	22	35
7	1	10	38	22	35
8	4	25	38	22	48
9	10	25	52	22	48
10	18	25	52	22	48
11	28	25	52	22	48
12	36	20	52	22	48
13	40	20	52	22	48
14	41	30	52	22	48
15	38	30	52	22	48
16	32	30	52	22	48
17	22	30	52	22	48
18	12	15	52	22	48
19	5	15	52	22	48
20	2	8	52	22	48
21	0	8	52	22	48
22	0	8	52	22	48
23	0	8	52	22	35



For $t = 0, 1, 2, ..., 23$, we need to find the optimal solution of the mathematical model described earlier

The objective function value represents the optimized net cost of electricity after the model schedules charging, discharging, buying, and selling. A higher net cost indicates a more economically efficient operation. The Amount Saved is computed by comparing this optimized cost to the baseline cost of running the building without solar and battery support.

$$Baseline\ Cost = \sum_{t=0}^{T} E_t^{demand} P_t^{buy} = 18672\ AMD$$

$$Amount\ Saved = Baseline\ Cost - Actual\ Cost$$

2.4.2 Results

Linear Case - Scenario 1: 22 AMD

t	x_t^{buy}	x_t^{sell}	x_t^{charge}	$x_t^{discharge}$	s_t	y_t^{charge}	$y_t^{discharge}$
0	5	0	0	0	15	0	0
1	5	0	0	0	15	0	0
2	5	0	0	0	15	0	0
3	5	0	0	0	15	0	0
4	5	0	0	0	15	0	0
5	17	0	12	0	26.4	1	0
6	13.79	0	3.79	0	30	1	0
7	0	0	0	9	20.53	0	1
8	20.5	0	0	0.5	20	0	1
9	3	0	0	12	7.37	0	1

10	0	0	0	7	0	0	1
11	0	3	0	0	0	0	0
12	0	16	0	0	0	0	0
13	0	9.42	10.58	0	10.05	1	0
14	0	0	11	0	20.5	1	0
15	0	0	8	0	28.1	1	0
16	0	0	2	0	30	1	0
17	0	0	0	8	21.58	0	1
18	0	0	0	3	18.42	0	1
19	0	0	0	10	7.89	0	1
20	0	0	0	6	1.58	0	1
21	6.5	0	0	1.5	0	0	1
22	8	0	0	0	0	0	0
23	8	0	0	0	0	0	0

Exported Amount: 28.4 kWh

Value of the objective function: -3774.74

Actual Cost: 3774.74 AMD

Amount Saved = Baseline Cost - Actual Cost = 18672 -3774.74= 14897.26 AMD

Linear Case - Scenario 2: 35 or 48 AMD

t	x_t^{buy}	x_t^{sell}	x_t^{charge}	$x_t^{discharge}$	s_t	y_t^{charge}	$y_t^{discharge}$
0	5	0	0	0	15	0	0
1	5	0	0	0	15	0	0
2	5	0	0	0	15	0	0
3	5	0	0	0	15	0	0
4	5	0	0	0	15	0	0
5	8.79	0	3.79	0	18.6	1	0
6	22	0	12	0	30	1	0
7	0	0	0	9	20.53	0	1
8	20.5	0	0	0.5	20	0	1
9	3	0	0	12	7.37	0	1
10	0	0	0	7	0	0	1
11	0	3	0	0	0	0	0
12	0	16	0	0	0	0	0
13	0	20	0	0	0	0	0
14	0	11	0	0	0	0	0

15	0	8	0	0	0	0	0
16	0	2	0	0	0	0	0
17	8	0	0	0	0	0	0
18	3	0	0	0	0	0	0
19	10	0	0	0	0	0	0
20	6	0	0	0	0	0	0
21	8	0	0	0	0	0	0
22	8	0	0	0	0	0	0
23	8	0	0	0	0	0	0

Exported Amount: 60.0 kWh

Value of the objective function: -3002.00 AMD

Actual Cost: -3002 AMD

Amount Saved = Baseline Cost - Actual Cost = 18672 - 3002 = 15670 AMD

Non-Linear Case - Scenario 1: 22 AMD

We will consider the same cases studied in the linear case for $k = 0.012$

t	x_t^{buy}	x_t^{sell}	x_t^{charge}	$x_t^{discharge}$	s_t	y_t^{charge}	$y_t^{discharge}$
0	5	0	0	0	15	0	0
1	5	0	0	0	15	0	0
2	5	0	0	0	15	0	0
3	5	0	0	0	15	0	0
4	12.96	0	7.96	0	22.5	1	0
5	5	0	0	0	22.5	0	0
6	17.96	0	7.96	0	30	1	0
7	1.9	0	0	7.1	22.47	0	1
8	13.9	0	0	7.1	14.94	0	1
9	7.9	0	0	7.1	7.42	0	1
10	0	0	0	7	0	0	1
11	0	0	3	0	2.84	1	0
12	0	8.8	7.2	0	9.63	1	0
13	0	12.8	7.2	0	16.42	1	0
14	0	3.8	7.2	0	23.21	1	0
15	0	0.8	7.2	0	30	1	0
16	0	2	0	0	30	0	0
17	2.93	0	0	5.07	24.63	0	1

18	0	0	0	3	21.47	0	1
19	4.93	0	0	5.07	16.1	0	1
20	0.93	0	0	5.07	10.73	0	1
21	2.93	0	0	5.07	5.37	0	1
22	2.93	0	0	5.07	0	0	1
23	8	0	0	0	0	0	0

Exported Amount: 28.19 kWh

Value of the objective function: -3801.52 AMD

Actual Cost: 3801.52 AMD

Amount Saved = Baseline Cost - Actual Cost = 18672 - 3801.52 = 14870.48 AMD

Non-Linear Case - Scenario 1: 35 or 48 AMD

t	x_t^{buy}	x_t^{sell}	x_t^{charge}	$x_t^{discharge}$	s_t	y_t^{charge}	$y_t^{discharge}$
0	12.96	0	7.96	0	22.5	1	0
1	12.96	0	7.96	0	30	1	0
2	5	0	0	0	30	0	0
3	5	0	0	0	30	0	0
4	5	0	0	0	30	0	0
5	5	0	0	0	30	0	0
6	10	0	0	0	30	0	0
7	6.16	0	0	2.84	27	0	1
8	18.16	0	0	2.84	24	0	1
9	12.16	0	0	2.84	21	0	1
10	4.16	0	0	2.84	18	0	1
11	0	3	0	0	18	0	0
12	0	16	0	0	18	0	0
13	0	20	0	0	18	0	0
14	0	11	0	0	18	0	0
15	0	8	0	0	18	0	0
16	0	2	0	0	18	0	0
17	5.16	0	0	2.84	15	0	1
18	0.16	0	0	2.84	12	0	1
19	7.16	0	0	2.84	9	0	1
20	3.16	0	0	2.84	6	0	1
21	5.16	0	0	2.84	3	0	1
22	5.16	0	0	2.84	0	0	1

23	8	0	0	0	0	0	0

Exported Amount: 60.0 kWh

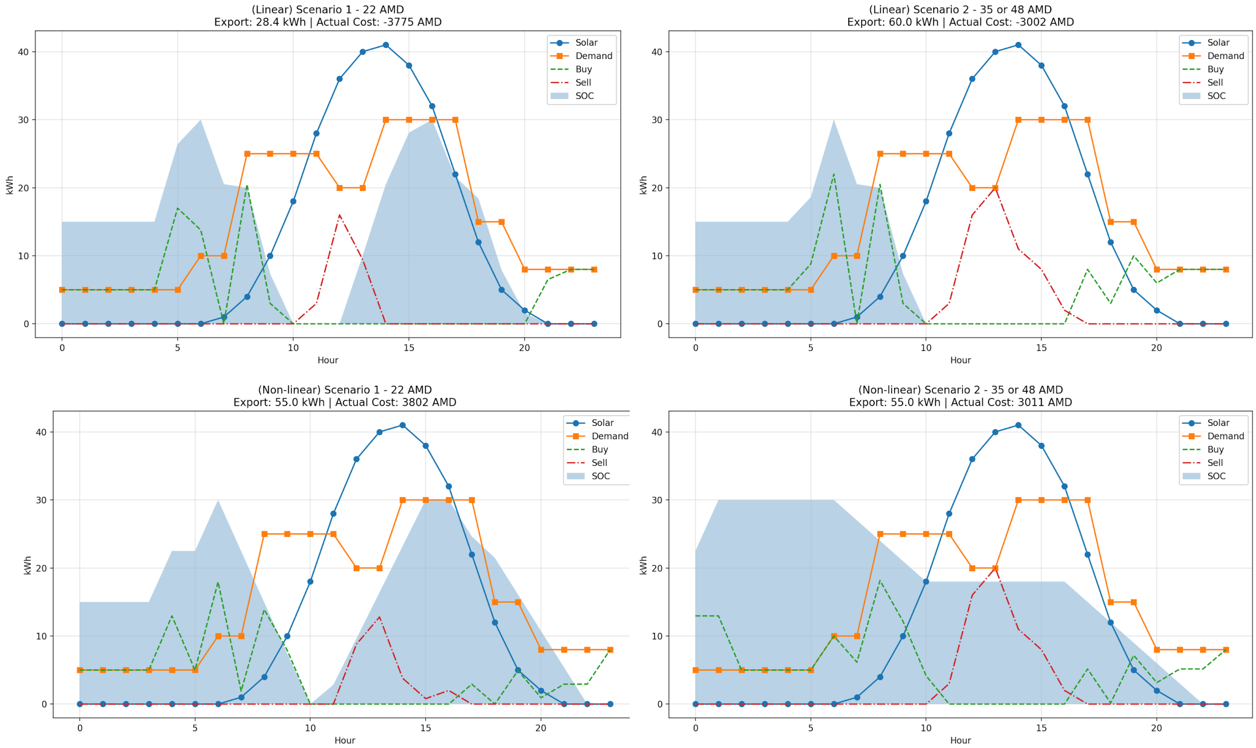
Value of the objective function: -3011.06 AMD

Actual Cost: 3011.06 AMD

Amount Saved = Baseline Cost - Actual Cost = 18672 - 3011.06 = 15660.94 AMD

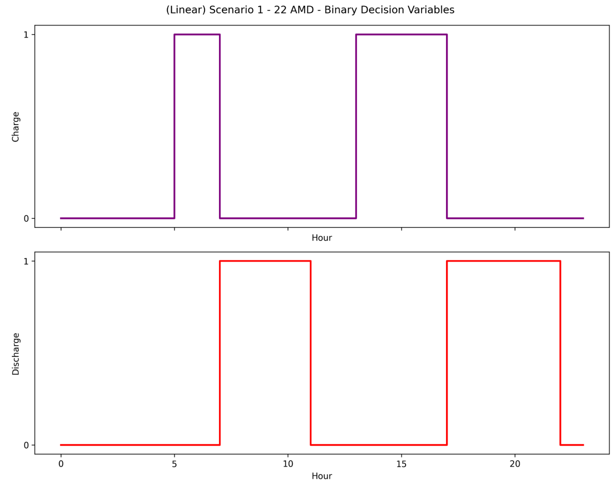
2.4.3 Discussion

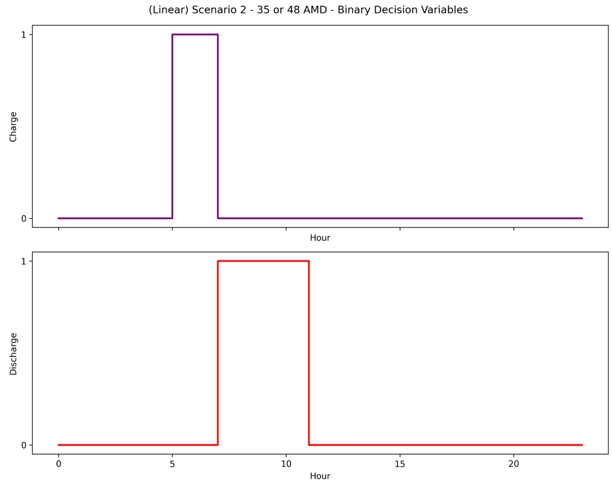
We compare two models (Linear MILP and Non-linear MINLP) on two export price scenarios.

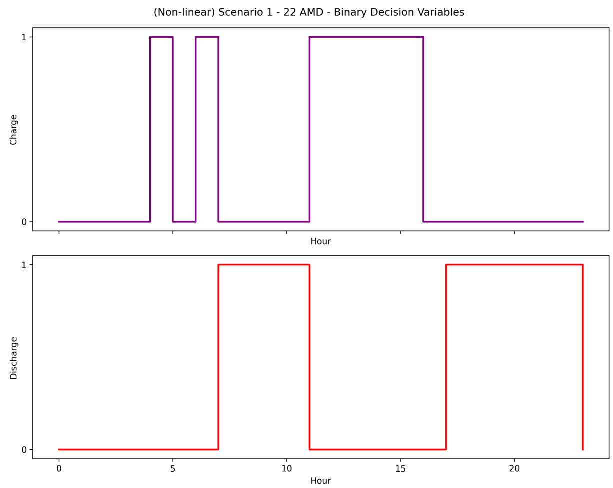


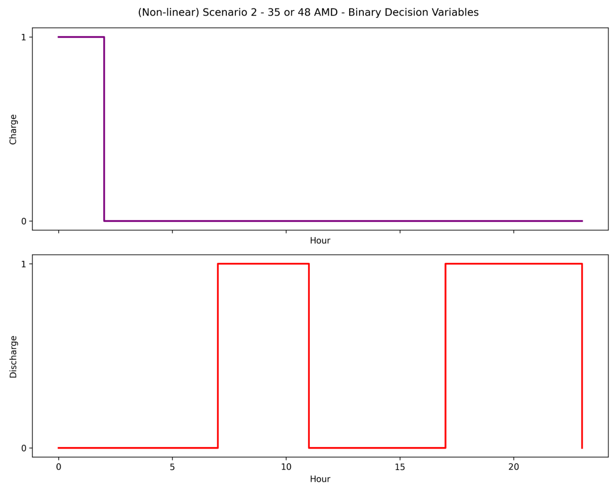
In scenario 2, the model prioritizes selling over charging the battery. The model exclusively sells during peak hours.

Compared to the linear case, in scenario 2, the battery SOC decreases slowly throughout the day. Whereas in other scenarios, the battery completely discharges at hour 10.



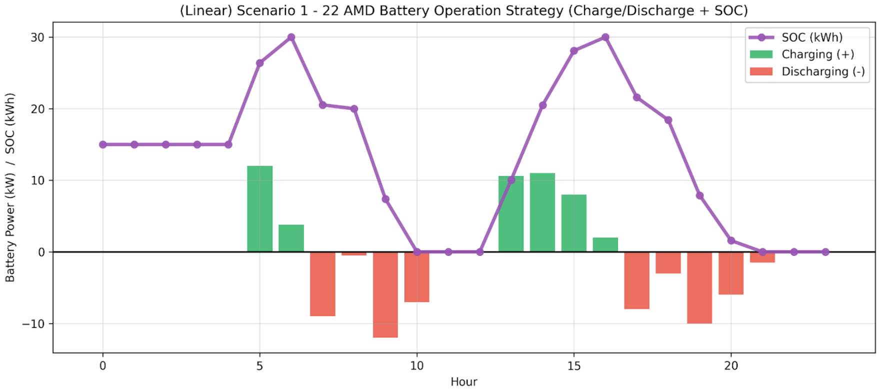


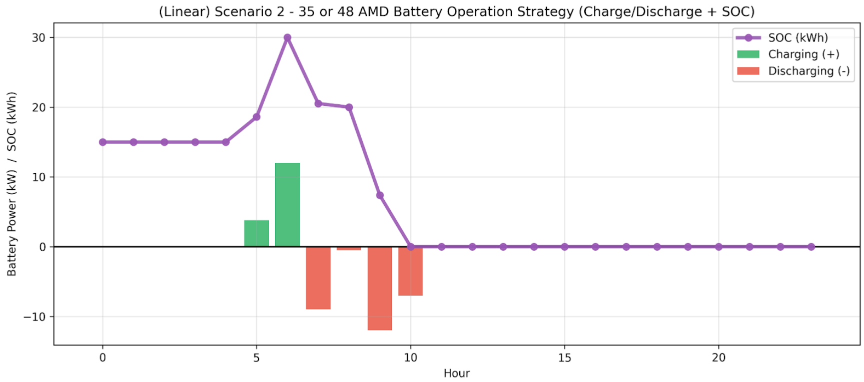


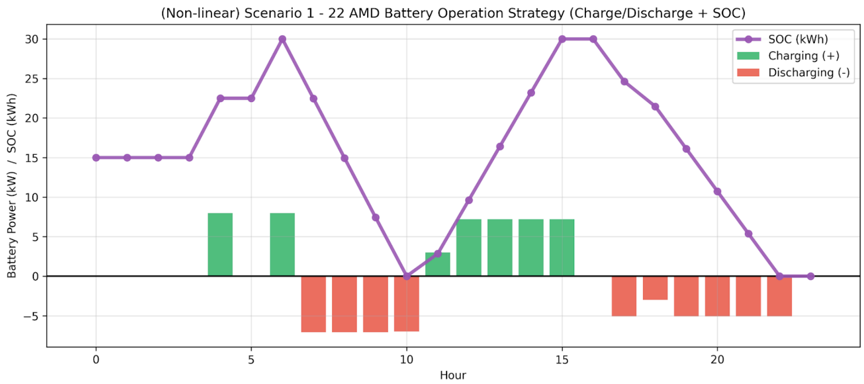


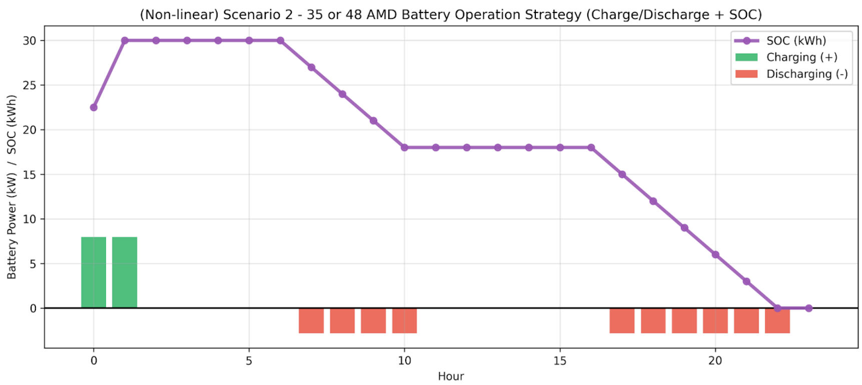
In scenario 1, the model charges the battery during hours when electricity is cheap or when solar generation is high. When solar output drops, the model discharges the battery to reduce grid purchases. This behaviour confirms that the optimization correctly prioritizes low-cost charging and high-value discharging to maximize overall savings.

In scenario 2, the model charges the battery only during non-peak hours. During peak hours, unlike in the first scenario, the optimizer does not charge the battery at all. Instead, it sells almost all excess solar energy directly to the grid because the high selling price makes exporting more profitable than storing the energy. This behavior is confirmed by the binary variables showing that the battery neither charges or discharges starting from hour 11 in the linear case and during hours 11-16 in the non linear case, meaning all exported energy comes from real-time solar production rather than stored energy during that time.

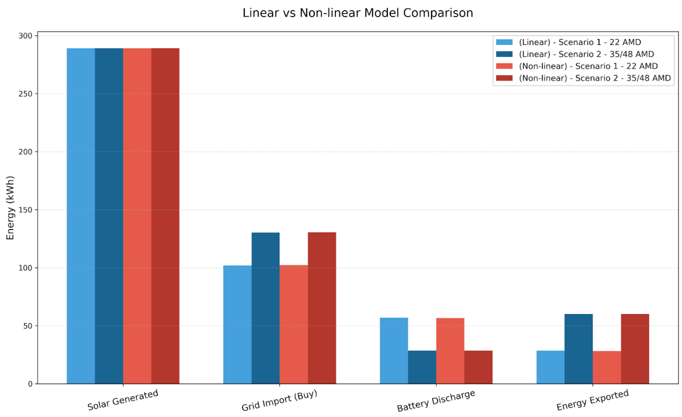

(Linear) Scenario 1 - 22 AMD Battery Operation Strategy (Charge/Discharge + SOC)
SOC (kWh)
Charging (+)
Discharging (-)
Battery Power (kW) / SOC (kWh)
Hour


(Linear) Scenario 2 - 35 or 48 AMD Battery Operation Strategy (Charge/Discharge + SOC)
SOC (kWh)
Charging (+)
Discharging (-)
Battery Power (kW) / SOC (kWh)
Hour

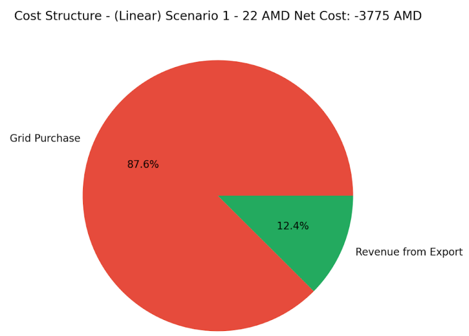

(Non-linear) Scenario 1 - 22 AMD Battery Operation Strategy (Charge/Discharge + SOC)
SOC (kWh)
Charging (+)
Discharging (-)
Battery Power (kW) / SOC (kWh)
Hour

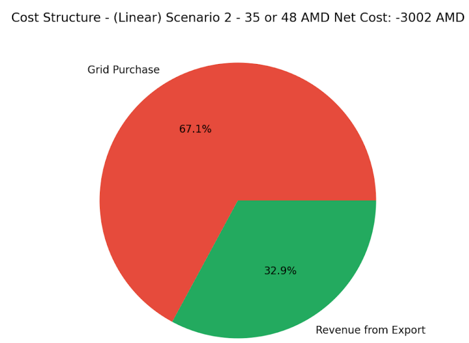

(Non-linear) Scenario 2 - 35 or 48 AMD Battery Operation Strategy (Charge/Discharge + SOC)
SOC (kWh)
Charging (+)
Discharging (-)
Battery Power (kW) / SOC (kWh)
Hour

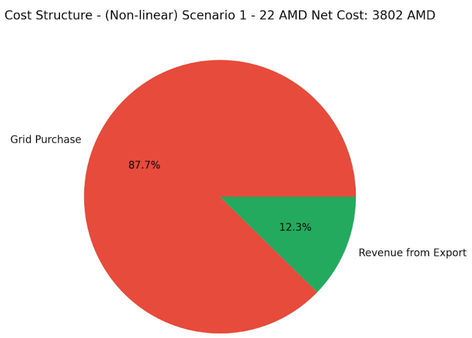
The two tariff scenarios show a clear relationship between the selling price and the optimal export behavior of the microgrid.

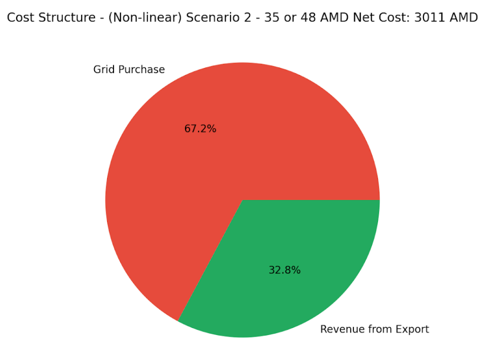


When the selling price is 22 AMD/kWh (Scenario 1), the optimizer exports 28 kWh to the grid. When the selling price increases to 35 or 48 AMD/kWh (Scenario 2), the exported energy rises to 60.0 kWh, indicating that higher selling prices cause the model to prioritize exporting energy rather than self-consumption. In scenario 2, the optimizer buys more energy from the grid at non-peak hours, charges the battery and then sells at peak hours when it is more profitable.

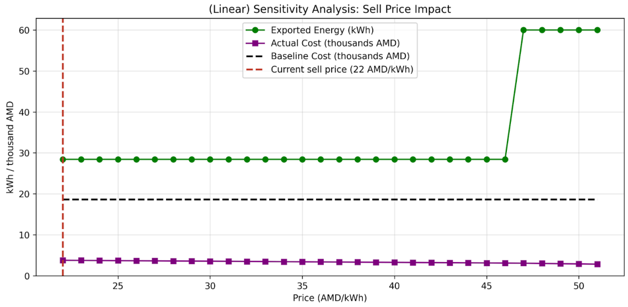


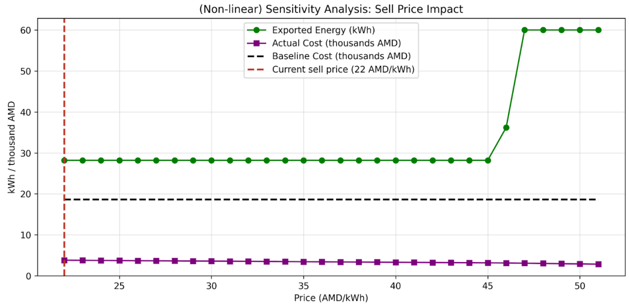






In Scenario 2, both models yield similar operating strategies and therefore the same gross exported energy (60 kWh). However, the non-linear model explicitly accounts for quadratic inverter and battery losses. These physical losses represent energy that is consumed internally and does not reach the building load or the grid. To satisfy the hourly energy balance under the presence of these additional losses, the non-linear model purchases more electricity from the grid compared to the linear case. This small increase in grid purchases directly demonstrates the real economic penalty of inverter and battery inefficiency that is completely invisible in traditional linear formulations. This difference is seen through the 0.1% shift in the cost structure chart between the linear and non-linear cases in both export-price scenarios.





Increasing the sell price leads to higher export levels. Once the sell price becomes greater than the buy price, exporting becomes more profitable than storing or self-consuming energy, resulting in a rise in exported energy.

In conclusion, the comparison shows that the system imports and exports more whenever the selling price approaches or exceeds the buy price.

Conclusion

The thesis developed two global optimisation models for daily energy management of a commercial building with rooftop PV and battery storage under 2025 Armenian tariffs.

The linear MILP model (PuLP + CBC) provides a fast benchmark, while the convex MINLP model (GEKKO + APOPT/IPOPT) accurately captures quadratic inverter and battery losses.

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