HW3 — Federated Learning & Differential Privacy (FL + DP)

This notebook combines all code for:

- **Part 1**: Federated Learning (FedAvg) - Serial and Ray-based implementations
- **Part 2**: Differential Privacy with Laplace Noise

Table of Contents

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- How to Run: Notebook should be in the same directory as 'HW3-data' folder.

Run as is to produce output folders 'output/part1' and 'output/part2' with all required plots.

1. Setup and Imports

```
import os
import sys
import math
import random
import csv
import json
from typing import List, Tuple, Dict
from pathlib import Path
from collections import defaultdict

import numpy as np
import torch
import torch.nn as nn
```

```python
from torch.utils.data import Dataset, DataLoader, TensorDataset
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix, classification_report
from scipy.stats import laplace

# Optional: Ray for parallel federated learning
try:
    import ray
    RAY_AVAILABLE = True
except ImportError:
    RAY_AVAILABLE = False
    print("Ray not available. Parallel FedAvg will be disabled.")

# Set random seeds for reproducibility
def set_seed(seed=42):
    random.seed(seed)
    np.random.seed(seed)
    torch.manual_seed(seed)
    if torch.cuda.is_available():
        torch.cuda.manual_seed_all(seed)

set_seed(42)

# Configuration
class Config:
    # Data paths
    train_data_path = "HW3-data/Assignment3-data/train_data.npy"
    test_data_path = "HW3-data/Assignment3-data/test_data.npy"

    # Federated Learning hyperparameters
    rounds = 50
    client_frac = 0.1
    local_epochs = 1
    batch_size = 64
    lr = 1e-3
    optimizer = "adam"  # "sgd", "adam", "adamw"
    weight_decay = 0.0
    label_smoothing = 0.0

    # Model architecture
    hidden_layers = [256, 128]
    dropout = 0.0
    use_bn = False
    standardize = True

    # Differential Privacy
    noise_scales = [0.0, 0.01, 0.05, 0.1]  # Laplace scale parameter b

    # Output directories
    output_dir = "output"
```

```python
    part1_dir = "output/part1"
    part2_dir = "output/part2"

    # Device
    use_gpu = torch.cuda.is_available()

# Create output directories
os.makedirs(Config.part1_dir, exist_ok=True)
os.makedirs(Config.part2_dir, exist_ok=True)

print(f"Device: {'CUDA' if Config.use_gpu else 'CPU'}")
print(f"Ray available: {RAY_AVAILABLE}")
```

```
/Users/victorapanpa/Documents/Learning/MachineLearning-DeepLearning-
DS_Coding/.venv/lib/python3.12/site-packages/tqdm/auto.py:21:
TqdmWarning: IProgress not found. Please update jupyter and
ipywidgets. See
https://ipywidgets.readthedocs.io/en/stable/user_install.html
  from .autonotebook import tqdm as notebook_tqdm
2025-11-19 13:41:02,342	INFO util.py:154 -- Missing packages:
['ipywidgets']. Run `pip install -U ipywidgets`, then restart the
notebook server for rich notebook output.
```

```
Device: CPU
Ray available: True
```

2. Helper Functions (Data Loading & Preprocessing)

```python
# -------------------------
# Data Loading Helpers
# -------------------------

def _as_ndarray(x):
    return x if isinstance(x, np.ndarray) else np.asarray(x)

def _unwrap_object_array(a):
    if isinstance(a, np.ndarray) and a.dtype == object and a.shape ==
(1,):
        try:
            return a.item()
        except Exception:
            return a
    return a

def _maybe_item(x):
    if isinstance(x, np.ndarray) and x.dtype == object and x.shape ==
():
        try:
            return x.item()
        except Exception:
```

```
            return x
    return x

def _xy_from_obj(obj) -> Tuple[np.ndarray, np.ndarray]:
    """Extract X and y from various container types."""
    obj = _maybe_item(obj)
    if isinstance(obj, dict):
        kx = next((k for k in obj.keys() if k.lower() in
("images","image","x","data","features")), None)
        ky = next((k for k in obj.keys() if k.lower() in
("labels","label","y","target","targets")), None)
        if kx is None or ky is None:
            raise ValueError("Dict missing image/label keys.")
        return _as_ndarray(obj[kx]), _as_ndarray(obj[ky])
    if isinstance(obj, (list, tuple)) and len(obj) == 2:
        X, y = obj
        return _as_ndarray(X), _as_ndarray(y)
    raise ValueError("Unknown (X,y) container.")

def _flatten2d(X: np.ndarray) -> np.ndarray:
    """Flatten images to 2D if needed."""
    return X.reshape(X.shape[0], -1) if X.ndim > 2 else X

def _cfloat32(a: np.ndarray) -> np.ndarray:
    """Convert to contiguous float32 array."""
    return np.ascontiguousarray(a.astype(np.float32, copy=False))

def load_clients_from_npy(train_path: str) -> List[Tuple[np.ndarray,
np.ndarray]]:
    """Load federated client data from .npy file."""
    a = np.load(train_path, allow_pickle=True)
    a = _unwrap_object_array(a)

    if isinstance(a, np.ndarray) and a.dtype == object and a.ndim == 1
and a.size > 1:
        clients = []
        for i in range(a.size):
            Xi, yi = _xy_from_obj(a[i])
            # Normalize images to [0, 1] if needed
            if Xi.max() > 1.0:
                Xi = Xi / 255.0
            clients.append((Xi, yi))
        return clients

    # Fallback: monolithic data, split into clients
    X, y = _xy_from_obj(a)
    if X.max() > 1.0:
        X = X / 255.0
    return split_into_clients(X, y, n_clients=100, seed=42)
```

```python
def load_test_from_npy(test_path: str) -> Tuple[np.ndarray,
np.ndarray]:
    """Load test data from .npy file."""
    b = np.load(test_path, allow_pickle=True)
    b = _unwrap_object_array(b)
    Xte, yte = _xy_from_obj(b)
    # Normalize images to [0, 1] if needed
    if Xte.max() > 1.0:
        Xte = Xte / 255.0
    return Xte, yte

def split_into_clients(X: np.ndarray, y: np.ndarray, n_clients: int,
seed: int = 42):
    """Split monolithic data into n_clients shards."""
    rng = np.random.default_rng(seed)
    idx = np.arange(len(y))
    rng.shuffle(idx)
    shards = np.array_split(idx, n_clients)
    return [(X[s], y[s]) for s in shards]

def remap_labels_global(clients: List[Tuple[np.ndarray, np.ndarray]],
test_y: np.ndarray):
    """Remap labels to consecutive integers starting from 0."""
    ys = [c[1].ravel() for c in clients if len(c[1]) > 0]
    ys.append(test_y.ravel())
    all_y = np.concatenate(ys)

    if all_y.dtype.kind in "fc":
        all_y = np.rint(all_y).astype(np.int64)
    uniq = np.unique(all_y)
    label2new = {int(lbl): i for i, lbl in enumerate(uniq.tolist())}

    def _remap(y):
        y = np.rint(y).astype(np.int64) if y.dtype.kind in "fc" else
y.astype(np.int64, copy=False)
        return np.vectorize(lambda t: label2new[int(t)])(y)

    new_clients = [(X, _remap(y)) for (X, y) in clients]
    test_y_new = _remap(test_y)
    K = len(uniq)
    return new_clients, test_y_new, K, label2new

def standardize_train_test(clients, Xte):
    """Standardize features using training data statistics."""
    Xs = [_flatten2d(c[0]) for c in clients if len(c[1]) > 0]
    bigX = np.concatenate(Xs, axis=0)
    mean = bigX.mean(axis=0)
    std = bigX.std(axis=0)
    std[std == 0] = 1.0
    new_clients = []
```

```python
    for Xc, yc in clients:
        X2 = _flatten2d(Xc)
        X2 = _cfloat32((X2 - mean) / std)
        new_clients.append((X2, yc))
    Xte2 = _flatten2d(Xte)
    Xte2 = _cfloat32((Xte2 - mean) / std)
    return new_clients, Xte2

def to_float_clients(clients):
    """Convert client data to float32 without standardization."""
    return [(_cfloat32(_flatten2d(X)), y) for (X, y) in clients]

def to_float_test(Xte):
    """Convert test data to float32 without standardization."""
    return _cfloat32(_flatten2d(Xte))

print("Data loading functions defined.")
```

```
Data loading functions defined.
```

```python
# -------------------------
# Model: Multi-Layer Perceptron (MLP)
# -------------------------

class MLP(nn.Module):
    def __init__(self, in_dim: int, hidden: List[int], out_dim: int,
dropout: float = 0.0, use_bn: bool = False):
        super().__init__()
        layers = []
        last = in_dim
        for h in hidden:
            layers.append(nn.Linear(last, h))
            if use_bn:
                layers.append(nn.BatchNorm1d(h))
            layers.append(nn.ReLU(inplace=True))
            if dropout and dropout > 0:
                layers.append(nn.Dropout(dropout))
            last = h
        layers.append(nn.Linear(last, out_dim))
        self.net = nn.Sequential(*layers)

    def forward(self, x):
        return self.net(x)

print("MLP model defined.")
```

```
MLP model defined.
```

4. Part 1: Federated Learning (FedAvg)

4.1 Serial FedAvg Implementation

```
# -------------------------
# Dataset and DataLoader
# -------------------------

class NumpyDataset(Dataset):
    def __init__(self, X, y):
        self.X = _cfloat32(_as_ndarray(X))
        self.y = _as_ndarray(y).astype(np.int64, copy=False)
        assert len(self.X) == len(self.y)

    def __len__(self):
        return len(self.y)

    def __getitem__(self, i):
        return self.X[i], self.y[i]

def make_loader(X, y, bs, shuffle=True, drop_last=False):
    if len(y) == 0:
        Xd = np.zeros((0, X.shape[1]), dtype=np.float32)
        yd = np.zeros((0,), dtype=np.int64)
        return DataLoader(NumpyDataset(Xd, yd), batch_size=bs,
shuffle=False, drop_last=False)
    return DataLoader(NumpyDataset(X, y), batch_size=bs,
shuffle=shuffle, drop_last=drop_last)

# -------------------------
# Local Training Function
# -------------------------

def local_train(global_state, model_ctor, X, y, local_epochs, lr,
optimizer_type, weight_decay,
                label_smoothing, batch_size, use_bn, device):
    """Train model locally on client data."""
    if len(y) == 0:
        return None

    model = model_ctor().to(device)
    model.load_state_dict(global_state, strict=True)

    if optimizer_type == "sgd":
        opt = torch.optim.SGD(model.parameters(), lr=lr, momentum=0.9,
weight_decay=weight_decay)
    elif optimizer_type == "adam":
        opt = torch.optim.Adam(model.parameters(), lr=lr,
weight_decay=weight_decay)
    else:
```

```python
        opt = torch.optim.AdamW(model.parameters(), lr=lr,
weight_decay=weight_decay)

    ce = nn.CrossEntropyLoss(label_smoothing=label_smoothing) if
label_smoothing > 0 else nn.CrossEntropyLoss()
    loader = make_loader(X, y, batch_size, shuffle=True,
drop_last=use_bn)

    model.train()
    for _ in range(local_epochs):
        for xb, yb in loader:
            # BN-safe: skip pathological 1-sample batch
            if use_bn and xb.size(0) == 1:
                continue
            xb = xb.to(device, non_blocking=True)
            yb = yb.to(device, non_blocking=True)
            logits = model(xb)
            loss = ce(logits, yb)
            opt.zero_grad(set_to_none=True)
            loss.backward()
            opt.step()

    st = {k: v.detach().cpu().clone() for k, v in
model.state_dict().items()}
    return (st, len(y))

# -------------------------
# Evaluation Functions
# -------------------------

@torch.no_grad()
def evaluate(model, Xte, yte, bs, device):
    """Evaluate model on test data."""
    model.eval()
    loader = make_loader(Xte, yte, bs, shuffle=False, drop_last=False)
    n, correct, total_loss = 0, 0, 0.0
    ce = nn.CrossEntropyLoss(reduction="sum")
    for xb, yb in loader:
        xb = xb.to(device)
        yb = yb.to(device)
        logits = model(xb)
        total_loss += ce(logits, yb).item()
        preds = logits.argmax(dim=1)
        correct += (preds == yb).sum().item()
        n += len(yb)
    if n == 0:
        return 0.0, float("nan")
    return correct / n, total_loss / n

@torch.no_grad()
```

```
def evaluate_on_clients(model, clients, bs, device):
    """Evaluate model on union of all client training data."""
    model.eval()
    Xs = [c[0] for c in clients if len(c[1]) > 0]
    ys = [c[1] for c in clients if len(c[1]) > 0]
    if not Xs:
        return 0.0, float("nan")
    X_all = np.concatenate(Xs, axis=0)
    y_all = np.concatenate(ys, axis=0)
    acc, loss = evaluate(model, X_all, y_all, bs, device)
    return acc, loss

# -------------------------
# FedAvg Aggregation
# -------------------------

def weighted_average_states(collected: List[Tuple[Dict[str,
torch.Tensor], int]]):
    """Weighted average of model states by sample count."""
    total = sum(n for _, n in collected)
    base = {k: v.clone() for k, v in collected[0][0].items()}
    for k in base.keys():
        if torch.is_floating_point(base[k]):
            base[k].mul_(collected[0][1] / total)
            for st, n_i in collected[1:]:
                base[k].add_(st[k] * (n_i / total))
        else:
            base[k] = collected[0][0][k]
    return base

# -------------------------
# Main FedAvg Training Loop
# -------------------------

def train_fedavg_serial(
    clients, Xte, yte, n_classes, in_dim,
    rounds=50, client_frac=0.1, local_epochs=1, batch_size=64, lr=1e-
3,
    optimizer="adam", weight_decay=0.0, label_smoothing=0.0,
    hidden=[256, 128], dropout=0.0, use_bn=False,
    device=None, seed=42, noise_scale=0.0
):
    """
    Train federated model using serial FedAvg.

    Parameters:
    -----------
    noise_scale : float
        Laplace noise scale b for DP (0 = no noise). Applied to client
data.
```

```
    """
    if device is None:
        device = torch.device("cuda:0" if torch.cuda.is_available()
else "cpu")

    set_seed(seed)

    # Apply Laplace noise if specified (for Part 2)
    if noise_scale > 0.0:
        print(f"Applying Laplace noise with scale b={noise_scale} to
all clients")
        for i in range(len(clients)):
            Xc, yc = clients[i]
            noise = laplace.rvs(loc=0.0, scale=noise_scale,
size=Xc.shape)
            Xc_noisy = np.clip(Xc + noise, 0.0, 1.0)
            clients[i] = (Xc_noisy, yc)

    def model_ctor():
        return MLP(in_dim=in_dim, hidden=hidden, out_dim=n_classes,
dropout=dropout, use_bn=use_bn)

    global_model = model_ctor()
    global_state = {k: v.detach().cpu().clone() for k, v in
global_model.state_dict().items()}

    best_acc = -1.0
    history = []

    for rnd in range(1, rounds + 1):
        # Select clients for this round
        m = max(1, int(math.ceil(client_frac * len(clients))))
        sel = np.random.choice(len(clients), size=m, replace=False)
        collected = []

        for idx in sel:
            Xc, yc = clients[idx]
            if len(yc) == 0:
                continue
            res = local_train(
                global_state, model_ctor, Xc, yc, local_epochs, lr,
optimizer,
                weight_decay, label_smoothing, batch_size, use_bn,
device
            )
            if res is not None:
                collected.append(res)

        if not collected:
            continue
```

```
        # Aggregate updates
        global_state = weighted_average_states(collected)
        global_model.load_state_dict(global_state, strict=True)

        # Evaluate
        train_acc, train_loss =
evaluate_on_clients(global_model.to(device), clients, bs=1024,
device=device)
        test_acc, test_loss = evaluate(global_model.to(device), Xte,
yte, bs=1024, device=device)

        print(f"[Round {rnd:03d}] train_acc={train_acc:.4f}
test_acc={test_acc:.4f} loss={test_loss:.4f}")

        history.append({
            "round": rnd,
            "train_acc": train_acc,
            "test_acc": test_acc,
            "loss": test_loss
        })

        if test_acc > best_acc:
            best_acc = test_acc

    print(f"Done. Best acc={best_acc:.4f}")
    return history, global_state, best_acc
```

4.2 Example: Run Part 1 (Serial FedAvg)

```
# Load data
print("Loading data...")
clients = load_clients_from_npy(Config.train_data_path)
Xte, yte = load_test_from_npy(Config.test_data_path)

# Preprocess
clients, yte, n_classes, label_map = remap_labels_global(clients, yte)
print(f"Number of classes: {n_classes}")

if Config.standardize:
    clients, Xte = standardize_train_test(clients, Xte)
    print("Data standardized.")
else:
    clients = to_float_clients(clients)
    Xte = to_float_test(Xte)
    print("Data converted to float32.")

# Get input dimension
sizes = [len(c[1]) for c in clients]
n_total = int(sum(sizes))
```

```python
in_dim = clients[next(i for i, s in enumerate(sizes) if s > 0)]
[0].shape[1] if any(s > 0 for s in sizes) else Xte.shape[1]

print(f"Clients: {len(clients)} | Total samples: {n_total}")
print(f"Input dimension: {in_dim} | Output classes: {n_classes}")
if sizes:
    print(f"Client sizes: min={min(sizes)} med={int(np.median(sizes))}
max={max(sizes)} empties={(np.array(sizes)==0).sum()}")
```

```
Loading data...
Number of classes: 62
Data standardized.
Clients: 100 | Total samples: 31825
Input dimension: 784 | Output classes: 62
Client sizes: min=118 med=331 max=393 empties=0
```

```python
# Run Part 1: Federated Learning (no noise)
device = torch.device("cuda:0" if Config.use_gpu else "cpu")
print(f"\n{'='*60}")
print("Part 1: Federated Learning (FedAvg) - Serial")
print(f"{'='*60}")

history_part1, global_state_part1, best_acc_part1 =
train_fedavg_serial(
    clients=clients,
    Xte=Xte,
    yte=yte,
    n_classes=n_classes,
    in_dim=in_dim,
    rounds=Config.rounds,
    client_frac=Config.client_frac,
    local_epochs=Config.local_epochs,
    batch_size=Config.batch_size,
    lr=Config.lr,
    optimizer=Config.optimizer,
    weight_decay=Config.weight_decay,
    label_smoothing=Config.label_smoothing,
    hidden=Config.hidden_layers,
    dropout=Config.dropout,
    use_bn=Config.use_bn,
    device=device,
    seed=42,
    noise_scale=0.0  # No noise for Part 1
)
```

```
============================================================
Part 1: Federated Learning (FedAvg) - Serial
============================================================
[Round 001] train_acc=0.0820 test_acc=0.0842 loss=4.1060
```

```
[Round 002] train_acc=0.1513 test_acc=0.1563 loss=3.9805
[Round 003] train_acc=0.1360 test_acc=0.1397 loss=3.7770
[Round 004] train_acc=0.1613 test_acc=0.1743 loss=3.4912
[Round 005] train_acc=0.1921 test_acc=0.2157 loss=3.2660
[Round 006] train_acc=0.2827 test_acc=0.3088 loss=3.1021
[Round 007] train_acc=0.3002 test_acc=0.3192 loss=2.9643
[Round 008] train_acc=0.3451 test_acc=0.3607 loss=2.8641
[Round 009] train_acc=0.3508 test_acc=0.3676 loss=2.7449
[Round 010] train_acc=0.4036 test_acc=0.4159 loss=2.6527
[Round 011] train_acc=0.4178 test_acc=0.4319 loss=2.5653
[Round 012] train_acc=0.4011 test_acc=0.4126 loss=2.5074
[Round 013] train_acc=0.4436 test_acc=0.4502 loss=2.4184
[Round 014] train_acc=0.4441 test_acc=0.4502 loss=2.3610
[Round 015] train_acc=0.4541 test_acc=0.4609 loss=2.3150
[Round 016] train_acc=0.4774 test_acc=0.4841 loss=2.2491
[Round 017] train_acc=0.4981 test_acc=0.5037 loss=2.1966
[Round 018] train_acc=0.4890 test_acc=0.4949 loss=2.1738
[Round 019] train_acc=0.5215 test_acc=0.5220 loss=2.1428
[Round 020] train_acc=0.5008 test_acc=0.5018 loss=2.1163
[Round 021] train_acc=0.5092 test_acc=0.5162 loss=2.0841
[Round 022] train_acc=0.5131 test_acc=0.5217 loss=2.0470
[Round 023] train_acc=0.5410 test_acc=0.5385 loss=2.0175
[Round 024] train_acc=0.5451 test_acc=0.5490 loss=1.9894
[Round 025] train_acc=0.5555 test_acc=0.5551 loss=1.9593
[Round 026] train_acc=0.5546 test_acc=0.5556 loss=1.9407
[Round 027] train_acc=0.5500 test_acc=0.5510 loss=1.9481
[Round 028] train_acc=0.5679 test_acc=0.5670 loss=1.8896
[Round 029] train_acc=0.5552 test_acc=0.5595 loss=1.9246
[Round 030] train_acc=0.5685 test_acc=0.5645 loss=1.9244
[Round 031] train_acc=0.5597 test_acc=0.5650 loss=1.9056
[Round 032] train_acc=0.5744 test_acc=0.5725 loss=1.8721
[Round 033] train_acc=0.5854 test_acc=0.5882 loss=1.8399
[Round 034] train_acc=0.5705 test_acc=0.5766 loss=1.8687
[Round 035] train_acc=0.5948 test_acc=0.5949 loss=1.8132
[Round 036] train_acc=0.5904 test_acc=0.5921 loss=1.8211
[Round 037] train_acc=0.5997 test_acc=0.6020 loss=1.8057
[Round 038] train_acc=0.6000 test_acc=0.6007 loss=1.7978
[Round 039] train_acc=0.5959 test_acc=0.5965 loss=1.7920
[Round 040] train_acc=0.5992 test_acc=0.6023 loss=1.7902
[Round 041] train_acc=0.6076 test_acc=0.6150 loss=1.7666
[Round 042] train_acc=0.6057 test_acc=0.6078 loss=1.7819
[Round 043] train_acc=0.6107 test_acc=0.6175 loss=1.7618
[Round 044] train_acc=0.6147 test_acc=0.6186 loss=1.7532
[Round 045] train_acc=0.6120 test_acc=0.6136 loss=1.7532
[Round 046] train_acc=0.6155 test_acc=0.6150 loss=1.7492
[Round 047] train_acc=0.6175 test_acc=0.6236 loss=1.7354
[Round 048] train_acc=0.6251 test_acc=0.6252 loss=1.7483
[Round 049] train_acc=0.6251 test_acc=0.6275 loss=1.7302
```

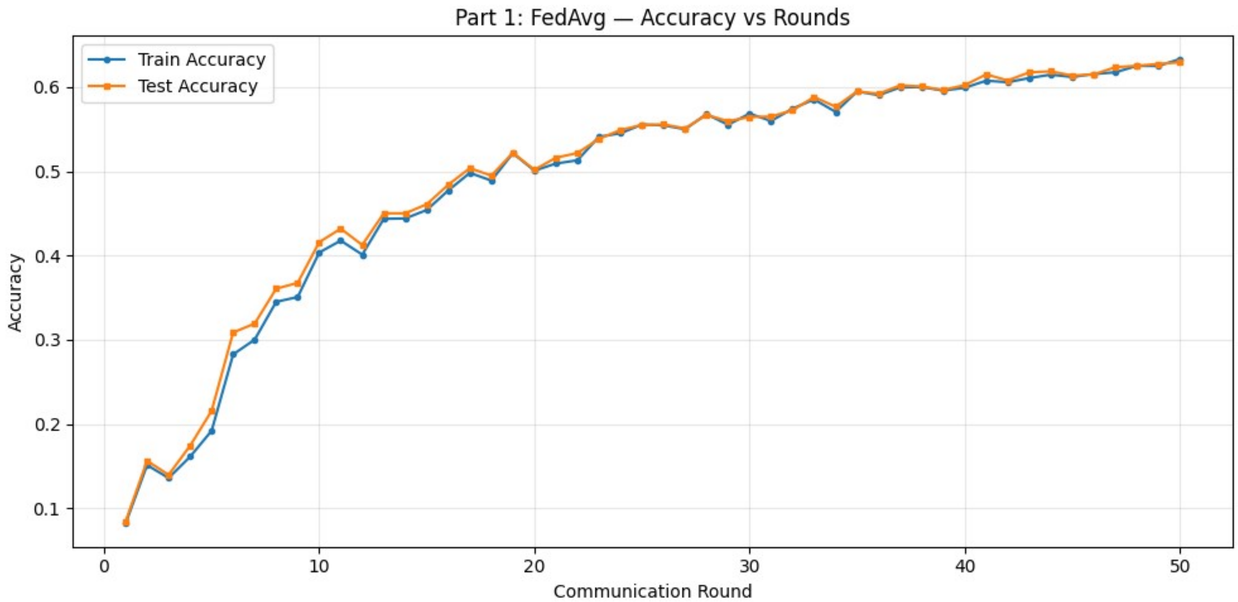
```
[Round 050] train_acc=0.6332 test_acc=0.6294 loss=1.7246
Done. Best acc=0.6294
```

4.3 Plotting Utilities for Part 1

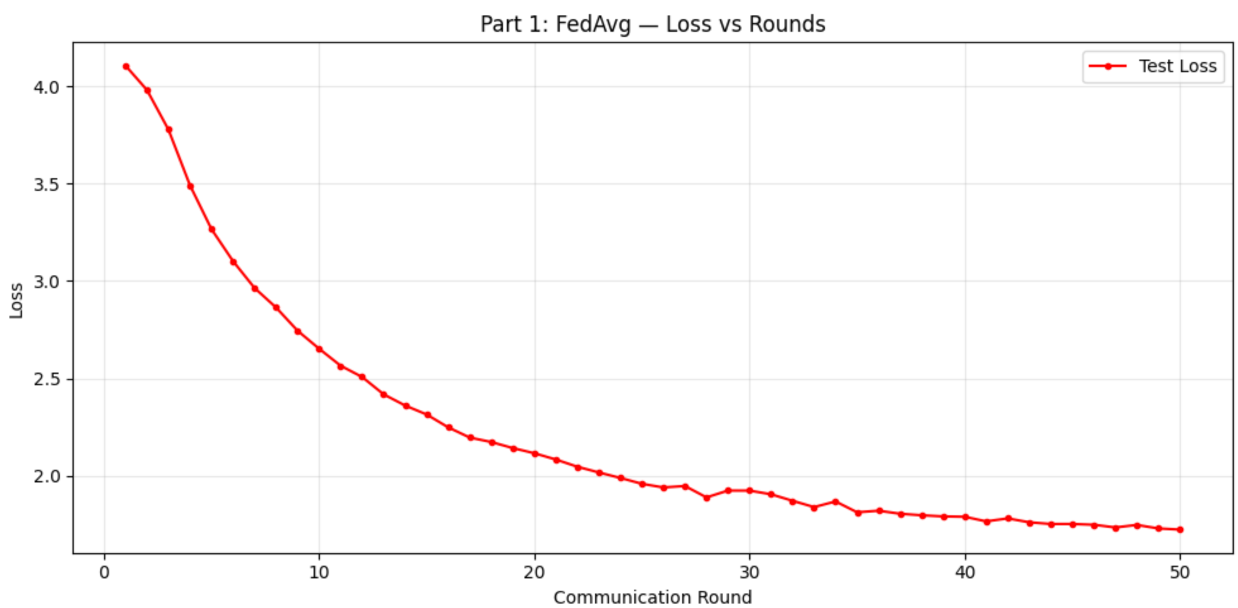
```
# -------------------------
# Plotting Functions
# -------------------------

def plot_training_history(history, title_prefix="Part 1",
save_path=None):
    """Plot accuracy and loss vs communication rounds."""
    rounds = [h["round"] for h in history]
    train_accs = [h["train_acc"] for h in history]
    test_accs = [h["test_acc"] for h in history]
    losses = [h["loss"] for h in history]

    # Accuracy plot
    plt.figure(figsize=(10, 5))
    plt.plot(rounds, train_accs, label='Train Accuracy', marker='o',
markersize=3)
    plt.plot(rounds, test_accs, label='Test Accuracy', marker='s',
markersize=3)
    plt.xlabel('Communication Round')
    plt.ylabel('Accuracy')
    plt.title(f'{title_prefix} – Accuracy vs Rounds')
    plt.legend()
    plt.grid(True, alpha=0.3)
    plt.tight_layout()
    if save_path:
        plt.savefig(f"{save_path}_acc.png", dpi=160,
bbox_inches="tight")
        print(f"Saved: {save_path}_acc.png")
    plt.show()

    # Loss plot
    plt.figure(figsize=(10, 5))
    plt.plot(rounds, losses, label='Test Loss', marker='o',
markersize=3, color='red')
    plt.xlabel('Communication Round')
    plt.ylabel('Loss')
    plt.title(f'{title_prefix} – Loss vs Rounds')
    plt.legend()
    plt.grid(True, alpha=0.3)
    plt.tight_layout()
    if save_path:
        plt.savefig(f"{save_path}_loss.png", dpi=160,
bbox_inches="tight")
        print(f"Saved: {save_path}_loss.png")
```

```
    plt.show()

def plot_label_histograms(clients, yte, save_dir=None,
num_clients_to_plot=5):
    """Plot label distributions for global and selected clients."""
    # Global label histogram
    all_labels = np.concatenate([c[1] for c in clients] + [yte])
    unique_labels, counts = np.unique(all_labels, return_counts=True)

    plt.figure(figsize=(12, 5))
    plt.bar(unique_labels, counts, alpha=0.7)
    plt.xlabel('Label')
    plt.ylabel('Count')
    plt.title('Global Label Distribution')
    plt.grid(True, alpha=0.3, axis='y')
    plt.tight_layout()
    if save_dir:
        plt.savefig(f"{save_dir}/part1_global_label_hist.png",
dpi=160, bbox_inches="tight")
        print(f"Saved: {save_dir}/part1_global_label_hist.png")
    plt.show()

    # Per-client label histograms
    for i in range(min(num_clients_to_plot, len(clients))):
        if len(clients[i][1]) > 0:
            client_labels = clients[i][1]
            unique, counts = np.unique(client_labels,
return_counts=True)
            plt.figure(figsize=(10, 4))
            plt.bar(unique, counts, alpha=0.7)
            plt.xlabel('Label')
            plt.ylabel('Count')
            plt.title(f'Client {i} Label Distribution
(n={len(client_labels)})')
            plt.grid(True, alpha=0.3, axis='y')
            plt.tight_layout()
            if save_dir:

plt.savefig(f"{save_dir}/part1_client_{i}_label_hist.png", dpi=160,
bbox_inches="tight")
            plt.show()

# Plot Part 1 results
plot_training_history(history_part1, title_prefix="Part 1: FedAvg",

save_path=f"{Config.part1_dir}/part1_cf{Config.client_frac:.3f}_e{Conf
ig.local_epochs}")

plot_label_histograms(clients, yte, save_dir=Config.part1_dir,
num_clients_to_plot=5)
```

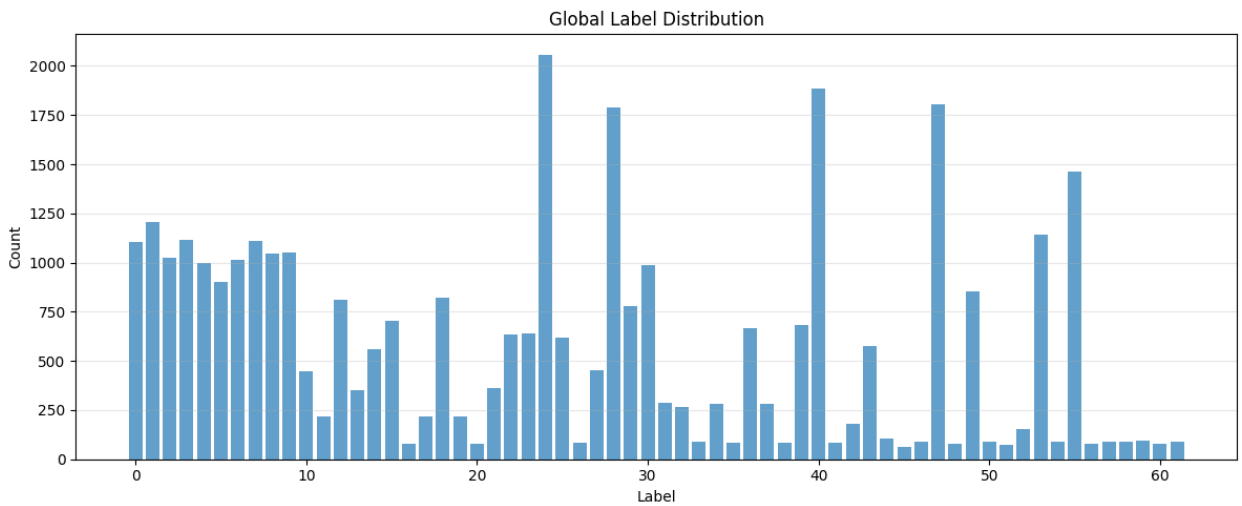
```
Saved: output/part1/part1_cf0.100_e1_acc.png
```

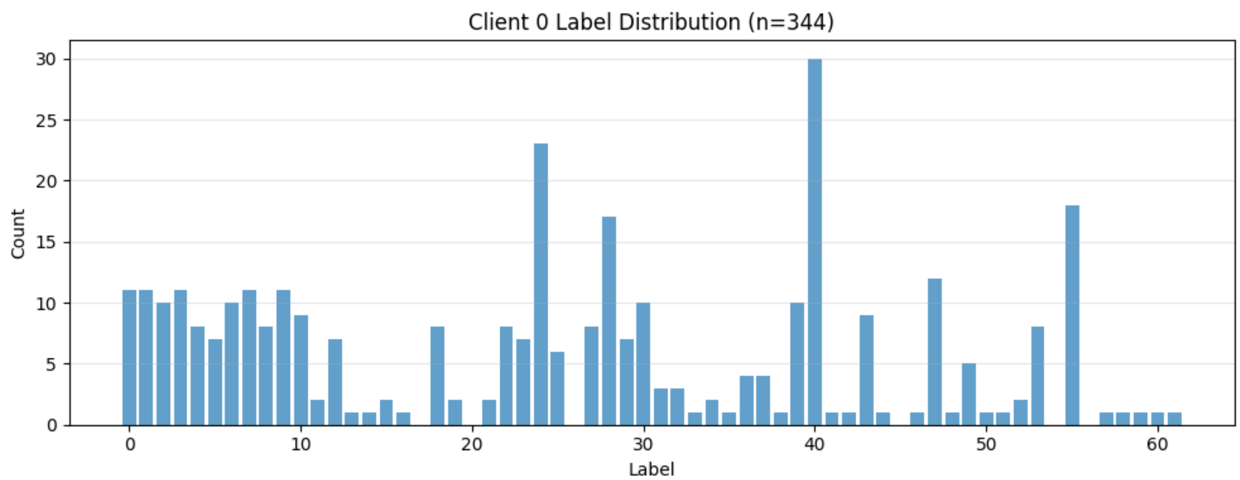


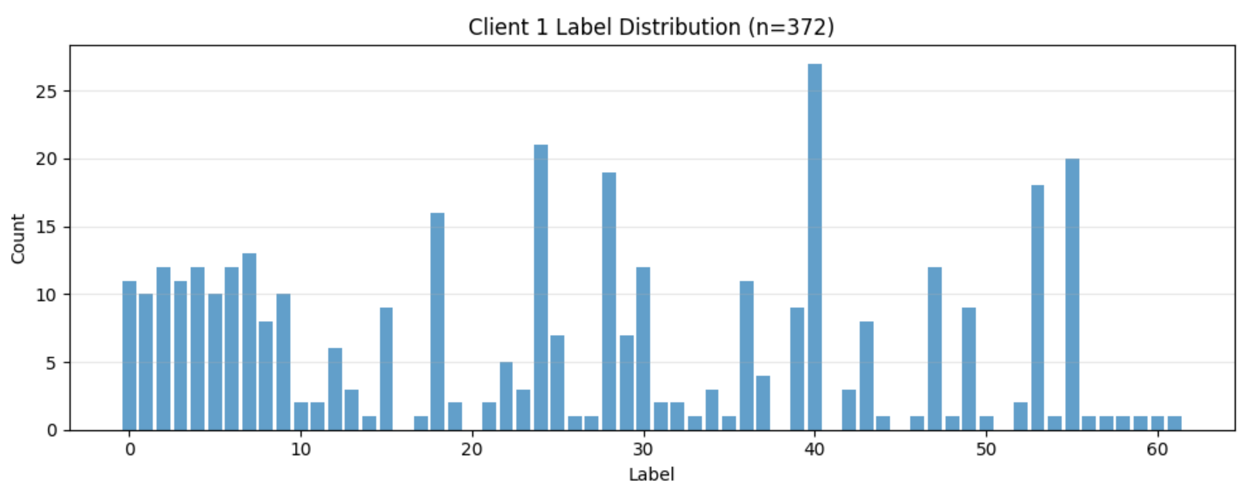

```
Saved: output/part1/part1_cf0.100_e1_loss.png
```

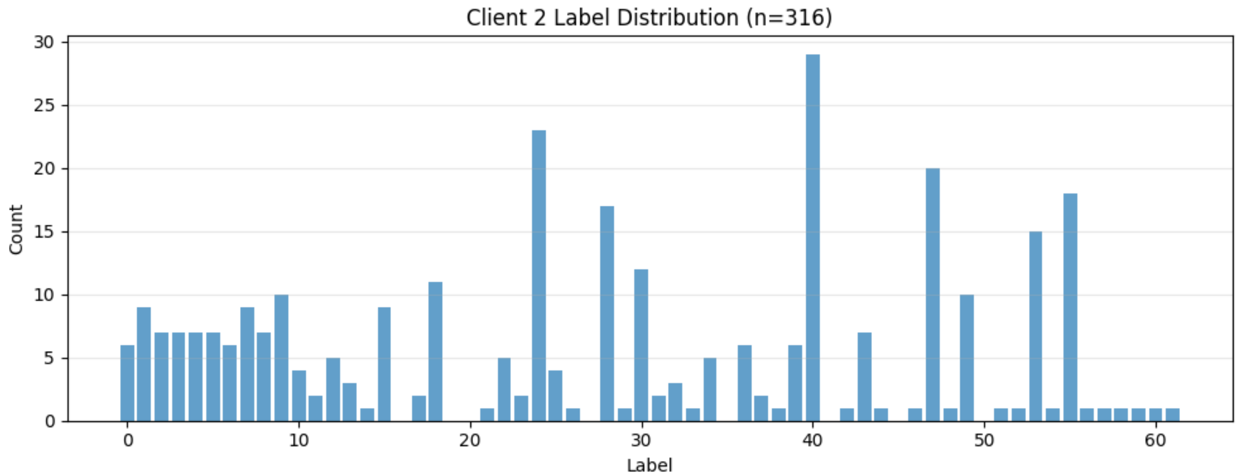


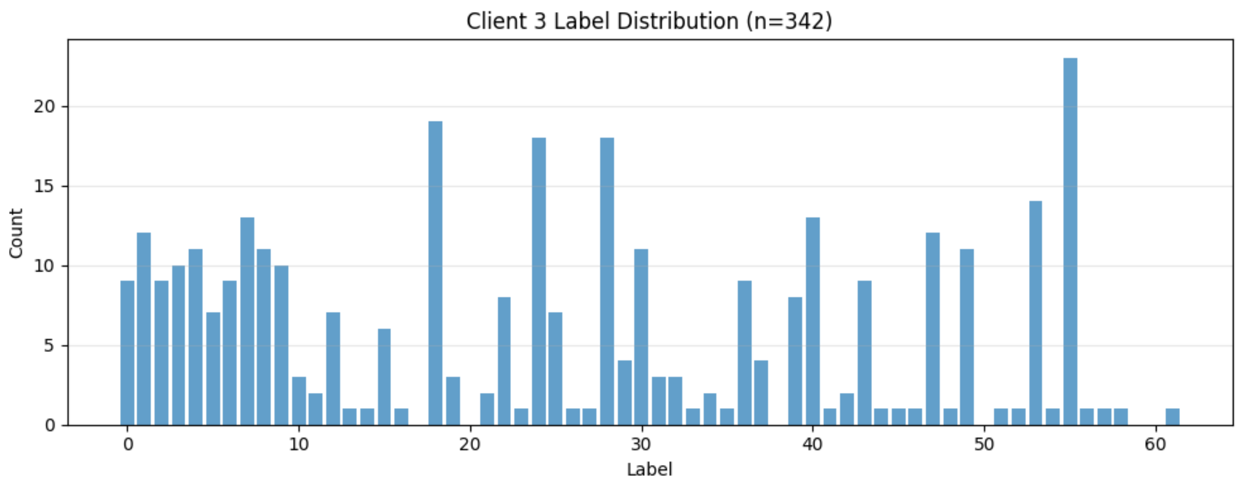

```
Saved: output/part1/part1_global_label_hist.png
```

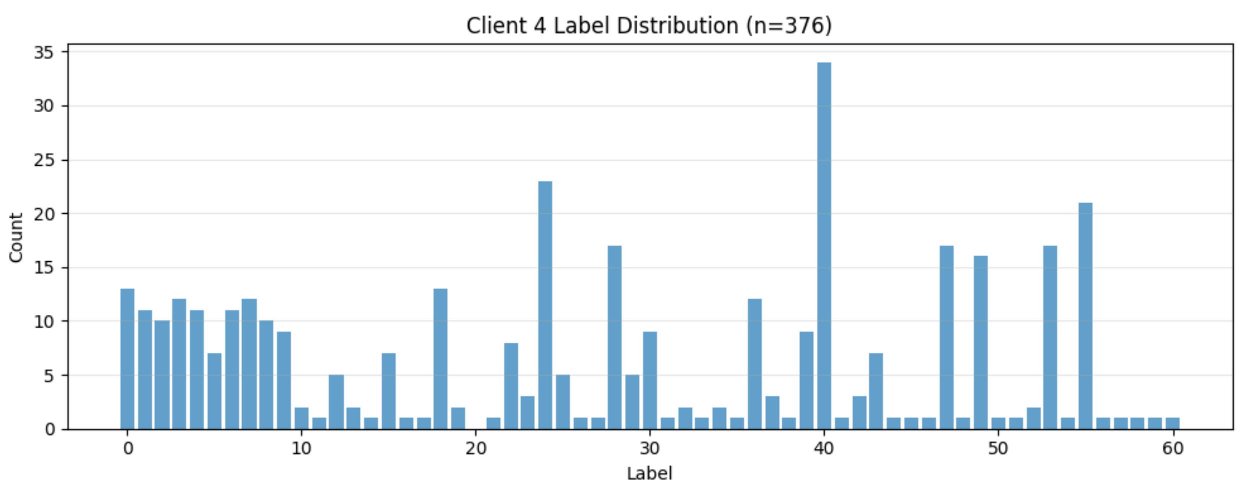

Global Label Distribution
Count
Label


Client 0 Label Distribution (n=344)
Count
Label


Client 1 Label Distribution (n=372)
Count
Label







4.4 Optional: Ray-based Parallel FedAvg

Note: Requires Ray installation. Skip this section if Ray is not available.

```
if RAY_AVAILABLE:
    # Ray-based parallel FedAvg implementation
    @ray.remote
    class ClientTrainer:
        def __init__(self, X, y, cfg):
            self.X = torch.from_numpy(X.copy())
            self.y = torch.from_numpy(y.copy())
            self.cfg = cfg
            self.device = torch.device("cpu")
            self.has_bn = bool(cfg.get("use_bn", False))

        def train(self, global_state):
            cfg = self.cfg
            model = MLP(
                cfg["input_dim"],
                cfg["hidden"],
                cfg["n_classes"],
                dropout=cfg["dropout"],
                use_bn=cfg["use_bn"],
            )
            model.load_state_dict(global_state)
            model.to(self.device)

            if cfg["label_smoothing"] > 0:
                ce =
nn.CrossEntropyLoss(label_smoothing=cfg["label_smoothing"])
            else:
                ce = nn.CrossEntropyLoss()

            if cfg["optimizer"] == "sgd":
                opt = torch.optim.SGD(model.parameters(),
lr=cfg["lr"], momentum=0.9,
                                     weight_decay=cfg["weight_decay"])
            elif cfg["optimizer"] == "adam":
                opt = torch.optim.Adam(model.parameters(),
lr=cfg["lr"],

weight_decay=cfg["weight_decay"])
            else:
                opt = torch.optim.AdamW(model.parameters(),
lr=cfg["lr"],

weight_decay=cfg["weight_decay"])

            ds = TensorDataset(self.X, self.y)
            loader = DataLoader(ds, batch_size=cfg["batch_size"],
shuffle=True, drop_last=False)

            model.train()
            for _ in range(cfg["local_epochs"]):
```

```
                for xb, yb in loader:
                    xb = xb.to(self.device)
                    yb = yb.to(self.device)
                    opt.zero_grad()
                    if self.has_bn and xb.size(0) == 1:
                        model.eval()
                        logits = model(xb)
                        model.train()
                    else:
                        logits = model(xb)
                    loss = ce(logits, yb)
                    loss.backward()
                    opt.step()

            return {k: v.cpu() for k, v in
model.state_dict().items()}, int(self.y.shape[0])

    def fedavg_ray(global_state, client_states, client_counts):
        """FedAvg aggregation for Ray."""
        total = float(sum(client_counts))
        new_state = {}
        for k in global_state.keys():
            agg = None
            for st, n in zip(client_states, client_counts):
                w = n / total
                v = st[k].float()
                if agg is None:
                    agg = w * v
                else:
                    agg += w * v
            new_state[k] = agg
        return new_state

    def train_fedavg_ray(clients, Xte, yte, n_classes, in_dim,
rounds=50, client_frac=0.1,
                        local_epochs=1, batch_size=64, lr=1e-3,
optimizer="adamw",
                        weight_decay=0.0, label_smoothing=0.0,
hidden=[256, 128],
                        dropout=0.0, use_bn=False, device=None,
seed=42, ray_cpus=None):
        """Train federated model using Ray-based parallel FedAvg."""
        if device is None:
            device = torch.device("cuda" if torch.cuda.is_available()
else "cpu")

        set_seed(seed)

        cfg = dict(
            input_dim=in_dim,
```

```
            n_classes=n_classes,
            hidden=hidden,
            dropout=dropout,
            use_bn=use_bn,
            local_epochs=local_epochs,
            batch_size=batch_size,
            lr=lr,
            optimizer=optimizer,
            weight_decay=weight_decay,
            label_smoothing=label_smoothing,
        )

        print("Initializing Ray...")
        ray.init(num_cpus=ray_cpus, ignore_reinit_error=True)

        print("Creating client actors...")
        actors = [ClientTrainer.remote(Xc, yc, cfg) for Xc, yc in
clients]

        model = MLP(in_dim, hidden, n_classes, dropout=dropout,
use_bn=use_bn).to(device)
        global_state = {k: v.cpu() for k, v in
model.state_dict().items()}

        history = []
        m_per_round = max(1, int(math.ceil(client_frac *
len(actors))))
        print(f"Using {m_per_round} clients per round
(client_frac={client_frac})")

        for r in range(1, rounds + 1):
            selected = random.sample(range(len(actors)), m_per_round)
            futures = [actors[i].train.remote(global_state) for i in
selected]
            results = ray.get(futures)
            client_states, client_counts = zip(*results)

            global_state = fedavg_ray(global_state, client_states,
client_counts)

            model.load_state_dict(global_state)
            model.to(device)

            acc, loss = evaluate(model, Xte, yte, bs=1024,
device=device)
            history.append({"round": r, "test_acc": acc, "loss":
loss})
            print(f"[Round {r:03d}] test_acc={acc:.4f}
loss={loss:.4f}")
```

```
        ray.shutdown()
        return history, global_state

    print("Ray-based FedAvg functions defined.")
    print("\nTo use Ray-based training, call:")
    print("history_ray, state_ray = train_fedavg_ray(clients, Xte, 
yte, n_classes, in_dim, ...)")
else:
    print("Ray not available. Skipping Ray-based implementation.")
```

```
Ray-based FedAvg functions defined.

To use Ray-based training, call:
history_ray, state_ray = train_fedavg_ray(clients, Xte, yte, 
n_classes, in_dim, ...)
```

5. Part 2: Differential Privacy

5.1 Laplace Noise Implementation

```
# -------------------------
# Differential Privacy: Laplace Noise
# -------------------------

def add_laplace_noise(X, b: float):
    """
    Add element-wise Laplace(0, b) noise to images.

    Parameters
    ----------
    X : np.ndarray
        Shape (n_samples, 28, 28) or (n_samples, 784).
        Values are expected in [0, 1].
    b : float
        Laplace scale parameter. If b <= 0, X is returned unchanged.

    Returns
    -------
    np.ndarray
        Noisy images, clipped back to [0, 1].
    """
    if b <= 0:
        return X

    eps = laplace.rvs(loc=0.0, scale=b, size=X.shape)
    X_noisy = X + eps
    # Keep images in valid range
    X_noisy = np.clip(X_noisy, 0.0, 1.0)
    return X_noisy
```

```
print("Laplace noise function defined.")
```

```
Laplace noise function defined.
```

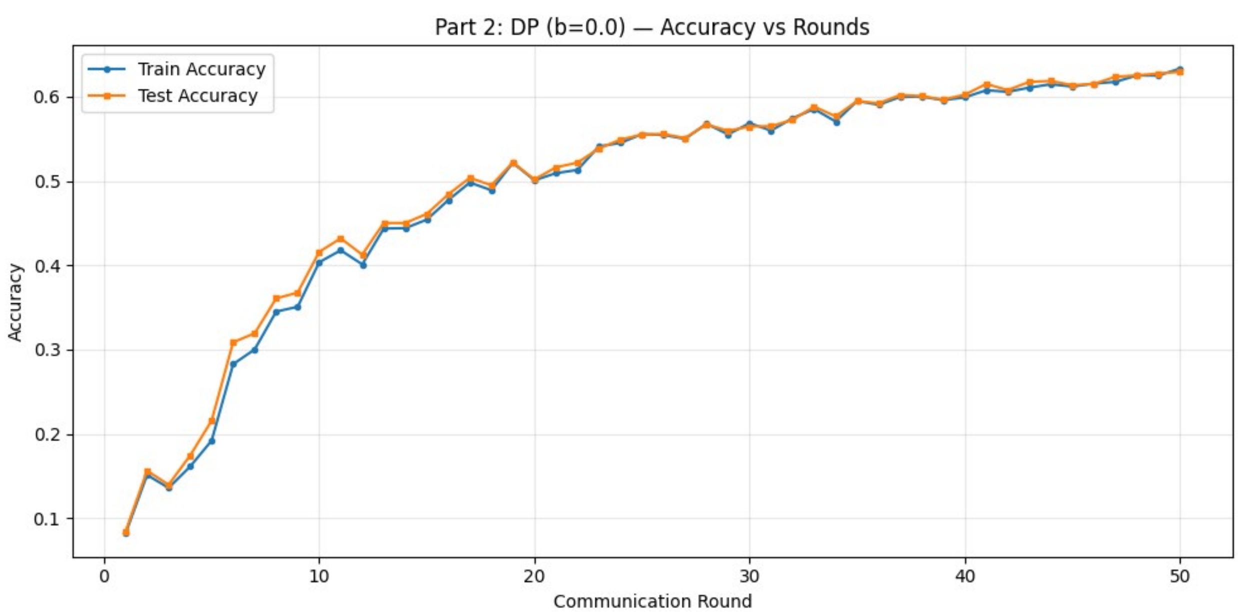
5.2 DP Experiments: Train with Different Noise Scales

```
# Run Part 2: Differential Privacy experiments
print(f"\n{'='*60}")
print("Part 2: Differential Privacy with Laplace Noise")
print(f"{'='*60}")

# Store results for all noise scales
dp_results = {}

for noise_scale in Config.noise_scales:
    print(f"\n{'='*60}")
    print(f"Training with noise scale b = {noise_scale}")
    print(f"{'='*60}")

    # Reload and preprocess data (to avoid modifying original)
    clients_dp = load_clients_from_npy(Config.train_data_path)
    Xte_dp, yte_dp = load_test_from_npy(Config.test_data_path)
    clients_dp, yte_dp, n_classes_dp, _ =
remap_labels_global(clients_dp, yte_dp)

    if Config.standardize:
        clients_dp, Xte_dp = standardize_train_test(clients_dp,
Xte_dp)
    else:
        clients_dp = to_float_clients(clients_dp)
        Xte_dp = to_float_test(Xte_dp)

    # Train with noise
    history_dp, state_dp, best_acc_dp = train_fedavg_serial(
        clients=clients_dp,
        Xte=Xte_dp,
        yte=yte_dp,
        n_classes=n_classes_dp,
        in_dim=in_dim,
        rounds=Config.rounds,
        client_frac=Config.client_frac,
        local_epochs=Config.local_epochs,
        batch_size=Config.batch_size,
        lr=Config.lr,
        optimizer=Config.optimizer,
        weight_decay=Config.weight_decay,
        label_smoothing=Config.label_smoothing,
        hidden=Config.hidden_layers,
        dropout=Config.dropout,
```

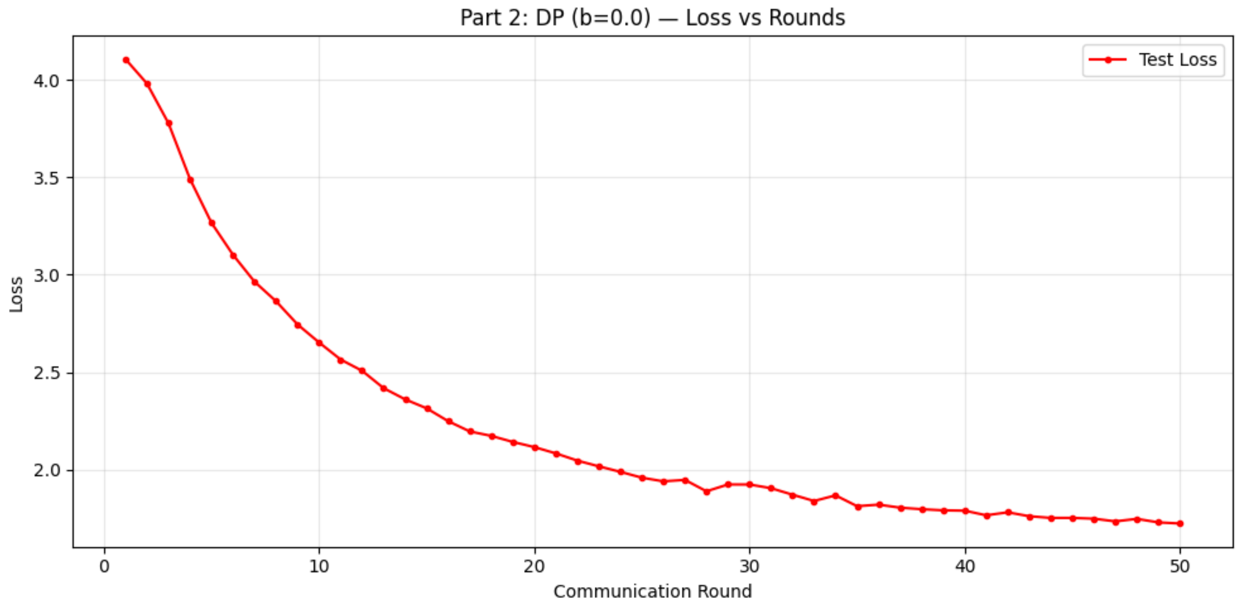
```python
        use_bn=Config.use_bn,
        device=device,
        seed=42,
        noise_scale=noise_scale
    )

    dp_results[noise_scale] = {
        'history': history_dp,
        'best_acc': best_acc_dp,
        'final_acc': history_dp[-1]['test_acc'] if history_dp else 0.0
    }

    # Plot individual results
    plot_training_history(
        history_dp,
        title_prefix=f"Part 2: DP (b={noise_scale})",
        save_path=f"{Config.part2_dir}/part2_b{noise_scale:03.0f}"
    )
```

```
============================================================
Part 2: Differential Privacy with Laplace Noise
============================================================

============================================================
Training with noise scale b = 0.0
============================================================
[Round 001] train_acc=0.0820 test_acc=0.0842 loss=4.1060
[Round 002] train_acc=0.1513 test_acc=0.1563 loss=3.9805
[Round 003] train_acc=0.1360 test_acc=0.1397 loss=3.7770
[Round 004] train_acc=0.1613 test_acc=0.1743 loss=3.4912
[Round 005] train_acc=0.1921 test_acc=0.2157 loss=3.2660
[Round 006] train_acc=0.2827 test_acc=0.3088 loss=3.1021
[Round 007] train_acc=0.3002 test_acc=0.3192 loss=2.9643
[Round 008] train_acc=0.3451 test_acc=0.3607 loss=2.8641
[Round 009] train_acc=0.3508 test_acc=0.3676 loss=2.7449
[Round 010] train_acc=0.4036 test_acc=0.4159 loss=2.6527
[Round 011] train_acc=0.4178 test_acc=0.4319 loss=2.5653
[Round 012] train_acc=0.4011 test_acc=0.4126 loss=2.5074
[Round 013] train_acc=0.4436 test_acc=0.4502 loss=2.4184
[Round 014] train_acc=0.4441 test_acc=0.4502 loss=2.3610
[Round 015] train_acc=0.4541 test_acc=0.4609 loss=2.3150
[Round 016] train_acc=0.4774 test_acc=0.4841 loss=2.2491
[Round 017] train_acc=0.4981 test_acc=0.5037 loss=2.1966
[Round 018] train_acc=0.4890 test_acc=0.4949 loss=2.1738
[Round 019] train_acc=0.5215 test_acc=0.5220 loss=2.1428
[Round 020] train_acc=0.5008 test_acc=0.5018 loss=2.1163
[Round 021] train_acc=0.5092 test_acc=0.5162 loss=2.0841
[Round 022] train_acc=0.5131 test_acc=0.5217 loss=2.0470
[Round 023] train_acc=0.5410 test_acc=0.5385 loss=2.0175
```

```
[Round 024] train_acc=0.5451 test_acc=0.5490 loss=1.9894
[Round 025] train_acc=0.5555 test_acc=0.5551 loss=1.9593
[Round 026] train_acc=0.5546 test_acc=0.5556 loss=1.9407
[Round 027] train_acc=0.5500 test_acc=0.5510 loss=1.9481
[Round 028] train_acc=0.5679 test_acc=0.5670 loss=1.8896
[Round 029] train_acc=0.5552 test_acc=0.5595 loss=1.9246
[Round 030] train_acc=0.5685 test_acc=0.5645 loss=1.9244
[Round 031] train_acc=0.5597 test_acc=0.5650 loss=1.9056
[Round 032] train_acc=0.5744 test_acc=0.5725 loss=1.8721
[Round 033] train_acc=0.5854 test_acc=0.5882 loss=1.8399
[Round 034] train_acc=0.5705 test_acc=0.5766 loss=1.8687
[Round 035] train_acc=0.5948 test_acc=0.5949 loss=1.8132
[Round 036] train_acc=0.5904 test_acc=0.5921 loss=1.8211
[Round 037] train_acc=0.5997 test_acc=0.6020 loss=1.8057
[Round 038] train_acc=0.6000 test_acc=0.6007 loss=1.7978
[Round 039] train_acc=0.5959 test_acc=0.5965 loss=1.7920
[Round 040] train_acc=0.5992 test_acc=0.6023 loss=1.7902
[Round 041] train_acc=0.6076 test_acc=0.6150 loss=1.7666
[Round 042] train_acc=0.6057 test_acc=0.6078 loss=1.7819
[Round 043] train_acc=0.6107 test_acc=0.6175 loss=1.7618
[Round 044] train_acc=0.6147 test_acc=0.6186 loss=1.7532
[Round 045] train_acc=0.6120 test_acc=0.6136 loss=1.7532
[Round 046] train_acc=0.6155 test_acc=0.6150 loss=1.7492
[Round 047] train_acc=0.6175 test_acc=0.6236 loss=1.7354
[Round 048] train_acc=0.6251 test_acc=0.6252 loss=1.7483
[Round 049] train_acc=0.6251 test_acc=0.6275 loss=1.7302
[Round 050] train_acc=0.6332 test_acc=0.6294 loss=1.7246
Done. Best acc=0.6294
Saved: output/part2/part2_b000_acc.png
```

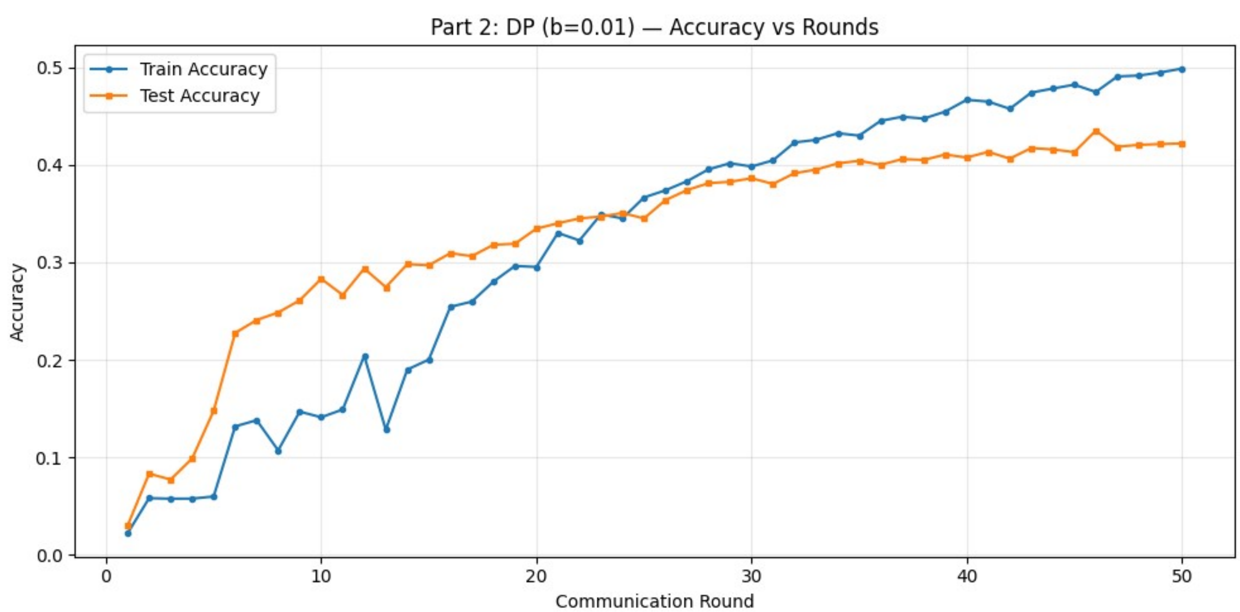


```
Saved: output/part2/part2_b000_loss.png
```

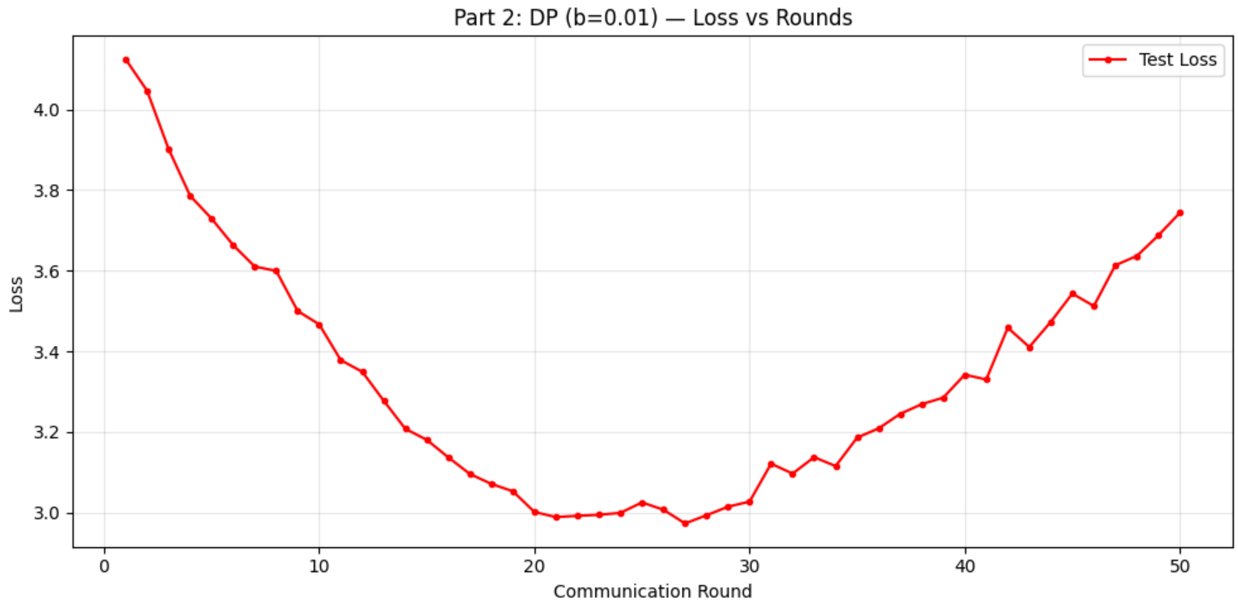



```
============================================================
Training with noise scale b = 0.01
============================================================
Applying Laplace noise with scale b=0.01 to all clients
[Round 001] train_acc=0.0222 test_acc=0.0301 loss=4.1248
[Round 002] train_acc=0.0583 test_acc=0.0834 loss=4.0463
[Round 003] train_acc=0.0577 test_acc=0.0773 loss=3.9018
[Round 004] train_acc=0.0578 test_acc=0.0989 loss=3.7858
[Round 005] train_acc=0.0600 test_acc=0.1477 loss=3.7292
[Round 006] train_acc=0.1318 test_acc=0.2276 loss=3.6636
[Round 007] train_acc=0.1382 test_acc=0.2408 loss=3.6105
[Round 008] train_acc=0.1073 test_acc=0.2486 loss=3.5995
[Round 009] train_acc=0.1470 test_acc=0.2610 loss=3.5000
[Round 010] train_acc=0.1411 test_acc=0.2831 loss=3.4669
[Round 011] train_acc=0.1492 test_acc=0.2665 loss=3.3783
[Round 012] train_acc=0.2041 test_acc=0.2936 loss=3.3494
[Round 013] train_acc=0.1284 test_acc=0.2745 loss=3.2776
[Round 014] train_acc=0.1900 test_acc=0.2980 loss=3.2080
[Round 015] train_acc=0.2003 test_acc=0.2969 loss=3.1806
[Round 016] train_acc=0.2543 test_acc=0.3093 loss=3.1371
[Round 017] train_acc=0.2597 test_acc=0.3063 loss=3.0960
[Round 018] train_acc=0.2801 test_acc=0.3179 loss=3.0713
[Round 019] train_acc=0.2962 test_acc=0.3190 loss=3.0531
[Round 020] train_acc=0.2952 test_acc=0.3344 loss=3.0018
[Round 021] train_acc=0.3301 test_acc=0.3400 loss=2.9891
[Round 022] train_acc=0.3224 test_acc=0.3449 loss=2.9922
[Round 023] train_acc=0.3490 test_acc=0.3469 loss=2.9947
```

```
[Round 024] train_acc=0.3449 test_acc=0.3505 loss=2.9995
[Round 025] train_acc=0.3664 test_acc=0.3449 loss=3.0255
[Round 026] train_acc=0.3738 test_acc=0.3637 loss=3.0073
[Round 027] train_acc=0.3831 test_acc=0.3739 loss=2.9732
[Round 028] train_acc=0.3952 test_acc=0.3811 loss=2.9936
[Round 029] train_acc=0.4015 test_acc=0.3825 loss=3.0148
[Round 030] train_acc=0.3981 test_acc=0.3861 loss=3.0274
[Round 031] train_acc=0.4045 test_acc=0.3803 loss=3.1214
[Round 032] train_acc=0.4228 test_acc=0.3913 loss=3.0969
[Round 033] train_acc=0.4255 test_acc=0.3949 loss=3.1376
[Round 034] train_acc=0.4322 test_acc=0.4013 loss=3.1155
[Round 035] train_acc=0.4298 test_acc=0.4040 loss=3.1861
[Round 036] train_acc=0.4449 test_acc=0.3999 loss=3.2091
[Round 037] train_acc=0.4491 test_acc=0.4057 loss=3.2449
[Round 038] train_acc=0.4472 test_acc=0.4049 loss=3.2691
[Round 039] train_acc=0.4544 test_acc=0.4104 loss=3.2851
[Round 040] train_acc=0.4665 test_acc=0.4073 loss=3.3422
[Round 041] train_acc=0.4646 test_acc=0.4131 loss=3.3302
[Round 042] train_acc=0.4573 test_acc=0.4062 loss=3.4587
[Round 043] train_acc=0.4738 test_acc=0.4170 loss=3.4107
[Round 044] train_acc=0.4780 test_acc=0.4156 loss=3.4733
[Round 045] train_acc=0.4820 test_acc=0.4129 loss=3.5432
[Round 046] train_acc=0.4745 test_acc=0.4350 loss=3.5124
[Round 047] train_acc=0.4904 test_acc=0.4184 loss=3.6136
[Round 048] train_acc=0.4913 test_acc=0.4203 loss=3.6366
[Round 049] train_acc=0.4945 test_acc=0.4212 loss=3.6876
[Round 050] train_acc=0.4984 test_acc=0.4217 loss=3.7438
Done. Best acc=0.4350
Saved: output/part2/part2_b000_acc.png
```

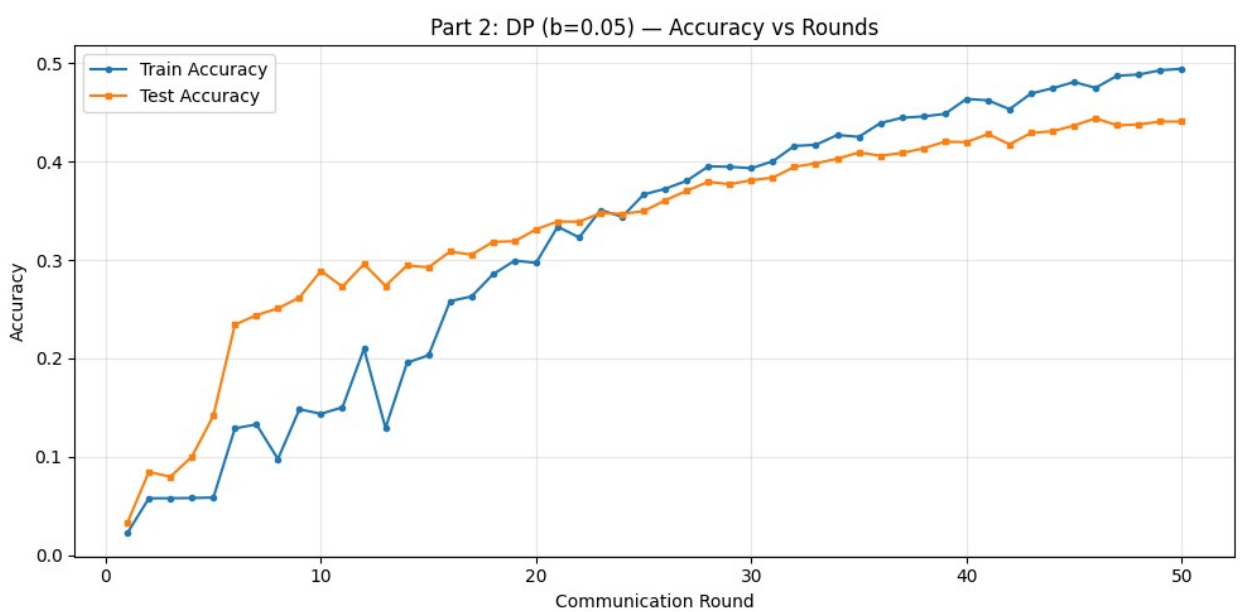


```
Saved: output/part2/part2_b000_loss.png
```

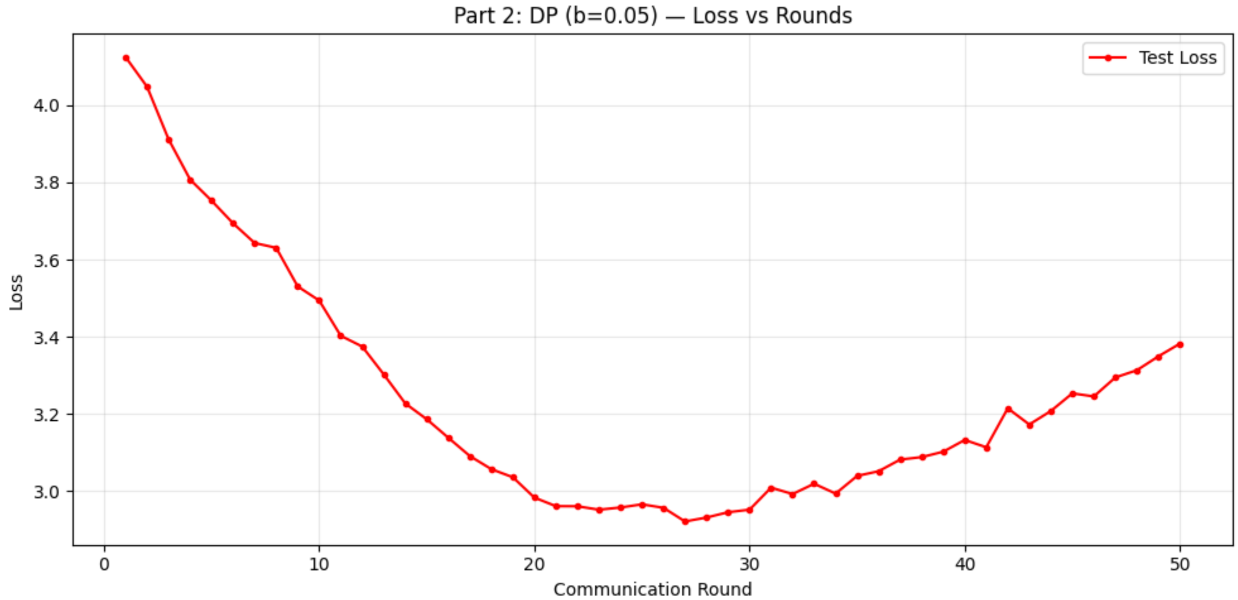



```
============================================================
Training with noise scale b = 0.05
============================================================
Applying Laplace noise with scale b=0.05 to all clients
[Round 001] train_acc=0.0222 test_acc=0.0326 loss=4.1250
[Round 002] train_acc=0.0577 test_acc=0.0845 loss=4.0479
[Round 003] train_acc=0.0577 test_acc=0.0795 loss=3.9117
[Round 004] train_acc=0.0580 test_acc=0.1000 loss=3.8074
[Round 005] train_acc=0.0584 test_acc=0.1417 loss=3.7525
[Round 006] train_acc=0.1288 test_acc=0.2342 loss=3.6938
[Round 007] train_acc=0.1328 test_acc=0.2439 loss=3.6429
[Round 008] train_acc=0.0977 test_acc=0.2508 loss=3.6305
[Round 009] train_acc=0.1482 test_acc=0.2618 loss=3.5304
[Round 010] train_acc=0.1436 test_acc=0.2889 loss=3.4938
[Round 011] train_acc=0.1502 test_acc=0.2729 loss=3.4025
[Round 012] train_acc=0.2097 test_acc=0.2958 loss=3.3745
[Round 013] train_acc=0.1291 test_acc=0.2737 loss=3.3022
[Round 014] train_acc=0.1955 test_acc=0.2944 loss=3.2270
[Round 015] train_acc=0.2031 test_acc=0.2925 loss=3.1860
[Round 016] train_acc=0.2581 test_acc=0.3085 loss=3.1381
[Round 017] train_acc=0.2629 test_acc=0.3054 loss=3.0907
[Round 018] train_acc=0.2855 test_acc=0.3184 loss=3.0566
[Round 019] train_acc=0.2993 test_acc=0.3190 loss=3.0357
[Round 020] train_acc=0.2971 test_acc=0.3311 loss=2.9829
[Round 021] train_acc=0.3339 test_acc=0.3391 loss=2.9607
[Round 022] train_acc=0.3229 test_acc=0.3389 loss=2.9605
[Round 023] train_acc=0.3507 test_acc=0.3477 loss=2.9515
```

```
[Round 024] train_acc=0.3439 test_acc=0.3469 loss=2.9573
[Round 025] train_acc=0.3666 test_acc=0.3496 loss=2.9655
[Round 026] train_acc=0.3724 test_acc=0.3607 loss=2.9565
[Round 027] train_acc=0.3805 test_acc=0.3703 loss=2.9210
[Round 028] train_acc=0.3951 test_acc=0.3795 loss=2.9313
[Round 029] train_acc=0.3948 test_acc=0.3770 loss=2.9450
[Round 030] train_acc=0.3933 test_acc=0.3811 loss=2.9516
[Round 031] train_acc=0.4003 test_acc=0.3836 loss=3.0087
[Round 032] train_acc=0.4159 test_acc=0.3946 loss=2.9921
[Round 033] train_acc=0.4171 test_acc=0.3982 loss=3.0188
[Round 034] train_acc=0.4271 test_acc=0.4029 loss=2.9933
[Round 035] train_acc=0.4252 test_acc=0.4093 loss=3.0392
[Round 036] train_acc=0.4392 test_acc=0.4060 loss=3.0513
[Round 037] train_acc=0.4446 test_acc=0.4087 loss=3.0815
[Round 038] train_acc=0.4459 test_acc=0.4134 loss=3.0879
[Round 039] train_acc=0.4485 test_acc=0.4203 loss=3.1019
[Round 040] train_acc=0.4637 test_acc=0.4198 loss=3.1323
[Round 041] train_acc=0.4623 test_acc=0.4281 loss=3.1134
[Round 042] train_acc=0.4533 test_acc=0.4176 loss=3.2144
[Round 043] train_acc=0.4692 test_acc=0.4292 loss=3.1724
[Round 044] train_acc=0.4746 test_acc=0.4308 loss=3.2073
[Round 045] train_acc=0.4809 test_acc=0.4366 loss=3.2531
[Round 046] train_acc=0.4750 test_acc=0.4441 loss=3.2452
[Round 047] train_acc=0.4873 test_acc=0.4369 loss=3.2944
[Round 048] train_acc=0.4885 test_acc=0.4377 loss=3.3131
[Round 049] train_acc=0.4929 test_acc=0.4408 loss=3.3491
[Round 050] train_acc=0.4944 test_acc=0.4408 loss=3.3820
Done. Best acc=0.4441
Saved: output/part2/part2_b000_acc.png
```

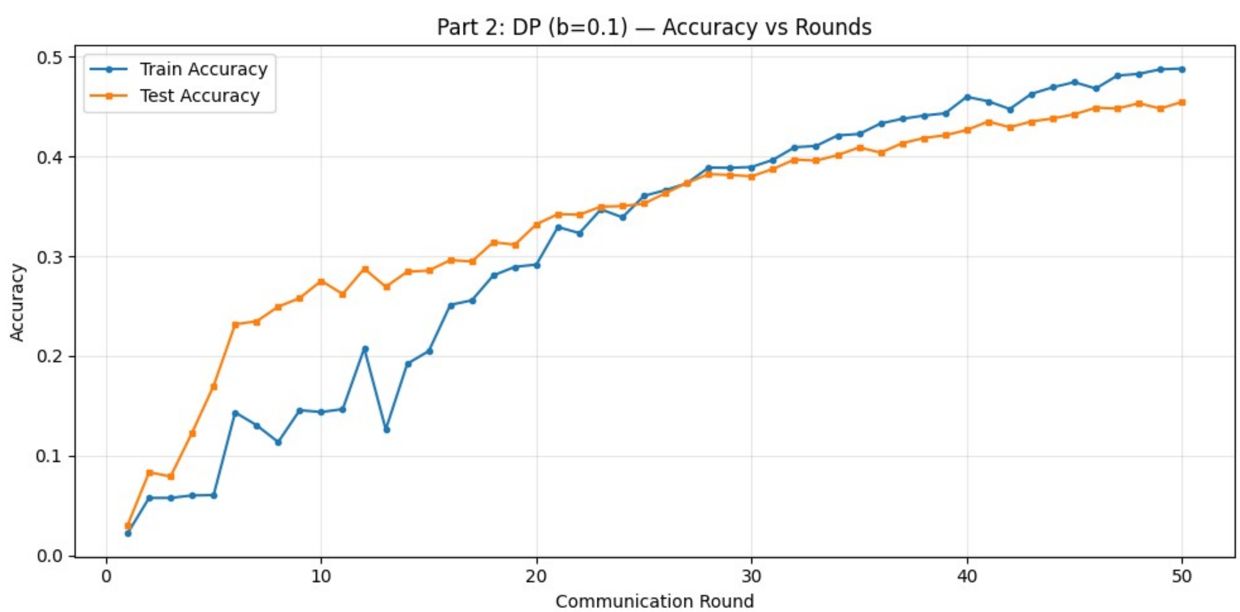


```
Saved: output/part2/part2_b000_loss.png
```

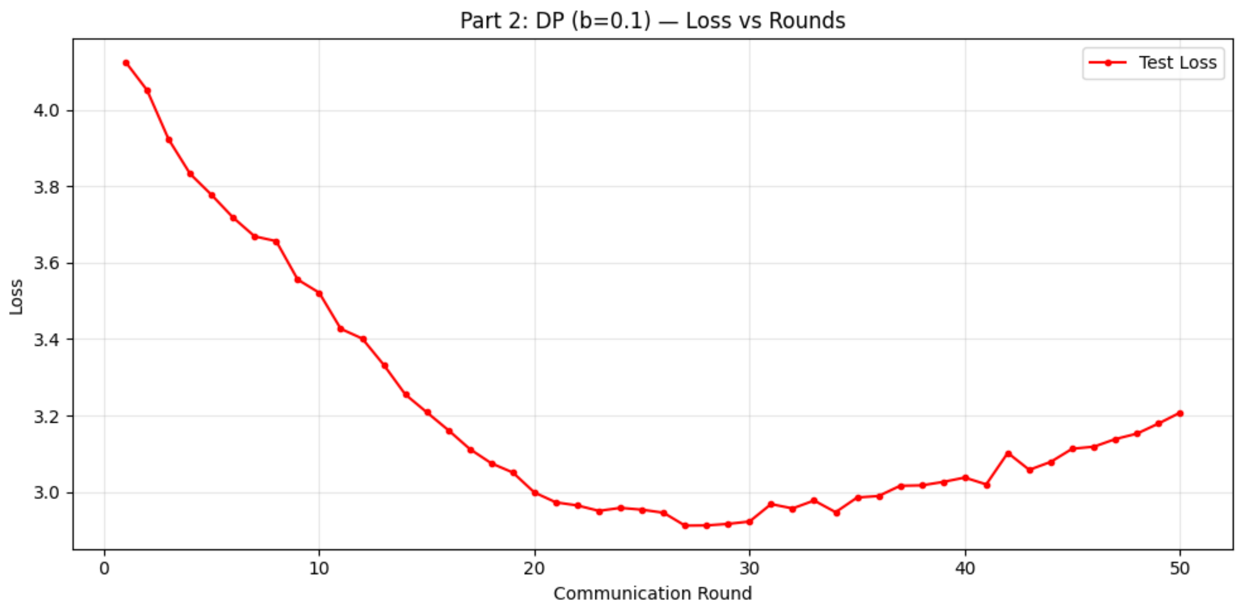



```
============================================================
Training with noise scale b = 0.1
============================================================
Applying Laplace noise with scale b=0.1 to all clients
[Round 001] train_acc=0.0222 test_acc=0.0304 loss=4.1249
[Round 002] train_acc=0.0577 test_acc=0.0834 loss=4.0510
[Round 003] train_acc=0.0577 test_acc=0.0793 loss=3.9226
[Round 004] train_acc=0.0602 test_acc=0.1232 loss=3.8322
[Round 005] train_acc=0.0606 test_acc=0.1701 loss=3.7769
[Round 006] train_acc=0.1434 test_acc=0.2317 loss=3.7177
[Round 007] train_acc=0.1305 test_acc=0.2347 loss=3.6686
[Round 008] train_acc=0.1137 test_acc=0.2494 loss=3.6562
[Round 009] train_acc=0.1456 test_acc=0.2579 loss=3.5555
[Round 010] train_acc=0.1438 test_acc=0.2753 loss=3.5211
[Round 011] train_acc=0.1467 test_acc=0.2621 loss=3.4268
[Round 012] train_acc=0.2076 test_acc=0.2875 loss=3.4013
[Round 013] train_acc=0.1263 test_acc=0.2693 loss=3.3324
[Round 014] train_acc=0.1922 test_acc=0.2845 loss=3.2552
[Round 015] train_acc=0.2049 test_acc=0.2856 loss=3.2084
[Round 016] train_acc=0.2512 test_acc=0.2961 loss=3.1617
[Round 017] train_acc=0.2557 test_acc=0.2947 loss=3.1122
[Round 018] train_acc=0.2807 test_acc=0.3140 loss=3.0749
[Round 019] train_acc=0.2893 test_acc=0.3115 loss=3.0508
[Round 020] train_acc=0.2916 test_acc=0.3320 loss=2.9986
[Round 021] train_acc=0.3292 test_acc=0.3422 loss=2.9728
[Round 022] train_acc=0.3231 test_acc=0.3416 loss=2.9647
[Round 023] train_acc=0.3469 test_acc=0.3496 loss=2.9505
```

```
[Round 024] train_acc=0.3389 test_acc=0.3502 loss=2.9584
[Round 025] train_acc=0.3604 test_acc=0.3527 loss=2.9536
[Round 026] train_acc=0.3662 test_acc=0.3632 loss=2.9457
[Round 027] train_acc=0.3730 test_acc=0.3734 loss=2.9119
[Round 028] train_acc=0.3888 test_acc=0.3822 loss=2.9125
[Round 029] train_acc=0.3885 test_acc=0.3814 loss=2.9164
[Round 030] train_acc=0.3893 test_acc=0.3800 loss=2.9227
[Round 031] train_acc=0.3966 test_acc=0.3875 loss=2.9682
[Round 032] train_acc=0.4091 test_acc=0.3969 loss=2.9569
[Round 033] train_acc=0.4105 test_acc=0.3957 loss=2.9776
[Round 034] train_acc=0.4210 test_acc=0.4013 loss=2.9470
[Round 035] train_acc=0.4224 test_acc=0.4090 loss=2.9854
[Round 036] train_acc=0.4330 test_acc=0.4038 loss=2.9894
[Round 037] train_acc=0.4376 test_acc=0.4131 loss=3.0161
[Round 038] train_acc=0.4408 test_acc=0.4184 loss=3.0172
[Round 039] train_acc=0.4432 test_acc=0.4212 loss=3.0262
[Round 040] train_acc=0.4597 test_acc=0.4264 loss=3.0376
[Round 041] train_acc=0.4552 test_acc=0.4350 loss=3.0198
[Round 042] train_acc=0.4475 test_acc=0.4292 loss=3.1021
[Round 043] train_acc=0.4626 test_acc=0.4350 loss=3.0578
[Round 044] train_acc=0.4693 test_acc=0.4380 loss=3.0787
[Round 045] train_acc=0.4743 test_acc=0.4421 loss=3.1131
[Round 046] train_acc=0.4680 test_acc=0.4488 loss=3.1183
[Round 047] train_acc=0.4810 test_acc=0.4479 loss=3.1383
[Round 048] train_acc=0.4827 test_acc=0.4532 loss=3.1525
[Round 049] train_acc=0.4873 test_acc=0.4479 loss=3.1788
[Round 050] train_acc=0.4879 test_acc=0.4546 loss=3.2073
Done. Best acc=0.4546
Saved: output/part2/part2_b000_acc.png
```



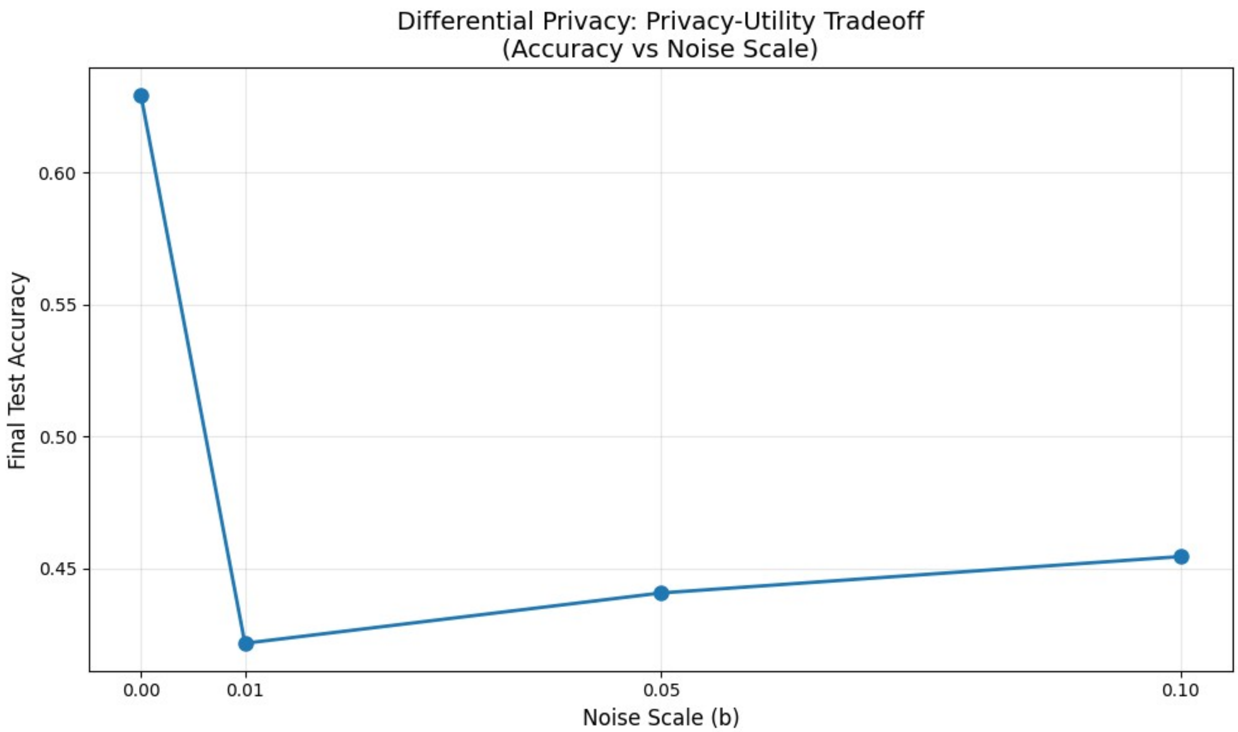
```
Saved: output/part2/part2_b000_loss.png
```



5.3 Privacy-Utility Tradeoff Analysis

```
# Plot accuracy vs noise scale (privacy-utility tradeoff)
noise_scales_sorted = sorted(dp_results.keys())
final_accs = [dp_results[b]['final_acc'] for b in noise_scales_sorted]

plt.figure(figsize=(10, 6))
plt.plot(noise_scales_sorted, final_accs, marker='o', markersize=8,
linewidth=2)
plt.xlabel('Noise Scale (b)', fontsize=12)
plt.ylabel('Final Test Accuracy', fontsize=12)
plt.title('Differential Privacy: Privacy-Utility Tradeoff\n(Accuracy
vs Noise Scale)', fontsize=14)
plt.grid(True, alpha=0.3)
plt.xticks(noise_scales_sorted)
plt.tight_layout()
plt.savefig(f"{Config.part2_dir}/part2_dp_final_acc_vs_b.png",
dpi=160, bbox_inches="tight")
print(f"Saved: {Config.part2_dir}/part2_dp_final_acc_vs_b.png")
plt.show()

# Print summary table
print("\n" + "="*60)
print("Differential Privacy Results Summary")
print("="*60)
print(f"{'Noise Scale (b)':<20} {'Final Accuracy':<20} {'Best
Accuracy':<20}")
print("-"*60)
for b in noise_scales_sorted:
```

```
    final = dp_results[b]['final_acc']
    best = dp_results[b]['best_acc']
    print(f"{b:<20.4f} {final:<20.4f} {best:<20.4f}")
print("="*60)

Saved: output/part2/part2_dp_final_acc_vs_b.png
```




```
============================================================
Differential Privacy Results Summary
============================================================
Noise Scale (b)     Final Accuracy      Best Accuracy
------------------------------------------------------------
0.0000              0.6294              0.6294
0.0100              0.4217              0.4350
0.0500              0.4408              0.4441
0.1000              0.4546              0.4546
============================================================
```

6. Additional Utilities

6.1 Save History to CSV

```
def save_history_to_csv(history, filepath):
    """Save training history to CSV file."""
    os.makedirs(os.path.dirname(filepath) if os.path.dirname(filepath)
```

```python
else '.', exist_ok=True)
    with open(filepath, "w", newline="") as f:
        writer = csv.DictWriter(f, fieldnames=["round", "train_acc",
"test_acc", "loss"])
        writer.writeheader()
        for row in history:
            writer.writerow(row)
    print(f"Saved history to {filepath}")

# Example: Save Part 1 history
save_history_to_csv(history_part1,
f"{Config.part1_dir}/part1_history.csv")

# Example: Save Part 2 histories
for noise_scale in Config.noise_scales:
    if noise_scale in dp_results:
        save_history_to_csv(
            dp_results[noise_scale]['history'],

f"{Config.part2_dir}/part2_b{noise_scale:03.0f}_history.csv"
        )
```

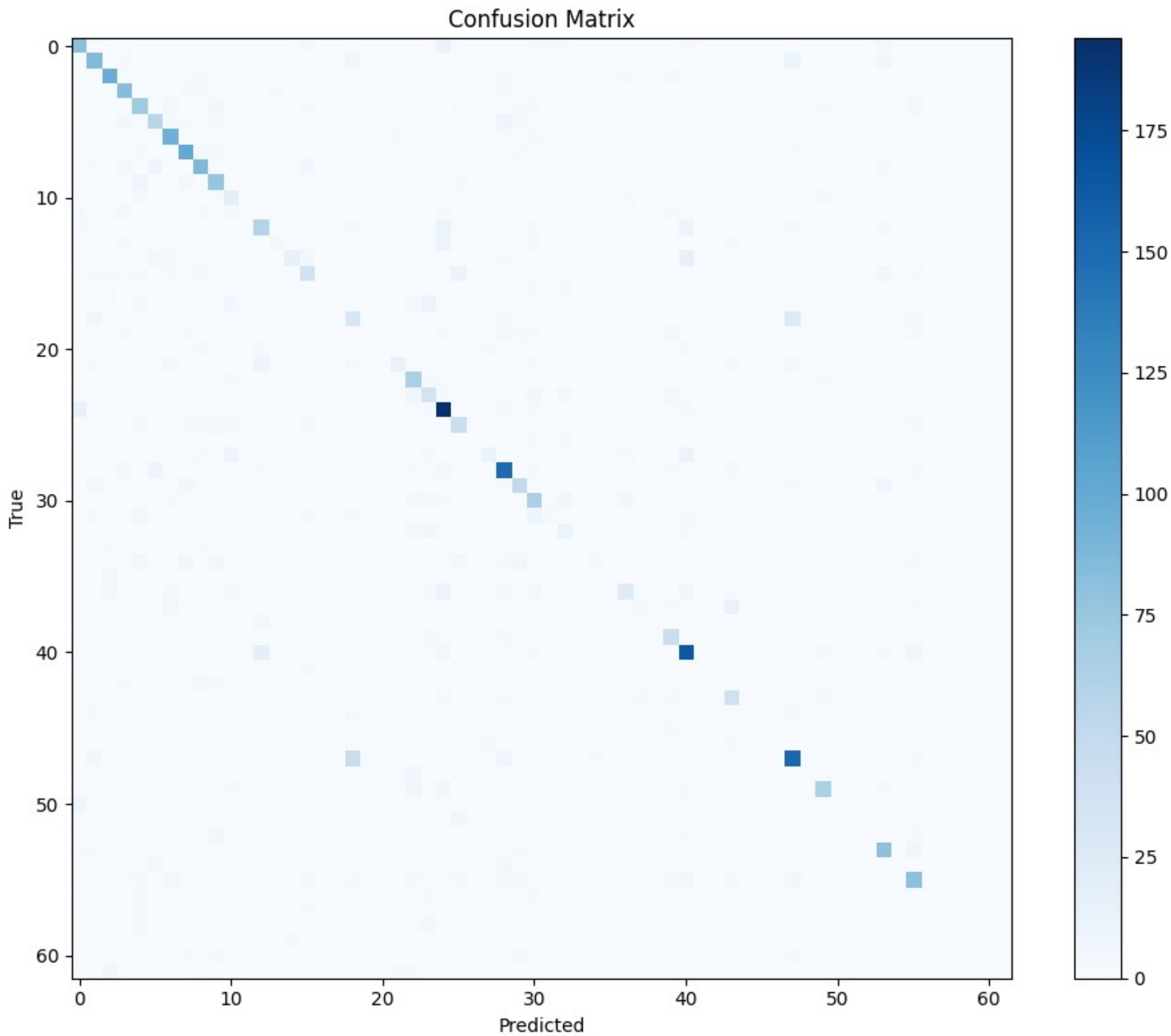
```
Saved history to output/part1/part1_history.csv
Saved history to output/part2/part2_b000_history.csv
Saved history to output/part2/part2_b000_history.csv
Saved history to output/part2/part2_b000_history.csv
Saved history to output/part2/part2_b000_history.csv
```

6.2 Model Evaluation (Confusion Matrix)

```python
def evaluate_model_detailed(model, Xte, yte, n_classes, device,
batch_size=1024):
    """Evaluate model and generate confusion matrix and classification
report."""
    model.eval()
    loader = make_loader(Xte, yte, batch_size, shuffle=False,
drop_last=False)

    all_preds, all_true = [], []
    with torch.no_grad():
        for xb, yb in loader:
            xb = xb.to(device)
            yb = yb.to(device)
            logits = model(xb)
            preds = logits.argmax(dim=1)
            all_preds.append(preds.cpu().numpy())
            all_true.append(yb.cpu().numpy())

    y_pred = np.concatenate(all_preds)
    y_true = np.concatenate(all_true)
```

```python
    acc = (y_pred == y_true).mean()
    print(f"Final Test Accuracy: {acc:.4f}")

    # Confusion matrix
    cm = confusion_matrix(y_true, y_pred, labels=np.arange(n_classes))
    plt.figure(figsize=(10, 8))
    plt.imshow(cm, interpolation="nearest", cmap='Blues')
    plt.title("Confusion Matrix")
    plt.xlabel("Predicted")
    plt.ylabel("True")
    plt.colorbar()
    plt.tight_layout()
    plt.show()

    # Classification report
    report = classification_report(y_true, y_pred,
labels=np.arange(n_classes), zero_division=0)
    print("\nClassification Report:")
    print(report)

    return acc, cm, report

# Example: Evaluate Part 1 model
print("Evaluating Part 1 model:")
model_part1 = MLP(in_dim=in_dim, hidden=Config.hidden_layers,
out_dim=n_classes,
                  dropout=Config.dropout,
use_bn=Config.use_bn).to(device)
model_part1.load_state_dict(global_state_part1, strict=True)
acc_part1, cm_part1, report_part1 =
evaluate_model_detailed(model_part1, Xte, yte, n_classes, device)
```

```
Evaluating Part 1 model:
Final Test Accuracy: 0.6294
```



Classification Report:

	precision	recall	f1-score	support
0	0.63	0.79	0.70	103
1	0.69	0.75	0.71	114
2	0.78	0.82	0.80	118
3	0.72	0.82	0.76	101
4	0.60	0.71	0.65	100
5	0.62	0.65	0.63	85
6	0.73	0.90	0.81	105
7	0.75	0.89	0.81	114
8	0.71	0.76	0.73	115
9	0.64	0.81	0.72	94
10	0.35	0.53	0.42	34
11	0.00	0.00	0.00	25

12	0.59	0.59	0.59	99
13	0.50	0.15	0.23	27
14	0.75	0.26	0.39	57
15	0.49	0.47	0.48	75
16	0.00	0.00	0.00	10
17	0.00	0.00	0.00	25
18	0.31	0.41	0.36	75
19	0.00	0.00	0.00	25
20	0.00	0.00	0.00	13
21	0.59	0.33	0.42	40
22	0.57	0.86	0.68	74
23	0.43	0.54	0.48	65
24	0.68	0.88	0.77	221
25	0.57	0.65	0.60	65
26	0.00	0.00	0.00	9
27	0.50	0.24	0.33	41
28	0.71	0.81	0.75	185
29	0.58	0.68	0.62	74
30	0.53	0.66	0.59	95
31	0.30	0.09	0.14	32
32	0.39	0.50	0.44	24
33	0.00	0.00	0.00	7
34	0.38	0.07	0.12	40
35	0.00	0.00	0.00	6
36	0.44	0.35	0.39	66
37	0.57	0.13	0.22	30
38	0.00	0.00	0.00	8
39	0.55	0.77	0.64	56
40	0.68	0.80	0.73	205
41	0.00	0.00	0.00	6
42	0.00	0.00	0.00	15
43	0.53	0.67	0.59	58
44	0.00	0.00	0.00	10
45	0.00	0.00	0.00	6
46	0.00	0.00	0.00	10
47	0.71	0.69	0.70	220
48	0.00	0.00	0.00	6
49	0.77	0.71	0.74	90
50	0.00	0.00	0.00	13
51	0.00	0.00	0.00	10
52	0.00	0.00	0.00	16
53	0.68	0.72	0.70	111
54	0.00	0.00	0.00	11
55	0.58	0.64	0.61	127
56	0.00	0.00	0.00	13
57	0.00	0.00	0.00	7
58	0.00	0.00	0.00	8
59	0.00	0.00	0.00	5
60	0.00	0.00	0.00	12

```
          61       0.00      0.00      0.00        10

    accuracy                           0.63      3621
   macro avg       0.35      0.36      0.34      3621
weighted avg       0.58      0.63      0.59      3621
```

Summary

This notebook combines all code for:

- **Part 1**: Federated Learning (FedAvg) - both serial and Ray-based implementations
- **Part 2**: Differential Privacy experiments with Laplace noise

Key Features:

1. **Data Loading**: Robust loading of federated EMNIST data from .npy files
2. **Preprocessing**: Label remapping, standardization, normalization
3. **Model**: Configurable MLP with batch normalization and dropout
4. **FedAvg**: Serial and parallel (Ray) implementations
5. **Differential Privacy**: Laplace noise addition and privacy-utility analysis
6. **Visualization**: Training curves, label histograms, confusion matrices
7. **Utilities**: CSV export, model evaluation, plotting functions

Outputs:

- Training history plots (accuracy/loss vs rounds)
- Label distribution histograms
- Privacy-utility tradeoff plots
- Confusion matrices
- CSV logs of training history

All outputs are saved to `output/part1/` and `output/part2/` directories.