

COMS3: Advanced Analysis of Algorithms Assignment

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Introduction

The game of peg solitaire is a one player game played on a 33 holed cross shaped board that involves jumping pegs over other pegs, in a manner similar to checkers. The rules are as follows:

1. A move consists of jumping a peg over an orthogonal neighbor into an empty space. The peg that was jumped over is then removed from the board.
2. Pegs can only jump onto an empty space.
3. The game is won if the final peg is in the centre space.
4. If no pegs can legally move or the final peg is not in the centre the game is lost.

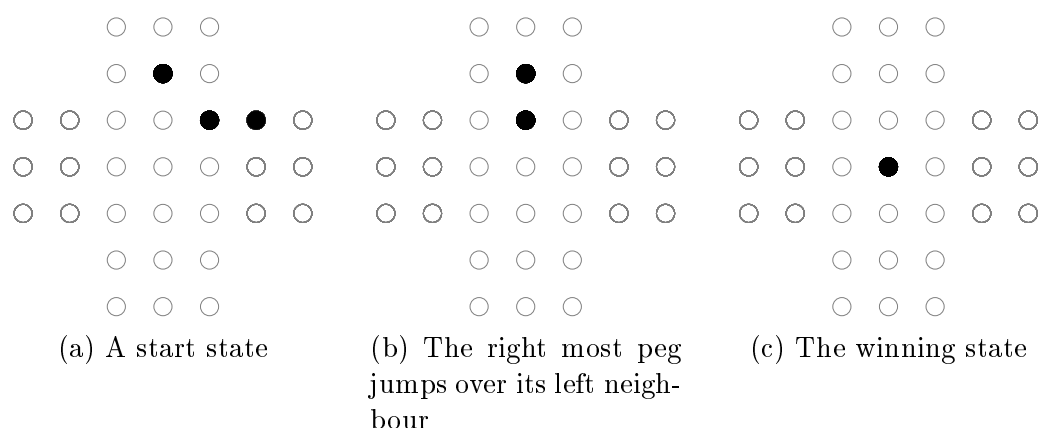


Figure 1: A winning set of valid moves

Backtracking Search

Recursive backtracking using depth first search was chosen as the method for state space exploration. The standard backtracking depth first algorithm is as follows:

```
1 Function backtrack
  input : An initial, possibly un-winnable, board state.
  output: A sequence of moves to get from the initial state to the
           winning state, if a winning state cannot be reached the
           sequence should be empty.

2  initialState ← The initial state
3  legalMoves ← A list of all the legal moves for initialState
4  result ← Empty list to store moves from the initial state to the
           winning state
5  foreach move in legalMoves do
6    state ← The state after playing move on initialState
7    if state is a winning state then
8      result ← move
9      break
10   end
11   childResult ← backtrack(state)
12   if childResult is not empty then
13     Prepend move to childResult
14     result ← childResult
15     break
16   end
17 end
18 return result
```

Algorithm 1: A standard recursive backtracking using DFS

But because pegs are indistinguishable, game states where pegs are in the same position are identical, regardless of the moves taken to arrive at that state. Thus exploring the state space looking for a sequence of states leading to the winning state will probably involve evaluating the same states many times. Thus a significant speed up can be achieved by saving states that are

known to not lead to the winning state as in the following algorithm:

```

1 Function dynamicBacktrack
    input : An initial, possibly un-winnable, board state.
    input : A set of all states that have been searched and are
           known to be un-winnable.
    output: A sequence of moves to get from the initial state to the
           winning state, if a winning state cannot be reached the
           sequence should be empty.

2  initialState ← The initial state
3  infeasibleSet ← The set of un-winnable states
4  legalMoves ← A list of all the legal moves for initialState
5  result ← Empty list to store moves from the initial state to the
           winning state
6  foreach move in legalMoves do
7      state ← The state after playing move on initialState
8      if state is in infeasibleSet then
9          continue
10     end
11     if state is a winning state then
12         result ← move
13         break
14     end
15     childResult ← dynamicBacktrack(state, infeasibleSet)
16     if childResult is not empty then
17         Prepend move to childResult
18         result ← childResult
19         break
20     else
21         Add state to infeasibleSet
22     end
23 end
24 return result

```

Algorithm 2: Recursive backtracking using DFS and dynamic programming methods

Parallel Implementation

To parallelise the above algorithms it was decided to make use of the OpenMP parallel for loop to allow the concurrent depth first exploration of multiple moves from the initial state. The OpenMP parallel for loop partitions the iteration space of the loop at [line 6](#) of Algorithm 2.

Static, dynamic and guided OpenMP load balancing constructs were all tried, but dynamic gave by far the best performance. This is because dynamic (with default chunk size) gives one element of the iteration space to a thread at a time, allocating new tasks to threads as they finish their allocated task in a round robin approach. This gives good load balancing in this particular problem since some elements of the iteration space may require exploring the game graph very deeply while others may quickly lead to previously seen infeasible states.

The infeasible set is an instance variable of the object and as such each thread can concurrently reference it. Simultaneously, concurrent threads writing to the infeasible set need to avoid data races, thus the function to add a state to the infeasible set must be an OpenMP critical region.

Nested parallelism was also tried and was found to be highly inefficient. Further various upper limits on the depth with which nested parallelism could take place were tried but the added overhead of parallelism within the recursive algorithm remained slower than no nested parallelism at all.

Results

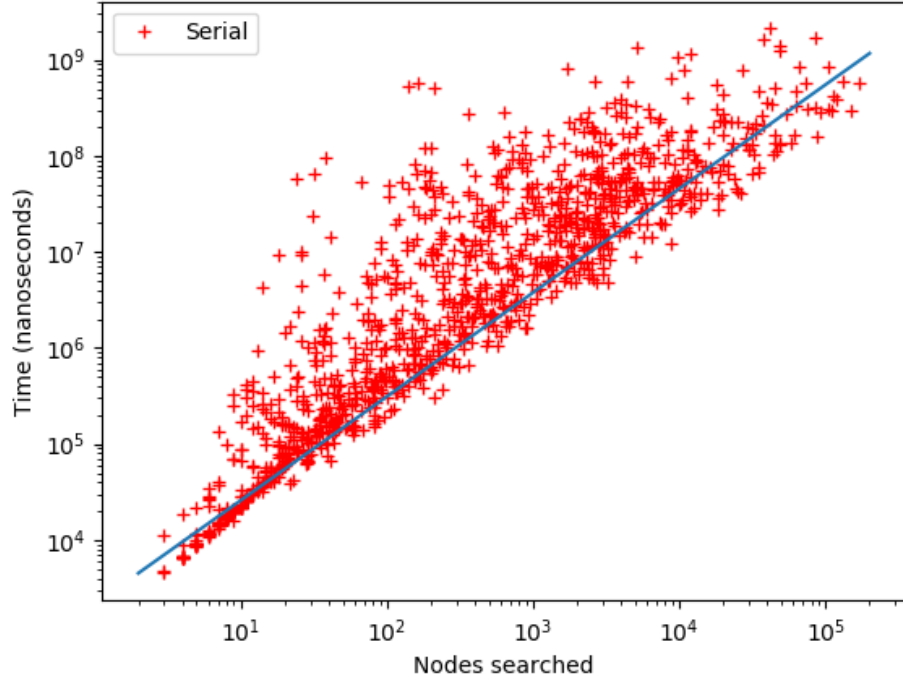


Figure 2: Graph comparing running time with the number of elements searched

Figure 2 shows that the running time can be fit to the line $2133.73x^{1.083}$ with an R^2 value of 0.81 meaning the algorithm is of $O(n^{0.81}) \approx O(n)$ complexity in the number of elements of the tree searched.

Figure 3 shows that the running time can be fit to the line $2000e^{0.6x}$ meaning the algorithm is of $O(e^n)$ complexity in the length of the solution.

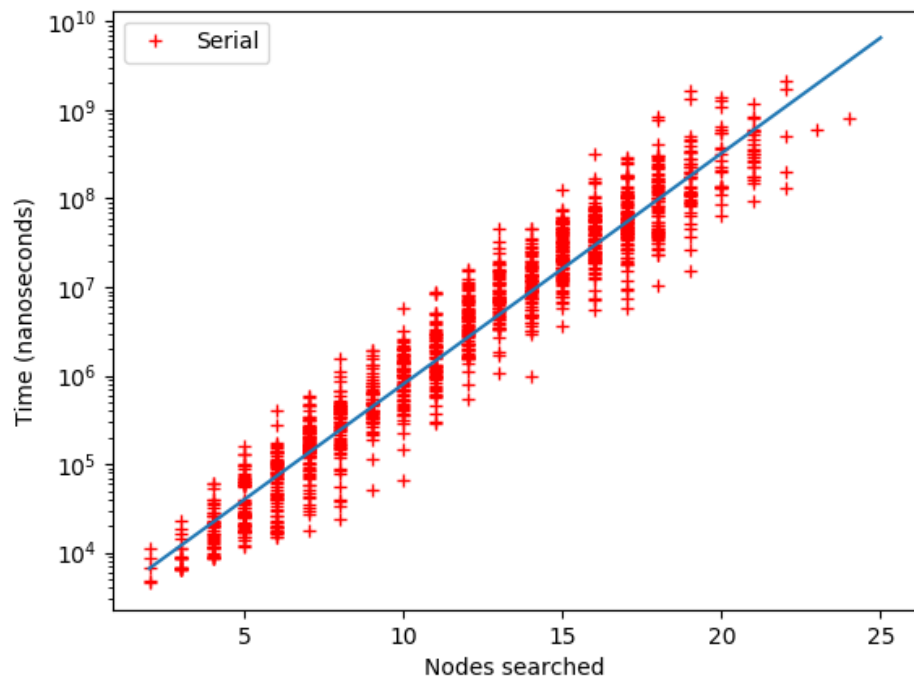


Figure 3: Graph comparing running time with the length of the solution found