Proofs and derivations from Calculus I

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Contents

1 Differential equations

1.1 Separable

1.1.1 Function of x as a multiplier

$$p_0(x)y' + p_1(x)y = 0 \qquad |\frac{1}{p_0(x)}$$

$$y' + p(x)y = 0 \qquad |p(x) = \frac{p_1(x)}{p_0(x)}$$

$$\frac{dy}{dx} + p(x)y = 0 \qquad |\frac{dx}{y}$$

$$\frac{dy}{y} + p(x)dx = 0$$

$$\int \frac{dy}{y} + \int p(x)dx = 0$$

$$\ln|y| = C_0 - \int p(x)dx$$

$$|y| = e^{C_0 - \int p(x)dx}$$

$$|y| = e^{C_0} e^{-\int p(x)dx}$$

$$y = Ce^{-\int p(x)dx} \qquad C = \pm e^{C_0}$$

1.2 Integration by parts

$$(uv)' = u'v + uv'$$

$$\int (uv)' = \int u'v + \int uv'$$

$$uv = \int u'v + \int uv'$$

$$\boxed{\int u'v = uv - \int uv'}$$