Proofs and derivations for Calculus I

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Contents

1 Differential equations

1.1 Separable

1.1.1 Function of x as a multiplier

$$p_0(x)y' + p_1(x)y = 0 \qquad \Big| \tfrac{1}{p_0(x)}$$

$$y' + p(x)y = 0 \qquad \Big| p(x) = \tfrac{p_1(x)}{p_0(x)}$$

$$\frac{dy}{dx} + p(x)y = 0 \qquad \Big| \tfrac{dx}{y}$$

$$\frac{dy}{y} + p(x)dx = 0$$

$$\int \frac{dy}{y} + \int p(x)dx = 0$$

$$\ln|y| = C_0 - \int p(x)dx$$

$$|y| = e^{C_0 - \int p(x)dx}$$

$$|y| = e^{C_0}e^{-\int p(x)dx}$$

$$y = Ce^{-\int p(x)dx} \qquad C = \pm e^{C_0}$$

1.2 Integration by parts

$$(uv)' = u'v + uv'$$

$$\int (uv)' = \int u'v + \int uv'$$

$$uv = \int u'v + \int uv'$$

$$\boxed{\int u'v = uv - \int uv'}$$

1.3 Linear

1.3.1 First order

1.3.1.1 Particular solution

$$y' + p(x)y = f(x) \tag{recall}$$

$$(e^{-\int p(x)dx})' = -p(x)e^{-\int p(x)dx} \tag{notice}$$

$$y_* = C(x)e^{-\int p(x)dx} \tag{consider}$$

$$y' + p(x)y = f(x)$$

$$(C(x)e^{-\int p(x)dx})' + p(x)C(x)e^{-\int p(x)dx} = f(x)$$

$$C'(x)e^{-\int p(x)dx} + C(x)(-p(x))e^{-\int p(x)dx} + p(x)C(x)e^{-\int p(x)dx} = f(x)$$

$$C'(x)e^{-\int p(x)dx} = f(x)$$

$$C(x) = \int f(x)e^{\int p(x)dx}$$

$$y_* = C(x)e^{-\int p(x)dx}$$

$$\boxed{y_* = e^{-\int p(x)dx} \int f(x)e^{\int p(x)dx}}$$