Gradient Calculations.

1] $p(x) = \sqrt{(x-x_{obs})^2 + (y-y_{obs})^2 + (z-z_{obs})^2}$ = distance.

$$\Rightarrow \nabla p = \frac{1}{2} \times \frac{1}{\text{distance}} \begin{bmatrix} 2(x-x_{obs}) \\ 2(y-y_{obs}) \\ 2(z-z_{obs}) \end{bmatrix} = \frac{X - X_{obs}}{\text{distance}}$$

2] $\cos\theta = \dfrac{v^T x}{\|v\| \, p(x)} \Rightarrow \nabla \cos\theta = \dfrac{1}{\|v\|} \nabla \left\{ \dfrac{v^T x}{p(x)} \right\}$

$$= \frac{1}{\|v\|} \times \frac{p(x) \times \nabla v^T x - v^T x \times \nabla p}{p^2(x)}$$

$$= \frac{1}{\|v\|} \times \frac{p(x) \cdot v^T - v^T x \nabla p}{p^2(x)}$$

$\nabla v^T x$

$= \nabla \{ v_1 x_1 + v_2 x_2 + v_3 x_3 \}$

$= \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = v^T$