

As part of this assignment, you will write the core parts of K-means algorithm. We will assume that the input is a $D \times N$ matrix (i.e., N D -dimension points), and the initial centers are represented by a $D \times K$ matrix (i.e., K D -dimensional points).

Task 1: Generate randomized input data (4 points)

Write a R function called `genData(N,D,K)` which returns a single R list with two components `"all_points"` and `"centers"`:

- `"all_points"` is a $D \times N$ matrix. Elements in `"all_points"` are selected uniformly at random from the range 1 to $D \times N$ (with replacement)
- `"centers"` is a $D \times K$ matrix. Elements in `"centers"` are K of the input points from `"all_points"` (i.e., D -dimensional points) selected uniformly at random (but without replacement).

Task 2: Calculate distance to closest center (15 points)

Write a R function called `closestCenters(all_points, centers)` which returns a single R list with two components `"new_centers"` ($D \times N$ matrix) and `"label"` ($1 \times N$ matrix).

- For each element in `"all_points"` find the closest point in `"centers"`, and store the center id in `label`. For example, if the first point in `"all_points"` is closest to center number three then `label[1]` will be 3. Use Euclidean distance measure. To obtain full points, you should use matrix multiplication and subtraction to calculate distances between points and centers (otherwise calculations may be too slow in R).
- `"new_centers"` is a $D \times K$ matrix. It is obtained by averaging the points that have the same label.

Task 3: Implement K-means algorithm (6 points)

Write a R function called `myKmeans(all_points, centers, niter)` which takes the arguments `"all_points"` ($D \times N$), initial center ($D \times K$), and `niter` (number of iterations), and return a single list with two components `"new_centers"` and `"label"`. The `"myKmeans"` function should successively refine the new cluster centers by calling the `"closestCenters"` function (developed in Task 2) for `"niter"` iterations.

Curious about correctness of your code?

R provides a built-in `"kmeans"` function. You can use the `kmeans` function (with `algorithm="Lloyd"`) to check the accuracy of the centers you have calculated. Note that there may be some differences between your results and what R's `kmeans` returns [There are no points for this step].