21-3-2020

Computer Graphics

Assignment - 1

Ques-1 Perform a 45° rotation of triangle A(0,0), B(1,2), C(5,2)

a) about the origin

b) about P(-1,-1).

Ans-1 a) using the formula

$$x_1 = x\cos(45°) - y\cos(45°)$$

$$y_1 = x\sin(45°) + y\cos(45°)$$

A(0,0) will mapped on $A_1(0,0)$

B(1,1) will mapped on $B_1(0,1.4)$

C(5,2) will mapped on $C_1(2.1,4.9)$

Represent the given triangle matrix form using homo-generous coordinates of the vertices

$$[A\ B\ C] = \begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$$

The matrix of rotation is $R_\theta = R_{45°} = \begin{bmatrix} \cos 45° & \sin 45° & 0 \\ -\sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\cos 45° = \sin 45° = \frac{\sqrt{2}}{2}$ thus $R_{45°} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The new coordinate A'B'C' of the rotated triangle ABC can be found as

$$A'B'C' = [A\ B\ C]\,R_{45°} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & \sqrt{2} & 1 \\ 3\sqrt{2}/2 & \frac{1\sqrt{2}}{2} & 1 \end{bmatrix}$$

Thus

$A' = (0,0)$, $B' = (0, \sqrt{2})$ & $C' = \left(\frac{3\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$ (Ans)

Ans 1 b) $T_m = T_v R(\theta) T_{-v}$

$$= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$= \quad t_x = -1 \quad t_y = -1 \quad \theta = 45°$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -(-1) \\ 0 & 1 & -(-1) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & -1 \\ \sqrt{2}/2 & \sqrt{2}/2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & -1 \\ \sqrt{2}/2 & \sqrt{2}/2 & (\sqrt{2}-1) \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & (3\sqrt{2}/2 - 1) \\ \sqrt{2}-1 & (2\sqrt{2}-1) & (\frac{9\sqrt{2}}{2} - 1) \\ 1 & 1 & 1 \end{bmatrix}$$

So, $A' = (-1, \sqrt{2}-1)$. $B' = (-1, 2\sqrt{2}-1)$ & $C' = \left(\frac{3\sqrt{2}}{2} - 1, \frac{9\sqrt{2}}{2} - 1\right)$

(Ans)

Ques-2 Magnify the triangle with vertices $A(0,0)$, $B(1,1)$ & $C(5,2)$ to twice its size while keeping $C(5,2)$ fixed.

Ans-2 Using the matrix formula

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix}$$

Represent a point P with coordinates (x,y) by the row vector $(x, y, 1)$, we have

$$\bar{A} = (0,0,1) \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = (-5,-2,1)$$

$$\bar{B} = (1\,1\,1) \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = (-3,0,1)$$

$$\bar{C} = (5\,2\,1) \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = (5,2,1)$$

So $\bar{A}' = (-5,-2)$, $\bar{B}' = (-3,0)$ and $\bar{C}'(5,2)$

Now, since triangle ABC is completely determined by its vertices, we could represent the vertices by using a 3×3 matrix

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix}$$

and apply $S_{2,2,C}$ this, so

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} S_{2,2,C} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -5 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 & 1 \\ -3 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix}$$

Ques-3 Reflect the diamond shaped polygon whose vertices are A(-1,0), B(0,-2), C(1,0) and D(0,2) about

a) the horizontal line $y=2$

b) the line $y=x+2$

Ans 3a) Given vertices are

A(-1,0), B(0,-2), C(1,0), & D(0,+2)

$P' = \{T(0,2) \cdot M_x T(0,-2)\} \cdot P$

$T(0,2) \cdot M_x \cdot T(0,-2)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

A(-1,0)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$$

$\therefore$ A'(x,y) = (-1,4)

B(0,-2)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$$

B'(x,y) = (0,6)

C(1,0)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

$C'(x,y) = (1,4)$

D(0,2)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$D'(x,y) = (0,2)$

$\therefore$ New positions are $A'(-1,4)$, $B'(0,6)$, $C'(1,4)$ and $D'(0,2)$.

Ans 3 b)

Given line equation $y = mx + c$

$\therefore m = 1, \; c = 2$

$\tan\theta = 1$

$\theta = 45°$

Then,

$$m_2 = \begin{bmatrix} \frac{1-m}{1-m^2} & \frac{2m}{1+m^2} & \frac{-2cm}{1+m^2} \\ \frac{2m}{4m^2} & \frac{m^2-1}{m^2+1} & \frac{2c}{1+m^2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$P' = M_2 \cdot P$

$$P' = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

A(-1,0)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$A'(x,y) = (-2,1)$

B(0,-2)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore B'(x,y) = (-4,2)$

C(1,0)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$\therefore C'(x,y) = (-2,3)$

D(0,2)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore D'(x,y) = (0,2)$.

$\therefore$ New positions are $A'(-2,1)$, $B'(-4,2)$, $C'(-2,3)$ and $D'(0,2)$. (Ans)

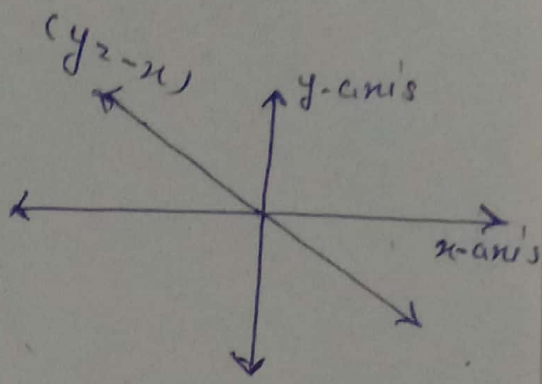
Ques-4 Show that transformation matrix for a reflection about $y=-x$ is equivalent to reflection relative to the y-axis followed by a counter clockwise rotation by $90°$.

Ans-4 equation of line $y = mx + c$

$c = 0, \theta = 90°$

$y = mx$

$\tan\theta = m$



1 Rotation

$\theta = -90°$ (clockwise)

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection matrix about y-axis $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotation matrix $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Reflection then rotation composite matrix is

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which is equal to the transformation matrix for reflection about the line $y = -x$