⑧ $v = \frac{(\sin x)\sinh(y)}{\cos(2x)+\cosh(2y)}$; $f(0)=1$ $\{N' \to$ Numerator

$v_x = [(D^r)(\cos x)\sinh(y)] - [(N^r)\{-2\sin(2x)\}]/(D')^2 \Rightarrow v_x(z,0)=0$

$v_y = [(D^r)(\sin x)\cosh(y)] - [(N^r)\{2\sinh(2y)\}]/(D^r)^2 \Rightarrow v_y(z,0)=\frac{1}{2}(\tan z)$

$f'(z) = v_y(z,0)+iv_x(z,0) = \frac{1}{2}(\tan z)(\sec z) \Rightarrow f(z)=\frac{1}{2}\sec(z)+$

Given: $f(0)=1 \Rightarrow 1=\frac{1}{2}\sec(0)+c' \Rightarrow \boxed{c'=\frac{1}{2}} \Rightarrow f(z)=\frac{1}{2}\sec(z)+\frac{1}{2}=\frac{1}{2}$

wt.

⑨ $v = (e^{-2y})[y\cos(2x)+x\sin(2x)]$

$v_x = (e^{-2y})[-2y\sin(2x)+2x\cos(2x)+\sin(2x)]$

$\Rightarrow v_x(z,0) = (e^0)[2z\cos(2z)+\sin(2z)] = 2z\cos(2z)+\sin(2z)$

$v_y = (e^{-2y})[\cos(2x)] - (2e^{-2y})[y\cos(2x)+x\sin(2x)]$

$v_y(z,0) = \cos(2z) - 2z\sin(2z)$

$f'(z) = v_y(z,0)+iv_x(z,0) = [\cos(2z)-2z\sin(2z)]+i[2z\cos(2z)+$

$= \cos(2z)[1+2iz] + \sin(2z)[i-2z$

$i[1+2z$

$\Rightarrow f'(z) = (1+2zi)e^{2iz}$ $\left[\because \cos(2z)+i\sin(2z) = e^{i2z}\right]$

$\Rightarrow f(z) = UV_1 - U'V_2$

$= (1+2zi)\left(\frac{e^{2iz}}{2i}\right) - \left(\frac{1}{2i}\right)(\cancel{2i})\left(\frac{e^{2iz}}{\cancel{2i}}\right) + c = \cancel{\frac{1}{2i}e^{2iz}} + ze^{2iz} - \cancel{\frac{1}{2i}e^{2iz}}$

$\Rightarrow \boxed{f(z) = ze^{2iz}+c}$

⑩ $3u+2v = y^2-x^2+16xy \to ①$; $\{u_x = v_y \ \& \ u_y = -v_x\}$

Diff. ① p.w.r.t. $x \Rightarrow 3u_x + 2v_x = -2x+16y \to ②$

..... p.w.r.t. $y \Rightarrow 3u_y + 2v_y = 2y+16x \to ③$

$\Rightarrow 2u_x - 3v_x = 2y+16x \to ④$

Solving ② & ④, we get $u_x = 2x+4y$ & $v_x = -4x+2y$

$u_x(z,0) = 2z$ & $v_x(z,0) = -4z$

$f'(z) = u_x(z,0) + iv_x(z,0) = 2z - i4z$

$\Rightarrow f(z) = z^2 - i(2z^2) + c$

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