Q. 3 Write short notes on : Properties of Fourier transforms. May 11. Dec. 11

Ans. :

Some of the important properties of the Fourier transform are listed as follows :

Linearity or superposition	Area under X (f)
Time scaling	Differentiation in time domain
Duality or symmetry	Integration in time domain
Time shifting	Conjugate function
Frequency shifting	Multiplication in time domain (Multiplication theorem)
Area under x (t)	Convolution theorem.

Let us understand these properties one-by-one.

Property 1 : Linearity or Superposition :

If $x_1(t) \overset{F}{\longleftrightarrow} X_1(f)$ and $x_2(t) \overset{F}{\longleftrightarrow} X_2(f)$ represent the Fourier transform pairs and if a_1 and a_2 are constants then we can write,

$$[a_1 x_1(t) + a_2 x_2(t)] \overset{F}{\longleftrightarrow} [a_1 X_1(f) + a_2 X_2(f)] \quad ...(1)$$

That means the linear combination of inputs gets transformed into linear combination of their Fourier transforms.

This property can be used to obtain the Fourier transform of a complicated function say x(t) by decomposing it in the form of sum of simpler functions, say $x_1(t)$ and $x_2(t)$.

Property 2 : Time Scaling :

1. $x(\alpha t)$ represents a time scaled signal and $X(f/\alpha)$ represents the frequency scaled signal or scaled frequency spectrum.
2. For $\alpha < 1$, $x(\alpha t)$ represents a compressed signal but $X(f/\alpha)$ represents an expanded version of X (f).
3. And for $\alpha > 1$, $x(\alpha t)$ will be an expanded signal in the time domain. But its Fourier transform $X(f/\alpha)$ represents a compressed version of X (f).

"α" being a constant, can be positive or negative. i.e. $\alpha > 0$ or $\alpha < 0$. Let us find the F.T. considering both the possibilities.

Property 3 : Duality or Symmetry Property :

This property states that, if $x(t) \overset{F}{\longleftrightarrow} X(-f)$

Then $X(t) \overset{F}{\longleftrightarrow} x(-f)$ ...(2)

i.e. t and f can be interchanged.

Meaning

1. In the term x(t), "x" represents the shape of the signal and "t" shows that the variable is time.
2. And in the term X (– f), "X" represents the shape of the spectrum and "f" shows that the variable is frequency.
3. The Duality theorem tell us that if $x(t) \overset{F}{\longleftrightarrow} X(-f)$ then the shape of the signal in the time domain and the shape of the spectrum can be interchanged.

Property 4 : Time Shifting

The time shifting property states that if x (t) and X (f) form a Fourier transform pair then,

$$x(t - t_d) \overset{F}{\longleftrightarrow} e^{-j2\pi f t_d} X(f) \quad ...(3)$$

Here the signal $x(t - t_d)$ is a time shifted signal. It is the same signal x (t) only shifted in time.

Property 5 : Area under x(t)

This property states that the area under the curve x(t) equals the value of its Fourier transform at f = 0.

i.e. if $x(t) \overset{F}{\longleftrightarrow} X(f)$ then,

$$\text{Area under } x(t) = \int_{-\infty}^{\infty} x(t)\, dt = X(0) \quad ...(4)$$

$$\text{Area under } x(t) = \int_{-\infty}^{\infty} x(t)\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi ft}\, dt \text{ at } f = 0.$$

$$= X(f) \text{ at } f = 0$$

$\therefore$ Area under x (t) = X (0). **...Proved.**

Property 6 : Area under X (f)

If $x(t) \overset{F}{\longleftrightarrow} X(f)$ then the area under X (f) is equal to the value of signal x(t) at t = 0.

That means if $x(t) \overset{F}{\longleftrightarrow} X(f)$ then,

$$\text{Area under } X(f) = \int_{-\infty}^{\infty} X(f) \cdot df = x(0) \quad ...(5)$$

Proof : By definition of inverse Fourier transform,

$$x(t) = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft}\, df$$

Substitute t = 0 in this equation to get,

$$x(0) = \int_{-\infty}^{\infty} X(f) \cdot e^{0}\, df = \int_{-\infty}^{\infty} X(f)\, dt$$

The RHS of this equation is the area under X (f). Hence the property is proved

Property 7 : Frequency Shifting

The frequency shifting characteristics states that if x(t) and X(f) form a Fourier transform pair then,

$$e^{j2\pi f_c t} x(t) \overset{F}{\longleftrightarrow} X(f - f_c) \quad ...(6)$$

Here f_c is a real constant.

Proof :

$$F\left[e^{j2\pi f_c t} x(t)\right] = \int_{-\infty}^{\infty} e^{j2\pi f_c t} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (f - f_c) t}\, dt$$

$$= X(f - f_c)$$

...Proved.

The time shifting property states that if x (t) and X (f) form a Fourier transform pair then,

$$x(t-t_d) \overset{F}{\longleftrightarrow} e^{-j2\pi f t_d} X(f) \qquad ...(3)$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi (f - f_c) t}\, dt$$

$$= X(f - f_c)$$

...Proved.

The term $X(f - f_c)$ represents a shifted frequency spectrum. The whole spectrum is thus shifted right by "f_c" in the frequency domain, when the signal x (t) is multiplied by $e^{j2\pi f_c t}$ in the time domain.

Property 8 : Differentiation in Time Domain

Some processing techniques involve differentiation and integration of the signal x(t). **This property is applicable if and only if the derivative of x (t) is Fourier transformable.**

Statement :

Let $x(t) \overset{F}{\longleftrightarrow} X(f)$ and let the derivative of x(t) be Fourier transformable. Then,

$$\frac{d}{dt} x(t) \overset{F}{\longleftrightarrow} j2\pi f\, X(f) \qquad ...(7)$$

Q. 4 Find the fourier transform of the decaying exponential pulse shown in Fig. 2.1. Dec. 12

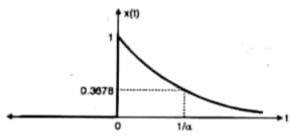

Fig. 2.1 : Decaying exponential pulse

Ans. : The exponential pulse shown in the Fig. 2.1 can be represented mathematically as follows :

$$x(t) = e^{-\alpha t} \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0 \qquad ...(1)$$

It can be represented in an alternate way as,

$$x(t) = e^{-\alpha t} u(t) \qquad ...(2)$$

The meaning of both the Equations (1) and (2) is the same. This is because u (t) = 1 for $t \geq 0$. So multiplying by does not affect the original function.

To find the fourier transform :

$$F[x(t)] = \int_{-\infty}^{\infty} e^{-\alpha t} u(t) \cdot e^{-j2\pi f t}\, dt$$

...as per definition of FT.

$$= \int_{-\infty}^{\infty} e^{-\alpha t} e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} e^{-(\alpha + j2\pi f) t}\, dt$$

$$= \frac{-1}{(\alpha + j2\pi f)} \left[e^{-(\alpha + j2\pi f) t}\right]_0^{\infty}$$

$$= \frac{-1}{(\alpha + j2\pi f)} \left[e^{-\infty} - e^{0}\right] = \frac{-1}{(\alpha + j2\pi f)} [0 - 1]$$

$$= \frac{1}{(\alpha + j2\pi f)}$$

This is the required result.

$$\therefore \quad e^{-\alpha t} u(t) \overset{F}{\longleftrightarrow} \frac{1}{(\alpha + j2\pi f)} \qquad ...(3)$$

Q. 5 Obtain the fourier transform of the delta function shown in Fig. 2.2(a). Dec. 09, Dec.12, Dec. 14

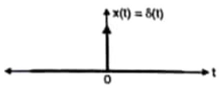

Fig. 2.2(a) : Delta function

Ans. :

1. By the definition of fourier transform.

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi f t}\, dt \qquad ...(1)$$

We cannot substitute the value of δ (t) directly in the Equation (1) because it is infinitely large at t = 0. Therefore let us use the sifting property of the delta function.

2. Sifting property of delta function : The shifting property states that

$$\int_{-\infty}^{\infty} f(t)\, \delta(t - t_d)\, dt = f(t_d) \qquad ...(2)$$

Let us use this property in Equation (1) as follows :

3. In Equation (2) assume that $t_d = 0$ and $f(t) = e^{-j2\pi f t}$

$$\therefore \quad X(f) = \int_{-\infty}^{\infty} e^{-j2\pi f t} \cdot \delta(t - 0)$$

∴ by using Equation (2).

$$X(f) = e^{-j2\pi f t_d} \quad \text{but } t_d = 0$$

$$\therefore \quad X(f) = e^{-j2\pi \cdot 0} = 1 \qquad ...(3)$$

$$\text{Thus } \delta(t) \overset{F}{\longleftrightarrow} 1$$

The amplitude spectrum of the delta function is as shown in the Fig. 2.2(b). This shows that the delta function contains all the frequencies from $-\infty$ to ∞ with equal amplitudes. The fourier transform of a delta function is a dc signal.

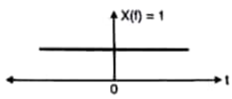

Fig. 2.2(b) : Amplitude spectrum of delta function

Q. 6 Obtain the fourier transform of a unit step function. Dec. 14

Ans. :

A unit step function is mathematically defined as,

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$= 0 \quad \text{elsewhere}$$

Using the definition of the fourier transform we get,

$$F[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot e^{-j2\pi f t}\, dt$$

But unit step function is present only for $t \geq 0$.

$$F[x(t)] = AT \frac{\sin(\pi f T)}{\pi f T} \qquad ...(3)$$

In the above equation,

$$\frac{\sin(\pi f T)}{\pi f T} = \text{sinc}(fT) \qquad ...(4)$$

$$\therefore \quad F[x(t)] = AT\, \text{sinc}(fT)$$

To find the fourier transform :

$$F[x(t)] = \int_{-\infty}^{\infty} e^{-\alpha t} u(t) \cdot e^{-j2\pi ft} dt$$

...as per definition of FT.

$$= \int_{-\infty}^{\infty} e^{-\alpha t} e^{-j2\pi ft} dt = \int_{-\infty}^{\infty} e^{-(\alpha + j2\pi f) t} dt$$

$$= \frac{-1}{(\alpha + j2\pi f)} \left[e^{-(\alpha + j2\pi f) t} \right]_0^{\infty}$$

$$= \frac{-1}{(\alpha + j2\pi f)} \left[e^{-\infty} - e^{0} \right] = \frac{-1}{(\alpha + j2\pi f)} [0 - 1]$$

$$= \frac{1}{(\alpha + j2\pi f)}$$

This is the required result.

$$\therefore \quad e^{-\alpha t} u(t) \overset{F}{\leftrightarrow} \frac{1}{(\alpha + j2\pi f)} \quad ...(3)$$

Q. 5 Obtain the fourier transform of the delta function shown in Fig. 2.2(a). Dec. 09. Dec.12 . Dec. 14

The amplitude spectrum of the delta function is as shown in the Fig. 2.2(b). This shows that the delta function contains all the frequencies from $-\infty$ to ∞ with equal amplitudes. The fourier transform of a delta function is a dc signal.

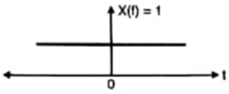

Fig. 2.2(b) : Amplitude spectrum of delta function

Q. 6 Obtain the fourier transform of a unit step function. Dec. 14

Ans. :

A unit step function is mathematically defined as,

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$= 0 \quad \text{elsewhere}$$

Using the definition of the fourier transform we get,

$$F[u(t)] = \int_{-\infty}^{\infty} u(t) \cdot e^{-j2\pi ft} dt$$

But unit step function is present only for $t \geq 0$.

$$\therefore F[u(t)] = \int_{0}^{\infty} 1 \cdot e^{-j2\pi ft} dt = \frac{1}{-j2\pi f} \left[e^{-j2\pi ft} \right]_0^{\infty}$$

$$= \frac{1}{-j2\pi f} \left[e^{-\infty} - e^{0} \right]$$

$$\therefore F[u(t)] = \frac{1}{-j2\pi f} [0 - 1] = \frac{1}{j2\pi f}$$

$$\therefore u(t) \overset{F}{\leftrightarrow} \frac{1}{j2\pi f} \quad ...\text{Ans.}$$

Q. 7 Obtain the fourier transform of a rectangular pulse of duration T and amplitude A as shown in Fig. 2.3(a). Dec. 10

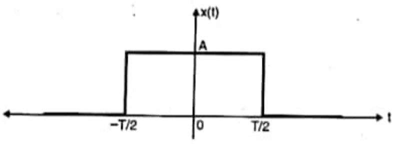

Fig. 2.3(a) : Rectangular pulse

Ans. : The rectangular pulse shown in Fig. 2.3(a) can be expressed mathematically as,

$$\text{rect}(t/T) = A \quad \text{for} -T/2 \leq t \leq T/2$$
$$= 0 \quad \text{elsewhere}$$

This is also known as the gate function. Therefore the fourier transform will be,

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} dt$$

...by definition of F. T.

$$= \int_{-T/2}^{T/2} A e^{-j2\pi ft} dt$$

$$= \frac{A}{-j2\pi f} \left[e^{-j2\pi ft} \right]_{-T/2}^{T/2}$$

$$= \frac{A}{-j2\pi f} \left[e^{-j\pi fT} - e^{j\pi fT} \right]$$

$$= \frac{A}{j2\pi f} \left[e^{j\pi fT} - e^{-j\pi fT} \right]$$

$$= \frac{A}{\pi f} \left[\frac{e^{j\pi fT} - e^{-j\pi fT}}{2j} \right] \quad ...(1)$$

As per the Euler's theorem, $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

Applying this to Equation (1),

$$\text{we get, } F[x(t)] = \frac{A}{\pi f} [\sin(\pi fT)] \quad ...(2)$$

Multiply and divide the RHS of Equation (2) by T to get,

$$F[x(t)] = AT \frac{\sin(\pi fT)}{\pi fT} \quad ...(3)$$

In the above equation,

$$\frac{\sin(\pi fT)}{\pi fT} = \text{sinc}(fT) \quad ...(4)$$

$$\therefore F[x(t)] = AT\,\text{sinc}(fT)$$

$$\therefore A\,\text{rect}(t/T) \overset{F}{\leftrightarrow} AT\,\text{sinc}(fT)$$

Thus the rectangular pulse transforms into a sinc function.

Amplitude spectrum

The amplitude spectrum of the rectangular function is as shown in Fig. 2.3(b).

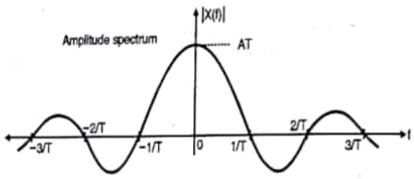

Fig. 2. 3(b) : Amplitude spectrum of a rectangular pulse

As we already know,

$$\text{sinc}(0) = 1 \quad \therefore \quad AT\,\text{sinc}(0) = AT$$

The sinc function will have zero value for the following values of "fT":

$$\text{Sinc}(fT) = 0 \quad \text{for } fT = \pm 1, \pm 2, \pm 3, \ldots\ldots$$

$$\text{i.e. for } f = \pm \frac{1}{T}, \pm \frac{2}{T}, \pm \frac{3}{T} \ldots\ldots$$

The phase spectrum has not been shown as it has zero value for all the values of f.

To absorb negative values of | X (f) | in the phase shift

The negative amplitude of the amplitude spectrum | X (f) | can be made positive by introducing a phase shift of $\pm 180°$ in the phase spectrum. This is as shown in Fig. 2.3(c). A negative phase shift for positive frequency and positive phase shift for the negative frequency is introduced in order to maintain symmetry of the phase spectrum.

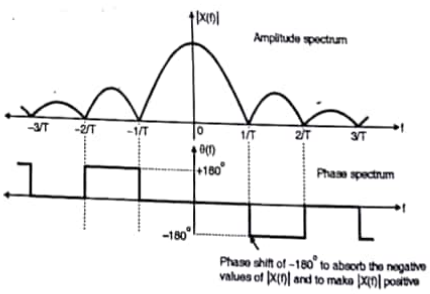

Fig. 2.3(c) : Amplitude and phase spectrums for a rectangular pulse. Negative values of | X (f) | have been absorbed in the additional phase shift of ± 180° in the phase spectrum

As per the Euler's theorem, $\sin\theta = \frac{\quad}{2j}$

Applying this to Equation (1),

$$\text{we get, } F[x(t)] = \frac{A}{\pi f}[\sin(\pi f T)] \qquad ...(2)$$

Multiply and divide the RHS of Equation (2) by T to get,

Fig. 2.3(c) : Amplitude and phase spectrums for a rectangular pulse. Negative values of | X (f) | have been absorbed in the additional phase shift of ± 180° in the phase spectrum

Q. 8 State and prove the properties of Fourier transform : Time shifting. May 09, May 10, Dec. 10, Dec. 13, Dec. 15, May 16

Ans. : The time shifting property states that if x (t) and X (f) form a Fourier transform pair then,

$$x(t-t_d) \overset{F}{\longleftrightarrow} e^{-j2\pi f t_d} X(f) \qquad ...(1)$$

Here the signal $x(t-t_d)$ is a time shifted signal. It is the same signal x (t) only shifted in time.

Proof :

$$F[x(t-t_d)] = \int_{-\infty}^{\infty} x(t-t_d)\cdot e^{-j2\pi f t}\,dt \qquad ...(2)$$

$$\text{Let } (t-t_d) = \tau,$$
$$\therefore \quad t = t_d+\tau$$
$$\therefore \quad dt = d\tau.$$

Substituting these values in Equation (1) we get,

$$F[x(t-t_d)] = \int_{-\infty}^{\infty} x(\tau)\cdot e^{-j2\pi f(t_d+\tau)}\,d\tau$$

$$= e^{-j2\pi f t_d}\int_{-\infty}^{\infty} x(\tau)\,e^{-j2\pi f\tau}\,d\tau$$

$$\therefore \quad F[x(t-t_d)] = e^{-j2\pi f t_d} X(f) \qquad \text{...Proved.}$$

This shows that the time shifting does not have any effect on the amplitude spectrum, but there is an additional phase shift of $-2\pi f t_d$, which is denoted by the term $e^{-j2\pi f t_d}$.

Significance of time shifting in communication systems

If signal $x(t)$ is transmitted by a transmitter, then due to the distance travelled, this signal becomes a time delayed signal $x(t-t_d)$ when it reaches the receiver.

The time delay "t_d" is dependent on the distance between the transmitter and the receiver.

The time shifting property explains the effect of such time shifting on the spectrum of the signal. It tells us that there is no effect of time shifting on the amplitude spectrum but there is an additional phase shift of $-2\pi f t_d$.

Q. 9 State and prove the differentiation in time domain property of the Fourier Transform. Dec. 15, Dec. 16

Ans. : Some processing techniques involve differentiation and integration of the signal x(t). **This property is applicable if and only if the derivative of x (t) is Fourier transformable.**

Statement : Let $x(t) \overset{F}{\longleftrightarrow} X(f)$ and let the derivative of x(t) be Fourier transformable. Then,

$$\frac{d}{dt}x(t) \overset{F}{\longleftrightarrow} j2\pi f X(f) \qquad ...(1)$$

Proof : By the definition of inverse Fourier transform,

$$x(t) = \int_{-\infty}^{\infty} X(f)\,e^{+j2\pi f t}\,df$$

Therefore

$$\frac{d}{dt}x(t) = \frac{d}{dt}\left[\int_{-\infty}^{\infty} X(f)\,e^{+j2\pi f t}\,df\right]$$

$$= \int_{-\infty}^{\infty} X(f)\left(\frac{d}{dt}e^{+j2\pi f t}\right)df$$

$$\frac{d}{dt}x(t) = \int_{-\infty}^{\infty} [X(f)\cdot j2\pi f]\,e^{+j2\pi f t}\,df$$

As per the definition of the inverse Fourier transform the term inside the square bracket must be the Fourier transform of $\frac{d}{dt}x(t)$.

$$\therefore \quad F\left[\frac{d}{dt}x(t)\right] = j2\pi f X(f)$$

$$\text{OR} \quad \frac{d}{dt}x(t) \overset{F}{\longleftrightarrow} j2\pi f X(f). \qquad \text{...Proved.}$$

Meaning :

Differentiating the signal in time domain is equivalent to multiplying its Fourier transform by $(j2\pi f)$ in the frequency domain. Thus differentiation will enhance the high frequency components since $|j2\pi f X(f)| > |X(f)|$.

Q. 10 Prove time convolution property of Fourier transform. May 09, May 10, Dec. 10, May 12, Dec. 13, May 16

Ans. : This property states that the convolution of signals in the time domain will be transformed into the multiplication of their Fourier transforms in the frequency domain.

$$\text{i.e. } [x_1(t) * x_2(t)] \overset{F}{\longleftrightarrow} X_1(f)X_2(f) \qquad ...(1)$$

Proof : The convolution of the two signals in the time domain is defined as,

$$X_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\lambda)\cdot x_2(t-\lambda)\,d\lambda \qquad ...(2)$$

Taking the Fourier transform of the convolution.

$$F[x_1(t)*x_2(t)] = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x_1(\lambda)\cdot x_2(t-\lambda)\,d\lambda\right]e^{-j2\pi f t}\,dt \qquad ...(3)$$

Multiply and divide the RHS of the Equation (2) by $e^{-j2\pi f\lambda}$ to get,

$$F[x_1(t)*x_2(t)] = \int_{-\infty}^{\infty} x_1(\lambda)\cdot e^{-j2\pi f\lambda}\,d\lambda\cdot\int_{-\infty}^{\infty} x_2(t-\lambda)\cdot e^{-j2\pi f t}\cdot e^{j2\pi f\lambda}\,dt$$

$$= \int_{-\infty}^{\infty} x_1(\lambda)\,e^{-j2\pi f\lambda}\,d\lambda\int_{-\infty}^{\infty} x_2(t-\lambda)\,e^{-j2\pi f(t-\lambda)}\,dt \qquad ...(4)$$

Let $(t-\lambda) = m$ in Equation (2)

$$\therefore \quad F[x_1(t)*x_2(t)] = \int_{-\infty}^{\infty} x_1(\lambda)\cdot e^{-j2\pi f\lambda}\,d\lambda\int_{-\infty}^{\infty} x_2(m)\cdot e^{-j2\pi f m}\,dm$$

only if the derivative of x (t) is Fourier transformable.

Statement : Let $x(t) \xleftrightarrow{F} X(f)$ and let the derivative of x(t) be Fourier transformable. Then,

$$\frac{d}{dt} x(t) \xleftrightarrow{F} j2\pi f\, X(f) \quad ...(1)$$

Proof : By the definition of inverse Fourier transform,

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi f t}\, df$$

$$F[x_1(t) * x_2(t)] = \int_{-\infty} x_1(\lambda) \cdot e^{-j2\pi f\lambda} d\lambda \cdot \int_{-\infty} x_2(t-\lambda) \cdot e^{-j2\pi f t} \cdot e^{j2\pi f\lambda} dt$$

$$= \int_{-\infty}^{\infty} x_1(\lambda)\, e^{-j2\pi f\lambda} d\lambda \int_{-\infty}^{\infty} x_2(t-\lambda)\, e^{-j2\pi f(t-\lambda)} dt \quad ...(4)$$

Let $(t-\lambda) = m$ in Equation (2)

$$\therefore\ F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\lambda) \cdot e^{-j2\pi f\lambda} d\lambda \int_{-\infty}^{\infty} x_2(m) \cdot e^{-j2\pi f m} dm$$

Using the definition of the Fourier transform to the RHS we get,

$$F[x_1(t) * x_2(t)] = X_1(f)\, X_2(f) \quad ...(5)$$

This is the required result.

Significance of convolution in a communication system:

Consider a communication system with input x(t), output y(t) and impulse response h(t), as shown in Fig. 2.4

Fig. 2.4

The relation between x(t), h(t) and y(t) is as follows,

$$y(t) = x(t) * h(t) \quad ...(6)$$

That means y(t) is obtained by taking convolution of x(t) and h (t).

Method I :

Output of a system y(t) can be obtained by taking the convolution of input x(t) and impulse response of the system h (t).

$$y(t) = x(t) * h(t)$$

Method II :

The output y(t) can be obtained using the Fourier transform.

Let $y(t) \xleftrightarrow{F} Y(f)$, $x(t) \xleftrightarrow{F} X(f)$ and $h(t) \xleftrightarrow{F} H(f)$.

Then taking the F.T. of Equation (16) we get,

$$Y(f) = X(f) \cdot H(f)$$

Now take IFT to get y (t) = IFT [Y (f)]

Multiplication and taking IFT is simpler than obtaining the convolution. Hence in practice we can use this method to obtain y (t) i.e. output of a system.

$$F\{Im[x(t)]\} = \frac{1}{2j}[x(f) - X^*(-f)] \quad ...(7)$$

A signal cannot be band limited and time limited simultaneously

To justify this statement let us first consider a time limited signal and its Fourier transform. A rectangular pulse of duration T and amplitude A is an excellent example of a time limited signal.

We know that its Fourier transform is a sinc pulse which extends from $f = -\infty$ to $f = +\infty$ even though the amplitude of the sinc function goes on decreasing with increase in frequency.

Thus a time limited signal results into an unlimited spectrum in frequency domain. Now consider a rectangular pulse of width "f" in the frequency domain. This is a band limited signal. Due to the principle of duality, the inverse Fourier transform of this rectangular pulse is a sinc function in time domain which extends from $t = -\infty$ to $t = +\infty$.

Thus band limiting in the frequency domain results in a "time unlimited" signal in the time domain. Thus a signal cannot be band limited or time limited simultaneously.

Q. 11 Obtain the fourier transform of a cosine wave having a frequency f_o and peak amplitude of unity and plot its spectrum. Refer Fig. 2.5(a).

Dec. 09

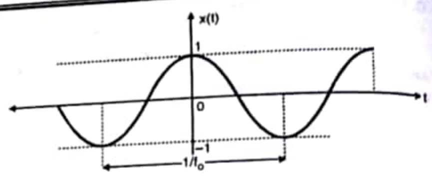

Fig. 2.5(a)

Ans. :

A cosine wave can be mathematically represented as,

$$x(t) = A\cos(2\pi f_o t)$$

But A = 1

$$\therefore\ x(t) = \cos(2\pi f_o t) \quad ...(1)$$

By Euler's identity we can write,

$$\therefore\ x(t) = \cos(2\pi f_o t) = \frac{e^{j2\pi f_o t} + e^{-j2\pi f_o t}}{2} \quad ...(2)$$

The fourier transform of x (t) is given by,

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t} dt$$

Substituting the value of x (t) we get,

$$X(f) = \int_{-\infty}^{\infty} \left[\frac{e^{j2\pi f_o t} + e^{-j2\pi f_o t}}{2}\right] e^{-j2\pi f t} dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} [e^{-j2\pi(f-f_o)t} + e^{-j2\pi(f+f_o)t}]\, dt$$

$$\therefore\ X(f) = \frac{1}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f-f_o)t} dt + \frac{1}{2}\int_{-\infty}^{\infty} e^{-j2\pi(f+f_o)t} dt \quad ...(3)$$

In which we have found the fourier transform of a dc signal. In that example we have proved that,

$$\int_{-\infty}^{\infty} e^{-j2\pi f t} dt = \delta(f) \quad ...(4)$$

Using this property for the RHS of Equation (3) we get,

$$X(f) = \frac{1}{2}\delta(f - f_o) + \frac{1}{2}\delta(f + f_o) \quad ...\text{Ans.}$$

The frequency spectrum is as shown in the Fig. 2.5(b) which shows that two impulses are present one at f_o and the other at $-f_o$.

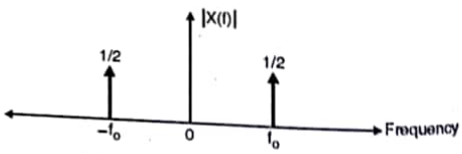

Fig. 2.5(b) : Spectrum of a cosine wave

Chapter 3 : Noise

Q. 1 Classify and explain the various noises that affect communications. Dec. 03, May 11

Ans. :

Noise can be divided into two broad categories :

1. External noise of uncorrelated noise.
2. Internal noise or correlated noise or fundamental noise.

The classification of noise sources is shown in Fig. 3.1

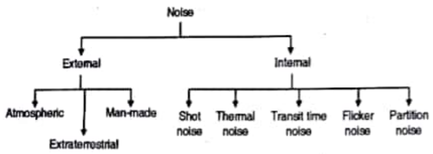

Fig. 3.1

External Noise (Uncorrelated Noise)

It is defined as the noise that is generated outside the device or circuit. As shown in Fig. 3.1., the external noise can be of three types :

1. Atmospheric noise
2. Extraterrestrial and
3. Man made noise

Fundamental or Internal Noise :

The fundamental sources of noise are within the electronic equipment. They are called fundamental sources because they are the integral part of the physical nature of the material used for making electronic components. This type of noise follows certain rules. Therefore it can be eliminated by properly designing the electronic circuits and equipments.

Q. 2 Classify and explain the various noises that affect communications. May 11

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It is defined as the noise that is generated outside the device or circuit. As shown in Fig. 3.1., the external noise can be of three types :

1. Atmospheric noise
2. Extraterrestrial and
3. Man made noise

1. Atmospheric noise

(i) This type of noise gets produced within the Earth's atmosphere. The common source of this type of noise is lightning. This type of noise is in the form of impulses or spikes which covers a wide frequency band typically upto 30 MHz.

(ii) The sputtering, cracking etc heard from the loud speakers of radio is due to atmospheric noise.

(iii) This type of noise becomes insignificant above 30 MHz.

2. Extraterrestrial noise

(i) This type of noise originates from the sources which exist outside the Earth's atmosphere. Hence this noise is also called as deep space noise.

(ii) The noise originating from the sun and the outer space is known as **Extraterrestrial Noise.**

(iii) The extraterrestrial noise can be sub-divided into two groups : (a) Solar noise (b) Cosmic noise.

(iv) Our sun being a large body at very high temperatures radiates a lot of noise. The noise radiation from sun varies with the temperature changes on its surface.

(v) The temperature changes follow a cycle of 11 years hence the cycle of great electrical disturbances (noise) also repeats after every 11 years.

(vi) The cosmic noise comes from the stars. This is identical to the noise radiated by sun because stars also are large hot bodies.

(vii) This noise is called as black body noise or thermal noise and it is distributed uniformly over the entire sky. The noise also gets originated from the center of our galaxy, other galaxies and special type of stars such as "Quasars" and "Pulsars".

3. Man made noise (Industrial noise)

The man made noise is generated due to the make and break process in a current carrying circuit. The examples are the electrical motors, welding machines, ignition system of the automobiles, thyristorised high current circuits, fluorescent lights, switching gears etc. This type of noise is also called as industrial noise.

Q. 3 What are the types of Internal Nosie ? May 15

Ans. : The fundamental noise sources produce different types of noise. They are as follows:

1. Thermal noise
2. Shot noise
3. Partition noise
4. Low frequency or flicker, noise.
5. High frequency or transit time, noise.
6. Avalanche noise.
7. Burst noise.

Let us know them one by one.

1. Shot Noise

(i) The shot noise is produced due to shot effect. Due to the shot effect, shot noise is produced in all the amplifying devices or for that matter in all the active devices.

(ii) The shot noise is produced due to the random variations in the arrival of electrons (or holes) at the output electrode of an amplifying device.

(iii) Therefore it appears as a randomly varying noise current superimposed on the output. The shot noise "sounds" like a shower of lead shots falling on a metal sheet if amplified and passed through a loud speaker.

[redacted]

(iv) The shot noise has a uniform spectral density like thermal noise. The exact formula for the shot noise can be obtained only for diodes.

(v) For all other devices an approximate equation is stated. The mean square shot noise current for a diode is given as :

5. High Frequency or Transit Time Noise

If the time taken by an electron to travel from the emitter to the collector of a transistor becomes comparable to the time period of the signal which is being amplified then the transit time effect will take place.

passed through a loud speaker.

(iv) The shot noise has a uniform spectral density like thermal noise. The exact formula for the shot noise can be obtained only for diodes.

(v) For all other devices an approximate equation is stated. The mean square shot noise current for a diode is given as :

$$I_n^2 = 2\, I_{dc}\, q\, B \text{ Amperes}^2 \quad ...(1)$$

where, I_{dc} = Direct current across the junction (in Amp)

q = Electron charge = 1.6×10^{-19} C.

B = Effective noise bandwidth in Hz.

For the amplifying devices the shot noise is :

1. Inversely proportional to the transconductance of the device.
2. Directly proportional to the output current.

2. Partition Noise

Partition noise is generated when the current gets divided between two or more paths. It is generated due to the random fluctuations in the division. Therefore the partition noise in a transistor will be higher than that in a diode. The devices like gallium arsenide FET draw almost zero gate bias current, hence keeping the partition noise to its minimum value.

3. Low Frequency or Flicker Noise :

The flicker noise will appear at frequencies below a few kilohertz. It is sometimes called as "1/f" noise. In the semiconductor devices, the flicker noise is generated due to the fluctuations in the carrier density (i.e. density of electrons and holes).

These fluctuations in the carrier density will cause the fluctuations in the conductivity of the material. This will produce a fluctuating voltage drop when a direct current flows through a device. This fluctuating voltage is called as flicker noise voltage.

The mean square value of flicker noise voltage is proportional to the square of direct current flowing through the device.

4. Thermal Noise or Johnson Noise

The free electrons within a conductor are always in random motion. This random motion is due to the thermal energy received by them. The distribution of these free electrons within a conductor at a given instant of time is not uniform.

It is possible that an excess number of electrons may appear at one end or the other of the conductor. The average voltage resulting from this non-uniform distribution is zero but the average power is not zero.

As this power has appeared as a result of the thermal energy, it is called as the "thermal noise power".

The average thermal noise power is given by,

$$P_n = k\,TB \text{ Watts} \quad ...(2)$$

where, k = Boltzmann's constant = 1.38×10^{-23} Joules/Kelvin.

B = Bandwidth of the noise spectrum (Hz).

T = Temperature of the conductor, °Kelvin

Equation (2) indicates that a conductor operated at a finite temperature can work as a generator of electrical energy. The thermal noise power P_n is proportional to the noise BW and conductor temperature.

5. High Frequency or Transit Time Noise

If the time taken by an electron to travel from the emitter to the collector of a transistor becomes comparable to the time period of the signal which is being amplified then the transit time effect will take place.

This effect is observed at very high frequencies typically in the VHF range. Due to the transit time effect some of the carriers may diffuse back to the emitter.

This gives rise to an input admittance, the conductance component of which increases with frequency.

The very small currents induced in the input of the device by means of the random fluctuations in the output current will create random noise at high frequencies. Once this noise appears, it goes on increasing with frequency at a rate of 6 dB per octave.

6. Correlated Noise

This is the form of internal noise. It is present in the circuit if and only if the signal is present. This is how it is correlated with the signal and hence called as the correlated noise.

In other words this type of noise will be absent if the signal is absent. Correlated noise is produced due to the nonlinear amplification process.

The types of correlated noise are :

1. Harmonic distortion 2. Intermodulation distortion

Both these are basically the nonlinear distortions.

All the electronic circuits exhibit a nonlinear behaviour and hence produce nonlinear distortions.

7. Impulse Noise

This type of noise is in the form of spikes of high amplitude and short duration, in the total noise spectrum. The shapes of these noise pulse are undefined as shown in Fig. 3.2

This type of noise is produced by electromechanical relays and solenoids, electric motors, fluorescent lights, ignition systems of automobiles and lightning.

The impulse noise produces a sharp popping or crackling sound in the audio circuits and produces errors in the data communication circuits.

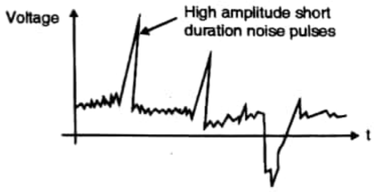

Fig. 3.2 : Impulse noise

8. Interference

This is a type of external noise. The meaning of the word "Interference" is "to disturb or detract from".

The interference is produced when information signal from one source produces frequencies that fall outside their allocated frequency band and interfere with the frequency band allocated to some other information signal as shown in Fig. 3.3. Interferences take place in the radio frequency spectrum when harmonics or cross-product frequencies from one source fall into the passband of the other source.

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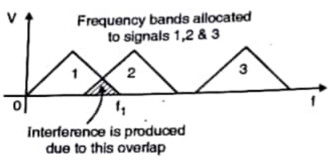

The mean square value of flicker noise voltage is proportional to the square of direct current flowing through the device.

Q. 7 Explain the term thermal noise.

Dec. 03, May 04, Dec. 07, May 15, May 16

Ans. : The free electrons within a conductor are always in random motion. This random motion is due to the thermal energy received by them. The distribution of these free electrons within a conductor at a given instant of time is not uniform. It is possible that an excess number of electrons may appear at one end or the

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$= 1.38 \times 10^{-23}$ Joules/Kelvin.

B = Bandwidth of the noise spectrum (Hz).

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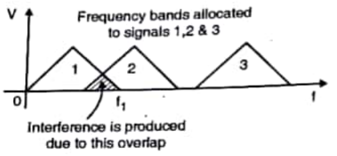

Fig. 3.3 : Interference

Q. 4 Explain the terms : Shot noise and Equivalent noise temperature. **Dec. 04, Dec. 13, May 15, Dec. 15**

Ans.: The shot noise is produced due to shot effect. Due to the shot effect, shot noise is produced in all the amplifying devices or for that matter in all the active devices.

The shot noise is produced due to the random variations in the arrival of electrons (or holes) at the output electrode of an amplifying device.

Therefore it appears as a randomly varying noise current superimposed on the output. The shot noise "sounds" like a shower of lead shots falling on a metal sheet if amplified and passed through a loud speaker.

The shot noise has a uniform spectral density like thermal noise. The exact formula for the shot noise can be obtained only for diodes. For all other devices an approximate equation is stated. The mean square shot noise current for a diode is given as :

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For the amplifying devices the shot noise is :

1. Inversely proportional to the trans conductance of the device.
2. Directly proportional to the output current.

Q. 5 Write a short note on : Partition Noise. **May 15**

Ans. :

Partition noise is generated when the current gets divided between two or more paths. It is generated due to the random fluctuations in the division. Therefore, the partition noise in a transistor will be higher than that in a diode.

The devices like gallium arsenide FET draw almost zero gate bias current, hence keeping the partition noise to its minimum value.

Q. 6 Write a short note on : Low Frequency or Flicker Noise. **May 15**

Ans. : The flicker noise will appear at frequencies below a few kilohertz. It is sometimes called as "1/f" noise.

In the semiconductor devices, the flicker noise is generated due to the fluctuations in the carrier density (i.e. density of electrons and holes).

These fluctuations in the carrier density will cause the fluctuations in the conductivity of the material. This will produce a fluctuating voltage drop when a direct current flows through a device. This fluctuating voltage is called as flicker noise voltage.

The mean square value of flicker noise voltage is proportional to the square of direct current flowing through the device.

Q. 7 Explain the term thermal noise. **Dec. 03, May 04, Dec. 07, May 15, May 16**

Ans. : The free electrons within a conductor are always in random motion. This random motion is due to the thermal energy received by them. The distribution of these free electrons within a conductor at a given instant of time is not uniform. It is possible that an excess number of electrons may appear at one end or the other of the conductor.

The average voltage resulting from this non-uniform distribution is zero but the average power is not zero.

As this power has appeared as a result of the thermal energy, it is called as the "thermal noise power".

The average thermal noise power is given by,

$$P_n = k\,TB \text{ Watts} \quad ...(1)$$

where, k = Boltzmann's constant

$= 1.38 \times 10^{-23}$ Joules/Kelvin.

B = Bandwidth of the noise spectrum (Hz).

T = Temperature of the conductor, °Kelvin

Equation (1) indicates that a conductor operated at a finite temperature can work as a generator of electrical energy.

The thermal noise power P_n is proportional to the noise BW and conductor temperature.

Q. 8 Write a short note on : High Frequency or Transit Time Noise **May 15**

Ans.: If the time taken by an electron to travel from the emitter to the collector of a transistor becomes comparable to the time period of the signal which is being amplified then the transit time effect will take place.

This effect is observed at very high frequencies typically in the VHF range. Due to the transit time effect some of the carriers may diffuse back to the emitter. This gives rise to an input admittance, the conductance component of which increases with frequency.

The very small currents induced in the input of the device by means of the random fluctuations in the output current will create random noise at high frequencies. Once this noise appears, it goes on increasing with frequency at a rate of 6 dB per octave.

Q. 9 An amplifier has a bandwidth of 4 MHz with 10 kΩ as the input resistor. Calculate the rms noise voltage at the input to this amplifier if the room temperature is 25°C. **Dec.16**

Ans. : Given : $B = 4$ MHz, $R_i = 10$ k Ω, $T = 25°C = 298°$ K

Rms noise voltage $V_n = \sqrt{4\,kTBR}$

$= \sqrt{4 \times 1.38 \times 10^{-23} \times 298 \times 4 \times 10^6 \times 10 \times 10^3}$

$V_n = 25.65\ \mu V$

Q. 10 Write a short note on : Correlated Noise. **May 15**

Ans. : This is the form of internal noise. It is present in the circuit if and only if the signal is present. This is how it is correlated with the signal and hence called as the correlated noise. In other words this type of noise will be absent if the signal is absent. Correlated noise is produced due to the nonlinear amplification process.

other harmonics.

with the message signal to degrade the system performance.

Q. 11 Write short notes on White noise.

May 12. Dec. 13

Ans. : The effect of "white" noise on the performance of different communication systems.

"White" noise is the noise whose power spectral density is uniform over the entire frequency range of interest, as shown in Fig. 3.6.

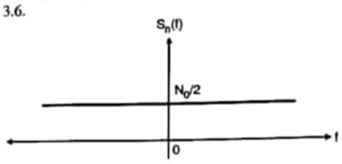


Fig. 3.6 : Power spectral density of white noise

Why is it called as white noise ?

The white noise contains all the frequency components in equal proportion. This is analogous with white light which is a superposition of all visible spectral components.

Why is it called as gaussian noise ?

The white noise has a Gaussian distribution. That means the PDF of white noise has the shape of Gaussian PDF. Therefore it is called as gaussian noise.

Power spectral density of white noise

As shown in Fig. 3.6 the power spectral density (psd) of a white noise is given by,

$$S_n(f) = \frac{N_0}{2} \quad ...(1)$$

This equation shows that the power spectral density of white noise is independent of frequency. As N_0 is constant, the psd is uniform over the entire frequency range including the positive as well as the negative frequencies. N_0 in Equation (3.3.11) is defined as :

$$N_0 = KT_e \quad ...(2)$$

where, K = Boltzmann's constant and

T_e = Equivalent noise temperature of the system.

Example of white noise

The best example of white noise is the thermal or Johnson noise.

Q. 12 Define the following term : Noise bandwidth.

Dec. 03. Dec. 12. May 15. May 16

Ans. : Assume that a white noise is present at the input of a receiver (filter). Let the filter have a transfer function H(f) as shown in Fig. 3.7.

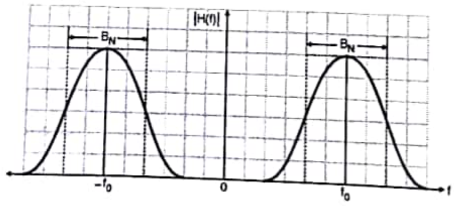


Fig. 3.7 : Noise bandwidth of a filter

This filter is being used to reduce the noise power actually passed on to the receiver. Now draw the frequency response of an ideal (rectangular) filter as shown by the dotted plot in Fig. 3.7. The center frequency of this ideal filter also is f_0.

Let the bandwidth "B_N" of the ideal filter be adjusted in such a way that the noise output power of the ideal filter is exactly equal to the noise output power of a real R-C filter.

Then B_N is called as the noise bandwidth of the real filter. Thus the noise bandwidth "B_N" is defined as the bandwidth of an ideal (rectangular) filter which passes the same noise power as passed by the real filter.

Q. 13 Define the following term : Signal to noise ratio.

Dec. 03. Dec. 04. May 06. May 09. Dec. 09. May 15. May 16

Ans. : In the communication systems the comparison of signal power with the noise power at the same point is important, to ensure that the noise at that point is not excessively large.

It is defined as the ratio of signal power to the noise power at the same point.

$$\therefore \quad \frac{S}{N} = \frac{P_s}{P_n} \quad ...(1)$$

Where, P_s = Signal power

P_n = Noise power at the same point.

The signal to noise ratio is normally expressed in dB and the typical values of S/N ratio range from about 10 dB to 90 dB. Higher the value of S/N ratio better the system performance in presence of noise.

$$S/N\ (dB) = 10 \log_{10} (P_s / P_n) \quad ...(2)$$

The powers can be expressed in terms of signal and noise voltages as follows :

$$P_s = \frac{V_s^2}{R} \text{ and } P_n = \frac{V_n^2}{R}$$

where V_s = Signal voltage and V_n = Noise voltage.

$$\therefore \quad \frac{S}{N} = \frac{V_s^2/R}{V_n^2/R} = \left(\frac{V_s}{V_n}\right)^2 \quad ...(3)$$

The signal to noise ratio in dB is given by,

$$(S/N)_{dB} = 10 \log_{10}\left[\frac{V_s}{V_n}\right]^2 = 20 \log_{10}\left[\frac{V_s}{V_n}\right] \quad ...(4)$$

All the possible efforts are made to keep the signal to noise ratio as high as possible, under all the operating conditions.

Although the signal to noise ratio is a fundamental characteristics of a communication system it is often difficult to measure. In practice instead of measuring S/N, another ratio called (S + N)/N is measured.

Q. 14 What is meant by signal to noise ratio ? Discuss the importance of SNR in radio receivers.

Dec. 04. May 06. May 15

Ans. : The given amplifier is shown in Fig. 3.8. Let us obtain the equivalent circuit for it.

The equivalent circuit is obtained as follows :

Remove the amplifier and calculate the open circuit voltage between points A and B. This is the Thevenin's voltage V_{oc} or V_{TH} and it is denoted by V_s. (See Fig. 3.9(a))

$$\therefore \quad V_s = V_{TH} = \frac{R_t}{R_t + R_s} \times E_s \quad ...(5)$$

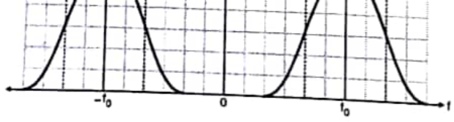


Fig. 3.7 : Noise bandwidth of a filter

The equivalent circuit is obtained as follows :

Remove the amplifier and calculate the open circuit volt between points A and B. This is the Thevenin's voltage V_{oc} or V and it is denoted by V_s. (See Fig. 3.9(a))

$$\therefore\ V_s = V_{TH} = \frac{R_i}{R_i + R_s} \times E_s \quad ...$$

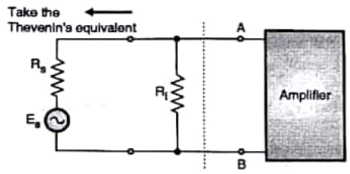
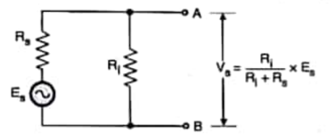
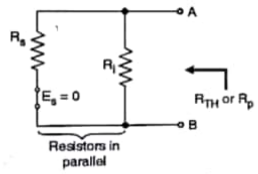


Fig. 3.8 : Given amplifier

Then calculate Thevenin's equivalent resistance R_{TH} or R_p, between A and B with E_s reduced to zero as shown in Fig. 3.9 (b).



(a) Calculation of Thevenin's voltage

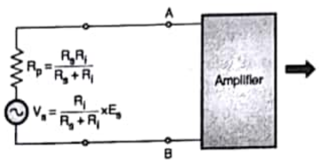


(b) Calculation of Thevenin's resistance

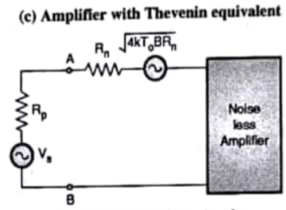
Fig. 3.9

From Fig. 3.9(b), $R_{TH} = R_p = R_s \parallel R_i = \frac{R_s R_i}{(R_s + R_i)}$ ...(6)

Hence the Thevenin's equivalent circuit of the amplifier is as shown in Fig. 3.9(c).



(c) Amplifier with Thevenin equivalent



(d) Final equivalent circuit

Fig. 3.9

Now complete the equivalent circuit as shown in Fig. 3.9 (d) by taking out the noise equivalent resistance R_n and the associated noise voltage, outside the amplifier.

Signal to noise ratio

The signal to noise ratio at the input is,

$$SNR_i = \frac{S_i}{N_i}$$

where S_i = Input signal power = V_s^2

and N_i = Input noise power = $V_n^2 = 4\,k\,T_0\,B\,(R_p + R_n)$

$$\therefore\ SNR_i = \frac{V_s^2}{4\,k\,T_0\,B\,(R_p + R_n)} \quad ...(7)$$

Since the amplifier is noiseless, the SNR at its input and output will be the same.

Then the signal to noise ratio at the amplifier output is,

$$\frac{S}{N} = \frac{V_s^2}{4\,(R_p + R_n)\,k\,T_0\,B} \quad ...(8)$$

Here $(R_p + R_n)$ represents the equivalent resistance producing noise at the input of the amplifier

Q. 15 Define the following term : Noise factor.

May 09, Dec. 10, May 12, Dec. 14, May 15

Ans. :

The noise factor (F) of an amplifier or any network is defined in terms of the signal to noise ratio at the input and the output of the system. It is defined as :

$$F = \frac{\text{S/N ratio at the input}}{\text{S/N ratio at the output}} \quad ...(1)$$

$$= \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}} \quad ...(2)$$

where P_{si} and P_{ni} = Signal and noise power at the input

and P_{so} and P_{no} = Signal and noise power at the output.

The temperature to calculate the noise power is assumed to be the room temperature. The S/N at the input will always be greater than that at the output. This is due to the noise added by the amplifier. Therefore the noise factor is the means to measure the amount of noise added and it will always be greater than one. The ideal value of the noise factor is unity.

The noise factor F is sometimes frequency dependent. Then its value determined at one frequency is known as the spot noise factor and the frequency must be stated along with the spot noise factor.

Q. 16 Define the following term : Noise figure.

Dec. 03, May 09, May 10, Dec. 10, Dec. 14, May 16

Ans. :

Sometimes the noise factor is expressed in decibels. When noise factor is expressed in decibels it is called noise figure.

$$\text{Noise figure} = F_{dB} = 10 \log_{10} F \quad ...(1)$$

Substituting the expression for the noise factor we get,

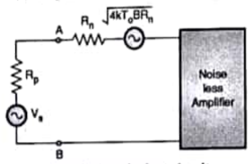
$$\text{Noise figure} = 10 \log_{10} \left[\frac{\text{S/N at the input}}{\text{S/N at the output}}\right]$$

$$\therefore\ F_{dB} = 10 \log_{10} (S/N)_i - 10 \log_{10} (S/N)_o$$

$$\therefore\ \text{Noise figure } F_{dB} = (S/N)_i\,dB - (S/N)_o\,dB \quad ...(2)$$

The ideal value of noise figure is 0 dB.

(c) Amplifier with Thevenin equivalent



(d) Final equivalent circuit

Fig. 3.9

Dec. 03, May 09, May 10, Dec. 10, Dec. 14, May 16

Ans. :

Sometimes the noise factor is expressed in decibels. When noise factor is expressed in decibels it is called noise figure.

$$\text{Noise figure} = F_{dB} = 10 \log_{10} F \quad ...(1)$$

Substituting the expression for the noise factor we get,

$$\text{Noise figure} = 10 \log_{10} \left[\frac{\text{S/N at the input}}{\text{S/N at the output}}\right]$$

$$\therefore F_{dB} = 10 \log_{10} (S/N)_i - 10 \log_{10} (S/N)_o$$

$$\therefore \text{Noise figure } F_{dB} = (S/N)_i \text{ dB} - (S/N)_o \text{ dB} \quad ...(2)$$

The ideal value of noise figure is 0 dB.

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Measure to improve the noise figure

To improve the noise figure of a receiver, the devices used for the amplifiers and mixer stages must produce low noise.

The diodes and FETs are therefore preferred. The receiver can be operated at low temperatures. This is done in satellite low noise receivers. Use high gain amplifiers. All this will collectively improve the noise figure of the receiver.

Q. 17 Define equivalent noise temperature.

May 03, Dec. 03, Dec. 04, May 08, May 09, May 12, Dec. 12, Dec. 13, Dec. 15, May 16

Ans. : The noise factor or noise figure can not always be used for measuring noise. Another way to represent the noise is by means of the equivalent noise temperature.

The equivalent noise temperature is used in dealing with the UHF and microwave low noise antennas, receivers or devices.

Definition : The equivalent noise temperature of a system is defined as the temperature at which the noisy resistor has to be maintained so that by connecting this resistor to the input of a noiseless version of the system, it will produce the same amount of noise power at the system output as that produced by the actual system.

Q. 18 Write short notes on : Friiss formula.

May 10, Dec. 10, May 12, Dec. 13, May 14, May 15, Dec. 15, Dec. 16

Ans. : In practice the filters or amplifiers are not used in isolated manner. They are used in the cascaded manner.

So in this section let us see the effect of cascading on the noise factor and noise temperature. The overall noise factor of such cascade connection can be determined as follows : Fig. 3.10 shows two amplifiers connected in cascade.

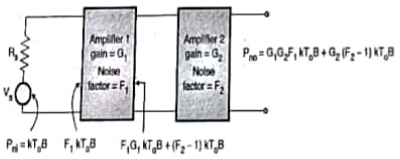

Fig. 3.10 : Two amplifiers connected in cascade

Let the power gains of the two amplifiers be G_1 and G_2 respectively and let their noise factors be F_1 and F_2 respectively.

The total noise power at the input of the first amplifier is given as :

$$P_{ni\,(total)} = F_1 k T_o B \quad ...(1)$$

The total noise power at the output of amplifier 1 will be the addition of two terms.

$$\therefore \text{Noise input to amplifier 2} = G_1 F_1 k T_o B + (F_2 - 1) kT_o B \quad ...(2)$$

The first term represents the amplified noise power (by G_1) and the second term represents the noise contributed by the second amplifier. The noise power at the output of the second amplifier is G_2 times the input noise power to amplifier 2.

$$\therefore P_{no} = G_2 \times (\text{Noise input to amplifier 2})$$

$$\therefore P_{no} = G_1 G_2 F_1 k T_o B + G_2 (F_2 - 1) k T_o B \quad ...(3)$$

The overall gain of the cascade connection is given by,

$$G = G_1 G_2 \quad ...(4)$$

The overall noise factor F is defined as follows :

$$F = \frac{P_{no}}{G_1 G_2 P_{si}} \quad ...(5)$$

Here P_{si} = Noise power supplied by the input source $= k T_o B$

Substituting the values of P_{no} and P_{si} we get,

$$F = \frac{G_1 G_2 F_1 k T_o B + G_2 (F_2 - 1) k T_o B}{G_1 G_2 k T_o B}$$

$$\therefore F = F_1 + \frac{(F_2 - 1)}{G_1} \quad ...(6)$$

The same logic can be extended for more number of amplifiers connected in cascade. Then the expression for overall noise factor F would be,

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \frac{(F_4 - 1)}{G_1 G_2 G_3} + \cdots \quad ...(7)$$

This formula is known as the **Friiss formula.**

Conclusion

This equation indicates that in the cascade configuration, the first stage is the most important stage in deciding the overall noise factor.

This is because due to presence of terms G_1, $G_1 G_2$ ...... in the denominators of second, third terms respectively ......, they become negligibly small as compared to the first term.

$$\therefore F \approx F_1 \quad ...(8)$$

Q.19 An amplifier with 10 dB noise figure and 4 dB power gain is cascaded with a second amplifier which has a 10 dB power gain. What is the overall noise figure and power gain ? May 16

Ans. : The given system is as shown in Fig. 3.11.

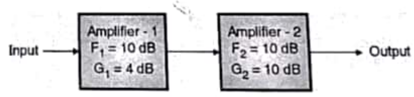

Fig. 3.11 : Given system

Step 1 : Convert dB into equivalent power ratios

$F_1 = 10$ dB $\quad \therefore F_1 = 10$

$F_2 = 10$ dB $\quad \therefore F_2 = 10$

$G_1 = 4$ dB $\quad \therefore 4 \text{ dB} = 10 \log G_1$

$\therefore G_1 = 2.5$

Step 2 : Calculate the overall noise factor

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

$$F = 10 + \frac{(10 - 1)}{2.5} = 13.6$$

Step 3 : Overall noise figure and power gain

Noise figure = 10 log (13.6) = 11.33 dB ...Ans.

Overall power gain = log (G_1 G_2) = log (2.5 × 10)

= 1.39 dB ...Ans.

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Q. Derive Equiv noise temp.

Q) Noise of 6dB. Calculate its equiv noise temp.

Let $T_0 = 27^\circ C = 300 K$

$\therefore T_{eq} = (F-1) T_0$

$= 1500^\circ K$

Q. → Explain term thermal noise.
→ Prove that the noise voltage
$V_n = \sqrt{4KTBR}$ For electronic device
operating at a temp of 17°C with a
bandwidth of 10KHz. determine
→ Thermal noise power in dBm.
→ RMS noise voltage for a 100Ω
internal resistance & a 100Ω load res.

Q.20 A three stage amplifier has the following power gains and noise figure (as ratios, not in dB) for each stage :

Calculate the power gain, noise figure and the noise temperature for the entire amplifier, assuming matched conditions.

May 04, Dec. 07, Dec. 11

Stage	Power gain	Noise figure
1	10	2
2	20	4
3	30	5

Ans. :

Given : $G_1 = 10, G_2 = 20, G_3 = 30,$

$F_1 = 2, F_2 = 4, F_3 = 5.$

1. Draw the block diagram of the system

The block diagram of the cascaded amplifiers is as shown in Fig. 3.12.

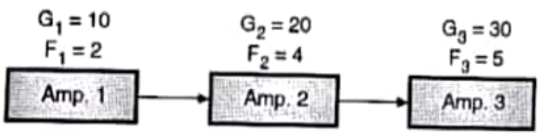

Fig. 3.12 : Cascaded amplifier system

2. Overall power gain

$$G = G_1 \times G_2 \times G_3$$

$$= 10 \times 20 \times 30 = 6000 \qquad \text{...Ans.}$$

3. Overall noise figure

The overall noise factor is given by,

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

$$\therefore F = 2 + \frac{(4-1)}{10} + \frac{(5-1)}{10 \times 20}$$

$$\therefore F = 2.32 \qquad \text{...(1)}$$

$\therefore$ Overall noise figure F dB = $10 \log_{10} (2.32)$

$$= 3.654 \text{ dB} \qquad \text{...Ans.}$$

4. Overall noise temperature

$$T_{eq} = (F-1) T_o$$

Let T_o = Room temperature = 300° K

$$\therefore T_{eq} = (2.32 - 1) \times 300$$

$$\therefore T_{eq} = 396°K \qquad \text{...Ans.}$$

Q. 21 Explain the term thermal noise. Prove that noise voltage $V_N = \sqrt{4 \text{ kTBR}}$. For electronic device operating at a temperature of 17°C with a bandwidth of 10 kHz, **Dec 07, May 11**

Determine :

1. **Thermal noise power in dBm.**
2. **rms noise voltage for a 100 Ω internal resistance and a 100 Ω load resistance.**

Ans.: For thermal noise, refer Q.7. For the derivation of noise voltage refer Q.7.

Solution of example :

Given : T = 17° C = 290° K, B = 10 kHz

1. Thermal noise power :

$$P = kTB$$

$$= 1.28 \times 10^{-23} \times 290 \times 10 \times 10^3$$

$$= 4.002 \times 10^{-17} \text{ W} \qquad \text{...Ans.}$$

$$P_{(dBm)} = 10 \log_{10} \left(\frac{4.002 \times 10^{-17}}{1 \times 10^{-3}} \right)$$

$$= -133.98 \text{ dBm} \qquad \text{...Ans.}$$

2. Rms noise voltage

Assume that the internal resistance of the device is in series with the load resistance.

$$\therefore R = 100 + 100$$

$$= 200\ \Omega$$

$$\therefore V_n = \sqrt{kTBR}$$

$$= \sqrt{4.002 \times 10^{-17} \times 200}$$

$$= 8.95 \times 10^{-8} \text{ V} \qquad \text{...Ans.}$$

Q. 22 For three cascaded amplifier stages, each with noise figure of 3dB and power gain of 10 dB, determine the total noise figure. **May 07**

Ans. :

$$F_1 = F_2 = F_3 = 3 \text{ dB},$$

$$G_1 = G_2 = G_3 = 10 \text{ dB}$$

Step 1 : Convert dB into equivalent power ratios

$F_1 = F_2 = F_3 = 3$ dB

$$\therefore 3 = 10 \log F_1$$

$$\therefore F_1 = 1.995 = F_2 = F_3$$

$G_1 = G_2 = G_3 = 10$ dB

$$\therefore 10 = 10 \log G_1$$

$$\therefore G_1 = 10$$

Step 2 : Calculate the overall noise factor

Using the Friiss formula,

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

$$= 1.995 + \frac{(1.995 - 1)}{10} + \frac{(1.995 - 1)}{100}$$

$$= 1.995 + 0.0995 + 9.95 \times 10^{-3}$$

$$F = 2.0594$$

Step 3 : Calculate the overall noise figure

$$F_{dB} = 10 \log_{10} (2.0594)$$

$$\text{Noise figure} = 3.1374 \text{ dB} \qquad \text{...Ans.}$$