MATHEMATICS FOR COMPUTER APPLICATIONS

ASSIGNMENT

01. A random variable X has the following probability function

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) find the value of 'a'
(ii) $P(2 \leq X \leq 5)$
(iii) Determine the distribution function of x.

Solu^n Given, probability function are countable
$\therefore$ It is DRV.

wkt, property of DRV
$\Rightarrow \Sigma P(x) = 1$

(i) $\Sigma P(x) = 1$

$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$

$81a = 1$

$a = 1/81$

given probability fun. becomes, sub $a = 1/81$

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a
P(x)	1/81	3/81	5/81	7/81	9/81	11/81	13/81	15/81	17/81

(ii) $P(2 \le x \le 5) = P(2) + P(3) + P(4) + P(5)$

$$= \frac{5}{81} + \frac{7}{81} + \frac{9}{81} + \frac{11}{81}$$

$$= \frac{32}{81}$$

(iii) distribution function of x

x	0	1	2	3	4	5	6	7	8
P(x)	1/81	3/81	5/81	7/81	9/81	11/81	13/81	15/81	17/81
F(x)	1/81	4/81	9/81	16/81	25/81	36/81	49/81	64/81	81/81

$F(x) = 0$ for $x < 0$

$F(0) = P(x=0) = a = 1/81 \quad (0 \le x < 1)$

$F(1) = P(x \le 1) = a + 3a = 4a = 4/81 \quad (1 \le x < 2)$

$F(2) = P(x \le 2) = 4a + 5a = 9a = 9/81 \quad (2 \le x < 3)$

$F(3) = P(x \le 3) = 9a + 7a = 16a = 16/81 \quad (3 \le x < 4)$

$F(4) = P(x \le 4) = 16a + 9a = 25a = 25/81 \quad (4 \le x < 5)$

$F(5) = P(x \le 5) = 25a + 11a = 36a = 36/81 \quad (5 \le x < 6)$

$F(6) = P(x \le 6) = 36a + 13a = 49a = 49/81 \quad (6 \le x < 7)$

$F(7) = P(x \le 7) = 49a + 15a = 64a = 64/81 \quad (7 \le x < 8)$

$F(8) = P(x \le 8) = 64a + 17 = 81a = 81/81 \quad (x \ge 8)$

2. Given the following bivariate probability distribution.

$X \backslash Y$	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Obtain

(I) $P(X \le 1, Y = 2)$

(II) $P(Y = 3)$

(III) $P(X < 3, Y < 4)$

(IV) Marginal distribution of X and Y.

Solu: Given, joint probability distribution of X & Y.

$\Rightarrow$ pdf is $\Sigma\Sigma P(x, y) = 1$

taking LCM of denominators

$\Rightarrow$ LCM = 64 LCM(32, 16, 8, 64) = 64

$\therefore$ given fun becomes

$1/32 \times 2/2 + 1/64 = 2/64$

$X \backslash Y$	1	2	3	4	5	6	total
0	0	0	2/64	4/64	4/64	6/64	16/64
1	6/64	4/64	8/64	8/64	8/64	8/64	40/64
2	2/64	2/64	1/64	1/64	0	2/64	8/64
total	6/64	6/64	11/64	13/64	12/64	16/64	1

(i) $P(X \leq 1, Y=2)$

find the probability of $X \leq 1$ & $Y=2$ where $X=0$ or $X=1$ & $Y=2$ ∴ Instead take direct table also

$P(X=0, Y=2) + P(X=1, Y=2)$

from the table

$P(X=0, Y=2) = 0$

$P(X=1, Y=2) = 4/64$

Thus $= 0 + \frac{4}{64} = \frac{4}{64} = \frac{1}{16}$ //

(ii) $P(Y=3)$

$P(Y=3) = P(X=0, Y=3) + P(X=1, Y=3) + P(X=2, Y=3)$

from the table

$P(X=0, Y=3) = 1/32$

$P(X=1, Y=3) = 1/8$

$P(X=2, Y=3) = 1/64$

(lcm table)

$1/32 = 2/64$, $1/8 = 8/64$, $1/64 = 1/64$

Adding them up,

$$P(Y=3) = \frac{2}{64} + \frac{8}{64} + \frac{1}{64} = \frac{11}{64}.$$

(iii) $P(X < 3, Y < 4)$

Since $X < 3$ includes all values in the table,
$Y < 4$ means $Y = 1, 2, 3$

$P(X=0, Y=1) + P(X=0, Y=2) + P(X=0, Y=3) + P(X=1, Y=1) + P(X=1, Y=2) + P(X=1, Y=3) + P(X=2, Y=1) + P(X=2, Y=2) + P(X=2, Y=3)$

from the table.

$0 + 0 + 1/32 + 1/16 + 1/16 + 1/8 + 1/32 + 1/32 + 1/32 + 1/64$

converting to denominator of 64

$0 + 0 + 2/64 + 4/64 + 4/64 + 8/64 + 2/64 + 2/64 + 2/64 + 1/64$

$$= \frac{2+4+4+8+2+2+1}{64} = \frac{23}{64}$$

(iv) Marginal distribution of X

X	0	1	2
$P(x)$	16/64	40/64	8/64

Marginal distribution of Y

Y	1	2	3	4	5	6
$P(x)$	6/64	6/64	11/64	13/64	12/64	16/64

8. Verify the function $f(x) = \begin{cases} 6x(1-x), & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

is PDF or not. Hence determine its mean.

Soluⁿ Given $f(x)$ is CRV & lies b/w 0 & 1

WKT, pdf is

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{1} 6x(1-x)\,dx = 1$$

$$6\int_{0}^{1} (x - x^2)\,dx = 1$$

$$6\left\{\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 d\right\} = 1$$

$$6\left\{\left[\frac{1}{2} - \frac{0}{2}\right] - \left[\frac{1}{3} - \frac{0}{3}\right]\right\} = 1$$

$$6\left\{\frac{1}{2} - \frac{1}{3}\right\} = 1$$

$$\cancel{6}\left(\frac{1}{\cancel{6}}\right) = 1$$

$$\boxed{1 = 1}$$

$\therefore f(x)$ is pdf

to determine mean

WKT, mean, $\bar{x} = \int_{-\infty}^{\infty} x f(x) dx$

$$\bar{x} = \int_0^1 x\,[6x(1-x)]\,dx.$$

$$= 6\int_0^1 (x^2 - x^3)\,dx$$

$$= 6\left\{\left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1\right\}$$

$$= 6\left\{\left[\frac{1}{3} - 0\right] - \left[\frac{1}{4} - 0\right]\right\}$$

$$= 6\left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= 6\left(\frac{1}{12}\right)$$

$$= \frac{1}{2}$$

$$\boxed{\bar{x} = 0.5}$$ //

Q. Given the following joint probability distribution

Y\X	0	1	2
0	0	1/27	2/27
1	2/27	3/27	4/27
2	4/27	5/27	6/27

obtain

(I) Marginal distribution of X and Y

(II) the conditional distribution of X given Y = 1.

Ans Given, joint probability distribution

Y\X	0	1	2	total
0	0	1/27	2/27	3/27
1	2/27	3/27	4/27	9/27
2	4/27	5/27	6/27	15/27
total	6/27	9/27	12/27	1

(1) marginal distribution of X

X	0	1	2
P(X)	6/27	9/27	12/27

marginal distribution of Y

Y	0	1	2
P(Y)	3/27	9/27	15/27

(ii) cond^{nal} distribution of x given $Y=1$

$$P(X, Y=1) = \frac{P(X \cap Y=1)}{P(Y=1)}$$

$x=0$,

$$P(X=0, Y=1) = \frac{P(X=0 \cap Y=1)}{P(Y=1)} = \frac{2/27}{9/27} = \frac{2}{9}$$

$x=1$

$$P(X=1, Y=1) = \frac{P(X=1 \cap Y=1)}{P(Y=1)} = \frac{3/27}{9/27} = \frac{1}{3}$$

$x=2$

$$P(X=2, Y=1) = \frac{P(X=2 \cap Y=1)}{P(Y=1)} = \frac{4/27}{9/27} = \frac{4}{9}$$

5. The number of accidents occuring in a city in a day is a poisson variate with mean 0.8. Find the probability that on a randomly selected day i) there are no accidents

(ii) there are accidents.

Sol: Given, mean = $\lambda = 0.8$

WKT, $P[X=x] = \frac{e^{-\lambda}\lambda}{x!}$ where $x = 0, 1, 2 \ldots$

(i) sub $\lambda = 0.8$ & $x = 0$

$$P[X=0] = \frac{e^{-0.8}(0.8)^0}{0!}$$

$$= \frac{e^{-0.8}.1}{1}$$

$$= e^{-0.8}$$

$$= 0.4493$$

(ii) there are accidents, sub $X \geq 1$ & $\lambda = 0.8$

WKT, $P[X \geq 1] = 1 - P[X=0]$

$$= 1 - \frac{e^{-0.8}(0.8)^0}{0!}$$

$$= 1 - 0.4493$$

$$= 0.5507$$

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6. The length of a telephone conversation has been found to have an exponential distribution with mean 3 minutes. what is the probability that a call may last more than 1 minute?

Sol? Given, mean $= \alpha = 3$

WKT, $f(x) = \frac{1}{\alpha} e^{-x/\alpha} = f(x) = \frac{1}{3} e^{-x/3}$, for $x > 0 \rightarrow$ ①

call may last more than 1 min, i,e $\rightarrow P(x \geq 1)$

$$P(x \geq 1) = 1 - P(x \leq 1)$$

$$= 1 - P(0 < x < 1)$$

$$= 1 - \int_0^1 f(x)dx$$

$$= 1 - \int_0^1 \frac{1}{3} e^{-x/3} dx$$

$$= 1 - \frac{1}{3} \left[\frac{e^{-x/3}}{-(1/3)} \right]_0^1$$

$$= 1 + (e^{-1/3} - e^0)$$

$$= 1 + e^{-1/3} - 1$$

$$= e^{-1/3}$$

$$= 0.7165_{//}$$

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7. A random variable X has the following probability function

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) find the value of k
(ii) $P(1<X<6)$
(iii) Determine the distribution function of x.

u? Given function is DRV

(i) WKT $\Sigma P(x) = 1$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$9k + 10k^2 = 1$$

$$10k^2 + 9k - 1 = 0$$

by factorizing $10k^2 + 10k - k - 1 = 0$

$$10k(k+1) - 1(k+1) = 0$$

$$(10k-1)(k+1) = 0$$

$$k = 1/10,\ k = 1$$

$\boxed{k = 1/10}$ ($\because$ prob. lies b/w 0 & 1)

Given probability fun becomes, sub $k = 1/10$

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$
P(x)	0	1/10	2/10	2/10	3/10	1/100	2/100	17/100

ii) $P(1 < X < 6) = P(2) + P(3) + P(4) + P(5)$

$$= \frac{2}{10} + \frac{2}{10} + \frac{3}{10} + \frac{1}{100}$$

$$= \frac{7}{10} + \frac{1}{100}$$

$$= \frac{71}{100}$$

iii) distribution fun of x

x	0	1	2	3	4	5	6	7
$P(x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$2/100$	$17/100$
$F(x)$	0	$1/10$	$3/10$	$5/10$	$8/10$	$81/100$	$83/100$	$100/100$

8 The number of person joining a cinema queue in a minute has poisson distribution with parameter 5.8. Find the probability that

i) no one joins the queue

ii) At least one person's join the queue

solu^n Given, parameters is 5.8, i,e, $\lambda = 5.8$

WKT $P[X = x] = \frac{e^{-\lambda} \lambda^{x}}{x!} \rightarrow (1)$ where $x = 0, 1, 2 \ldots$

(i) no one join the queue i,e $X = 0$ & $\lambda = 5.8$

$$P[X = 0] = \frac{e^{-5.8} (5.8)^{\cancel{0}\,1}}{\cancel{0!}\,1}$$

$$= 0.00302$$

(ii) atleast 1 joins, i,e $X \geq 1$ & λ 5.8

where, $P[X \geq 1] = 1 - P[X = 0]$

$$= 1 - \frac{e^{-5.8} (5.8)^{0}}{\cancel{0!}\,1}$$

$$= 1 - e^{-5.8}$$

$$= 1 - 0.00302$$

$$= 0.99698$$

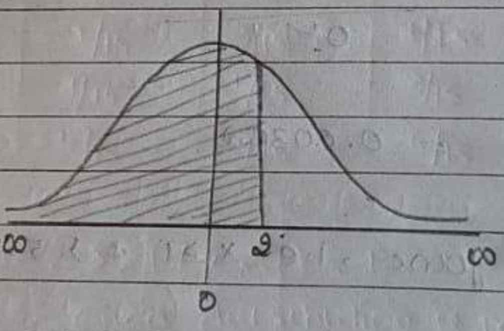
9. Let X is normal variate with mean 42 and standard deviation 4. Find the probability that a value taken by X is
(i) less than 50
(ii) between 43 and 46.

solu^n Given, mean $= \mu = 42$, $SD = \sigma = 4$

WKT, $z = \frac{x-\mu}{\sigma} = z = \frac{x-42}{4} \rightarrow$ ①.

(i) $P(x < 50) = P\left(z < \frac{50-42}{4}\right)$

$= P(z < 2)$



$=$ area $(-\infty$ to $2)$

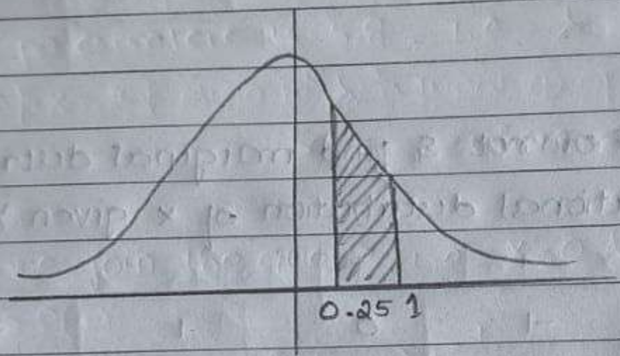
$=$ area $(-\infty$ to $0) +$ area $(0$ to $2)$

$= 0.5 + 0.4772$

$= 0.9772$

(ii) $P(43 < x < 46) = P\left[\frac{43-42}{4} < z < \frac{46-42}{4}\right]$

$= P[0.25 < Z < 1]$



$= \text{area}(0.25 \text{ to } 1)$

$= \text{area}(0 \text{ to } 1) - \text{area}(0 \text{ to } 0.25)$

$= 0.3413 - 0.0987$

$= 0.2426.$

10. Given the following bivariate probability distribution

$Y \backslash X$	-1	0	1
0	$3/15$	$1/15$	$1/15$
1	$1/15$	$3/15$	$1/15$
2	$1/15$	$1/15$	$3/15$

obtain

(i) verify PDF or not & find marginal distribution of x and y

(ii) the conditional distribution of x given $Y=2$.

solu^n

(i)

$Y \backslash X$	-1	0	1	total
0	$3/15$	$1/15$	$1/15$	$5/15$
1	$1/15$	$3/15$	$1/15$	$5/15$
2	$1/15$	$1/15$	$3/15$	$5/15$
total	$5/15$	$5/15$	$5/15$	$15/15=1$ $15/15=1$

here, $\sum\sum P(x,y) = 1$

$\therefore$ Given distribution is pdf

marginal distribution of X

X	-1	0	1
$P(x)$	$5/15$	$5/15$	$5/15$

marginal distribution of Y

Y	0	1	2
$P(Y)$	$5/15$	$5/15$	$5/15$

(ii) cond^{n} dist of x given $Y = 2$

$x = -1$

$$P(X/Y=2) = \frac{P(X \cap Y = 2)}{P(Y = 2)}$$

sub $X = -1$ =

$$= \frac{P(X = -1 \cap Y = 2)}{P(Y = 2)}$$

$$P(X = -1, Y = 2) = \frac{1/15}{5/15}$$

$$= \frac{1}{5}$$

$x = 0$

$$P(x = 0, Y = 2) = \frac{P(X = 0 \cap Y = 2)}{P(Y = 2)}$$

$$= \frac{\frac{1}{15}}{5/15} = 1/15$$

$X = 1$

$$P(X = 1, Y = 2) = \frac{P(X = 1 \cap Y = 2)}{P(Y = 2)}$$

$$= \frac{3/15}{5/15} = \frac{3}{5}$$

11. Let X is normal variate with mean 45 and standard deviation 4. Find the probability that a value taken by X is between 42 and 48

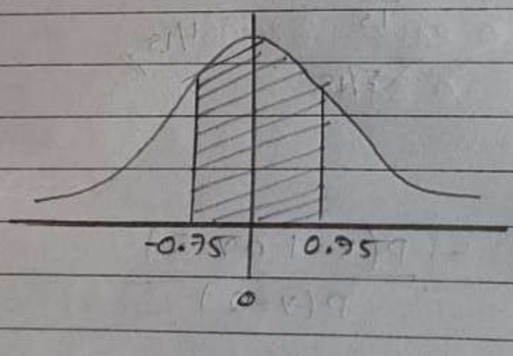
Solu: Given, mean $\mu = 45$ & $SD = \sigma = 4$

WKT, $z = \frac{x-\mu}{\sigma} = z = \frac{x-45}{4} \rightarrow ①$

x is between 42 & 48

$$P[42 < x < 48] = P\left[\frac{42-45}{4} < z < \frac{48-45}{4}\right]$$

$$= P[-0.75 < z < 0.75]$$



$= \text{area}(-0.75 \text{ to } 0.75)$

$= \text{area}(-0.75 \text{ to } 0) + \text{area}(0 \text{ to } 0.75)$

$= \text{area}(0 \text{ to } 0.75) + \text{area}(0 \text{ to } 0.75)$

$= 0.2734 + 0.2732$

$= 0.5468$

18. The life time of a certain kind of battery is a random variable which has exponential distribution with mean of 250 hours, find the probability that such a battery will last anywhere between 300 & 500 hours.

Solu:

Given, mean $= \alpha = 250$

WKT $f(x) = \frac{1}{\alpha} e^{-x/\alpha} \rightarrow f(x) = \frac{1}{250} e^{-x/250}$, for $x>0 \rightarrow$ (1).

last anywhere between 300 & 500

$$P[300 < X < 500) = \int_{300}^{500} \frac{1}{250} e^{-x/250}$$

$$= \frac{1}{250} \left[\frac{e^{-x/250}}{(-1/250)} \right]_{300}^{500}$$

$$= - \left[e^{-x/250} \right]_{300}^{500}$$

$$= e^{-500/250} - e^{-300/250}$$

$$= e^{-1.2} - e^{-2}$$

$$= 0.3012 - 0.1353$$

$$= 0.1659.$$