

MATHEMATICS ASSIGNMENT-4

1. 500 articles from a factory are examined and found to be 2% defective. 800 similar articles from a second factory are found to have only 1.5% defectives. Can it be reasonable concluded that the products of the first factory are inferior to those of the second. Use 5% level of significance.

Sol:- Given $n_1 = 500$ $n_2 = 800$
 $P_1 = 2\%$ $P_2 = 1.5\%$
 $= 0.02$ $= 0.015$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{(500)(0.02) + (800)(0.015)}{500 + 800} = 0.0169$$

$$Q = 1 - P = 1 - 0.0169 = 0.9831$$

Step 1:- H_0 First factory is inferior to second factory.

Step 2:- H_1 first factory is not inferior to second factory.

Step 3:-
$$\frac{|P_1 - P_2|}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{0.02 - 0.015}{\sqrt{(0.0169)(0.9831) \left(\frac{1}{500} + \frac{1}{800} \right)}} = 0.6084$$

Step 4 :- 5% $K = 1.98$

Step 5:- $|Z| = 0.6084 < K = 1.98$

$\therefore H_0$ is accepted

first factory is not inferior to second factory.

2. It is known that an IQ of boys has SD 10 & that an IQ of girls has SD 12. Mean IQ of 200 randomly selected boys is 99 & mean IQ of 300 randomly selected girls is 97. Can it be concluded that on an average boys and girls have the same IQ? Use 1% of significance.

Sol: Given $\sigma_1 = 10$ $\bar{x}_1 = 99$ $n_1 = 200$
 $\sigma_2 = 12$ $\bar{x}_2 = 97$ $n_2 = 300$

Step 1: H_0 boys & girls have same IQ

Step 2: H_1 boys & girls does not have same IQ

Step 3: $|Z| = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
 $= \frac{|99 - 97|}{\sqrt{\frac{10^2}{200} + \frac{12^2}{300}}}$
 $= 2.0$

Step 4: 1% $K = 2.58$

Step 5: $|Z| = 2.0 < K = 2.58$

$\therefore H_0$ is accepted.

Conclusion: Boys and girls have same IQ.

3. It is required test the hypothesis that on average height of Punjabi's is 180 cm tall for this random sample containing 50 Punjabi's are considered. The mean and standard deviation of heights of those are found to be 178.9 cms & 3.3 cm. Based on this data that would you conclude? (Use 5% level of significance).

Sol:- Given $n=50$, $\bar{x}=178.9$, $\sigma=3.3$, $\mu=180$

Step 1: H_0 on average height of Punjabi's is 180cm

Step 2: H_1 on average height of Punjabi's is not 180cm

$$\text{Step 3: } |z| = \frac{|\bar{x} - \mu|}{\frac{\sigma}{\sqrt{n}}} = \frac{178.9 - 180}{\frac{3.3}{\sqrt{50}}} = +2.3$$

Step 4: 5% of significance $k=1.98$

Step 5: $|z| = 2.3 > k = 1.98$

$\therefore H_1$ is accepted.

Conclusion: On average height of Punjabi's is not 180cm.

4. A machine is designed so as to fill bottles that with 200ml of a medicine. A sample of 100 bottles when measured had a mean content of 201.5ml. If the standard deviation of the filling is known to be 5ml, test whether the machine is functioning properly. Use 1% level of significance.

Sol:- Given $\mu=200$, $n=100$, $\bar{x}=201.5$, $\sigma=5$

Step 1: $H_0 \Rightarrow$ The machine is functioning properly.

Step 2: $H_1 \rightarrow$ The machine is not functioning properly.

$$\text{Step 3: } |z| = \frac{|\bar{x} - \mu|}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{|201.5 - 200|}{\frac{5}{\sqrt{100}}} = 3$$

Step 4: 1% of significance $k=2.58$

Step 5: $|Z| = 3$
 $k = 2.58$
 $|Z| > k$
 $3 > 2.58$
 $\therefore H_1$ is accepted

Conclusion: The machine is not functioning properly.

5. Intelligence test on two groups of males and females gave the following results.

Marks	Mean	SD	Sample Size
Females	75	15	150
Males	70	20	250

Is there significance difference in mean marks obtained by the males & females? Test at 1% level of significance.

Sol:- Given $\sigma_1 = 15$ $\bar{x}_1 = 75$ $n_1 = 150$
 $\sigma_2 = 20$ $\bar{x}_2 = 70$ $n_2 = 250$

Step 1: H_0 . There is significant difference in the mean marks obtained by boys and girls.

Step 2: H_1 There is no significant difference in the mean obtained by the boys & girls.

Step 3: $|Z| = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{|75 - 70|}{\sqrt{\frac{15^2}{150} + \frac{20^2}{250}}}$
 $= 2.84$

Step 4: 1% $k = 2.58$

Step 5: $|Z| = 2.84 > 2.58 = k$.

$\therefore H_1$ is accepted

Conclusion: There is no significant difference in the mean marks obtained by the boys & girls.

6. Write a short note of the following

i) Sample: A finite subset of a population is called a sample (sampling).

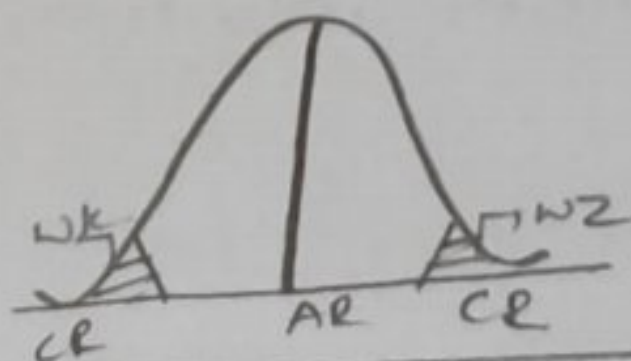
* The number of objects in a sample is called the sample size.

Eg:- Heights & weights of the students based on marks scored in different subjects.

ii) Hypothesis: Hypothesis is a statement about the value of the population parameter. It is made on the basis of the information obtained by experimentation.

Testing of Hypothesis:- In order to arrive at a decision regarding the population through a sample of the population we have to make certain assumption referred to as hypothesis which may be true or false. i.e Null Hypothesis, Alternate Hypothesis.

(iii) two tail test: While testing H_0 if the critical region is considered at both the tails of the sampling distribution of the test statistics, the test is called two-tailed test



(iv) Type I and Type II error

Type I error:

Instead of accepting the null hypothesis if we reject H_0 then we say type I error has occurred.

Type II error:

Instead of rejecting the null hypothesis if we accept H_0 then we say type II error has occurred.

	Accepting the hypothesis	Rejecting the hypothesis
Hypothesis is true	Correct Decision	Wrong decision (Type I error)
Hypothesis is false	Wrong Decision (Type II error)	Correct decision.

7. The following data are got from an investigation

Samples	Mean	SD	Sample Size
Sample 1	47.4	3.1	400
Sample 2	50.3	3.3	900

Find out whether the two mean differ significantly? Test at 1% level of significance?

Sol:- Given $\sigma_1 = 3.1$ $\bar{x}_1 = 47.4$ $n_1 = 400$
 $\sigma_2 = 3.3$ $\bar{x}_2 = 50.3$ $n_2 = 900$

Step 1: H_0 : There is a significant difference in the mean marks obtained by sample 1 & sample 2.

Step 2: H_1 : There is no significant difference in the mean obtained by sample 1 & Sample 2

$$\text{Step 3: } |z| = \frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{|47.4 - 50.3|}{\sqrt{\frac{(3.1)^2}{400} + \frac{(3.3)^2}{900}}} = \underline{2.43}$$

$$= 2.4311$$

$$\text{Step 4: } 1\% K = 2.58$$

$$\text{Step 5: } |z| = 2.43 < K = 2.58$$

$\therefore H_0$ is accepted.

Conclusion: There is a significant difference in the mean obtained by sample 1 & sample 2.

9. From the following data, test whether the difference b/w the proportions in the two samples are significant.

	Size	Proportion
Sample I	1000	0.02
Sample II	1200	0.01

Sol:- Given Sample 1 size: $n_1 = 1000$
 Sample 1 proportion: $p_1 = 0.02$
 Sample 2 size: $n_2 = 1200$
 Sample 2 proportion: $p_2 = 0.01$

Step 1: H_0