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Wednesday

Mathematics

Unit-1
Set-Theory

The theory of set was developed by German mathematics Georg Cantor [1845 - 1918]

Set :- A well defined number of collection of objects.
Eg :- $A = \{1, 2, 3, 4, 5 \ldots\}$ $B = \{a, b, c\}$ $C = \{1, 2, a, b, c\}$

There are 2 different type of methods.
(1) Roaster method :-
Eg :- $A = \{1, 2, 3, \ldots 10\}$
The set of all vowels in english alphabet.
The set of all odd natural number $\{1, 3, 5 \ldots\}$

(2) Set builder method :-
Eg :- $A = \{x / 1 \le x \le 10, x \in N\}$
$B = \{y : y$ is a vowel english alphabet$\}$
$C = \{z / z$ is an odd natural number$\}$

Element :- Every number [object] of the set is called element of the set.
Eg :- $A = \{1, 2, 3\}$: Here $1 \in A$
[1 belongs to A set]
$4 \notin A$ [4 doesn't belongs to A set]

Type of Sets
(1) Null Set :- A set doesnot contain any element called as null set.
Eg :- $\phi = \{\}$

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(2) **Finite set** :- A set which containing a countable number of elements called finite set

Eg :- $A = \{1, 2, 3, \ldots 100\} \Rightarrow n(A) = 100$.
$B = \{a, b, c \ldots z\} \Rightarrow n(B) = z$.

(3) **Infinite set** :- A set having uncountable number of elements called infinite set.

Eg :- $A = \{1, 2, 3, \ldots \infty\}$

(4) **Sub set** :- If every element of A is also an element of B then the set A is said to be subset of B denoted by $\subseteq$

Eg :- $A \subseteq B = \{x / x \in A \rightarrow x \in B, \forall x\}$
$A = \{1, 2\}, B = \{1, 2, 3\}$
Here :- $1 \in A \rightarrow 1 \in B$
$2 \in A \rightarrow 2 \in B$
Every element of A is also element of B.

Eg :- $C = \{3\}, D = \{2, 3\}$
$3 \in C \rightarrow 3 \in D$
$C \subseteq D$

Eg : $C = \{3\}, B = \{1, 2, 3\}$
Here :- $3 \in C \rightarrow 3 \in B$
So :- $C \subseteq B$

(5) **Equal set** : If A & B be the two sets & A is the subset B & also B is subset of A then A & B are equal $A = B$

Eg. $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$

$\left.\begin{array}{l} 1 \in A \rightarrow 1 \in B \\ 2 \in A \rightarrow 2 \in B \\ 3 \in A \rightarrow 3 \in B \end{array}\right\} \therefore A \subseteq B$

$2 \in B \rightarrow 2 \in A$
$3 \in B \rightarrow 3 \in A$
$1 \in B \rightarrow 1 \in A$
$\therefore B \subseteq A$

So : $A = B$

(6) **Proper subset** : If A is a subset of B & $A \neq B$ then A is called proper subset of B

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Eg: $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$
$A \subseteq B$ but $A \neq B \Rightarrow A \subset B$

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(7) Equivalence set :- If A & B be the two set & $A \not\subset B$ & $B \not\subset A$ but both the sets have equal number of elements then the set A & B called equivalent.

Eg: $A = \{1, 2, 3\}$, $B = \{a, b, c\}$
$A \not\subset B$ & $B \not\subset A$ but $n(A) = n(B) = 3$ $[A = B]$

(8) Power set :- Given set A, the power set of A is the set of all subset of set A & element by 2^A or $p(A)$

Eg:- $A = \{1, 2\}$
$2^A = p(A) = \{\{\}, \{1\}, \{2\}, \{1, 2\}\}$

Operations on sets

(1) Union of two sets '$\cup$', 'or', '$\vee$'
i.e. $A \cup B = \{x / x \in A$ or $x \in B, \forall x\}$

Eg: $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$
$A \cup B = \{1, 2, 3\} \cup \{3, 4, 5\}$
$= \{1, 2, 3, 4, 5\}$
$B \cup A = \{1, 2, 3, 4, 5\}$ $\therefore A \cup B = B \cup A$

(2) Intersection of two sets '$\cap$', 'and', '$\wedge$'
i.e. $A \cap B = \{x / x \in A$ and $x \in B, \forall x\}$

Eg: $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$
$= \{3\}$ $\therefore A \cap B = B \cap A$

③ **Difference of two sets:**

$A - B = \{x / x \in A \text{ and } x \notin B, \forall x\}$

Eg. $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

$A - B = \{1, 2\}$

$B - A = \{3, 4, 5\} - \{1, 2, 3\}$

$= \{4, 5\}$ $\therefore A - B \neq B - A$

④ **Symmetric difference of two sets** '$+$', '$\oplus$', 'Δ'

$A + B = A \oplus B = A \Delta B$

i.e. $A + B = \{x / x \in (A - B) \text{ or } x \in (B - A), \forall x\}$

OR

$A + B = (A - B) \cup (B - A)$

Eg: $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

$A - B = \{1, 2\}$

$B - A = \{4, 5\}$

$A + B = (A - B) \cup (B - A)$

$= \{1, 2\} \cup \{4, 5\}$

$= \{1, 2, 4, 5\}$

$B + A = (B - A) \cup (A - B)$

$= \{4, 5\} \cup \{1, 2\}$

$= \{1, 2, 4, 5\}$

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⑤ **Complement of A** ($\bar{A}$, A')

$\bar{A} = A' = \{x / x \in U \text{ and } x \in A, \forall x\}$

$\bar{A} = U - A$

Eg: $U = \{1, 2, 3, 4, 5, 6\}$ $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $C = \{2, 4, 6\}$

then find $\bar{A}$, $\bar{B}$, $\bar{C}$.

$\bar{A} = U - A = \{1, 2, 3, 4, 5, 6\} - \{1, 2, 3\} = \{4, 5, 6\}$

$\bar{B} = U - B = \{1, 2, 3, 4, 5, 6\} - \{3, 4, 5\} = \{1, 2, 6\}$

$\bar{C} = U - C = \{1, 2, 3, 4, 5, 6\} - \{2, 4, 6\} = \{1, 3, 5\}$

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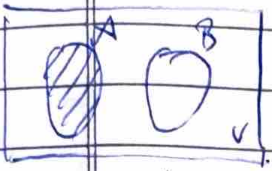
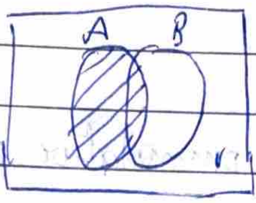
Venn diagram :- Sets can be represented graphically using venn diagram. [English mathematician Venn]


A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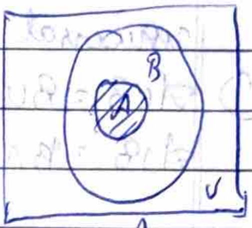
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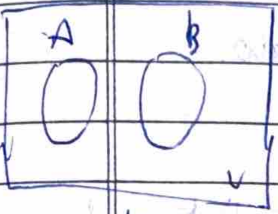
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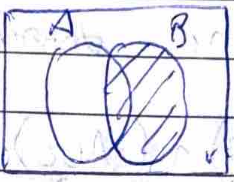
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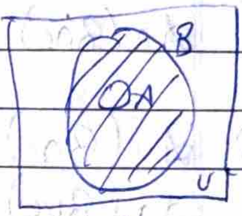
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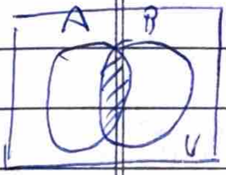
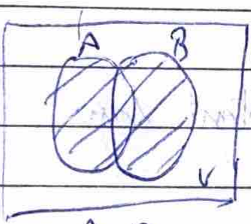
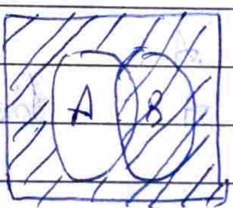
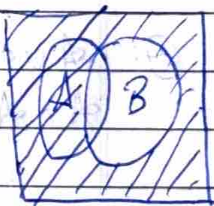
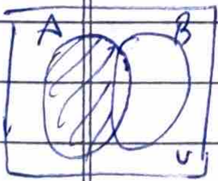
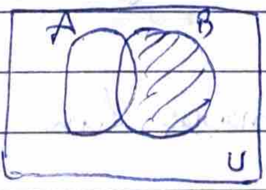

$A \cap B$

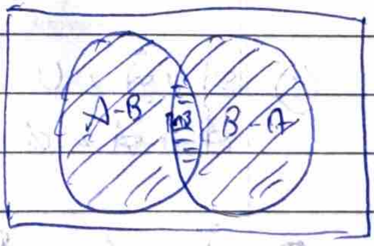

$A \cup B$


A'


B'


$A - B$


$B - A$

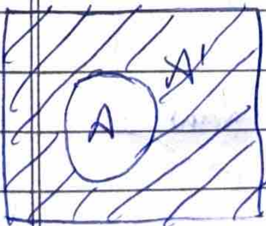


$A = (A-B) \cup (A \cap B)$

$B = (B-A) \cup (A \cap B)$

$A \cup B = (A-B) \cup (A \cap B) \cup (B-A)$

U



$A' = U - A$

$U = A \cup A'$

$A \cap A' = \phi$

$\bar{\cup}$ [union bar mean intersection & get $\cap$]

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Basic Set Identity

For any set A, B & C taken from a universal set U.

① $A \cup B = B \cup A$
$A \cap B = B \cap A$ Commutative Laws

② $A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$ Associative Law.

③ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Distributive Law

④ $A \cup A = A$
$A \cap A = A$ Idempotent Law

⑤ $A \cup \phi = A$
$A \cap U = A$ Identity Law

⑥ $A \cup \bar{A} = U$
$A \cap \bar{A} = \phi$ Inverse Law

⑦ $\bar{\bar{A}} = A$ Double complement law

⑧ $\overline{(A \cup B)} = A' \cap B'$
$\overline{(A \cap B)} = A' \cup B'$ Demorgan's Law

⑨ $A \cup U = U$
$A \cap \phi = \phi$ Domination Law

⑩ $A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$ Absorption Law

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Set Identities Applications

(11) $A-B = A\cap B'$ $\quad C-A = C\cap A'$
$B-A = B\cap A'$

(12) $A+B = (A-B)\cup(B-A)$
$= (A\cup B)-(A\cap B)$

Problems

(1) Simplify $(A-B)\cup(A\cap B)$

Sol:- $= (A-B)\cup(A\cap B)$
$= (A\cap B')\cup(A\cap B)$ $\quad [\because A-B = A\cap B']$
$= A\cap(B'\cup B)$ $\quad [\because B'\cup B = U]$
$= A\cap U$ $\quad [\because A\cap U = A]$
$= A$

(2) Simplify $A\cap(B-A)$

Sol: $A\cap(B-A)$
$= A\cap(B\cap A')$
$= A\cap(A'\cap B)$ $\quad [\because$ Commutative Law$]$
$= (A\cap A')\cap B$ $\quad [\because$ Inverse Law$]$
$= \emptyset\cap B$ $\quad [\because$ Domination Law$]$
$= \emptyset$

(3) Show that $A-(A\cap B) = A-B$

Sol:- $LHS = A-(A\cap B)$
$= A\cap(A\cap B)'$
$= A\cap(A'\cup B')$
$= (A\cap A')\cup(A\cap B')$
$= \emptyset\cup(A\cap B')$
$= (A\cap B')$
$= A-B$ $\therefore$ RHS

(4) Show that $A-(A-B) = A \cap B$

Sol:- LHS $= A-(A-B)$

$= A-(A \cap B')$

$= A-(A \cap B')$

$= A \cap (\bar{A} \cup \bar{\bar{B}})$

$= A \cap (\bar{A} \cup B)$

$= (A \cap \bar{A}) \cup (A \cap B)$

$= \phi \cup (A \cap B)$

$= (A \cap B)$ ∴ RHS

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Cardinal Number:- A set have 'A' countable number of elements that number is called CN.

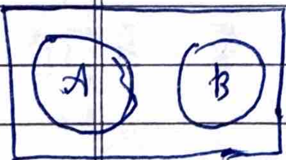
Denoted by $n(A)$ or $|A|$

Eg:- If $A = \{a,b,c,d,e\}$ then $|A| = n(A) = 5$

If $B = \{1,2,\ldots 10\}$ then $|B| = n(B) = 10$

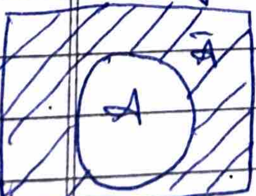
The principle of inclusion - exclusion

(1) If A & B are disjoint sets ($A \cap B = \phi$) then



$n(A \cup B) = n(A) + n(B)$

(2) If A is subset of U & $\bar{A}$ is the complementary set of A.



$A \cup \bar{A} = U$

$n(A \cup \bar{A}) = n(U)$

$n(A) + n(\bar{A}) = n(U)$

$\boxed{\text{Note (1) } n(\bar{A}) = n(U) - n(A)}$

(2) $n(A) = n(U) - n(\bar{A})$

③ $n(\overline{A \cup B}) = n(U) - n(A \cup B)$

④ $n(\overline{A \cap B}) = n(U) - n(A \cap B)$

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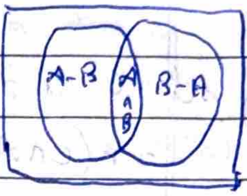
③ If A & B are any two sets then

$A = (A - B) \cup (A \cap B)$



$\therefore n(A) = n(A-B) + n(A \cap B)$

OR $n(A-B) = n(A) - n(A \cap B)$

$n(B-A) = n(B) - n(A \cap B)$

$B = (B-A) \cup (A \cap B)$

$n(B) = n(B-A) + n(A \cap B)$

$n(B-A) = n(B) - n(A \cap B)$

Only set A elements: $n(A-B) = n(A) - n(A \cap B)$

Only set B elements: $n(B-A) = n(B) - n(A \cap B)$

④ If A & B are any two sets then

$(A \cup B) = (A-B) \cup (A \cap B) \cup (B-A)$

$\therefore n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$

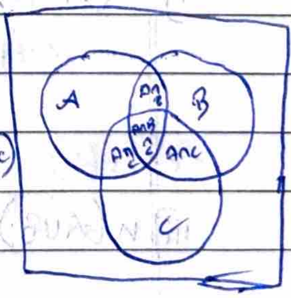
$n(A \cup B) = n(A) - \underline{n(A \cap B)} + \underline{n(A \cap B)} + n(B) - n(A \cap B)$

$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$ → Generalized formula.

$\therefore \boxed{n(A \cap B) = n(A) + n(B) - n(A \cup B)}$

⑤ If A, B & C are any three sets then

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C)$
$- n(B \cap C) + n(A \cap B \cap C)$



$\therefore n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Only set A elements: $n[A - (B \cup C)]$
$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

Only set B elements: $n[B - (C \cup A)] =$
$= n(B) - n(B \cap C) - n(B \cap A) + n(B \cap C \cap A)$

Only set C elements: $n[C - (A \cup B)] =$
$= n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C)$

Only set A & B elements : $n[(A \cap B) - C] =$
$= n(A \cap B) - n(A \cap B \cap C)$

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Only set B & C elements : $n[(B \cap C) - A]$
$= n(B \cap C) - n(A \cap B \cap C)$

Only set C & A elements : $n[(C \cap A) - B] =$
$= n(C \cap A) - n(A \cap B \cap C)$

Problems on Cardinal number

① In a survey of 5000 persons, it was found that 2800 read Indian express & 2300 ~~used~~ read times of India while 400 read both.

i] How many read only Indian Express?
ii] How many read only Times of India
iii] How many read Indian Express & Times of India
iv] How many read neither Indian Express nor Times of India.

Sol:
Let A be set of Indian Express, $n(A) = 2800$
Let B be set of Times of India, $n(B) = 2300$
Let both be, $n(A \cap B) = 400$
Total survey, $n(U) = 5000$

i] $n(A-B) = n(A) - n(A \cap B)$
$= 2800 - 400$
$= 2400$

ii] $n(B-A) = n(B) - n(A \cap B)$
$= 2300 - 400$
$= 1900$

iii] $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 2800 + 2300 - 400$
$= 4700$

iv] $n(\overline{A \cup B}) = n(U) - n(A \cap B)$
$= 5000 - \cancel{400}\ 4700$
$= \cancel{4}\ 300$

[illegible] 29/24

at least on → consider union
and → consider intersection

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② A group of 80 people, 42 like coffee, 60 likes tea & each person like atleast one of the two drinks. Find how many people like both cfe & tea

Sol: $n(A) = 42$, $n(B) = 60$, $n(U) = 80$, $n(A \cup B) = 80$

$n(A \cap B) = n(A) + n(B)\ n(A \cap B)$

$= 42 + 60 - 80$

$= \underline{\underline{22}}$

③ A survey of 500 television watcher produced the following information: 285 watches football games, 195 watches hockey games, 115 watches basketball games, 45 watches football & basketball games, 70 watches football & hockey game, 50 watches hockey & basketball game & 50 do not watch any of these three games. Find how many people in the survey watch.

i] all those kinds of games

ii] Only football games

iii] Only hockey games

iv] Only basketball game

v] Only watch football & hockey

vi] Only watch hockey & basketball

vii] Only watch basketball & football game.

Sol:- $n(U) = 500$, $n(A) = 285$, $n(B) = 195$, $n(C) = 115$

$n(A \cap C) = 45$, $n(A \cap B) = 70$, $n(B \cap C) = 50$

$n(\overline{A \cup B \cup C}) = 50$

i] $n(A \cup B \cup C) = n(U) - \cancel{s}\ n(\overline{A \cup B \cup C})$

$= 500 - 50$

$= 450$

To find how many people watch ~~at least one~~ all three game

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$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C)$
$\qquad - n(C\cap A) + n(A\cap B\cap C)$

$450 = 285 + 195 + 115 - 70 - 50 - 45 + n(A\cap B\cap C)$

$450 = 430 + n(A\cap B\cap C)$

$\Rightarrow n(A\cap B\cap C) = 450 - 430$

$= 20$

160

ii] $n[A-(B\cup C)]$

$= n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

$= 285 - 70 - 45 + 20$

$= \cancel{200}\ 190$

305
105
200

iii] $n[B-(A\cap C)]$

$= n(B) - n(B\cap A) - n(B\cap C) + n(A\cap B\cap C)$

$= 195 - 70 - 50 + 20$

$= 95$

145
120
720
115

iv] $n[C-(A\cap B)]$

$= n(C) - n(C\cap A) - n(C\cap B) + n(A\cap B\cap C)$

$= 115 - 45 - 50 + 20$

$= 40$

v] $n[(A\cap B)-C]$

$= n(A\cap B) - n(A\cap B\cap C)$

$= 70 - 20$

$= 50$

vi] $n[(B\cap C)-A]$

$= n(B\cap C) - n(A\cap B\cap C)$

$= 50 - 20$

$= 30$

vii] $n[(C\cap A)-B]$

$= n(C\cap A) - n(A\cap B\cap C)$

$= 45 - 20$

$= 25$

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Properties of Relations

① Reflexive :- A relation 'R' on a set 'A' is called reflexive is $(a,a) \in R$, $\forall a \in A$.

$A = \{1,2,3\}$

$R = \{(1,1),(2,2),(3,3)\}$

$\therefore (a,a) \in R \quad \forall a \in A$

$\therefore R$ is reflexive

(ii) $R_2 = \{(3,3),(1,1),(2,2)\}$ here $(a,a) \in R$, $\forall a \in R$

R_2 is a reflexive

(iii) $R_3 = \{(2,2),(3,3),(1,3),(1,1)\}$

here $(a,a) \in R_3$; $\forall a \in A$ R_3 is reflexive

(iv) $R_4 = \{(1,1),(1,2),(2,2)\}$

here $(3,3) \notin R_4$, R_4 is not a reflexive.

(v) $R_5 = \{(1,1),(2,3),(3,2),(3,3)\}$

here $(2,2) \notin R_5$, R_5 is not a reflexive.

(vi) $R_6 = \{(3,3),(1,1),(2,2),(2,4)\}$

It not a reflexive.

② Symmetric Relation:-

A revolution 'R' on a set A is called Symmetric relation is $(a,b) \in R$ $(b,a) \in R$ $\forall a,b \in A$.

Eg: $A = \{1,2,3\}$

$R = \{(1,1),(1,2),(2,1)\}$

$R_1 \leq (A \times A)$

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$(1,1) \in R_1 \Rightarrow (1,1) \in R_1$
$(1,2) \in R_1 \Rightarrow (2,1) \in R_1$
$(2,1) \in R_1 \Rightarrow (1,2) \in R_1$
So every R_1 is satisfied, R is symmetric

(ii) $R_2 = \{(1,2), (2,3), (3,3), (2,1), (1,1)\}$
here $R = \leq A \times A$
If symmetric $(a,b) \in R \Rightarrow (b,a) \in R, \forall a,b$
$(1,2) \in R_2 \Rightarrow (2,1) \in R_2$
$(2,3) \in R_2 \Rightarrow (3,2) \notin R_2$
R_2 is not a symmetric.

(iii) $R_3 = \{(3,1), (3,2), (3,3), (1,3), (2,3)\}$
here $R_2 \leq A \times A$
$(3,1) \in R_3 \Rightarrow (1,3) \in R_3$
$(3,2) \in R_3 \Rightarrow (2,3) \in R_3$
$(3,3) \in R_3 \Rightarrow (3,3) \in R_3$
R_3 is symmetric

(iv) $R_4 = \{(1,1), (2,2), (2,3), (3,3), (3,2)\}$
here $(a,a) \in R_1, \forall a \in A$
R_4 is Reflexive.
here $(a,b) \in R_4, (b,a) \in R_4$
R_4 is symmetric.

(3) Transitive :-
A relation 'R' on a set 'A' is called Transitive is whenever $(a,b) \in R$ & $(b,c) \in R$. Then $(a,c) \in R, \forall a,b,c$.

$\boxed{(a,b) \ \& \ (b,c) \Rightarrow (a,c) \in R, \forall a,b,c}$

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$A = \{1,2,3\}$

$R = \{(1,1),(1,2),(2,2)\}$

here $R_1 \leq A \times A$

$(1,1)$ & $(1,2) \Rightarrow (1,2) \in R_1$

$(1,2)$ & $(2,2) \Rightarrow (1,2) \in R$

$(2,2)$ & .... R_1 is transitive.

22/01/25 Thursday

Universal Relation:-

The relation $R = A \times A$ is called universal relation "U".

Eg:- $A = \{1,2\}$

$\therefore A \times A = \{(1,1),(1,2),(2,1),(2,2)\}$

& $R = \{(1,1),(1,2),(2,1),(2,2)\}$

here $R = A \times A$ = Universal relation

$\therefore U = R$

Inverse relation R^{-1}:-

Let R be a relation from the set A to set B, i.e., $R \leq A \times B$. The inverse relation denoted by R^{-1} from set B to set A & denoted by $R^{-1} \leq B \times A$.

$R = \{(a,b) / a \in A, b \in A, \forall a,b\}$

$R^{-1} = \{(a,b) / b \in A, a \in A, \forall b,a\}$

Eg:- $A = \{1,2,3\}$

$R = \{(1,2),(3,2),(1,3)\}$

$R^{-1} = \{(2,1),(2,3),(3,1)\}$

Complement of Relation $\bar{R}$

$\boxed{\bar{R} = U - R}$

Eg: $A = \{1,2\}$ $R \leq A \times A$

here $= \{(1,1),(2,1)\}$

Universal Relation $U = A \times A$

$U = \{(1,1),(1,2),(2,1),(2,2)\}$

$\bar{R} = U - R$

$= \{(1,2),(2,2)\}$

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Relation representing by i] zero-one matrix & ii] graph

Definition :- A relation from a finite set of 'A' to a finite set of 'B' can be represented by a matrix [0 & 1 matrix] called relation matrix of R.

Its denoted by $M_R = [m_{ij}]$

$$M_R = m_{ij} = \begin{cases} 0 & (a,b) \notin R \\ 1 & (a,b) \in R \end{cases}$$

Problems

① Let $A = \{1,2,3,4\}$ and $R = \{(1,1),(1,2),(1,3),(2,3),(4,1),(4,4)\}$ then write the relation matrix & draw the relation graph.

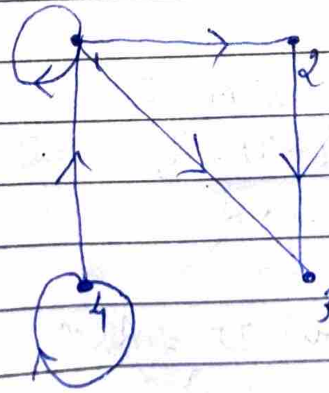
Sol:- $A = \{1,2,3,4\}$

$R = \{(1,1),(1,2),(1,3),(2,3),(4,1),(4,4)\}$

A\A	1	2	3	4
1	1	1	1	0
2	0	0	1	0
3	0	0	0	0
4	1	0	0	1

$$\therefore M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Relation graph.



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(2) Let $A = \{a, b, c\}$ & $R \subseteq A \times A$ i.e., $R = \{(a,a), (a,b), (b,a), (b,b), (b,c), (c,b), (c,c)\}$, find the relation matrix & graph.

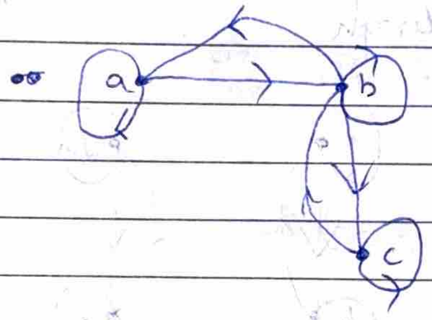
Sol: $A = \{a, b, c\}$

$R = \{(a,a), (a,b), (b,a), (b,b), (b,c), (c,b), (c,c)\}$

A/A	a	b	c
a	1	1	0
b	1	1	1
c	0	1	1

$$\therefore M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Relation graph



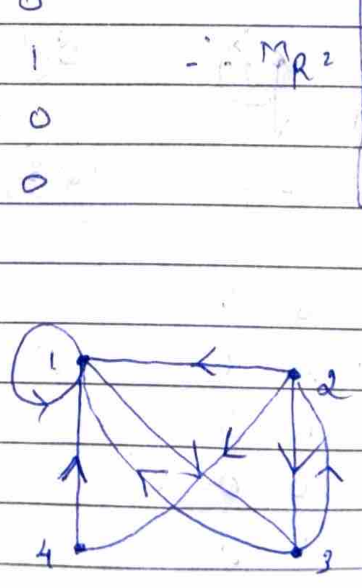
(3) Let $A = \{1, 2, 3, 4\}$ & $R \subseteq A \times A$ i.e., $R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$ find the relation matrix & graph

Sol:

A/A	1	2	3	4
1	1	0	1	0
2	1	0	1	1
3	1	1	0	0
4	1	0	0	0

$$\therefore M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Relation graph



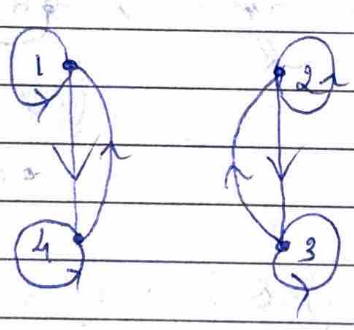
(4) If $x = \{1, 2, 3, 4\}$ & its relation matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ find matrix & graph

Sol :-

A/A	1	2	3	4
1	1	0	0	1
2	0	1	1	0
3	0	1	1	0
4	1	0	0	1

$\therefore R = \{(1,1), (1,4), (2,2), (2,3), (3,2), (3,3), (4,1), (4,4)\}$

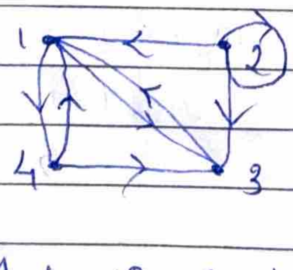
Relation graph



03/01/25
Friday

(5) Find the relation & its matrix for the following

i]

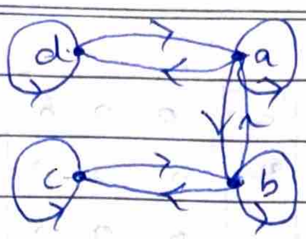


Sol :- $A = \{1, 2, 3, 4\}$
$R = \{(1,3), (1,4), (2,2), (2,1), (2,3), (3,1), (4,1), (4,3)\}$

A/A	1	2	3	4
1	0	0	1	1
2	1	1	1	0
3	1	0	0	0
4	1	0	1	0

$\therefore M_R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Sol:

ii] 

$A = \{a, b, c, d\}$

$R = \{(a,a), (a,b), (a,d), (b,b), (b,a), (b,c), (c,c), (c,b), (d,d), (d,a)\}$

A\A	a	b	c	d
a	1	1	0	1
b	1	1	1	0
c	0	1	1	0
d	1	0	0	1

$$\therefore M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Union, Intersection of relation matrix

1] $M_{R\cup S} = M_R \vee M_S \rightarrow \left.\begin{matrix} 1\vee 1 \\ 1\vee 0 \\ 0\vee 1 \end{matrix}\right\} \rightarrow 1$, $\quad 0\vee 0 \rightarrow 0$

2] $M_{R\cap S} = M_R \wedge M_S \rightarrow 1\wedge 1 \rightarrow 1$, $\quad \left.\begin{matrix} 1\wedge 0 \\ 0\wedge 1 \\ 0\wedge 0 \end{matrix}\right\} = 0$

Problems:- If $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ & $M_S = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ find $M_{R\cup S}$ & $M_{R\cap S}$

Sol,

$$M_{R\cup S} = M_R \vee M_S = \begin{bmatrix} 1\vee 1 & 0\vee 0 & 1\vee 1 \\ 1\vee 0 & 0\vee 1 & 0\vee 1 \\ 0\vee 1 & 1\vee 0 & 0\vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$M_{R\cap S} = M_R \wedge M_S = \begin{bmatrix} 1\wedge 1 & 0\wedge 0 & 1\wedge 1 \\ 1\wedge 0 & 0\wedge 1 & 0\wedge 1 \\ 0\wedge 1 & 1\wedge 0 & 0\wedge 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) If $M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ & $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ find $M_{R\cup S}$ & $M_{R\cap S}$.

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Sol:-

$$M_{R\cap S} = M_R \wedge M_S = \begin{bmatrix} 1\wedge 0 & 0\wedge 1 & 1\wedge 0 \\ 1\wedge 0 & 1\wedge 0 & 0\wedge 1 \\ 0\wedge 1 & 0\wedge 0 & 0\wedge 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{R\cup S} = M_R \vee M_S = \begin{bmatrix} 1\vee 0 & 0\vee 1 & 1\vee 0 \\ 1\vee 0 & 1\vee 0 & 0\vee 1 \\ 0\vee 1 & 0\vee 0 & 0\vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(3) If $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ & $M_S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ find $M_{R\cup S}$ & $M_{R\cap S}$

Sol:-

$$M_{R\cup S} = M_R \vee M_S = \begin{bmatrix} 1\vee 1 & 1\vee 0 & 0\vee 0 & 1\vee 1 \\ 1\vee 0 & 1\vee 1 & 1\vee 1 & 0\vee 0 \\ 0\vee 0 & 1\vee 1 & 1\vee 1 & 0\vee 0 \\ 1\vee 1 & 0\vee 0 & 0\vee 0 & 1\vee 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{R\cap S} = M_R \wedge M_S = \begin{bmatrix} 1\wedge 1 & 1\wedge 0 & 0\wedge 0 & 1\wedge 1 \\ 1\wedge 0 & 1\wedge 1 & 1\wedge 1 & 0\wedge 0 \\ 0\wedge 0 & 1\wedge 1 & 1\wedge 1 & 0\wedge 1 \\ 1\wedge 1 & 0\wedge 1 & 0\wedge 0 & 1\wedge 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Composition of Relation:-

Let $R \subseteq A\times B$ & $S \subseteq B\times C$. Then composition of R and S is denoted by $R\circ S$

$$R\circ S = \{(a,c) \mid (a,b)\in R \text{ \& } (b,c)\in S, \forall a,b,c\}$$

$\boxed{\text{Note} = R^2 = R\circ R,\ S^3 = S^2\circ S}$

Problem 1: Let $R = \{(1,2),(3,4),(2,2)\}$ & $S = \{(4,2),(2,5),(3,1),(1,3)\}$. Find RoS, SoR, R^2, S^2, $Ro(RoS)$, $So(RoS)$, R^3, S^3.

Sol:

$RoS =$

R	o S	⇒ RoS
(1,2)	(2,5) →	(1,5)
(3,4)	(4,2) →	(3,2)
(2,2)	(2,5) →	(2,5)

$\therefore RoS = \{(1,5),(3,2),(2,5)\}$

$SoR =$

S	o R	⇒ SoR
(4,2)	(2,2) →	(4,2)
(2,5)	– →	–
(3,1)	(1,2) →	(3,2)
(1,3)	(3,4) →	(1,4)

$\therefore SoR = \{(4,2),(3,2),(1,4)\}$

$R^2 =$

R	o R	⇒ R^2
(1,2)	(2,2) →	(1,2)
(3,4)	– →	–
(2,2)	(2,2) →	(2,2)

$\therefore R^2 = \{(1,2),(2,2)\}$

$S^2 =$

S	o S	⇒ S^2
(4,2)	(2,5) →	(4,5)
(2,5)		
(3,1)	(1,3) →	(3,3)
(1,3)	(3,1) →	(1,1)

$\therefore S^2 = \{(4,5),(3,3),(1,1)\}$

$Ro(RoS) =$

R	o RoS	⇒ Ro(RoS)
(1,2)	(2,5) →	(1,5)
(3,4)	– →	–
(2,2)	(2,5) →	(2,5)

$\therefore Ro(RoS) = \{(1,5),(2,5)\}$

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$So(RoS) = S$	o	$RoS \Rightarrow So(RoS)$
$(4,2)$		$(2,5) \to (4,5)$
$(2,5)$		$- \to -$
$(3,1)$		$(1,5) \to (3,5)$
$(1,3)$		$(3,2) \to (1,2)$

$\therefore So(RoS) = \{(4,5), (3,5), (1,2)\}$

$R^3 = R^2$	o	$R \Rightarrow R^3$
$(1,2)$		$(2,2) \to (1,2)$
$(2,2)$		$(2,2) \to (2,2)$

$\therefore R^3 = \{(1,2), (2,2)\}$

$S^3 = S^2$	o	$S \Rightarrow S^3$
$(4,5)$		$- \to -$
$(3,3)$		$(3,1) \to (3,1)$
$(1,1)$		$(1,3) \to (1,3)$

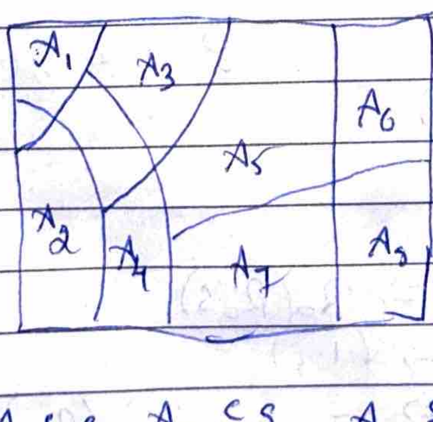
$\therefore S^3 = \{(3,1), (1,3)\}$

06/01/25
Monday

① Covering set:

Let S be a given set of $A = \{A_1, A_2, A_3\}$ where each A_i's is a subset of S.

i.e.,



i.e., $A_1 \subseteq S$, $A_2 \subseteq S$, $A_3 \subseteq S$

here $A_1 \cup A_2 \cup \ldots \cup A_n = S$

i.e., $\cup A_i = S$

Then set A is called a covering of set S.

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② Partsion set :

Let S be a given set & $A = \{A_1, A_2 \dots A_n\}$ where each A_i same subset of S & $A_i \cap A_j = \phi$ $\forall i, j$.

Eg:

A_1	A_3	A_4	A_5
A_2	A_6	A_7	A_8

here $A_1 \cup A_2 \dots \cup A_8 = S$

i.e., $\bigcup_{i=1} A_i = S$

Then $A_1, A_2 ; A_8$ are called porlision set of S.

③ Partially ordered set [POSET]

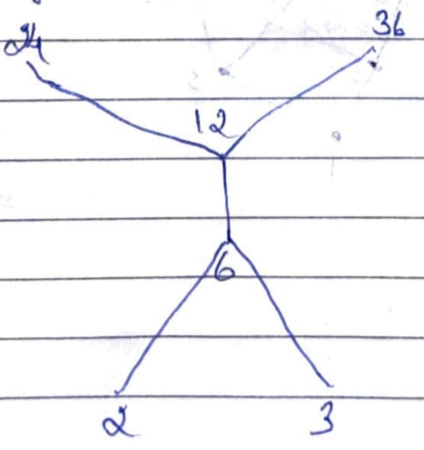
A binary relation in a set A is called partially ordered relation $\boxed{OR}$ a partially ordering in P iff R^{-1} is Reflexive & antiSymmetric & transitive. It is denoted by symbol '$\leq$' $\boxed{OR}$ '$\subseteq$'.

If '$\leq$' partially ordering on 'P' then the ordered pair $(P, \leq)$ is called POST.

Problems

① Let $A = \{2, 3, 6, 12, 24, 36\}$ the relation '$\leq$' such that a divides 'b'. Draw a Hasse diagram of $(A, \leq)$

Sol:-



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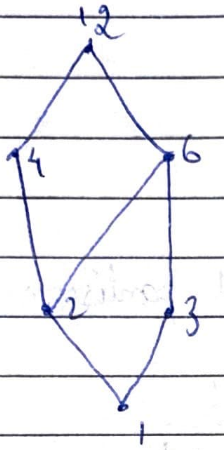
(2) Let 'A' be the set of factors of a particular positive integers 'n' and '≤' be relation divides

$\leq = \{(a,b) / a \text{ divides } b \ \forall a\}$

i] n = 12, m = 45, n = 50

m = 12 factor = {1, 2, 3, 4, 6, 12}



'S' Hasse diagram.

08/01/25
Wednesday

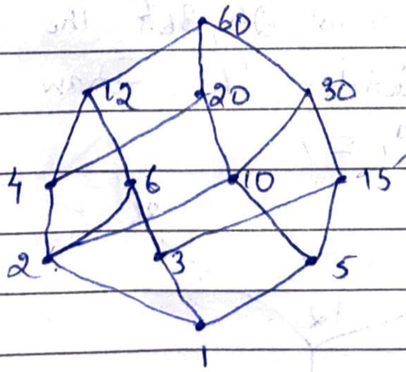
(3) Let 'A' be the set of factors positive integer 'm'

i] m = 12, ii] m = 45, iii] m = 50 iv] m = 60

m = 60

factor {1, 2, 3, 5, 4, 6, 10, 15, 12, 20, 30, 60}

2	60
2	30
3	15
5	5
	1



Step 1 - always 1

step 2 - alway prime

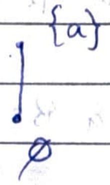
step 3 - multy

(4) Let A be the finite set & $P(A)$ is the power set, let $\subseteq$ be relation of the element $P(A)$. Draw an Hasse diagram & $(P(A), \subseteq)$

i] $A = \{a\}$ ii] $A = \{a, b\}$ iii] $A = \{a, b, c\}$

Sol: $A = \{a\}$

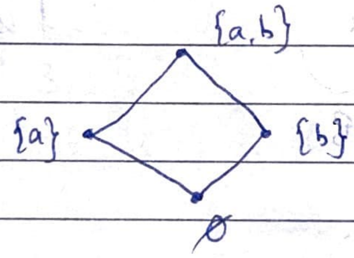
$\therefore P(A) = \{\emptyset, \{a\}\}$



$(P(A), \subseteq)$ & Hasse diagram.

$A = \{a, b\}$

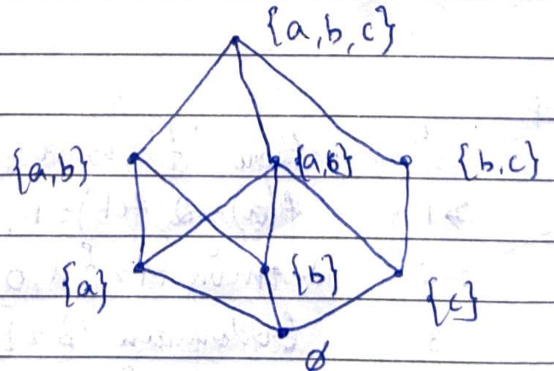
$\therefore P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$



$(P(A), \subseteq)$ & Hasse diagram

$A = \{a, b, c\}$

$\therefore P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$



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Function :-

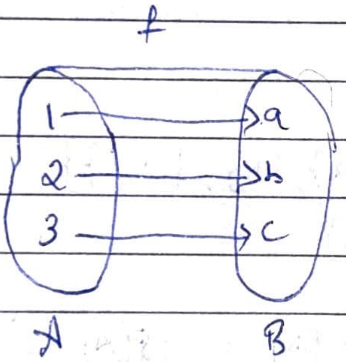
Let A & B be two non empty sets then a function (or mapping) from the set A into set B, is a relative from A & B.

i] $\forall a \in A$, $\exists b \in B$, such that $(a,b) \in f$

ii] $(a,b) \in f$ and $(a,b_1) \in f \Rightarrow b = b_1$, $\forall a \in A$, $b, b_1 \in B$.

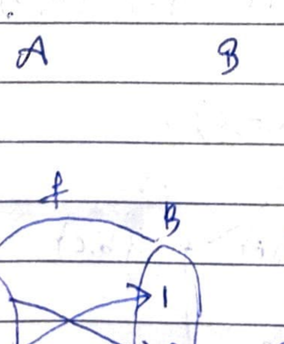
The function is defined by $f : A \to B$.

Eg:- If $A = \{1,2,3\}$ & $B = \{a,b,c\}$ are two sets.



here clearly $f : A \to B$

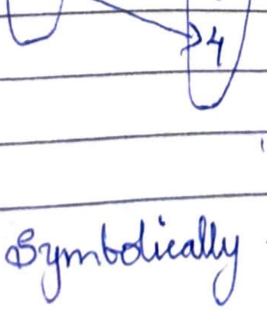
$\therefore f$ is a function.



here clearly $f : A \to B$

$\therefore f$ is a function

09/01/25
Thursday



Here 'f' is a function

$f(a) = 2$, $f(b) = 1$, $f(c) = 4$

Domain $A = \{a,b,c\}$

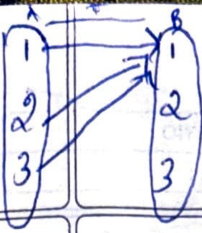
Codomain $B = \{1,2,3,4\}$

Range $= \{1,2,4\} \subseteq B$

'b' is called image of 'a'

'a' is called preimage of 'b'

Symbolically $f = \{(a, f(a)) / \forall a \in A, \exists f(a) \in B\}$



Here $f(1)=1$, $f(2)=1$, $f(3)=1$

Domain $A=\{1,2,3\}$

Codomain $B=\{1,2,3\}$

Range $=\{1\}\subseteq B$

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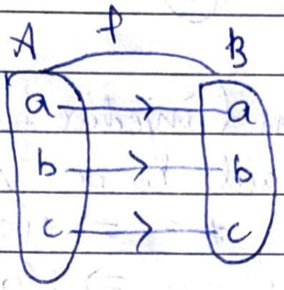
Types of functions

⇒ Identity function :- A function $f: A\to A$ such that $f(a)=a\ \forall a\in A$ is called the I.f

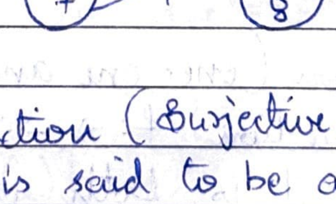
Eg:- If $A=\{a,b,c\}$ then $f: A\to A$ defined $f(a)=a$, $f(b)=b$, $f(c)=c$.



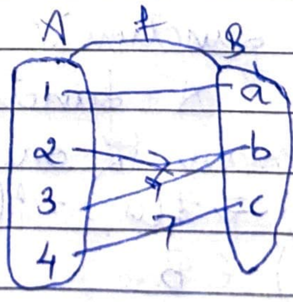
⇒ Constant function :- A function $f: A\to B$ such that $f(a)=k\ \forall a\in A$ where k is a fixed element of B is called C.f.

Eg:- Let $A=\{1,3,5,7\}$ & $B=\{2,4,6,8\}$

Define $f(1)=6$, $f(3)=6$, $f(5)=6$ & $f(7)=6$

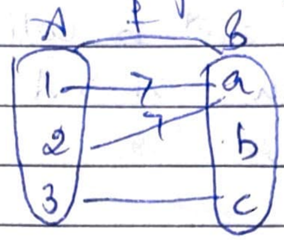


⇒ Onto function (Surjective function) :- A function $f: A\to B$ is said to be an onto function, if $\forall b\in B$, $\exists$ at least one element of $a\in A$ such that $f(a)=b$ {Codomain = Range}



A function $f: A\to B$ is said to be onto iff $R(f)=B$

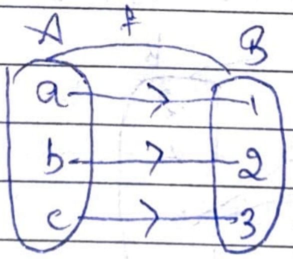
⇒ Into function :- A function $f: A \to B$ is said to be into function if $R(f)$ is a proper subset of B, i.e., $R(f) \subset B$



⇒ One to one function (injective function) :- A function $f: A \to B$ is said to be injective if
$\forall a_1, a_2 \in A \; a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$

OR $\forall a_1, a_2 \in A \; a_1 = a_2 \Rightarrow f(a_1) = f(a_2)$

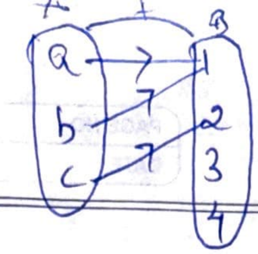
other :- The function f is said to be one to one if the distinct elementary of A have distinct image of B.



⇒ Bijective function (one-one and onto) :
A function $f: A \to B$ is said to be bijective if it is both one-one & onto.

Eg - If $f(x) = 3x + 2, \; \forall x \in R$ is a bijective function

⇒ Many to one function :-
A function $f: A \to B$ is said to be many to one function such that $a_1 \neq a_2 \Rightarrow f(a_1) = f(a_2) \; \forall a_1 \in A$ & $f(a_1), f(a_2) \in B$



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⇒ Inverse function :- A function $f: A \to B$ be a bijective function then the function $f: B \to A$ defined by

$f^{-1}(b) = a$ such that $f(a) = b$ is called if
$f^{-1}(b) = a, \forall b \in B$, such that $f(a) = b$.

Composite function
Let $f: A \to B$ and $g: B \to C$ be the two function, then the composite function of f and g denoted by, fog
$\therefore fog = A \to C$

Note : $fog = fog(x)$
$f[g(x)]$

Prob 1) $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$, $\forall x \in R$ then find fog, foh, gof, goh, hof, hog, $fo(goh)$, $go(foh)$, f^2, g^2

Sol :- Given $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$

$fog = fog(x)$
$= f[g(x)]$
$= g(x)+2$
$= x - \cancel{2} + \cancel{2}$
$\boxed{fog = x}$

$foh = foh(x)$
$= f[h(x)]$
$= h(x)+2$
$\boxed{foh = 3x+2}$

$gof = gof(x)$
$= g[f(x)]$
$= f(x) - 2$
$= x + \cancel{2} - \cancel{2}$
$\boxed{gof = x}$

$goh = goh(x)$
$= g[h(x)]$
$\boxed{goh = 3x - 2}$

$hof = hof(x)$
$= h[f(x)]$
$= x+2$

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Recursive function

Sometimes it is difficult to define an object explicity. However it maybe easy to define the object in Terms of itself. This process called RF.

Eg: $S_n = n^2 - 1 \quad [n \geq 1]$

$S_1 = 1^2 - 1$

$S_2 = 2^2 - 1$

$= 4 - 1$

$\boxed{S_2 = 3}$

Problems

① Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ & $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ then find a_3, a_4, a_5 & a_6.

Sol: Substitute $n = 3$

$a_3 = a_{3-1} + a_{3-2} + a_{3-3}$

$= a_2 + a_1 + a_0$

$= 3 + 2 + 1$

$\boxed{a_3 = 6}$

Substitute $n = 4$

$a_4 = a_{4-1} + a_{4-2} + a_{4-3}$

$= a_3 + a_2 + a_1$

$= 6 + 3 + 2$

$\boxed{a_4 = 11}$

Substitute $n = 5$

$a_5 = a_{5-1} + a_{5-2} + a_{5-3}$

$= a_4 + a_3 + a_2$

$= 11 + 6 + 3$

$\boxed{a_5 = 20}$

Substitute $n = 6$

$a_6 = a_{6-1} + a_{6-2} + a_{6-3}$

$= a_5 + a_4 + a_3$

$= 20 + 11 + 6$

$\boxed{a_6 = 37}$

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(2) Let $f(0)=3$ & $f(n+1)=2f(n)+3$ find $f(1), f(2), f(3)$ & $f(4)$

Sol:- Substitute $n=0$

$f(n+1)=2f(n)+3$

$f(0+1)=2f(0)+3$

$f(1)=2(3)+3$

$=6+3$

$\boxed{f(1)=9}$

$n=1$

$f(1+1)=2f(1)+3$

$f(2)=2(9)+3$

$f(2)=18+3$

$\boxed{f(2)=21}$

$n=2$

$f(2+1)=2f(2)+3$

$f(3)=2(21)+3$

$f(3)=42+3$

$\boxed{f(3)=45}$

$n=3$

$f(3+1)=2f(3)+3$

$f(4)=2(45)+3$

$f(4)=90+3$

$\boxed{f(4)=93}$

(3) Let $F_0=0$, $F_1=1$ & $F_n=F_{n-1}+F_{n-2}$ $[n>2]$
find F_2, F_3, F_4, F_5 [first & ~~sq~~ sequence]

Sol:-

$n=2$

$F_n=F_{n-1}+F_{n-2}$

$F_2=F_{2-1}+F_{2-2}$

$F_2=F_1+F_0$

$=1+0$

$\boxed{F_2=1}$

$n=3$

$F_3=F_{3-1}+F_{3-2}$

$=F_2+F_1$

$=1+1$

$=2$

$n=4$

$F_4 = F_{4-1} + F_{4-2}$

$= F_3 + F_2$

$= 2+1$

$= 3$

$n=5$

$F_5 = F_{5-1} + F_{5-2}$

$= F_4 + F_3$

$= 3+2$

$= 5$

(4) If $L_0 = 2$, $L_1 = 1$ & $L_n = L_{n-1} + L_{n-2}$ $n > 2$ find first 8 sequence

Sol:-

$n=2$

$L_2 = L_{2-1} + L_{2-2}$

$= L_1 + L_0$

$= 1+2$

$= 3$

$n=3$

$L_3 = L_{3-1} + L_{3-2}$

$= L_2 + L_1$

$= 3+1$

$= 4$

$n=4$

$L_4 = L_{4-1} + L_{4-2}$

$= L_3 + L_2$

$= 4+3$

$= 7$

$n=5$

$L_5 = L_{5-1} + L_{5-2}$

$= L_4 + L_3$

$= 7+4$

$= 11$

$n=6$

$L_6 = L_{6-1} + L_{6-2}$

$= L_6 + L_4$

$= 11+7$

$= 18$

$n=7$

$L_7 = L_{7-1} + L_{7-2}$

$= L_6 + L_5$

$= 18+11$

$= 29$

$n=8$

$L_8 = L_{8-1} + L_{8-2}$

$= L_7 + L_6$

$= 29+18$

$= 47$