

First name: _____ Last name: _____ Zid: _____

1.
 - 1) Draw the recursion tree for $T(n) = 4T(n/2) + n$ and give a tight asymptotic bound on its solution.
 - 2) Verify your bound by induction.
 - 3) Use the master's method to solve above recurrence.
2. Use the master method to solve $T(n) = aT(n/2) + cn^3$ for three cases $a = 7, 8, 9$ and a constant $c > 0$.
3. You are given an unsorted array of n integers.
 - 1) What is the time cost to find the maximum or minimum?
 - 2) Assume the time cost to find each of the maximum and minimum is $T(n)$. A straightforward way to find both of the maximum and minimum costs $2T(n)$. Please give a better approach using divide-and-conquer. Describe the recurrence and solve it. You can assume n is power of 2.
4. Banks often record transactions on an account in order of the times of the transactions, but many people like to receive their bank statement with checks listed in order by check number. People usually write checks in order by check number, and merchants usually cash them with reasonable dispatch. The problem of converting time-of-transaction ordering to check-number ordering is therefore the problem of sorting almost-sorted input. Argue that insertion sort would tend to beat quick sort on this problem.
5. Let $A[1..n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion.
 - 1) List the inversions in the array $\langle 2, 3, 8, 6, 1 \rangle$.
 - 2) What array with elements from the set $\{1, 2, \dots, n\}$ has the most number of inversions? How many does it have?