DEPARTMENT OF MASTER OF

COMPUTER APPLICATION

Mathematical Foundation for Computer Applications
Activity - 3

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Activity - 3

1Q) Let p be the statement "Maria Learns discrete Mathematics" and q the statement "Maria will find a good job" express the statement $p \to q$ as a statement in english.

Sol:- From the defination of Conditional Statements, we see that when p is the statement "Maria Learns discrete Mathematics" and q is the statement "Maria will find a good job." $p \to q$ represents the statement.

"If maria Learns discrete Mathematics, then she will find a good job".

There are many other ways to express this conditional statement in english. Among the most natural of these are

"Maria will find a good job when she learns the discrete mathematics".

"For Maria to get a good job, it is sufficient for her to learn discrete Mathematics."

and

"Maria will find a good job unless she does not learn discrete mathematics."

2Q) Construct the truth table of the Compound proposition
$$(p \vee \sim q) \to (p \wedge q)$$

Sol:- Because this truth table involves two propositional variable p and q, there are four rows in this truth table, one for each of the pairs of truth values TT, TF, FF, FF. The first two columns are used for the truth values of p and q respectively. In the third column we find the truth values of $\sim q$, needed to find the truth values of $p \vee \sim q$, found in the fourth column. The fifth column gives the truth value of $p \wedge q$. Finally, the truth value of $(p \vee \sim q) \to (p \wedge q)$

is found in the last column. The resulting truth table is shown table 1.

Table 1 The truth table of $(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$p \vee \sim q$	$p \wedge q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

3) Show that $\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent.

Sol: The truth tables for these compound propositions are displayed in table 2. Because the truth values of the compound propositions $\sim(p \vee q)$ and $\sim p \wedge \sim q$ agree for all possible combination of the truth values of p and q, it follows that $\sim(p \vee q) \longleftrightarrow (\sim p \wedge \sim q)$ is a tantology and that these compound propositions are logically equivalent.

Table 2 Truth Tables for $\sim(p \vee q)$ and $\sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

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4) Prove by PMI that $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Sol:- Let $P(n)$ be the statement that $1+2+3\dots+n = \frac{n(n+1)}{2}$

Step (1):- Put $n=1$

$LHS = 1$

$RHS = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$\therefore LHS = RHS$ $\therefore P(1)$ is true

Step (2):- Assume that $P(n)$ is true for $n=K$

Let $P(K)$ is true

$1+2+3+\dots+K = \frac{K(K+1)}{2}$

Step (3):- Now, we have to prove that $f(n)$ is true for $n=(K+1)$ LHS of $P(K+1)$

$1+2+3\dots+K+K+1 = \frac{(K+1)(K+2)}{2}$

Now

$1+2+3+\dots+K+K+1$

$= \frac{K(K+1)}{2} + K+1$

$= (K+1)\left[\frac{K}{2}+1\right]$

$= \frac{(K+1)(K+2)}{2} = RHS$

$\therefore P(n)$ is true for $n \in N$

$\therefore$ By PMI $P(A)$ is true $\forall n \in N$

= Prove

5) Find the Contrapositive, the Converse, and the inverse of the Conditional statement.

"The home team wins whenever it is raining".

Sol:-

Because "q whenever p" is one of the ways to express the Conditional statement $p \rightarrow q$, the original statement can be rewritten as.

"It is raining, then the home team wins"

Consequently, the Contrapositive of this Conditional Statement is

"It is home team does not win, then it is not raining".

The Converse is

"If the home team wins, then it is raining".

The inverse is

"If it is not raining, then the home team does not win"

Only the Contrapositive is equivalent to the original statement.

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6) What is the truth value of $\forall x P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

Sol: The statement $\forall x P(x)$ is the same as the conjuction.
$P(1) \land P(2) \land P(3) \land P(4)$.

Because the domain consists of the integers 1, 2, 3, and 4. Because $P(4)$, which is the statement "$4^2 < 10$", is false, it follows that $\forall x\, P(x)$ is false.

Similarly, when the elements of the domain are $x_1, x_2, \ldots, x_n$. where n is a positive integer, the existential quantification $\exists x P(x)$ is the same as the disjunction.

$P(x_1) \lor P(x_2) \lor \cdots \lor P(x_n)$.

Because this disjunction is true if and only if at least one of $P(x_1), P(x_2) \ldots P(x_n)$ is true.

7) What do the statements $\forall x < 0\ (x^2 > 0)$, $\forall y \neq 0\ (y^3 \neq 0)$, and $\exists z > 0\ (z^2 = 2)$ mean, where the domain in each case consists of the real numbers?

Sol:

The statement $\forall x < 0\ (x^2 > 0)$ statement that for every real number x with $x < 0$, $x^2 > 0$. That is, it startes "The square of a negative real number is positive." This statement is the same as $\forall x\ (x < 0 \rightarrow x^2 > 0)$.

The statement $\forall y \neq 0\ (y^3 \neq 0)$ states that for every real number y with $y \neq 0$, we have $y^3 \neq 0$. That is, it states "The cube of every nonzero real number is nonzero." This statement is equivalent to $\forall y\ (y \neq 0 \rightarrow y^3 \neq 0)$.

Finally, the statement $\exists z > 0\ (z^2 = 2)$ states that there exists a real number z with $z > 0$ such that $z^2 = 2$. That is, it states "There is a positive square root of 2". This statement is equivalent to $\exists z\ (z > 0 \land z^2 = 2)$.

Note that the of a universal qualification is the same as the universal quantification of a conditional statement. For instance, $(\forall x < 0 / x^2 > 0)$ is another way of expressing same as the existential qualification of a conjuction. For instance $\exists z > 0\ (z^2 = 2)$ is another way of expressing $\exists z\ (z > 0 \land z^2 = 2)$.

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8) Express the statement "Every student in this class has studied Calculus using predicates and quantifiers.

Sol:

First, we rewrite the statement so that we can clearly identify the appropriate quantifiers to use. Doing so, we obtain.

"For every student in this class, that student has studied calculus."

Next, we introduce a variable x so that our statement becomes.
"For every student x in this class, x has studied calculas".

Continuing, We introduce $C(x)$, which is the statement "x has studied calculus." Consequently, if the domain for x consists of the students in the class, we can translate our statement as $\forall x\, C(x)$.

However, there are other correct approaches different domain of discourse and other predicts can be used. The approach we select depends on the subsequent reasoning we want to carry out. For example, we may be interested in a wider group of people than only those in this class. If we change the domain to consist of all people we will need to express our statement as.

Finally, when we are interested in the background of people in subjects besides calculus, we may prefer to use the two variable (x, y) for the statement "Student x has studied subject y." Then we should replace $C(x)$ by $Q(x, \text{Calculate})$ in both approaches to obtain $\forall x\, Q(x, \text{Calculus})$.

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9) Combinations. A farmer buys 3 cows, 2 Pigs & 4 hens from a man who has 6 cow, 5 pig and 8 hens how many choices does the farmer have.

Sol:-

$^6C_3$ = Cows

$^3C_2$ = Pigs

$^4C_8$ = hens

According to principle of Counting

$$^6C_3 \times {}^3C_2 \times {}^4C_8 = \frac{6!}{3!\,3!} \times \frac{5!}{2!\,3!} \times \frac{8!}{4! \times 4!}$$

$$= \frac{6\times5\times4}{3\times2} \times \frac{5\times4}{1\times2} \times \frac{8\times7\times6\times5}{4\times3\times2}$$

$$= 20\times10\times70$$

$$= 14000$$

10) 6 men, 5 women To form the Committee of 5 members you should have 2 womens.

Sol:- Given

Total = 11

Members = 5

According to Combinations

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$^3C_2 \times {}^6C_3 = \frac{5!}{2!\,3!} \times \frac{6!}{3!\,3!}$$

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$$= \frac{5\times4\times3\times2\times1}{2\times1\times3\times2\times1} \times \frac{6\times5\times4\times3\times2\times1}{3\times2\times1\times3\times2\times1}$$

$$= \frac{5\times4}{2\times1} \times \frac{6\times5\times4}{3\times2\times1}$$

$$= \frac{20}{2} \times \frac{120}{6}$$

$$= 10\times20$$

$$= 200$$