	Pennies	Nickels	Total
ratio	2	1	3
multiply by	8	8	8
actual numbers	16	8	24

Therefore, of the 24 coins, 16 are pennies and 8 are nickels.

Let’s try a GRE example.

Flour, eggs, yeast, and salt are mixed by weight in the ratio of 11 : 9 : 3 : 2, respectively. How many pounds of yeast are there in 20 pounds of the mixture?

◯ $1\frac{3}{5}$

◯ $1\frac{4}{5}$

◯ 2

◯ $2\frac{2}{5}$

◯ $8\frac{4}{5}$

Need More Math Review?

Check out *Math Workout for the GRE*. If you’re in a hurry, pick up *Crash Course for the GRE*.

Here's How to Crack It

The minute you see the word *ratio*, draw a ratio box on your scratch paper and fill in what you know.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	
multiply by					
actual numbers					**20**

First, add all of the numbers in the ratio to get the ratio total.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	**25**
multiply by					
actual numbers					**20**

Now, what do we multiply 25 by to get 20?

$$25x = 20$$

$$\frac{25x}{25} = \frac{20}{25}$$

$$x = \frac{20}{25}$$

$$x = \frac{4}{5}$$

So $\frac{4}{5}$ is our "multiply by" number. Let's fill it in.

	Flour	Eggs	Yeast	Salt	Total
ratio	11	9	3	2	**25**
multiply by	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$	$\frac{4}{5}$
actual numbers					**20**

The question asks for the amount of yeast, so we don't have to worry about the other ingredients. Just look at the yeast column. All we have to do is multiply 3 by $\frac{4}{5}$ to get the answer: $3 \times \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$, which is (D).

What Is a Proportion?

So you know that a fraction is a relationship between part and whole, and that a ratio is a relationship between part and part. A **proportion** is an equivalent relationship between two fractions or ratios. Thus, $\frac{1}{2}$ and $\frac{4}{8}$ are proportionate because they are equivalent fractions. But $\frac{1}{2}$ and $\frac{2}{3}$ are not in proportion because they are not equal ratios.

The GRE often contains problems in which you are given two proportional, or equal, ratios from which one piece of information is missing. These questions take a relationship or ratio, and project it onto a larger or smaller scale. Proportion problems are recognizable because they always give you three values and ask for a fourth value. Here's an example:

If the cost of a one-hour telephone call is $7.20, what would be the cost in dollars of a 10-minute telephone call at the same rate?

The key to proportions is setting them up correctly.

Here's How to Crack It

It's very important to set up proportion problems correctly. That means placing your information on your scratch paper. Be especially careful to label *everything*. It takes only an extra two or three seconds, but doing this will help you catch lots of errors.

Relationship Review

You may have noticed a trend in the preceding pages. Each of the major topics covered—fractions, percents, ratios, and proportions—described a particular relationship between numbers. Let's review:

- A fraction expresses the relationship between a part and the whole.
- A percent is a special type of fraction, one that expresses the relationship of part to whole as a fraction with the number 100 in the denominator.
- A ratio expresses the relationship between part and part. Adding the parts of a ratio gives you the whole.
- A proportion expresses the relationship between equal fractions, percents, or ratios.

- Each of these relationships shares all the characteristics of a fraction. You can reduce them, expand them, multiply them, and divide them using the exact same rules you used for working with fractions.

For this question, let's express the ratios as dollars over minutes, because we're being asked to find the cost of a 10-minute call. That means that we have to convert 1 hour to 60 minutes (otherwise it wouldn't be a proportion).

$$\frac{\$}{\text{min}} = \frac{\$7.20}{60} = \frac{x}{10}$$

Now cross multiply.

$$60x = (7.20)(10)$$
$$60x = 72$$
$$\frac{60x}{60} = \frac{72}{60}$$
$$x = \frac{6}{5}$$

On the test, you'll enter 1.20 into the answer box.

AVERAGES

The **average** (arithmetic mean) of a list of numbers is the sum, or total value, of all the numbers in the list divided by the number of numbers in the list. The average of the list 1, 2, 3, 4, 5 is equal to the total of the numbers (1 + 2 + 3 + 4 + 5, or 15) divided by the number of numbers in the list (which is 5). Dividing 15 by 5 gives us 3, so 3 is the average of the list.

> GRE average problems always give you two of the three numbers needed.

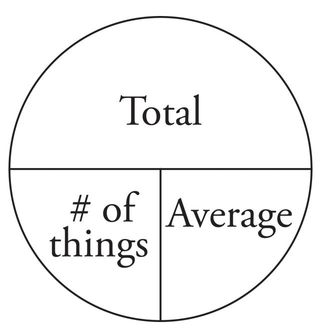
ETS always refers to an average as an “average (arithmetic mean).” This confusing parenthetical remark is meant to keep you from being confused by other more obscure kinds of averages, such as geometric and harmonic means. You’ll be less confused if you simply ignore the parenthetical remark and know that average means total of the elements divided by the number of elements.

Think Total

Don’t try to solve average problems all at once. Do them piece by piece. The key formula to keep in mind when doing problems that involve averages is

$$\text{Average} = \frac{\text{Total}}{\text{\# of things}}$$

Drawing an Average Pie will help you organize your information.



> The minute you see the word *average*, draw an Average Pie on your scratch paper.

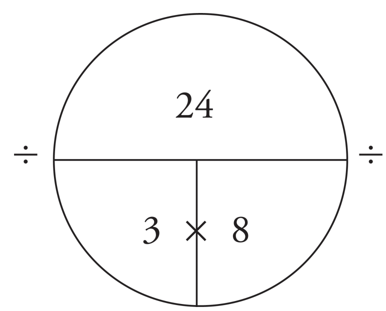
Here's how the Average Pie works. The *total* is the sum of the numbers being averaged. The *number of things* is the number of different elements that you are averaging. And the *average* is, naturally, the average.

Say you wanted to find the average of 4, 7, and 13. You would add those numbers to get the total and divide that total by three.

$$4 + 7 + 13 = 24$$

$$\frac{24}{3} = 8$$

Mathematically, the Average Pie works like this:



> Which two pieces of the pie do you have?

The horizontal bar is a division bar. If you divide the *total* by the *number of things*, you get the *average*. If you divide the total by the *average*, you get the *number of things*. If you have the *number of things* and the *average*, you can simply multiply them together to find the *total*. This is one of the most important things you need to be able to do to solve GRE average problems.

Using the Average Pie has several benefits. First, it's an easy way to organize information. Furthermore, the Average Pie makes it clear that if you have two of the three pieces, you can always find the third. This makes it easier to figure out how to approach the problem. If you fill in the number of things, for example, and the question wants to know the average, the Average Pie shows you that the key to unlocking that problem is finding the total.

Try this one.

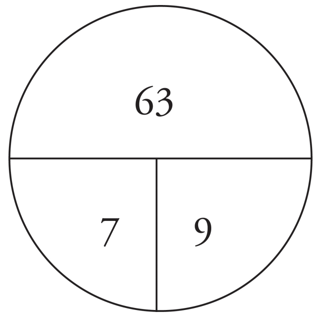
The average (arithmetic mean) of seven numbers is 9 and the average of three of these numbers is 5. What is the average of the other four numbers?

- ◯ 4
- ◯ 5
- ◯ 7
- ◯ 10
- ◯ 12

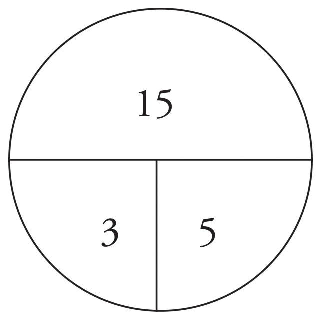
> Draw a new Average Pie each time you encounter the word average in a question.

Here's How to Crack It

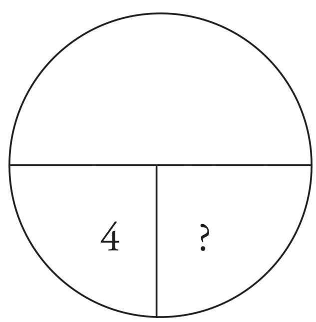
Let's take the first sentence. You have the word *average*, so draw an Average Pie and fill in what you know. We have seven numbers with an average of 9, so plug those values into the Average Pie and multiply to find the total.



Now we also know that three of the numbers have an average of 5, so draw another Average Pie, plug those values into their places, and multiply to find the total of those three numbers.

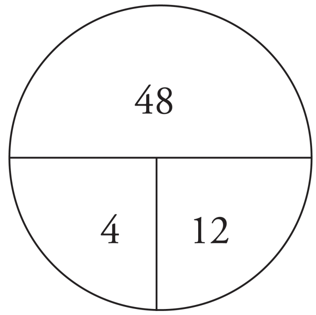


The question is asking for the average of the four remaining numbers. Draw one more Average Pie and Plug In 4 for the number of things.



In order to solve for the average, we need to know the total of those four numbers. How do we find this? From our first Average Pie we know that the total of all seven numbers is 63. The second Average Pie tells us that the total of three of those numbers was 15. Thus, the total of the remaining four has to be 63 – 15, which is 48. Plug

48 into the last Average Pie, and divide by 4 to get the average of the four numbers.



The average is 12, which is (E).

Let's try one more.

The average (arithmetic mean) of a set of 6 numbers is 28. If a certain number, y, is removed from the set, the average of the remaining numbers in the set is 24.

Quantity A	**Quantity B**
y	48

○ Quantity A is greater.

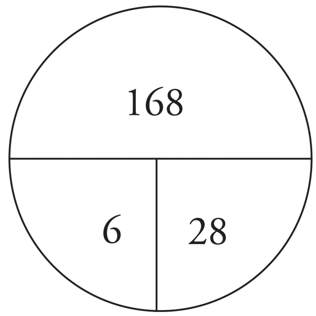
○ Quantity B is greater.

○ The two quantities are equal.

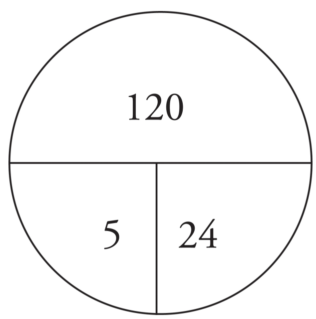
○ The relationship cannot be determined from the information given.

Here's How to Crack It

All right, let’s attack this one. The problem says that the average of a set of six numbers is 28, so let’s immediately draw an Average Pie and calculate the total.



If a certain number, y, is removed from the set, there are now five numbers left. We already know that the new average is 24, so draw another Average Pie.



The difference between the totals must be equal to y: 168 – 120 = 48. Thus, the two quantities are equal, and the answer is (C).

Up and Down

Averages are very predictable. You should make sure you automatically know what happens to them in certain situations. For example, suppose you take three tests and earn an average score of 90. Now you take a fourth test. What do you know?

If your average goes up as a result of the fourth score, then you know that your fourth score was higher than 90. If your average stays the same as a result of the fourth score, then you know that your fourth score was exactly 90. If your average goes down as a result of the fourth score, then you know that your fourth score was less than 90.

MEDIAN, MODE, AND RANGE

> Don't confuse median and mode!

The **median** is the middle value in a list of numbers; above and below the median lie an equal number of values. For example, in the list of numbers (1, 2, 3, 4, 5, 6, 7), the median is 4, because it's the middle number (and there are an odd number of numbers in the list). If the list contained an even number of integers such as (1, 2, 3, 4, 5, 6), the median is the average of 3 and 4, or 3.5. When looking for the median, sometimes you have to put the numbers in order yourself. What is the median of the list of numbers (13, 5, 6, 3, 19, 14, 8)? First, put the numbers in order from least to greatest, (3, 5, 6, 8, 13, 14, 19). Then take the middle number. The median is 8. Just think *median* = *middle* and always make sure the numbers are in order.

> When you see the word *median* in a question, put the numbers in the problem in order.

The **mode** is the number in a list of numbers that occurs most frequently. For example, in the list (2, 3, 4, 5, 3, 8, 6, 9, 3, 9, 3), the mode is 3, because 3 shows up the most. Just think *mode* = *most*.

The **range** is the difference between the greatest and the least numbers in a list of numbers. So, in the list of numbers (2, 6, 13, 3, 15, 4, 9), the range is 15 (the greatest number in the list) – 2 (the least number in the list), or 13.

Here's an example:

Set $F = \{4, 2, 7, 11, 8, 9\}$

Quantity A	**Quantity B**
The range of Set F	The median of Set F

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here's How to Crack It

Let's put the numbers in order first, so it'll be easier to see what we have: {2, 4, 7, 8, 9, 11}. First, let's look at Quantity A. The range is the greatest number, or 11, minus the least number, or 2. That's 9. Now let's look at Quantity B. The minute you see the word *median*, be sure to put the numbers in order. The median is the middle number of the set, but because there are two middle numbers, 7 and 8, we have to find the average. Or do we? Isn't the average of 7 and 8 clearly going to be smaller than the number in Quantity A, which is 9? Yes, in quant comp questions, we compare, not calculate. The answer is (A).

STANDARD DEVIATION

Standard deviation is one of those phrases that people uncomfortable with math shy away from. In truth, standard deviation is fairly straightforward and with some understanding and a little practice, you can conquer standard deviation on the GRE.

The prevalence of standard deviation questions on the GRE is small, and though the GRE might ask you questions about standard deviation, you'll never have to actually calculate it; instead, you'll just need a basic understanding of what standard deviation is and how it's tested on the GRE. Generally, the GRE treats standard deviation as a measure of spread.

A Question of Spread

The first thing to know is what is meant by standard deviation on the GRE. Standard deviation is a measure of the amount of spread, or variation, of a set of data values. A low standard deviation indicates that the data values tend to be close to the mean (thus, to have little spread), while a high standard deviation indicates that the values are spread out over a wider range. So, the further the distance between the members of a set and the set's average, the greater the standard deviation of the set.

Consider two sets of numbers, {4, 4, 4} and {3, 4, 5}. The first set, {4, 4, 4}, has a mean value of 4, as $\left(\frac{4+4+4}{3}\right)$ is equal to 4. However, since each member of the set is equal to the mean (and thus to each other member of the set), there is no distance between the members of the set and the mean of the set, so there is no spread amongst the numbers. Now look at the second set, {3, 4, 5}. This set also has a mean of 4, as $\left(\frac{3+4+5}{3}\right)$ is equal to 4. However, in this group, instead of all numbers being equal to the mean, two members of the set have some distance from the mean (3 and 5 are not equal to the mean of 4). Therefore, since the second set has more spread (the members of the set have more distance from the mean), the second set has a greater standard deviation.

Here's an example of how ETS might test standard deviation:

Quantity A	**Quantity B**
The standard deviation of a list of data consisting of 10 integers ranging from −20 to −5	The standard deviation of a list of data consisting of 10 integers ranging from 5 to 20

◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

Here's How to Crack It

ETS is hoping you'll make a couple of wrong turns on this problem. The first trap they set is that one of the list of numbers contains negative integers while the other doesn't—but this doesn't mean that one list has a negative standard deviation. Standard deviation is defined as the distance a point is from the mean, so it can never be negative. The second trap is that ETS hopes you'll waste a lot of time trying to calculate standard deviation based on the information given. But you know better than to try to do that. Remember that ETS won't ask you to calculate standard deviation; it's a complex calculation. Plus, as you know, you need to know the mean in order to calculate the standard deviation and there's no way we can find it based on the information here. Thus, we have no way of comparing these two quantities, so the answer is (D).

Now, let's try a question that deals with standard deviation differently.

For a certain distribution, the value 12.0 is one standard deviation above the mean and the value 15.0 is three standard deviations above the mean. What is the mean of the data set?

- 9.0
- 9.5
- 10.0
- 10.5
- 11.0

Here's How to Crack It

This one's a little tougher than the earlier standard deviation questions. The question provides neither the individual data points nor the mean, and ETS is hoping that this will throw you off. But remember that standard deviation deals with the distance from the mean. Start by determining the size of one standard deviation. Since 12.0 is one standard deviation above the mean and 15.0 is three standard deviations above the mean, the difference between the data values represents the difference in the number of standard deviations. Therefore,

$$15.0 - 12.0 = 3.0 \text{ st dev} - 1.0 \text{ st dev, so } 3.0 = 2.0 \text{ st dev}$$

Now set up a proportion to find the size of one standard deviation:

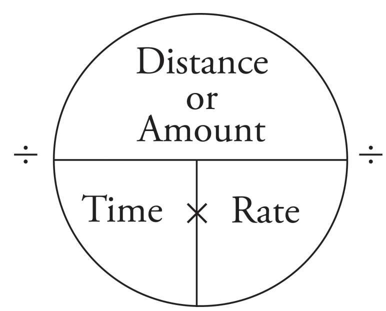
$$\frac{3.0}{2.0\,\mathit{st\ dev}} = \frac{x}{1\,\mathit{st\ dev}}$$

Solve to find that $x = 1.5$, which means that one standard deviation for this data set is equal to 1.5. Since 12.0 is one standard deviation above the mean, the mean of the data set is 12.0 – 1.5 = 10.5, so the answer is (D).

RATE

Rate problems are similar to average problems. A rate problem might ask for an average speed, distance, or the length of a trip, or how long a trip (or a job) takes. To solve rate problems, use the Rate Pie.

> A rate problem is really just an average problem.



The Rate Pie works exactly the same way as the Average Pie. If you divide the *distance* or *amount* by the *rate*, you get the *time*. If you divide the *distance* or *amount* by the *time*, you get the *rate*. If you multiply the *rate* by the *time*, you get the *distance* or *amount*.

Let's take a look.

It takes Carla three hours to drive to her brother's house at an average speed of 50 miles per hour. If she takes the same route home, but her average speed is 60 miles per hour, what is the time, in hours, that it takes her to drive home?

◯ 2 hours

◯ 2 hours and 14 minutes

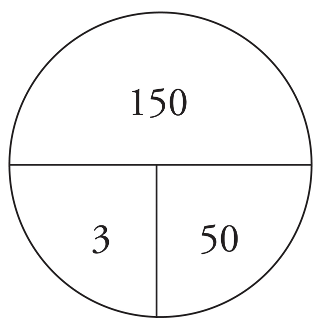
◯ 2 hours and 30 minutes

◯ 2 hours and 45 minutes

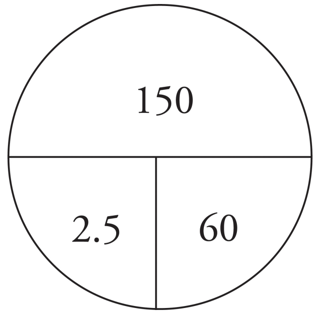
◯ 3 hours

Here's How to Crack It

The trip to her brother's house takes three hours, and the rate is 50 miles per hour. Plug those numbers into a Rate Pie and multiply to find the distance.



So the distance is 150 miles. On her trip home, Carla travels at a rate of 60 miles per hour. Draw another Rate Pie and Plug In 150 and 60. Then all you have to do is divide 150 by 60 to find the time.



So it takes Carla two and a half hours to get home. That’s (C).

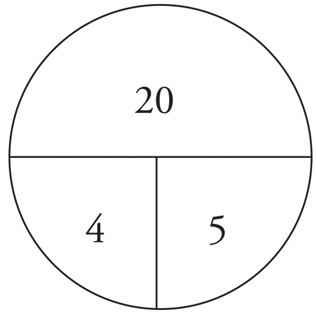
Try another one.

A machine can stamp 20 envelopes in 4 minutes. How many of these machines, working simultaneously, are needed to stamp 60 envelopes per minute?

- ◯ 5
- ◯ 10
- ◯ 12
- ◯ 20
- ◯ 24

Here’s How to Crack It

First, we have to find the rate per minute of one machine. Plug 20 and 4 into a Rate Pie and divide to find the rate.



The rate is 5. If one machine can stamp 5 envelopes per minute, how many machines do you need to stamp 60 per minute? 60 ÷ 5 = 12, or (C).

CHARTS

Every GRE Math section has a few questions that are based on a chart or graph (or on a group of charts or graphs). But don't worry; the most important thing that chart questions test is your ability to interpret the information provided in the chart. Remember, the information is already there for you! You just need to go find it.

Chart Questions

There are usually two or three questions per chart or per set of charts. Chart questions appear on split screens. Be sure to click on the scroll bar and scroll down as far as you can; there may be additional charts underneath the top one, and you want to make sure you've seen all of them.

Chart problems mostly recycle the basic arithmetic concepts we've already covered: fractions, percentages, and so on. This means you can use the techniques we've discussed for each type of question, but there are two additional techniques that are especially important to use when doing chart questions.

Get Your Bearings

Before you start the questions, spend a few seconds looking over the charts.

- Scroll down to find any data or notes that aren't visible.
- Read titles and legends.
- Check units.

Work the Questions

1. Read the questions and determine
    - Which chart the question deals with
    - What information you need to find
    - What calculations you need to perform
2. Work the chart(s)
    - Get the information you need from the chart(s).
    - Write down what you found on your scratch paper.
3. Approximate or calculate
    - Are the numbers in the answers far apart? If so, you can probably estimate the answer.
    - Use the calculator!

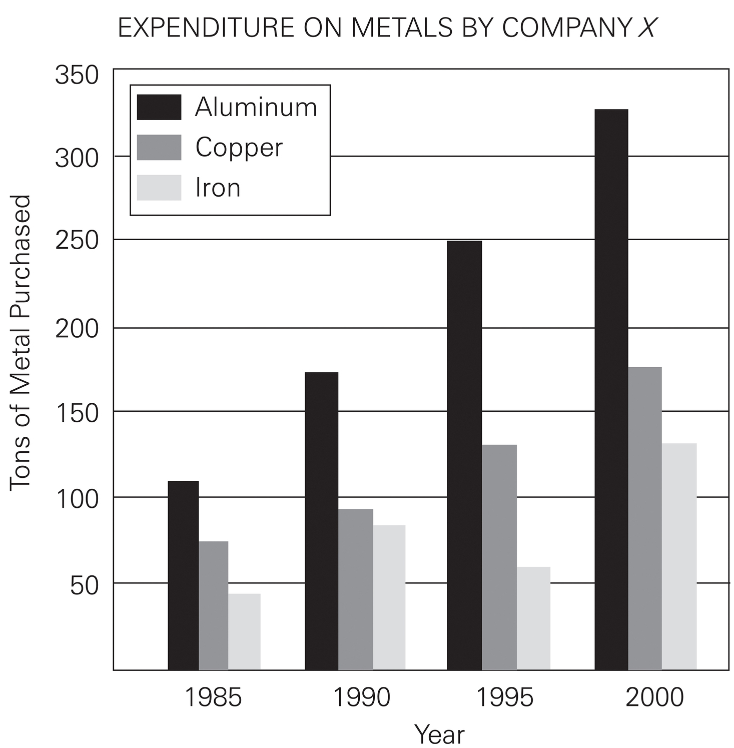
Notice that, when working the questions, the final step is to determine whether to approximate or to calculate the answer. While the calculator is available to you, it's important to remember that the calculator is only a tool and is only as smart as you are. Additionally, you can save yourself precious seconds by knowing the "friendly" percentages and their fractions (from back in Chapter 10) by heart.

Nonetheless, with chart questions it is occasionally easier to estimate percentages and round to whole numbers. This is most effective when the numbers in the answers are far apart and the values in the chart are difficult to work with. By estimating and rounding numbers under these conditions, often you can determine the correct answer based on the proximity of the answer to your estimated values.

Chart Problems

Make sure you've read everything on the chart carefully before you try the first question.

> Chart questions frequently test percents, percent change, ratios, proportions, and averages.



Year	Price of Aluminum Per Ton
1985	$1,900
1990	$2,200
1995	$2,700
2000	$3,400

Approximately how many tons of aluminum and copper combined were purchased in 1995 ?

- ◯ 125
- ◯ 255
- ◯ 325
- ◯ 375
- ◯ 515

How much did Company *X* spend on aluminum in 1990 ?

- ◯ $675,000
- ◯ $385,000
- ◯ $333,000
- ◯ $165,000
- ◯ $139,000

Approximately what was the percent increase in the price of aluminum from 1985 to 1995 ?

- ◯ 8%
- ◯ 16%
- ◯ 23%
- ◯ 30%
- ◯ 42%

Here’s How to Crack the First Question

As you can see from the graph on the previous page, in 1995, the black bar (which indicates aluminum) is at 250, and the dark gray bar (which indicates copper) is at approximately 125. Add those figures and you get the number of tons of aluminum and copper combined that were purchased in 1995: 250 + 125 = 375. That's (D). Notice that the question says "approximately." Also notice that the numbers in the answer choices are pretty far apart.

Here's How to Crack the Second Question

We need to use the chart and the graph to answer this question, because we need to find the number of tons of aluminum purchased in 1990 and multiply it by the price per ton of aluminum in 1990 in order to figure out how much was spent on aluminum in 1990. The bar graph tells us that 175 tons of aluminum was purchased in 1990, and the little chart tells us that aluminum was $2,200 per ton in 1990: 175 × $2,200 = $385,000. That's (B).

Here's How to Crack the Third Question

Remember that percent increase formula from Chapter 10, Math Fundamentals?

$$\text{Percent change} = \frac{\text{Difference}}{\text{Original}} \times 100$$

We'll need to use the little chart for this one. In 1985, the price of aluminum was $1,900 per ton. In 1995, the price of aluminum was $2,700 per ton. Now let's use the formula: 2,700 – 1,900 = 800, so that's the difference. This is a percent increase problem, so the original number is the smaller one. Thus, the original is 1,900, and our formula looks like this: Percent change = $\frac{800}{1{,}900} \times 100$. By canceling the 0's in the fraction, you get $\frac{8}{19} \times 100$, and multiplying gives you $\frac{800}{19}$. At this point you could divide 800 by 19 to get the exact answer, but because they're looking for an approximation, let's round 19 to 20. What's 800 ÷ 20? That's 40, and (E) is the only one that's close.

Real-World Math Drill

Click here to download a PDF of Real-World Math Drill.

Now it's time to try out what you have learned on some practice questions. Try the following problems and then check your answers in Part V.

1 of 12

Sadie sells half the paintings in her collection, gives one-third of her paintings to friends, and keeps the remaining paintings for herself. What fraction of her collection does Sadie keep?

2 of 12

$$5x - 2y = 2y - 3x$$

Quantity A	**Quantity B**
x	y

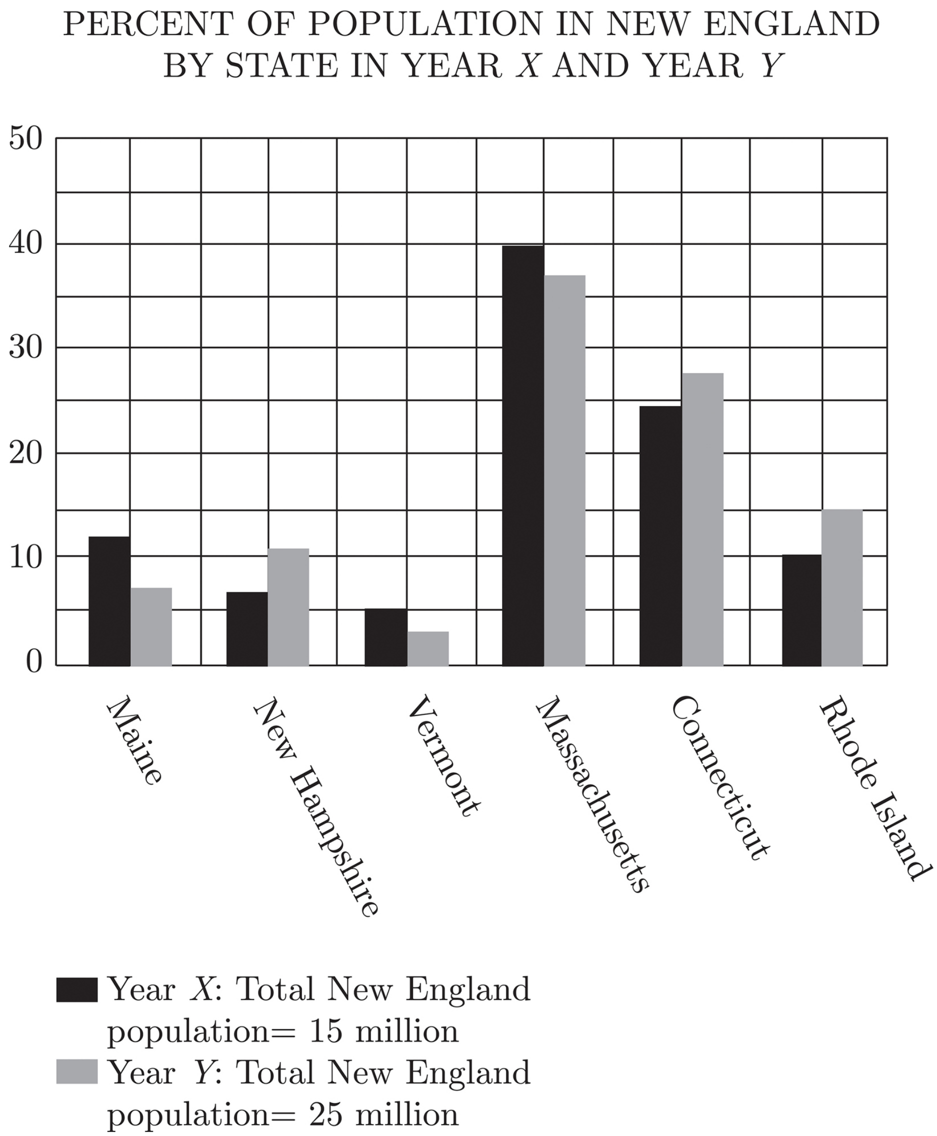
◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

Questions 3 through 5 refer to the following graph.



If the six New England states are ranked by population in Year *X* and Year *Y*, how many states would have a different ranking from Year *X* to Year *Y* ?

◯ None

◯ One

◯ Two

◯ Three

○ Four

4 of 12

In Year *X*, the population of Massachusetts was approximately what percent of the population of Vermont?

○ 50%

○ 120%

○ 300%

○ 800%

○ 1,200%

5 of 12

By approximately how much did the population of Rhode Island increase from Year *X* to Year *Y*?

○ 750,000

○ 1,250,000

○ 1,500,000

○ 2,250,000

○ 3,375,000

6 of 12

A water jug with a capacity of 20 gallons is 20 percent full. At the end of every third day, water is added to the jug. If the amount of water added is equal to 50 percent of the water in the jug at the beginning of that day, how many days does it take for the jug to be at least 85% full?

○ 4

○ 6

○ 12

○ 15

○ 20

7 of 12

Towns *A, B, C*, and *D* are all in the same voting district. Towns *A* and *B* have 3,000 people each who support referendum *R* and the referendum has an average (arithmetic mean) of 3,500 supporters in towns *B* and *D* and an average of 5,000 supporters in Towns *A* and *C*.

Quantity A	**Quantity B**
The average number of supporters of Referendum *R* in Towns *C* and *D*	The average number of supporters of Referendum *R* in Towns *B* and *C*

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

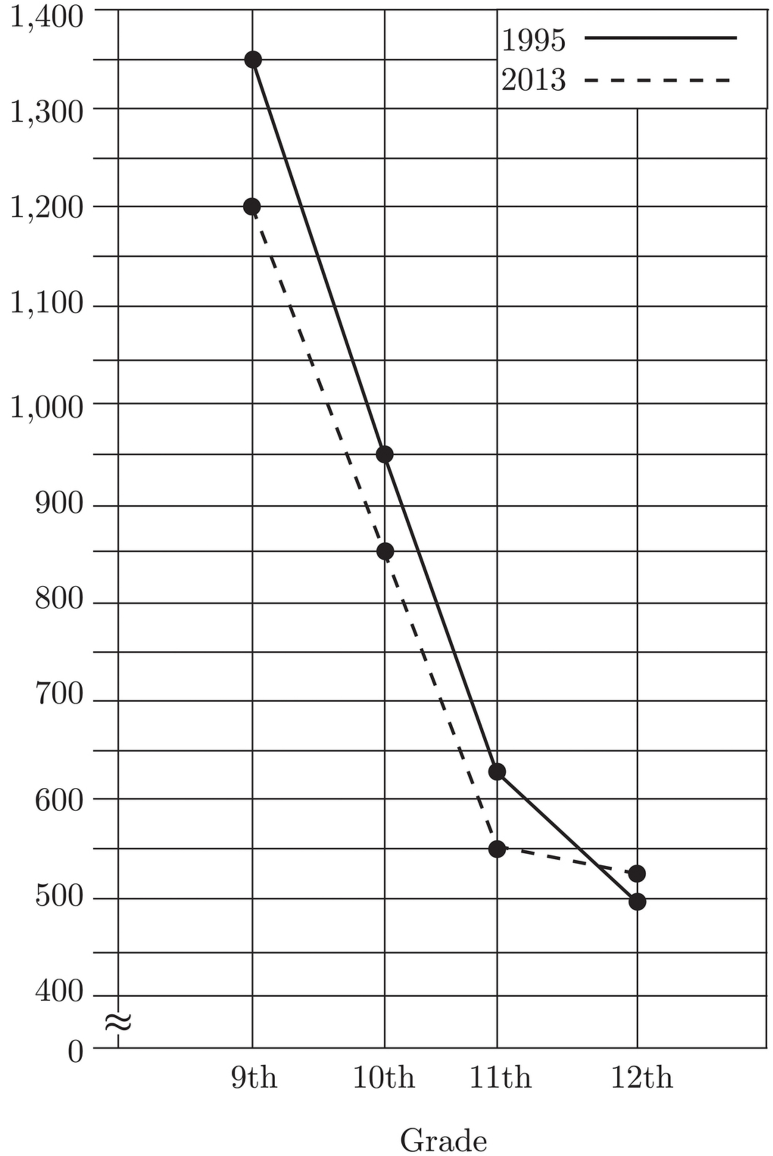
8 of 12

A company paid $500,000 in merit raises to employees whose performances were rated *A*, *B*, or *C*. Each employee rated *A* received twice the amount of the raise that was paid to each employee rated *C*; and each employee rated *B* received one-and-a-half times the amount of the raise that was paid to each employee rated *C*. If 50 workers were rated *A*, 100 were rated *B*, and 150 were rated *C*, how much was the raise paid to each employee rated *A* ?

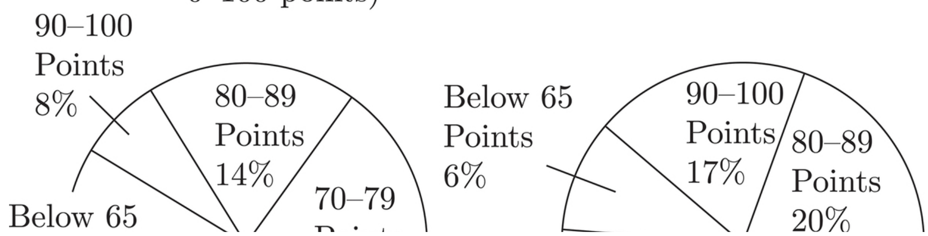
- ◯ $370
- ◯ $625
- ◯ $740
- ◯ $1,250
- ◯ $2,500

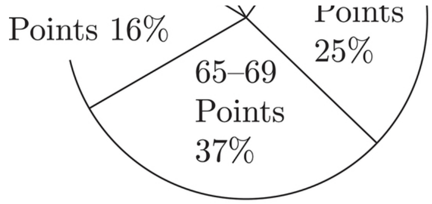
Questions 9 through 11 refer to the following graphs.

NUMBER OF STUDENTS IN GRADES
9 THROUGH 12 FOR SCHOOL DISTRICT
X IN 1995 AND 2013

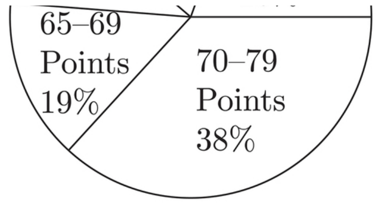


DISTRIBUTION OF READING TEST
SCORES* FOR SCHOOL DISTRICT *X*
STUDENTS IN 2013
(*Reading Test scores can range from
0–100 points)





9th Grade Students



10th–12th Grade Students

9 of 12

In 2013, the median reading test score for ninth-grade students was in which score range?

- ◯ Below 65 points
- ◯ 65–69 points
- ◯ 70–79 points
- ◯ 80–89 points
- ◯ 90–100 points

10 of 12

If the number of students in grades 9 through 12 in School District X in 1995 comprised 35 percent of the number of students in City Y in 1995, then approximately how many students were in City Y in 1995 ?

- ◯ 9,700
- ◯ 8,700
- ◯ 3,400
- ◯ 3,000
- ◯ 1,200

11 of 12

Assume that all students in School District X took the reading test each year. In 2013, approximately how many more ninth-grade students had reading test scores in the 70–79 point range than in the 80–89 point range?

- ◯ 470
- ◯ 300
- ◯ 240
- ◯ 170
- ◯ 130

12 of 12

One ounce of Solution X contains only ingredients a and b in a ratio of $2:3$. One ounce of Solution Y contains only ingredients a and b in a ratio of $1:2$. If Solution Z is created by mixing solutions X and Y in a ratio of $3:11$, then 630 ounces of Solution Z contains how many ounces of a ?

- ◯ 68
- ◯ 73
- ◯ 89
- ◯ 219
- ◯ 236

Summary

- A ratio expresses a part-to-part relationship. The key to ratio problems is finding the total. Use the ratio box to organize ratio questions.
- A proportion expresses the relationship between equal fractions, percents, or ratios. A proportion problem always provides you with three pieces of information and asks you for a fourth.
- Use the Average Pie to organize and crack average problems.
- The median is the middle number in a set of values. The mode is the value that appears most frequently in a set. The range of a set is the difference between the largest and smallest values in the set.
- You will never have to calculate standard deviation on the GRE.
- Standard deviation problems are really average and percent problems. Make sure you know the percentages associated with the bell curve: 34%, 14%, 2%.
- Use the Rate Pie for rate questions.
- On chart questions, make sure you take a moment to understand what information the chart is providing. Estimate answers to chart questions whenever possible.

Chapter 13
Geometry

Chances are you probably haven't used the Pythagorean Theorem recently or had to find the area of a circle in quite a while. However, you'll be expected to know geometry concepts such as these on the GRE. This chapter reviews all the important rules and formulas you'll need to crack the geometry problems on the GRE. It also provides examples of how such concepts will be tested on the GRE Math section.

WHY GEOMETRY?

Good question. If you're going to graduate school for political science or linguistics or history or practically anything that doesn't involve math, you might be wondering why the heck you have to know the area of a circle or the Pythagorean Theorem for this exam. While we may not be able to give you a satisfactory answer to that question, we can help you do well on the geometry questions on the GRE.

> Expect to see a handful of basic geometry problems on each of your Math sections.

WHAT YOU NEED TO KNOW

The good news is that you don't need to know much about actual geometry to do well on the GRE; we've boiled down geometry to the handful of bits and pieces that ETS actually tests.

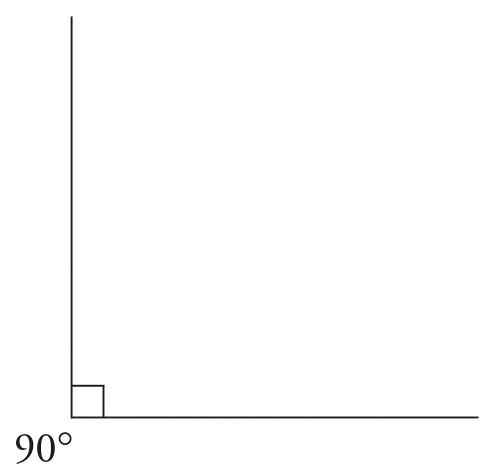
Before we begin, consider yourself warned. Since you'll be taking your test on a computer screen, you'll have to be sure to transcribe all the figures onto your scrap paper accurately. All it takes is one mistaken angle or line and you're sure to get the problem wrong. So make ample use of your scratch paper and always double-check your figures. Start practicing now, by using scratch paper with this book.

Another important thing to know is that you cannot necessarily trust the diagrams ETS gives you. Sometimes they are very deceptive and are intended to confuse you. Always go by what you read, not what you see.

DEGREES, LINES, AND ANGLES

For the GRE, you will need to know that

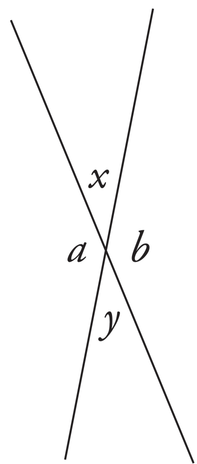
1. A line is a 180-degree angle. In other words, a line is a perfectly flat angle.
2. When two lines intersect, four angles are formed; the sum of these angles is 360 degrees.
3. When two lines are perpendicular to each other, their intersection forms four 90-degree angles. Here is the symbol ETS uses to indicate perpendicular lines: ⊥.
4. Ninety-degree angles are also called *right angles*. A right angle on the GRE is identified by a little box at the intersection of the angle's arms.



5. The three angles inside a triangle add up to 180 degrees.
6. The four angles inside any four-sided figure add up to 360 degrees.
7. A circle contains 360 degrees.
8. Any line that extends from the center of a circle to the edge of the circle is called a *radius* (plural is *radii*).

Vertical Angles

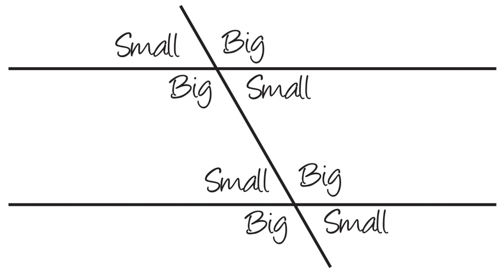
Vertical angles are the angles that are across from each other when two lines intersect. Vertical angles are always equal. In the drawing below, angle *x* is equal to angle *y* (they are vertical angles) and angle *a* is equal to angle *b* (they are also vertical angles).

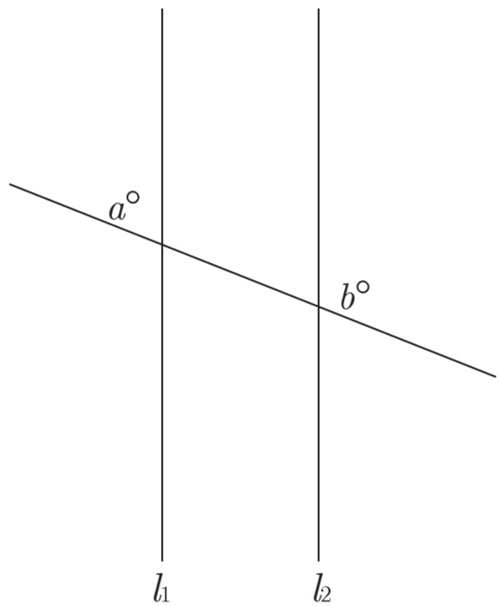


> On the GRE, the measure of only one of the vertical angles is typically shown. But usually you'll need to use the other angle to solve the problem.

Parallel Lines

Parallel lines are lines that never intersect. When a pair of parallel lines is intersected by a third, two types of angles are formed: big angles and small angles. Any big angle is equal to any big angle, and any small angle is equal to any small angle. The sum of any big angle and any small angle will always equal 180. When ETS tells you that two lines are parallel, this is what is being tested. The symbol for parallel lines and the word *parallel* are both clues that tell you what to look for in the problem. The minute you see either of them, immediately identify your big and small angles; they will probably come into play.





l_1 and l_2 are parallel.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$a + b$	180

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

Here’s How to Crack It

Notice that you're told that these lines are parallel. Here's one very important point: you need to be told that. You can't assume that they are parallel just because they look like they are.

Okay, so as you just learned, only two angles are formed when two parallel lines are intersected by a third line: a big angle (greater than 90 degrees) and a small one (smaller than 90 degrees). More importantly, you learned that all the big angles are equal, and all the small angles are equal. Therefore, the angle directly across l_1 from angle a is the same as angle b. These now form a straight line, so $a + b = 180$, and the correct answer is (C).

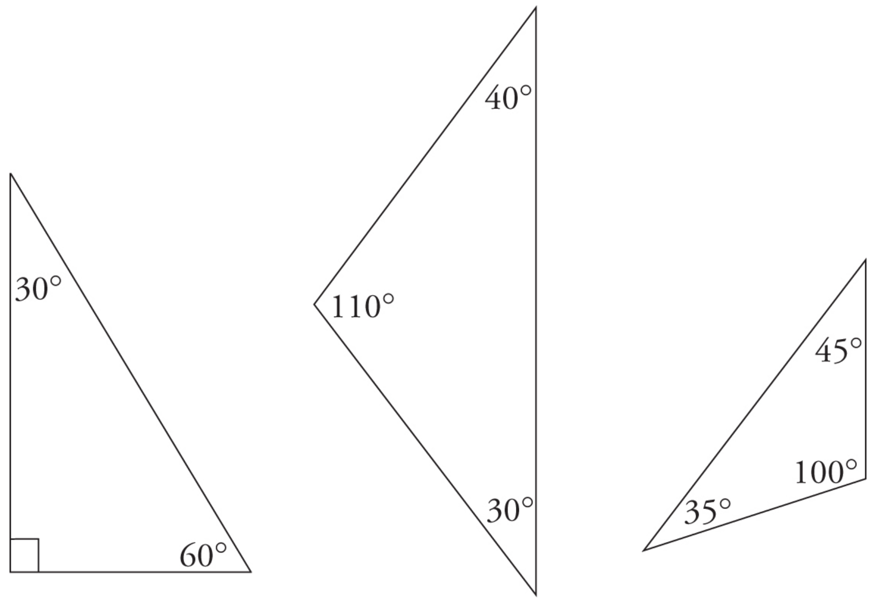
TRIANGLES

Triangles are perhaps ETS's favorite geometrical shape. Triangles have many properties, which make them great candidates for standardized test questions. Make sure you familiarize yourself with the following triangle facts.

> Triangles are frequently tested on the GRE.

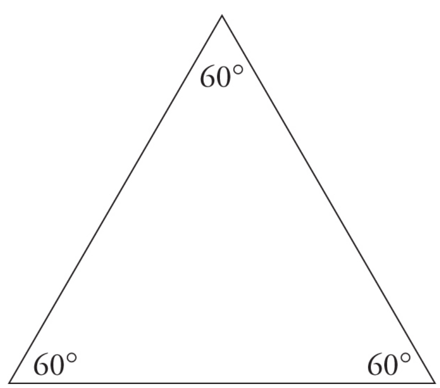
The Rule of 180°

Every triangle contains three angles that add up to 180 degrees. You must know this fact cold for the exam. This rule applies to every triangle, no matter what it looks like. Here are some examples:



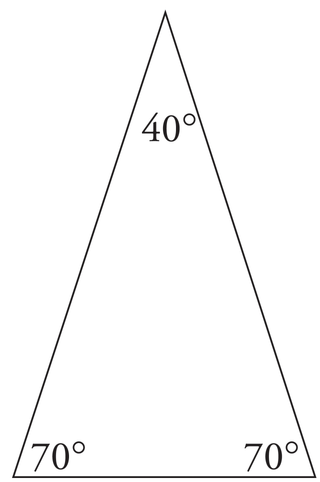
Equilateral Triangles

An **equilateral triangle** is a triangle in which all three sides are equal in length. Because all of the sides are equal in these triangles, all of the angles are equal. Each angle is 60 degrees because 180 divided by 3 is 60.



Isosceles Triangles

An **isosceles triangle** is a triangle in which two of the three sides are equal in length. This means that two of the angles are also equal.



Angle/Side Relationships in Triangles

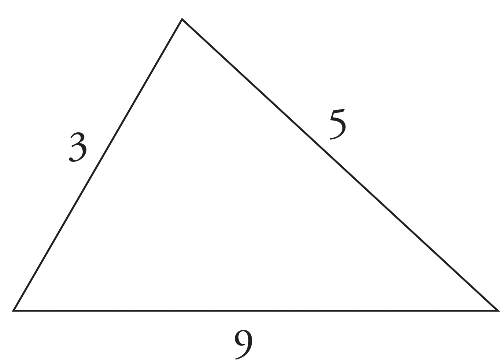
In any triangle, the longest side is opposite the largest interior angle; the shortest side is opposite the smallest interior angle. That's why the hypotenuse of a right triangle is its longest side—there couldn't be another angle in the triangle bigger than 90 degrees. Furthermore, equal sides are opposite equal angles.

Perimeter of a Triangle

The perimeter of a triangle is simply a measure of the distance around it. All you have to do to find the perimeter of a triangle is add up the lengths of the sides.

The Third Side Rule

Why is it impossible for the following triangle to exist?



This triangle could not exist because the length of any one side of a triangle is limited by the lengths of the other two sides. This can be summarized by the **third side rule**:

> The length of any one side of a triangle must be less than the sum of the other two sides and greater than the difference between the other two sides.

This rule is not tested frequently on the GRE, but when it is, it's usually the key to solving the problem. Here's what the rule means in application: take the lengths of any two sides of a triangle. Add them together, then subtract one from the other. The length of the third side must lie between those two numbers.

Take the sides 3 and 5 from the triangle above. What's the longest the third side could measure? Just add and subtract. It could not be as long as 8 (5 + 3), and it could not be as short as 2 (5 – 3).

Therefore, the third side must lie between 2 and 8. It's important to remember that the third side cannot be equal to either 2 or 8. It must be greater than 2 and less than 8.

Try the following question:

A triangle has sides 4, 7, and *x*. Which of the following could be the perimeter of the triangle?

Indicate <u>all</u> such perimeters.

☐ 13

☐ 16

☐ 17

☐ 20

☐ 22

Here's How to Crack It

Remember the third side rule of triangles here, which is how to find possible lengths of the third side of a triangle when given the two other sides. The third side rule dictates that the length of the third side of a triangle must be greater than the difference, but less than the sum, of the length of the two known sides. In this particular problem, the two known sides are 4 and 7. The difference between 4 and 7 is 3, and the sum of 4 and 7 is 11, so the third side of the triangle must be greater than 3 and less than 11. This can be represented by the expression $3 < x < 11$. Use these values to create a range for the possible perimeter of the triangle. If the third side of the triangle is 3, and the other two sides are 4 and 7, the perimeter is $3 + 4 + 7 = 14$. If the third side of the triangle is 11, and the other two sides are 4 and 7, then the perimeter is $11 + 4 + 7 = 22$. Because the third side of the triangle is *greater* than 3 and *less* than 11, the perimeter of the triangle must be *greater* than 14 and *less* than 22. This can be represented by the expression $14 < perimeter < 22$. The only answer choices that fall in that range are (B), (C), and (D), which are the correct answers.

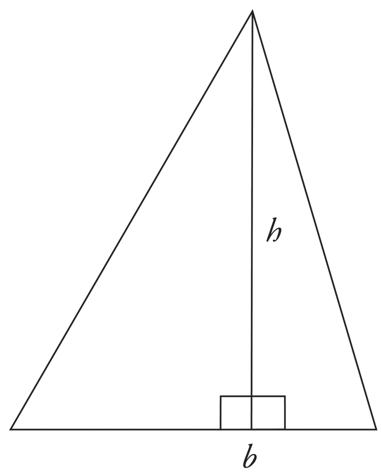
Area of a Triangle

The area of any triangle is equal to its height (or altitude) multiplied by its base, divided by 2, so

$$A = \frac{1}{2}bh$$

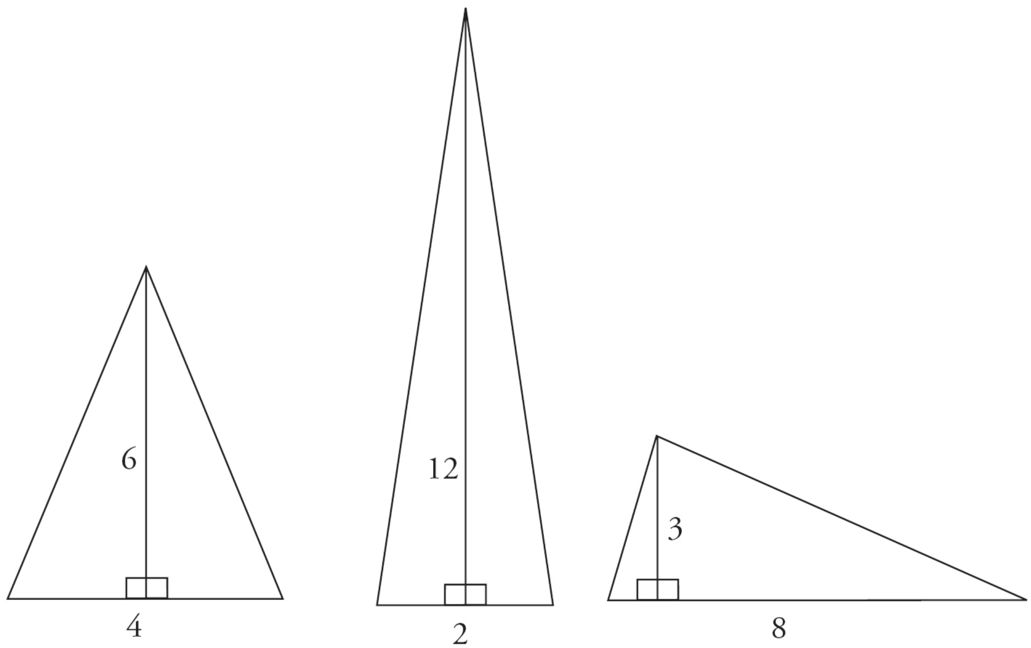
> Any time you see the word *area* or any other word that indicates that a formula is to be used, write the formula on your scratch paper and park the information you're given directly underneath.

The height of a triangle is defined as the length of a perpendicular line drawn from the point of the triangle to its base.

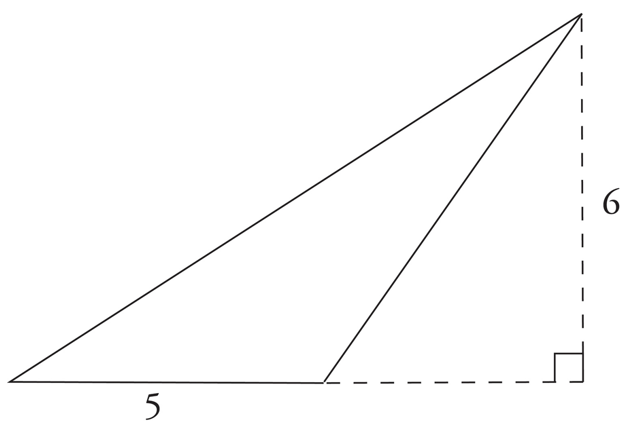


> The height of a triangle must be perpendicular to the base.

This area formula works on any triangle.



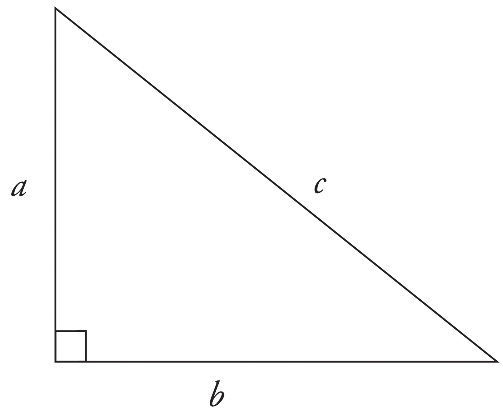
area = 12 *area* = 12 *area* = 12



area = 15

The Pythagorean Theorem

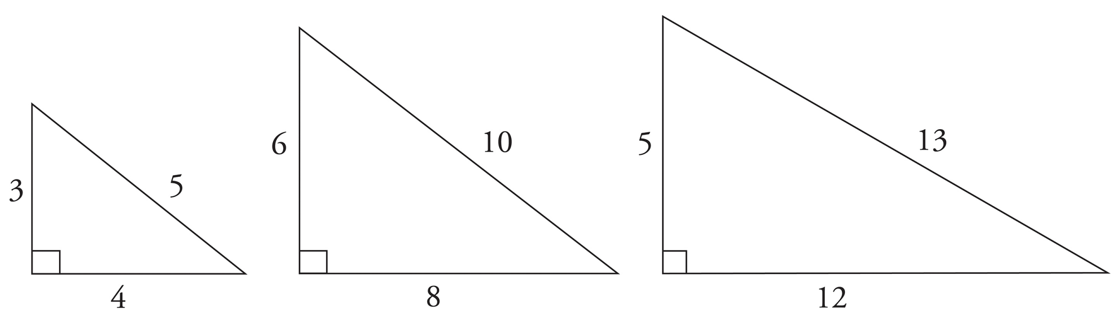
The Pythagorean Theorem applies only to right triangles. This theorem states that in a right triangle, the square of the length of the hypotenuse (the longest side, remember?) is equal to the sum of the squares of the lengths of the two other sides. In other words, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the other sides. (The two sides that are not the hypotenuse are called the legs.)



You can always use the Pythagorean Theorem to calculate the third side of a right triangle.

> ETS will sometimes try to intimidate you by using multiples of the common Pythagorean triples. For example, you might see a 10-24-26 triangle. That's just a 5-12-13 in disguise.

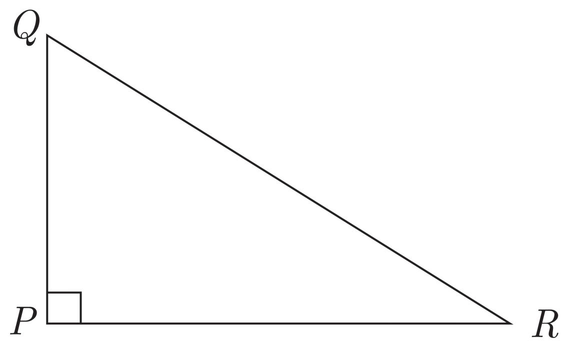
Here are the most common right triangles:



Note that a triangle could have sides with actual lengths of 3, 4, and 5, or 3 : 4 : 5 could just be the ratio of the sides. If you double the ratio, you get a triangle with sides equal to 6, 8, and 10. If you triple it, you get a triangle with sides equal to 9, 12, and 15.

Let's try an example.

Write everything down on scratch paper! Don't do anything in your head.



In the figure above, if the distance from point P to point Q is 6 miles and the distance from point Q to point R is 10 miles, what is the distance from point P to point R?

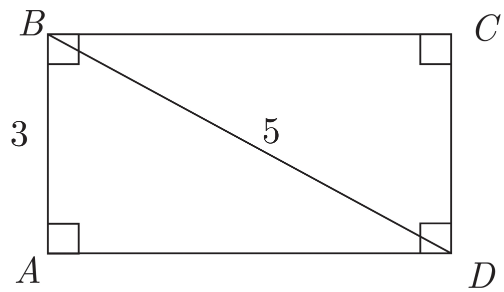
- 4
- 5
- 6
- 7
- 8

Here's How to Crack It

Once you've sensitized yourself to the standard right triangles, this problem couldn't be easier. When you see a right triangle, be suspicious. One leg is 6. The hypotenuse is 10. The triangle has a ratio of 3:4:5. Therefore, the third side (the other leg) must be 8.

The Pythagorean Theorem will sometimes help you solve problems that involve squares or rectangles. For example, every rectangle or square can be divided into two right triangles. This means that if you know the length and width of any rectangle or square, you also know the length of the diagonal—it's the shared hypotenuse of the hidden right triangles.

Here's an example:



In the rectangle above, what is the area of triangle *ABD* ?

Here's How to Crack It

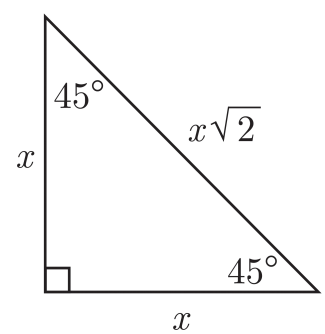
We were told that this is a rectangle (remember that you can never assume!), which means that triangle *ABD* is a right triangle. Not only that, but it's a 3:4:5 right triangle (with a side of 3 and a hypotenuse of 5, it must be), with side *AD* = 4. So, the area of triangle *ABD* is $\frac{1}{2}$ the base (4) times the height (3). That's $\frac{1}{2}$ of 12, or 6. Enter that value into the box.

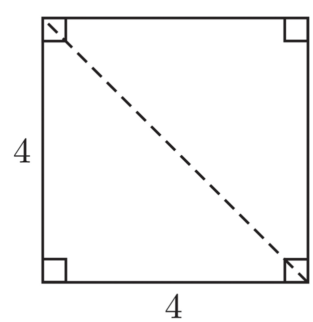
Right Isosceles Triangles

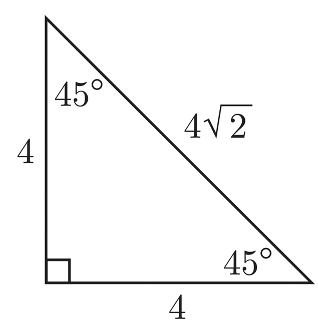
If you take a square and cut it in half along its diagonal, you will create a right isosceles triangle. The two sides of the square stay the same. The 90-degree angle will stay the same, and the other two angles that were 90 degrees each get cut in half and are now 45 degrees. The ratio of sides in a right isosceles triangle is $x : x : x\sqrt{2}$.

This is significant for two reasons. First, if you see a problem with a right triangle and there is a $\sqrt{2}$ anywhere in the problem, you know what to look for. Second, you always know the length of the diagonal of a square because it is one side times the square root of two.

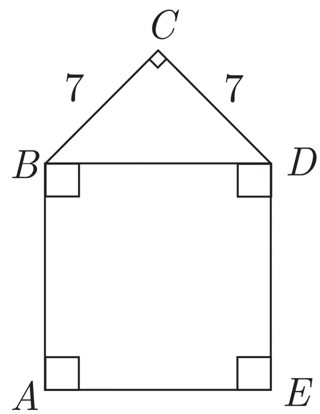
> You always know the length of the diagonal of a square because it is one side of the square times $\sqrt{2}$.







Let’s try an example involving a special right triangle.



In the figure above, what is the area of square *ABDE* ?

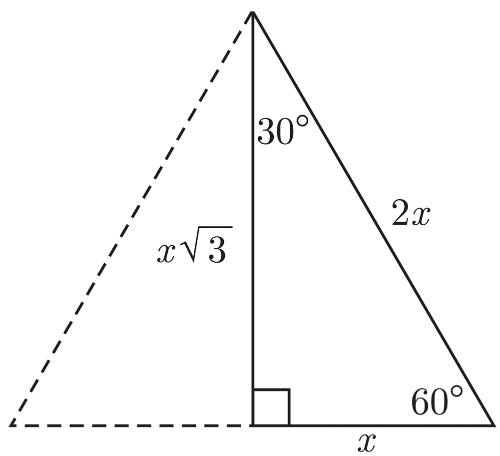
- ○ $28\sqrt{2}$
- ○ 49
- ○ $49\sqrt{2}$
- ○ 98
- ○ $98\sqrt{2}$

Here's How to Crack It

In order to figure out the area of square *ABDE*, we need to know the length of one of its sides. We can get the length of *BD* by using the isosceles right triangle attached to it. *BD* is the hypotenuse, which means its length is $7\sqrt{2}$. To get the area of the square, we have to square the length of the side we know, or $(7\sqrt{2})(7\sqrt{2})$ = (49)(2) = 98. That's (D).

30:60:90 Triangles

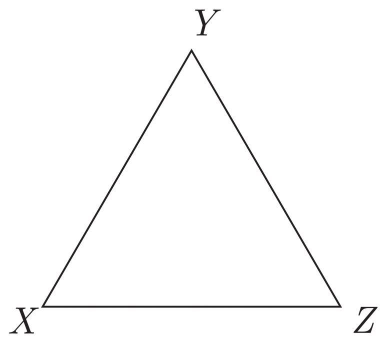
If you take an equilateral triangle and draw in the height, you end up cutting it in half and creating a right triangle. The hypotenuse of the right triangle has not changed; it's just one side of the equilateral triangle. One of the 60-degree angles stays the same as well. The angle where the height meets the base is 90 degrees, naturally, and the side that was the base of the equilateral triangle has been cut in half. The smallest angle, at the top, opposite the smallest side, is 30 degrees. The ratio of sides on a 30:60:90 triangle is $x : x\sqrt{3} : 2x$. Here's what it looks like:



> You can always calculate the area of an equilateral triangle because you know that the height is one half of one side times $\sqrt{3}$.

This is significant for two reasons. The first is that if you see a problem with a right triangle and one side is double the other or there is a $\sqrt{3}$ anywhere in the problem, you know what to look for. The second is that you always know the area of an equilateral triangle because you always know the height. The height is one half of one side times the square root of 3.

Here's one more:



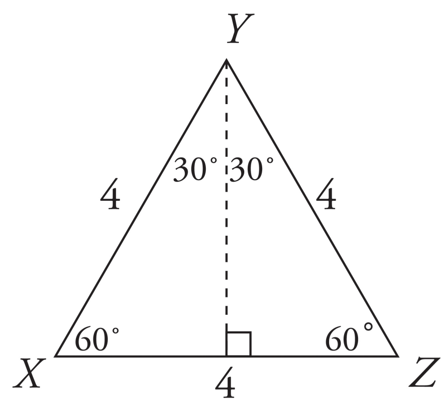
Triangle *XYZ* in the figure above is an equilateral triangle. If the perimeter of the triangle is 12, what is its area?

- ◯ $2\sqrt{3}$
- ◯ $4\sqrt{3}$
- ◯ 8
- ◯ 12
- ◯ $8\sqrt{3}$

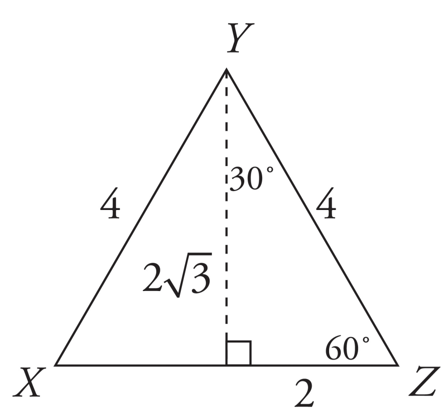
> If you see $\sqrt{2}$ or $\sqrt{3}$ in the answer choices of the problem, it's a tip-off that the problem is testing special right triangles.

Here's How to Crack It

Here we have an equilateral triangle with a perimeter of 12, which means that each side has a length of 4 and each angle is 60 degrees. Remember that in order to find the area of a triangle, we use the triangle area formula: $A = \frac{1}{2}bh$, but first we need to know the base and the height of the triangle. The base is 4, which now gives us $A = \frac{1}{2}4h$, and now the only thing we need is the height. Remember that the height always has to be perpendicular to the base. Draw a vertical line that splits the equilateral triangle in half. The top angle is also split in half, so now we have this:



What we've done is create two 30:60:90 right triangles, and we're going to use one of these right triangles to find the height. Let's use the one on the right. We know that the hypotenuse in a 30:60:90 right triangle is always twice the length of the short side. Here we have a hypotenuse (*YZ*) of 4, so our short side has to be 2. The long side of a 30:60:90 right triangle is always equal to the short side multiplied by the square root of 3. So if our short side is 2, then our long side must be $2\sqrt{3}$. That's the height.



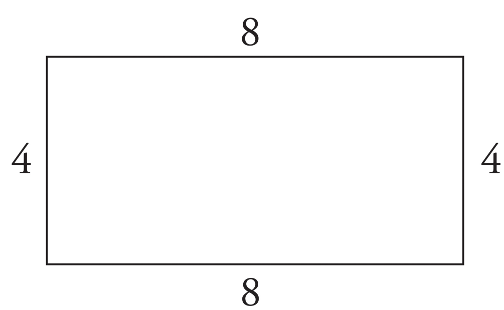
Finally, we return to our area formula. Now we have $A = \frac{1}{2} \times 4 \times 2\sqrt{3}$. Multiply it out and you get $A = 4\sqrt{3}$. The answer is (B).

FOUR-SIDED FIGURES

The four angles inside any figure that has four sides add up to 360 degrees. That includes rectangles, squares, and parallelograms. Parallelograms are four-sided figures made out of two sets of parallel lines whose area can be found with the formula $A = bh$, where h is the height of a line drawn perpendicular to the base.

Perimeter of a Rectangle

The perimeter of a rectangle is just the sum of the lengths of its four sides.



perimeter = 4 + 8 + 4 + 8

Area of a Rectangle

The area of a rectangle is equal to its length times its width. For example, the area of the rectangle above is 32 (or 8 × 4).

Squares

A square has four equal sides. The perimeter of a square is, therefore, four times the length of any side. The area of a square is equal to the length of any side times itself, or in other words, the length of any side, squared. The diagonal of a square splits it into two 45:45:90, or isosceles, right triangles.

The World of Pi

You may remember being taught that the value of pi (π) is 3.14, or even 3.14159. On the GRE, π = 3ish is a close enough approximation. You don't need to be any more precise than that when doing GRE problems.

What you might not recall about pi is that pi (π) is the ratio between the circumference of a circle and its diameter. When we say that π is a little bigger than 3, we're saying that every circle is about three times as far around as it is across.

CIRCLES

Circles are a popular test topic for ETS. There are a few properties that the GRE likes to test over and over again, and problems with circles also always seem to use that funny little symbol π. Here's all you need to know about circles.

Chord, Radius, and Diameter

A **chord** is a line that connects two points on the circumference of a circle. The **radius** of a circle is any line that extends from the center of the circle to a point on the circumference of the circle. The **diameter** of a circle is a line that connects two points on the circumference of the circle and that goes through the center of the circle. Therefore, the diameter of a circle is twice as long as its radius. Notice as well that the diameter of a circle is also the longest chord and that a radius is not a chord.

The radius is always the key to circle problems.

Circumference of a Circle

The **circumference** of a circle is like the perimeter of a triangle: it's the distance around the outside. The formula for finding the circumference of a circle is 2 times π times the radius, or π times the diameter.

$$\text{circumference} = 2\pi r \text{ or } \pi d$$

Circumference is just a fancy way of saying perimeter.

If the diameter of a circle is 4, then its circumference is 4π, or roughly 12+. If the diameter of a circle is 10, then its circumference is 10π, or a little more than 30.

An **arc** is a section of the outside, or circumference, of a circle. An angle formed by two radii is called a **central angle** (it comes out to the edge from the center of the circle). There are 360 degrees in a circle, so if there is an arc formed by, say, a 60-degree central angle, and 60 is $\frac{1}{6}$ of 360, then the arc formed by this 60-degree central angle will be $\frac{1}{6}$ of the circumference of the circle.

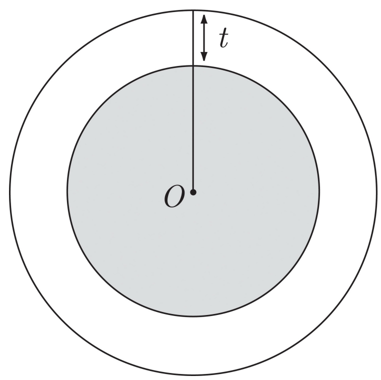
Area of a Circle

The area of a circle is equal to π times the square of its radius.

$$\text{area} = \pi r^2$$

When working with π, leave it as π in your calculations. Also, leave $\sqrt{3}$ as $\sqrt{3}$. The answer will have them that way.

Let’s try an example of a circle question.

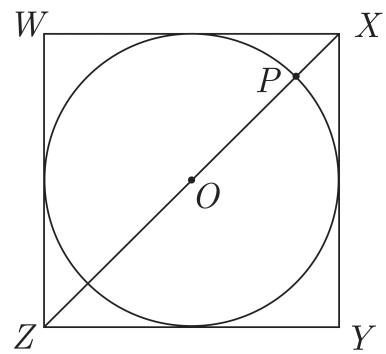


In the wheel above, with center O, the area of the entire wheel is 169π. If the area of the shaded hubcap is 144π, then t =

Here’s How to Crack It

We have to figure out what t is, and it’s going to be the length of the radius of the entire wheel minus the length of the radius of the hubcap. If the area of the entire wheel is 169π, the radius is $\sqrt{169}$, or 13. If the area of the hubcap is 144π, the radius is $\sqrt{144}$, or 12. $13 - 12 = 1$.

Let’s try another one.



In the figure above, a circle with the center *O* is inscribed in square *WXYZ*. If the circle has radius 3, then *PZ* =

- ◯ 6
- ◯ $3\sqrt{2}$
- ◯ $6 + \sqrt{2}$
- ◯ $3 + \sqrt{3}$
- ◯ $3\sqrt{2} + 3$

Ballparking answers will help you eliminate choices.

Here's How to Crack It

Inscribed means that the edges of the shapes are touching. The radius of the circle is 3, which means that *PO* is 3. If *Z* were at the other end of the diameter from *P*, this problem would be easy and the answer would be 6, right? But *Z* is beyond the edge of the circle, which means that *PZ* is a little more than 6. Let's stop there for a minute and glance at the answer choices. We can eliminate anything that's "out of the ballpark"—in other words, any answer choice that's less than 6, equal to 6 itself, or a lot more than 6. Remember when we told you to memorize a few of those square roots?

Let's use them:

(A) Exactly 6? Nope.

(B) That's 1.4 × 3, which is 4.2. Too small.

(C) That's 6 + 1.4, or 7.4. Not bad. Let's leave that one in.

(D) That's 3 + 1.7, or 4.7. Too small.

(E) That's (3 × 1.4) + 3, which is 4.2 + 3, or 7.2. Not bad. Let's leave that one in too.

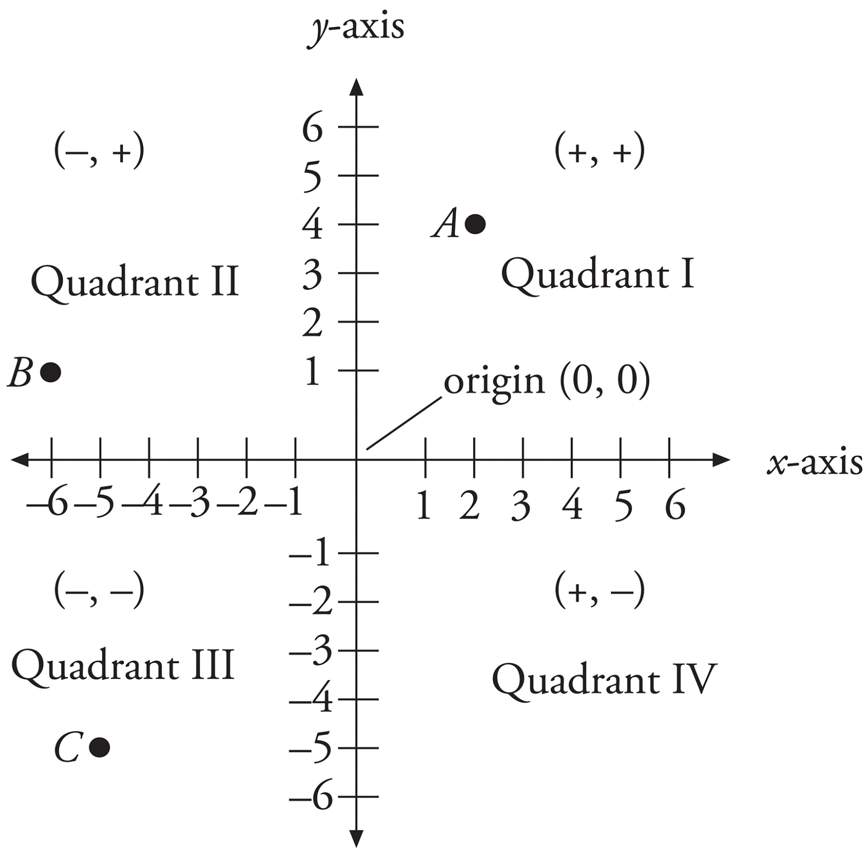
So we eliminated three choices with Ballparking. We're left with (C) and (E). You could take a guess here if you had to, but let's do a little more geometry to find the correct answer.

Because this circle is inscribed in the square, the diameter of the circle is the same as a side of the square. We already know that the diameter of the circle is 6, so that means that *ZY*, and indeed all the sides of the square, are also 6. Now, if *ZY* is 6, and *XY* is 6, what's *XZ*, the diagonal of the square? Well, *XZ* is also the hypotenuse of the isosceles right triangle *XYZ*. The hypotenuse of a right triangle with two sides of 6 is $6\sqrt{2}$. That's approximately 6 × 1.4, or 8.4.

The question is asking for *PZ*, which is a little less than *XZ*. It's somewhere between 6 and 8.4. The pieces that aren't part of the diameter of the circle are equal to 8.4 – 6, or 2.4. Divide that in half to get 1.2, which is the distance from the edge of the circle to *Z*. That means that *PZ* is 6 + 1.2, or 7.2. Check your remaining answers: Choice (C) is 7.4, and (E) is 7.2. Bingo! The answer is (E).

THE COORDINATE SYSTEM

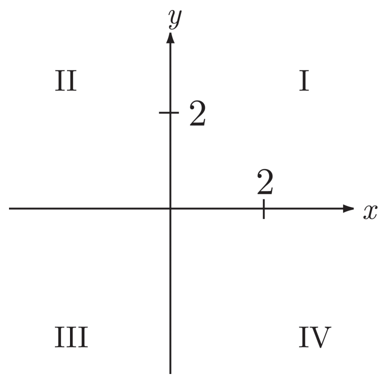
On a coordinate system, the horizontal line is called the ***x*-axis** and the vertical line is called the ***y*-axis**. The four areas formed by the intersection of these axes are called **quadrants**. The point where the axes intersect is called the **origin**. This is what it looks like:



Coordinate geometry questions often test basic shapes such as triangles and squares.

To express any point in the coordinate system, you first give the horizontal value, then the vertical value, or (x, y). In the diagram above, point A can be described by the coordinates (2, 4). That is, the point is two spaces to the right of the origin and four spaces above the origin. Point B can be described by the coordinates (−6, 1). That is, it is six spaces to the left and one space above the origin. What are the coordinates of point C? Right, it's (−5, −5).

Here's a GRE example:



Points (x, 5) and (–6, y), not shown in the figure above, are in Quadrants I and III, respectively. If $xy \neq 0$, in which quadrant is point (x, y) ?

◯ IV

◯ III

◯ II

◯ I

◯ It cannot be determined from the information given.

Here's How to Crack It

If point (x, 5) is in Quadrant I, that means x is positive. If point y is in Quadrant III, then y is negative. The quadrant that would contain coordinate points with a positive x and a negative y is Quadrant IV. That's (A).

Slope

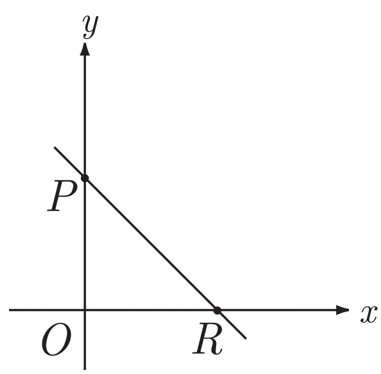
Trickier questions involving the coordinate system might give you the equation for a line on the grid, which will involve something called the slope of the line. The equation of a line is

$$y = mx + b$$

In this equation, x and y are both points on the line, b stands for the y-intercept, or the point at which the line crosses the y-axis, and m is the slope of the line. **Slope** is defined as the vertical change divided by the horizontal change, often called "the rise over the run" or the "change in y over the change in x."

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Sometimes on the GRE, m is written instead as a, as in $y = ax + b$. You'll see all this in action in a moment.



The line $y = -\frac{8}{7}x + 1$ is graphed on the rectangular coordinate axes.

Quantity A	**Quantity B**
OR	*OP*

○ Quantity A is greater.

○ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

Here's How to Crack It

The y-intercept, or b, in this case is 1. That means the line crosses the y-axis at 1. So the coordinates of point P are (0, 1). Now we have to determine the coordinates of point R. We know the y-coordinate is 0, so let's stick that into the equation (the slope and the y-intercept are constant; they don't change).

$$y = mx + b$$

$$0 = -\frac{8}{7}x + 1$$

Now let's solve for x.

$$0 = -\frac{8}{7}x + 1$$

$$0 - 1 = -\frac{8}{7}x + 1 - 1$$

$$-1 = -\frac{8}{7}x$$

$$\left(-\frac{7}{8}\right)(-1) = \left(-\frac{7}{8}\right)\left(-\frac{8}{7}\right)x$$

$$\frac{7}{8} = x$$

So the coordinates of point R are $(\frac{7}{8}, 0)$. That means OR, in Quantity A, is equal to $\frac{7}{8}$, and OP, in Quantity B, is equal to 1. The answer is

(B).

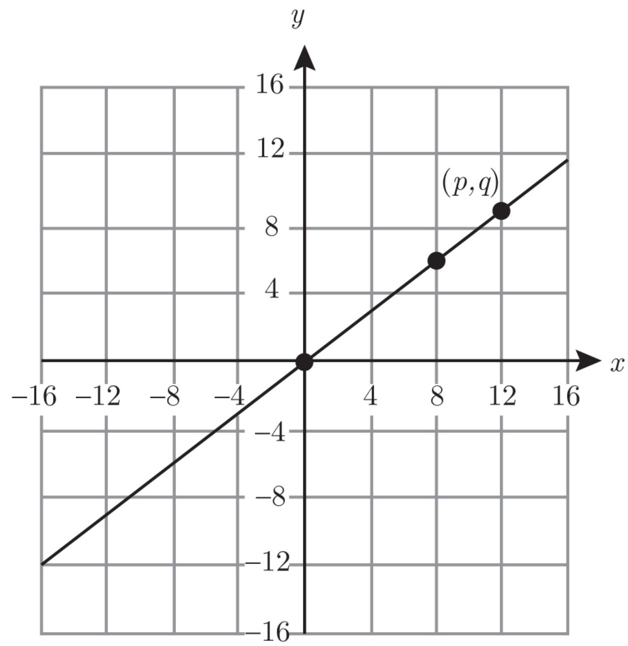
Another approach to this question would be to focus on the meaning of slope. Because the slope is $-\frac{8}{7}$, that means the vertical change is 8 and the horizontal change is 7. In other words, you count up 8 and over 7. Clearly the rise is more than the run; thus *OP* is more than *OR*.

Incidentally, if you're curious about the difference between a positive and negative slope, any line that rises from left to right has a positive slope. Any line that falls from left to right has a negative slope. (A horizontal line has a slope of 0, and a vertical line is said to have "no slope.")

Finding a Point

Another way coordinate systems are tested on the GRE is finding a point on a line. Knowing the rise over run equation for computing the slope can also help to solve these problems.

Here's a GRE example:



In the rectangular coordinate system above, $q = 9$. If the line passes through the origin, what is the value of p ?

- ◯ 6
- ◯ 8
- ◯ 9
- ◯ 12
- ◯ 13

Here's How to Crack It

You know the line passes through the origin (0, 0) and point (8, 6). The slope is equal to $\frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$. That will also be the slope using point (p, 9) and the origin. So $\frac{9-0}{p-0} = \frac{3}{4}$, which means that $\left(\frac{9}{p}\right) = \frac{3}{4}$ and $p = 12$. The correct answer is (D).

Finding the Intercept

Coordinate system questions may also ask you to solve for the intercept of a line. The intercept of a line is the point at which the line crosses another. Typically, on the GRE, the intercept is where the line crosses the *x*- or *y*-axis.

Knowing the slope-intercept form of a linear equation can help to solve these types of questions.

What is the *x*-intercept of the line defined by the equation $y = 2x + 3$?

- ○ $\left(\frac{2}{3}, \frac{2}{3}\right)$
- ○ $\left(2, -\frac{3}{2}\right)$
- ○ $\left(-\frac{3}{2}, 0\right)$
- ○ $(-2, 0)$
- ○ $(0, -3)$

Note the question asks for the *x*-intercept, not the *y*-intercept.

Here's How to Crack It

The *x*-intercept is the point where the line crosses the *x*-axis, and where $y = 0$. So eliminate (A), (B), and (E). Set $y = 0$ and solve for x.

The correct answer is (C).

Perpendicular Lines

Coordinate systems sometimes have more than two lines on a single plot. The GRE may ask you to determine the relationship between the two lines.

Many times, the relationship is determined by the slope of the lines. Parallel lines have the same slope. Perpendicular lines have slopes that are negative reciprocals of each other. In other words, if the slope of one line is 2, then the slope of a line perpendicular to it would be $-\frac{1}{2}$.

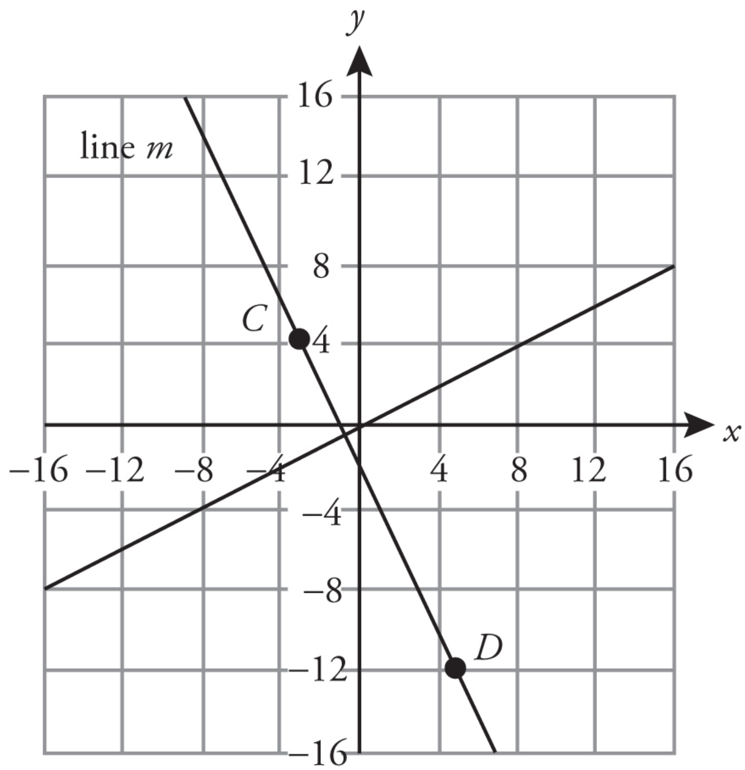
Here's a GRE example:

Points C and D are located at (−3, 4) and (5, −12), respectively. Line m passes through points C and D. What is the slope of a line that is perpendicular to line m ?

- ○ −4
- ○ −2
- ○ $-\frac{1}{2}$
- ○ $\frac{1}{2}$
- ○ 2

Here's How to Crack It

The slope of line m is equal to $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(-12-4)}{(5-(-3))} = \frac{-16}{8} = -2$. The negative reciprocal of −2 is $\frac{1}{2}$, so the correct answer is (D). Another way to solve this would be to plot line m using points C and D.



Line m slopes rather sharply down from left to right, so it has a negative slope. A line perpendicular to line m would have a positive slope, so (A), (B), and (C) can be eliminated. Finally, a line perpendicular to line m would have a rather gradual positive slope, so eliminate (E).

VOLUME

You can find the volume of a three-dimensional figure by multiplying the area of a two-dimensional figure by its height (or depth). For example, to find the volume of a rectangular solid, you would take the area of a rectangle and multiply it by the depth. The formula is lwh (length × width × height). To find the volume of a circular cylinder, take the area of a circle and multiply by the height. The formula is πr^2 times the height (or $\pi r^2 h$).

DIAGONALS IN THREE DIMENSIONS

There's a special formula that you can use if you are ever asked to find the length of a diagonal (the longest distance between any two corners) inside a three- dimensional rectangular box. It is $a^2 + b^2 + c^2 = d^2$, where a, b, and c are the dimensions of the figure (kind of looks like the Pythagorean Theorem, huh?).

> Questions that ask about diagonals are really about the Pythagorean Theorem.

Take a look:

What is the length of the longest distance between any two corners in a rectangular box with dimensions 3 inches by 4 inches by 5 inches?

- ○ 5
- ○ $5\sqrt{2}$
- ○ 12
- ○ $12\sqrt{2}$
- ○ 50

Here's How to Crack It

Let's use our formula, $a^2 + b^2 + c^2 = d^2$. The dimensions of the box are 3, 4, and 5.

$$\begin{aligned} 3^2 + 4^2 + 5^2 &= d^2 \\ 9 + 16 + 25 &= d^2 \\ 50 &= d^2 \\ \sqrt{50} &= d \\ \sqrt{25 \times 2} &= d \\ \sqrt{25} \times \sqrt{2} &= d \\ 5\sqrt{2} &= d \end{aligned}$$

That's (B).

SURFACE AREA

The surface area of a rectangular box is equal to the sum of the areas of all of its sides. In other words, if you had a box whose dimensions were 2 × 3 × 4, there would be two sides that are 2 by 3 (this surface would have an area of 6), two sides that are 3 by 4 (area of 12), and two sides that are 2 by 4 (area of 8). So, the total surface area would be 6 + 6 + 12 + 12 + 8 + 8, which is 52.

Don't confuse surface area with volume.

Key Formulas and Rules

Here is a review of the key rules and formulas to know for the GRE Math section.

Lines and angles

- All straight lines have 180 degrees.
- A right angle measures 90 degrees.
- Vertical angles are equal.

- Parallel lines cut by a third line have two kinds of angles: big angles and small angles. All of the big angles are equal and all of the small angles are equal. The sum of a big angle and a small angle is 180 degrees.

Triangles

- All triangles have 180 degrees.
- The angles and sides of a triangle are in proportion—the largest angle is opposite the largest side and the smallest side is opposite the smallest angle.
- The Pythagorean Theorem is $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.
- The area formula for a triangle is $A = \frac{bh}{2}$.

Quadrilaterals

- All quadrilaterals have 360 degrees.
- The area formula for squares and rectangles is bh.

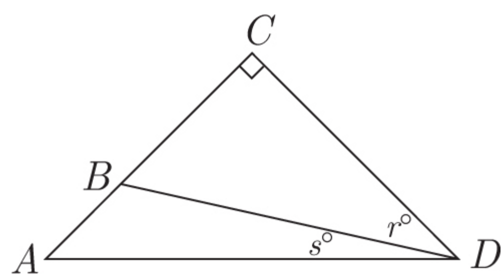
Circles

- All circles have 360 degrees.
- The radius is the distance from the center of the circle to any point on the edge.
- The area of a circle is πr^2.
- The circumference of a circle is $2\pi r$.

PLUGGING IN ON GEOMETRY PROBLEMS

Remember, whenever you have a question that has answer choices, like a regular multiple-choice question or a multiple-choice, multiple-answer question that has variables in the answer choices, Plug In. On geometry problems, you can Plug In values for angles or lengths as long as the values you plug in don't contradict either the wording of the problem or the laws of geometry (you can't have the interior angles of a triangle add up to anything but 180, for instance).

Here's an example:



In the drawing above, if $AC = CD$, then $r =$

- ◯ $45 - s$
- ◯ $90 - s$
- ◯ s
- ◯ $45 + s$
- ◯ $60 + s$

Here's How to Crack It

See the variables in the answer choices? Let's Plug In. First of all, we're told that *AC* and *CD* are equal, which means that *ACD* is an isosceles right triangle. So both angles *A* and *D* have to be 45 degrees. Now it's Plugging In time. The smaller angles, *r* and *s*, must add up to 45 degrees, so let's make $r = 40$ degrees and $s = 5$ degrees.

The question asks for the value of *r*, which is 40, so that's our target answer. Now eliminate answer choices by plugging in 5 for *s*.

(A) 45 – 5 = 40. Bingo! Check the other choices to be sure.

(B) 90 – 5 = 85. Nope.

(C) 5. Nope.

(D) 45 + 5 = 50. Eliminate it.

(E) 60 + 5 = 65. No way.

> Don't forget to Plug In on geometry questions. Just pick numbers according to the rules of geometry.

By the way, we knew that the correct answer couldn't be greater than 45 degrees, because that's the measure of the entire angle *D*, so you could have eliminated (D) and (E) right away.

DRAW IT YOURSELF

When ETS doesn't include a drawing with a geometry problem, it usually means that the drawing, if supplied, would make ETS's answer obvious. In cases like this, you should just draw it yourself. Here's an example of a time when ETS doesn't provide a drawing because it would make the correct answer too obvious—so go ahead and draw one yourself!

Quantity A	Quantity B
The diameter of a circle with area 49π	14

◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

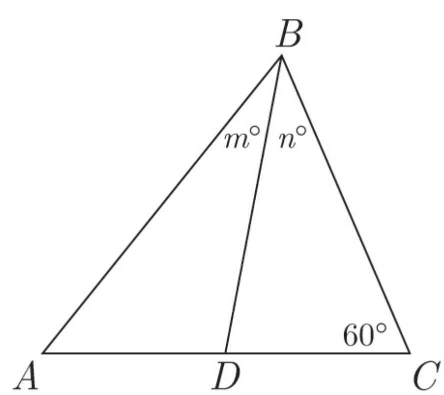
◯ The relationship cannot be determined from the information given.

Here’s How to Crack It

Visualize the figure. If the area is 49π, what’s the radius? Right: 7. And if the radius is 7, what’s the diameter? Right: 14. The answer is (C).

Redraw

On tricky quant comp questions, you may need to draw the figure once, eliminate two answer choices, and then draw it another way to try to disprove your first answer and to see if the answer is (D). Here’s an example of a problem that might require you to do this:



D is the midpoint of *AC*.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
m	n

◯ Quantity A is greater.

◯ Quantity B is greater.

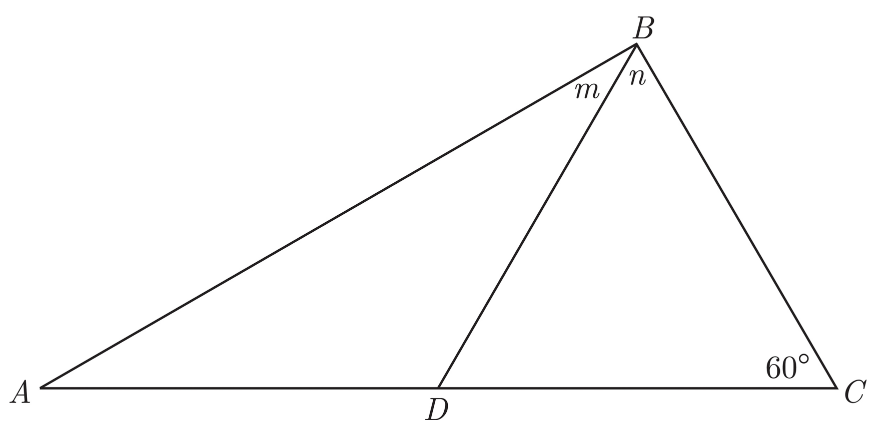
◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

For quant comp geometry questions, draw, eliminate, and REDRAW; it's like Plugging In twice.

Here's How to Crack It

Are you sure that the triangle looks exactly like this? Nope. We know only what we are told—that the lengths of *AD* and *DC* are equal; from this figure, it looks like angles *m* and *n* are also equal. Because this means that it's possible for them to be, we can eliminate (A) and (B). But let's redraw the figure to try to disprove our first answer.



Try drawing the triangle as stretched out as possible. Notice that *n* is now clearly greater than *m*, so you can eliminate (C), and the answer is (D).

Geometry Drill

Click here to download a PDF of Geometry Drill.

Think you have mastered these concepts? Try your hand at the following problems and check your work after you have finished. You can find the answers in Part V.

1 of 15

Which of the following could be the degree measures of two angles in a right triangle?

Indicate all such angles.

☐ 20° and 70°

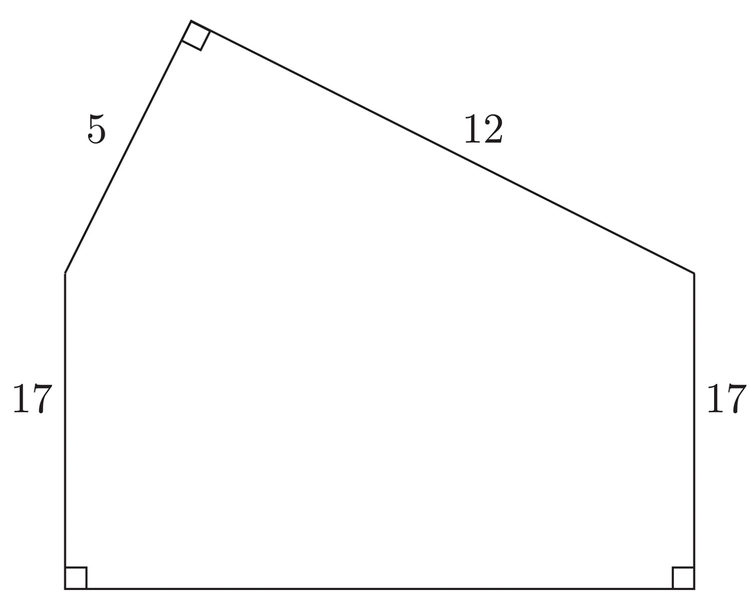
☐ 30° and 60°

☐ 45° and 45°

☐ 55° and 55°

☐ 75° and 75°

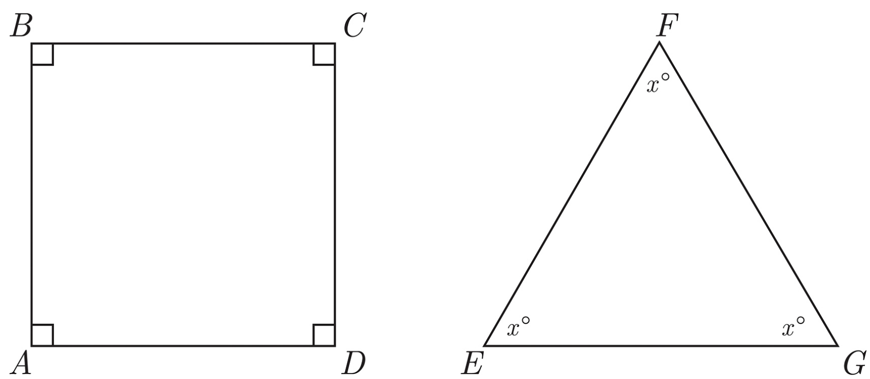
2 of 15



What is the perimeter of the figure above?

- 51
- 64
- 68
- 77
- 91

3 of 15



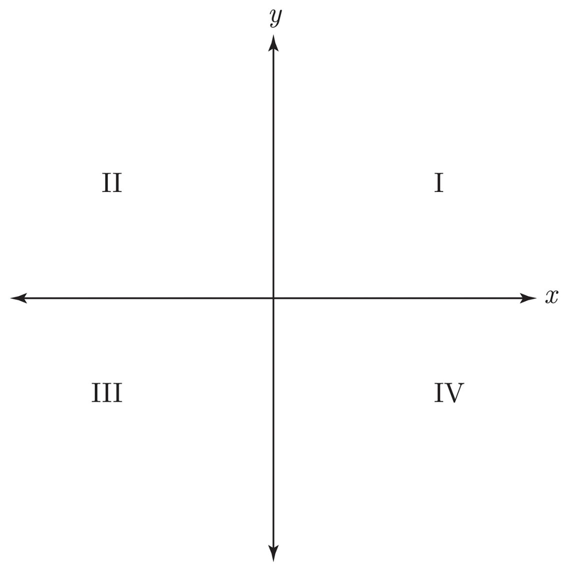
$AB = BC = EG$

$FG = 8$

Quantity A	**Quantity B**
The area of square *ABCD*	32

- Quantity A is greater.
- Quantity B is greater.
- The two quantities are equal.
- The relationship cannot be determined from the information given.

4 of 15



$(a, 6)$ is a point (not shown) in Quadrant I.

$(-6, b)$ is a point (not shown) in Quadrant II.

Quantity A	**Quantity B**
a	b

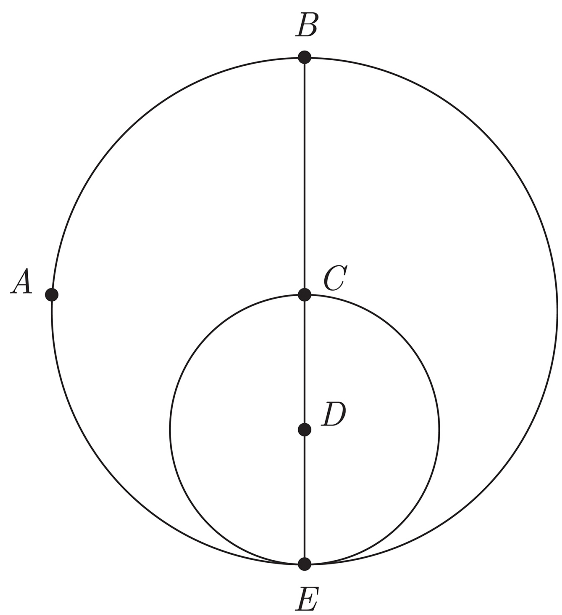
- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

5 of 15

A piece of twine with length of t is cut into two pieces. The length of the longer piece is 2 yards greater than 3 times the length of the shorter piece. Which of the following is the length, in yards, of the longer piece?

- ○ $\frac{t+3}{3}$

- $\dfrac{3t+2}{3}$
- $\dfrac{t-2}{4}$
- $\dfrac{3t+4}{4}$
- $\dfrac{3t+2}{4}$



The circle with center *D* is drawn inside the circle with center *C*, as shown in the figure above. If $CD = 3$, what is the area of semicircle *EAB* ?

- $\dfrac{9}{2}\pi$
- 9π
- 12π
- 18π
- 36π

7 of 15

For the final exam in a scuba diving certification course, Karl navigates from one point in a lake to another. Karl begins the test x meters directly beneath the boat and swims straight down toward the bottom of the lake for 8 meters. He then turns to his right and swims in a straight line parallel to the surface of the lake and swims 24 meters, at which point he swims directly from his location, in a straight line, back to the boat. If the distance that Karl swims back to the boat is 26 meters, what is the value of x ?

8 of 15

Quantity A	**Quantity B**
The circumference of a circular region with radius r	The perimeter of a square with side r

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

9 of 15

Triangle ABC is contained within a circle with center C. Points A and B lie on the circle. If the area of circle C is 25π, and the measure of angle ACB is $60°$, which of the following are possible lengths for side AB of triangle ABC ?

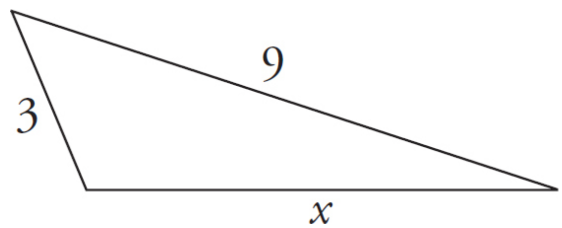
Indicate <u>all</u> such lengths.

- ◯ 3
- ◯ 4
- ◯ 5

○ 6

○ 7

10 of 15



Quantity A	Quantity B
x	5.9

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

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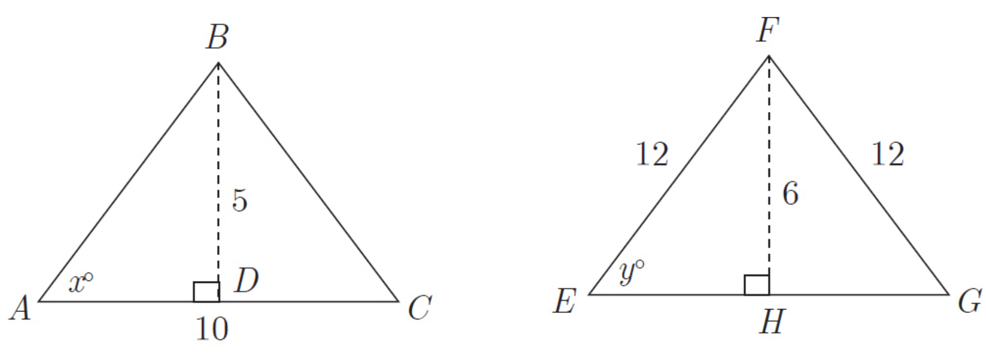

$AB = BC$

Quantity A	Quantity B
x	y

○ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

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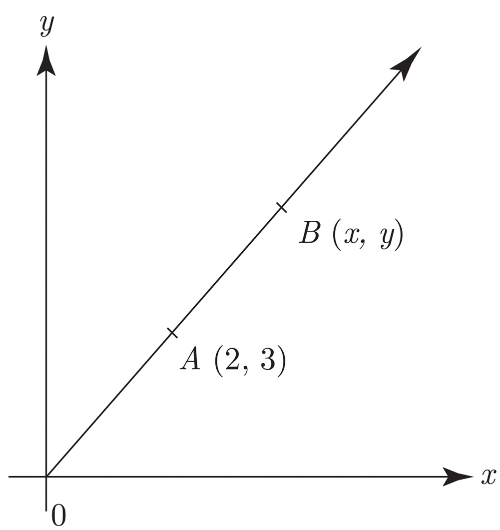

Given points $A(2, 3)$ and $B(x, y)$ in the rectangular coordinate system above, if $y = 4.2$, then $x =$

○ 2.6

○ 2.8

○ 2.9

○ 3.0

○ 3.2

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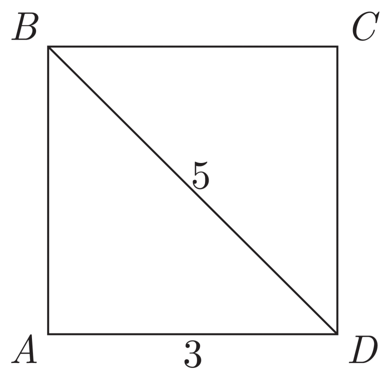

In rectangle *ABCD* above, which of the following is the area of the triangle *ABD* ?

- ○ 6
- ○ 7.5
- ○ 10
- ○ 12
- ○ 15

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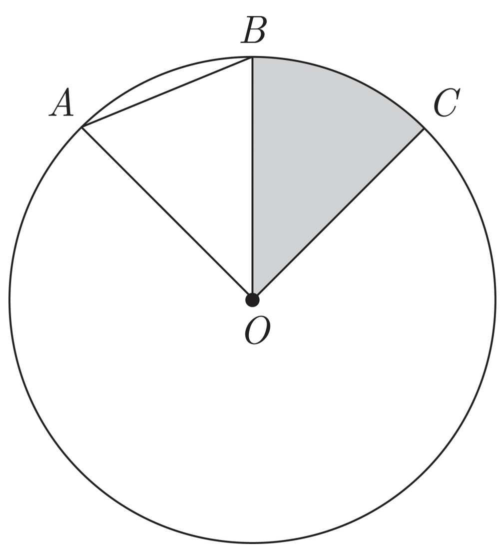
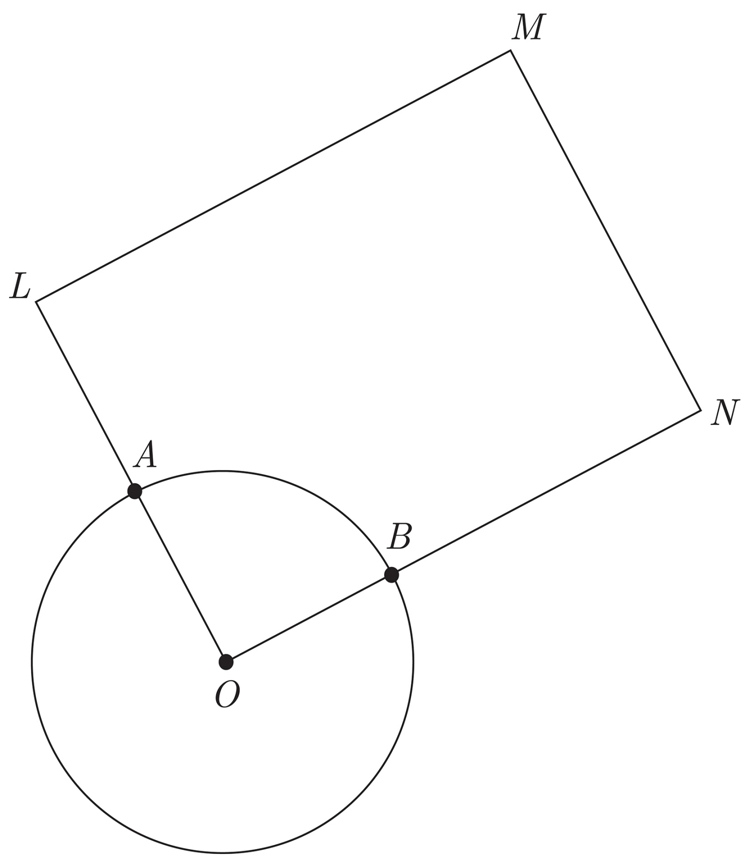


The circle above has a center *O*. $\angle AOB = \angle BOC$

Quantity A	**Quantity B**

The area of triangle *AOB*	The area of the shaded region

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.



The circumference of the circle with center O shown above is 15π. $LMNO$ is a parallelogram and $\angle OLM = 108°$. What is the length of minor arc AB ?

- ◯ 15π
- ◯ 9π

- ◯ 3π
- ◯ 2π
- ◯ π

Summary

- There may be only a handful of geometry questions on the GRE, but you'll be expected to know a fair number of rules and formulas.
- Line and angle problems typically test your knowledge of vertical angles, parallel lines, right angles, and straight angles.
- Triangles are a popular geometry topic on the GRE. Make sure you know your triangle basics, including the total degrees of a triangle, the relationship between the angles and sides of a triangle, and the third side rule.
- Right triangle problems frequently test the Pythagorean Theorem and the common Pythagorean triples 3:4:5 and 5:12:13.
- Be aware of the two special right triangles that ETS likes to torture test takers with: the 45:45:90 triangle and 30:60:90 triangle.
- Know the area formulas for triangles, rectangles, squares, and circles.
- Problems involving the *xy*-coordinate plane frequently test common geometry concepts such as the area of a triangle or a square. Other plane geometry questions will test you on slope and the equation of a line.
- Slope is defined as "rise over run." Find it by finding the change in y-coordinates (the rise) and the change in x-coordinates (the run).
- The equation of a line is $y = mx + b$, where x and y are the coordinates of any point on the line, m is the slope and b is the

y-intercept, the point at which the line crosses the y-axis.

- Don't forget to Plug In on geometry problems!

Chapter 14
Math Et Cetera

There are a few more math topics that may appear on the GRE that don't fit nicely into the preceding chapters. This chapter looks at some of these leftover topics, including probability, permutations and combinations, and factorials. The topics in this chapter are not essential to your GRE Math score, because these areas are not tested as frequently as the topics detailed earlier. However, if you feel confident with the previous math topics, and you're looking to maximize your GRE Math score, this chapter will show you all you need to know to tackle these more obscure GRE problems.

OTHER MATH TOPICS

The bulk of the GRE Math section tests your knowledge of fundamentals, basic algebra, and geometry. However, there are a few other topics that may appear. These "et cetera" concepts usually show up only once or twice per test (although at higher scoring levels they may appear more frequently) and often cause anxiety among test takers. Many test takers worry excessively about probability problems, for example, even though knowledge of more familiar topics such as fractions and percents will be far more important in determining your GRE math score. So tackle these problems only after you've mastered the rest. If you find these concepts more difficult, don't worry—they won't make or break your GRE score.

> These topics show up rarely on the GRE, but if you're going for a very high score, they are useful to know.

PROBABILITY

If you flip a coin, what's the probability that it will land heads up? The probability is equal to one out of two, or $\frac{1}{2}$. What is the probability that it won't land heads up? Again, one out of two, or $\frac{1}{2}$. If you flip a coin nine times, what's the probability that the coin will land on heads on the tenth flip? Still 1 out of 2, or $\frac{1}{2}$. Previous flips do not affect the outcome of the current coin flip.

You can think of probability as just another type of fraction. **Probabilities** express a special relationship, namely the chance of a certain outcome occurring. In a probability fraction, the denominator is the total number of possible outcomes that may occur, while the numerator is the number of outcomes that would satisfy the criteria. For example, if you have 10 shirts and 3 of them are black, the probability of selecting a black shirt from your closet without looking is $\frac{3}{10}$.

Since probability is expressed as a fraction, it can also be expressed as a decimal or a percentage. A probability of one-half is equivalent to a probability of 0.5, or 50%.

Think of probability in terms of fractions:

- If it is impossible for something to happen—if no outcomes satisfy the criteria—then the numerator of the probability fraction is 0 and the probability is equal to 0.
- If something is certain to happen—if all possible outcomes satisfy the criteria—then the numerator and denominator of the fraction are equal and the probability is equal to 1.
- If it is possible for something to occur, but it will not definitely occur, then the probability of it occurring is between 0 and 1.

$$\text{probablity} = \frac{\text{number of possible outcomes that satisfy the condition}}{\text{number of total possible outcomes}}$$

Let's see how it works.

At a meeting of 375 members of a neighborhood association, $\frac{1}{5}$ of the participants have lived in the community for less than 5 years and $\frac{2}{3}$ of the attendees have lived in the neighborhood for at least 10 years. If a member of the meeting is selected at random, what is the probability that the person has lived in the neighborhood for at least 5 years but less than 10 years?

- ○ $\frac{2}{15}$
- ○ $\frac{4}{15}$
- ○ $\frac{3}{10}$
- ○ $\frac{1}{2}$
- ○ $\frac{8}{15}$

Here's How to Crack It

In order to solve this problem, we need to put together our probability fraction. The denominator of our fraction is going to be 375, the total number of people from which we are selecting. Next we need to figure out how many attendees satisfy the criteria of having lived in the neighborhood for more than 5 years but fewer than 10 years.

What number goes on the bottom of the probability fraction?

First, we know that $\frac{1}{5}$ of the participants have lived in the neighborhood for less than 5 years. $\frac{1}{5}$ of 375 is 75 people, so we can take them out of the running. Also, $\frac{2}{3}$ of the attendees have lived in the neighborhood for at least 10 years. $\frac{2}{3}$ of 375 (be careful not to use 300 as the total!) is 250, so we can also remove them from consideration. Thus, if 75 people have lived in the neighborhood for less than 5 years and 250 have lived for at least 10, the remaining people are the ones we want. 250 + 75 is 325, so that leaves us with 50 people who satisfy the criteria. We need to make 50 the numerator of our fraction, which gives us $\frac{50}{375}$. This reduces to $\frac{2}{15}$, so (A) is the answer.

Two Important Laws of Probability

When you want to find the probability of a series of events in a row, you multiply the probabilities of the individual events. What is the probability of getting two heads in a row if you flip a coin twice? The probability of getting a head on the first flip is $\frac{1}{2}$. The probability is also $\frac{1}{2}$ that you'll get a head on the second flip, so the combined probability of two heads is $\frac{1}{2} \times \frac{1}{2}$, which equals $\frac{1}{4}$. Another way to look at it is that there are four possible outcomes: HH, TT, HT, TH. Only one of those outcomes consists of two heads in a row. Thus, $\frac{1}{4}$ of the outcomes consist of two heads in a row. Sometimes the number of outcomes is small enough that you can list them and calculate the probability that way.

Probability of A and B = Probability of A × Probability of B

Occasionally, instead of finding the probability of one event AND another event happening, you'll be asked to find the probability of either one event OR another event happening. In this situation, instead of multiplying the probabilities, you add them. Let's say you have a normal deck of 52 cards. If you select a card at random, what's the probability that you select a 7 or a 4? The probability of selecting a 7 is $\frac{4}{52}$, which reduces to $\frac{1}{13}$. The probability of selecting a 4 is the same; $\frac{1}{13}$. Therefore, the probability of selecting a 7 or a 4 is $\frac{1}{13} + \frac{1}{13} = \frac{2}{13}$.

Probability of A or B = Probability of A + Probability of B

(The full formula for the Probability of A or B includes subtracting the Probability of *both* A and B, but the GRE uses only mutually exclusive events, so both A and B can't happen, and you don't need to worry about it!)

Let's look at a problem:

When a pair of six-sided dice, each with faces numbered 1 to 6, is rolled once, what is the probability that the result is either a 3 and a 4 or a 5 and a prime number?

Give your answer as a fraction.

Here's How to Crack It

Probability is fundamentally about counting. You need to be able to count all the things that can happen and count all the situations that meet the conditions of the problem. Sometimes, the easiest way to count both everything that can happen and the situations that meet the condition is to write everything out. In this case, let's use a table:

	1	2	3	4	5	6
1	✘	✘	✘	✘	✘	✘
2	✘	✘	✘	✘	✓	✘
3	✘	✘	✘	✓	✓	✘
4	✘	✘	✓	✘	✘	✘
5	✘	✓	✓	✘	✓	✘
6	✘	✘	✘	✘	✘	✘

Each cell of this table represents a result when the dice are rolled. For example, the cell at the intersection of the row shown as 1 and the column shown as 1 would represent that 1 was showing on each of the two die. This cell has been marked with an ✘ because it does not meet either condition of the problem.

The cells marked with a ✓ are the only dice rolls that meet one of the conditions of the problem. To finish, just count the ✓ marks—there are 7. (Remember that 1 is not prime. That's why combinations such as 5 and 1 are not checked.) Next, count the total possibilities—there are 36. So, the probability of rolling either a 3 and a 4 or a 5 and prime number is $\frac{7}{36}$.

One last important thing you should know about probabilities is that the probability of an event happening and the probability of an event not happening must add up to 1. For example, if the probability of snow falling on one night is $\frac{2}{3}$, then the probability of no snow falling must be $\frac{1}{3}$. If the probability that it will rain is 80%, then the probability that it won't rain must be 20%. The reason this is useful is that, on some GRE probability problems, it will be easier to find the probability that an event doesn't occur; once you have that, just subtract from 1 to find the answer.

Let's look at the following example.

Dipak has a 25% chance of winning each hand of blackjack he plays. If he has \$150 and bets \$50 a hand, what is the probability that he will still have money after the third hand?

- ◯ $\frac{1}{64}$
- ◯ $\frac{3}{16}$
- ◯ $\frac{27}{64}$
- ◯ $\frac{37}{64}$
- ◯ $\frac{3}{4}$

> Since probabilities are just fractions, they can also be expressed as percents.

Here's How to Crack It

If Dipak still has money after the third hand, then he must have won at least one of the hands, and possibly more than one. However, directly calculating the probability that he wins at least one hand is tricky because there are so many ways it could happen (for example, he could lose-lose-win, or W-W-L or W-L-W or L-W-L, and so on). So think about it this way: the question asks for the probability that he will win at least one hand. What if he doesn't? That would mean that he doesn't win any hands at all. If we calculate the probability that he loses every hand, we can then subtract that from 1 and find the corresponding probability that he wins at least one hand. Since Dipak has a 25% chance of winning each hand, this means that he has a 75% chance of losing it, or $\frac{3}{4}$ (the answers are in fractions, so it's best to work with fractions). To find the probability that he loses all three hands, simply multiply the probabilities of his losing each individual hand. $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$, so there is a $\frac{27}{64}$ probability that he will lose all three hands. Subtracting this from 1 gives you the answer you're looking for: $1 - \frac{27}{64} = \frac{37}{64}$. The answer is (D).

Given events A and B, the probability of

- A and B = (Probability of A) × (Probability of B)
- A or B = Probability of A + Probability of B

Given event A

- Probability of A + Probability of Not A = 1

FACTORIALS

The **factorial** of a number is equal to that number times every positive whole number smaller than that number, down to 1. For example, the factorial of 6 is equal to 6 × 5 × 4 × 3 × 2 × 1, which equals 720. The symbol for a factorial is !, so 4! doesn't mean we're really excited about the number 4: it means 4 × 3 × 2 × 1, which is equal to 24. (0! is equal to 1, by the way.) When factorials show up in GRE problems, always look for a shortcut like canceling or factoring. The point of a factorial problem is not to make you do a lot of multiplication. Let's try one.

Quantity A	Quantity B
$\frac{12!}{11!}$	$\frac{4!}{2!}$

◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

Here's How to Crack It

Let's tackle Quantity A. We definitely don't want to multiply out the factorials since that would be pretty time-consuming: 12! and 11! are both huge numbers. Instead let's look at what they have in common. What we're really talking about here is $\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$. Now it's clear that both factorials share everything from 11 on down to 1. The entire bottom of the fraction will cancel and the only thing left on top will be 12, so the value of Quantity A is 12. For Quantity B, we can also write out the factorials and get $\frac{4 \times 3 \times 2 \times 1}{2 \times 1}$. The 2 and the 1 in the bottom cancel, and the only thing left on top will be 4 × 3, which is equal to 12. The two quantities are equal, so the answer is (C).

PERMUTATIONS AND COMBINATIONS

The basic definition of a **permutation** is an arrangement of things in a particular order. Suppose you were asked to figure out how many different ways you could arrange five statues on a shelf. All you have to do is multiply 5 × 4 × 3 × 2 × 1, or 120. (Yes, this is another application of factorials.) You have five possible statues that could fill the first slot on the shelf; then, once the first slot is filled, there are four remaining statues that could fill the second slot, three that could fill the third slot, and so on, down to one.

> Permutation problems often ask for arrangements, orders, schedules, or lists.

Now suppose that there are five people running in a race. The winner of the race will get a gold medal, the person who comes in second will get a silver medal, and the person who comes in third will get a bronze medal. You're asked to figure out how many different orders of gold-silver-bronze winners there can be. (Notice that this is a permutation because the order definitely matters.)

First, ask yourself how many of these runners can come in first? Five. Once one of them comes in first, she's out of the picture, so