Be careful: Don't confuse odd and even with positive and negative!

Consecutive Integers

Consecutive integers are integers listed in order of value without any integers missing in between them. Here are some examples:

- 0, 1, 2, 3, 4, 5
- −6, −5, −4, −3, −2, −1, 0
- −3, −2, −1, 0, 1, 2, 3

By the way, fractions and decimals cannot be consecutive; only integers can be consecutive. However, you can have different types of consecutive integers. For example consecutive even integers could be 2, 4, 6, 8, 10. Consecutive multiples of four could be 4, 8, 12, 16.

Absolute Value

The **absolute value** of a number is equal to its distance from 0 on the number line, which means that the absolute value of any number is always positive, whether the number itself is positive or negative. The symbol for absolute value is a set of double lines: $| \ |$. Thus $|-5| = 5$, and $|5| = 5$ because both −5 and 5 are a distance of 5 from 0 on the number line.

FACTORS, MULTIPLES, AND DIVISIBILITY

Now let's look at some ways that integers are related to each other.

Factors

A **factor** of a particular integer is a number that will divide evenly into the integer in question. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each number divides evenly into 12. In order to find all the factors of a particular integer, write down the factors systematically in pairs of integers that, when multiplied together, make 12, starting with 1 and the integer itself:

- 1 and 12
- 2 and 6
- 3 and 4

If you always start with 1 and the integer itself and work your way up, you'll make sure you get them all.

Multiples

The **multiples** of an integer are all the integers for which the original integer is a factor. For example, the multiples of 8 are all the integers of which 8 is a factor: 8, 16, 24, 32, 40, and so on. Note that there are an infinite number of multiples for any given number. Also, zero is a multiple of every number, although this concept is rarely tested on the GRE.

> There are only a few factors of any number; there are many multiples of any number.

Prime Numbers

A **prime number** is an integer that has only two factors: itself and 1. Thus, 37 is prime because the only integers that divide evenly into it are 1 and 37, while 10 is not prime because its factors are 1, 2, 5, and 10.

Here is a list of all the prime numbers that are less than 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

Here are some other facts about primes that are important to remember:

- 0 is not a prime number.

- 1 is not a prime number.
- 2 is the only even prime number.
- Prime numbers are positive integers. There's no such thing as a negative prime number or a prime fraction.

> 1 is not prime!

Divisibility

An integer is always divisible by its factors. If you're not sure if one integer is divisible by another, a surefire way to find out is to use the calculator. However, there are also certain rules you can use to determine whether one integer is a factor of another.

- An integer is divisible by 2 if its units digit is divisible by 2. For example, we know just by glancing at it that 598,447,896 is divisible by 2, because the units digit, 6, is divisible by 2.
- An integer is divisible by 3 if the sum of its digits is divisible by 3. For example, we know that 2,145 is divisible by 3 because 2 + 1 + 4 + 5 = 12, and 12 is divisible by 3.
- An integer is divisible by 4 if its last two digits form a number that's divisible by 4. For example, 712 is divisible by 4 because 12 is divisible by 4.
- An integer is divisible by 5 if its units digit is either 0 or 5. For example, 23,645 is divisible by 5 because its units digit is 5.
- An integer is divisible by 6 if it's divisible by both 2 and 3. For example, 4,290 is divisible by 6 because it is divisible by 2 (it's even) and by 3 (4 + 2 + 9 = 15, which is divisible by 3).
- An integer is divisible by 8 if its last three digits form a number that's divisible by 8. For example, 11,640 is divisible by 8 because 640 is divisible by 8.

- An integer is divisible by 9 if the sum of its digits is divisible by 9. For example, 1,881 is divisible by 9 because 1 + 8 + 8 + 1 = 18, which is divisible by 9.
- An integer is divisible by 10 if its units digit is 0. For example, 1,590 is divisible by 10 because its units digit is 0.

Remainders

If one integer is not divisible by another—meaning that the second integer is not a factor of the first number—you'll have an integer left over when you divide. This left-over integer is called a **remainder**; you probably remember working with remainders in grade school.

For example, when 4 is divided by 2, there's nothing left over, so there's no remainder. In other words, 4 is divisible by 2. You could also say that the remainder is 0.

> If a question asks about a remainder, don't use the calculator. Use long division.

On the other hand, 5 divided by 2 is 2, with 1 left over; 1 is the remainder. Also, 13 divided by 8 is 1, with 5 left over as the remainder.

Note that remainders are always less than the number that you are dividing by. For example, the remainder when 13 is divided by 7 is 6. What happens if you divide 14, the next integer, by 7? The remainder is 0.

Here's one more thing to know about remainders. What's the remainder when 5 is divided by 6? The remainder is 5 because 5 can be divided by 6 zero times and the amount that remains is 5. When the positive integer you are dividing by is greater than the integer being divided, the remainder will always be the number being divided.

MORE MATH VOCABULARY

In a way, the Math section is almost as much of a vocabulary test as the Verbal section. Below, you'll find some more standard terms that you should commit to memory before you do any practice problems.

Term	**Meaning**
sum	the result of addition
difference	the result of subtraction
product	the result of multiplication
quotient	the result of division
divisor	the number you divide by
numerator	the top number in a fraction
denominator	the bottom number in a fraction
consecutive	in order from least to greatest
terms	the numbers and expressions used in an equation

BASIC OPERATIONS WITH NUMBERS

Now that you've learned about numbers and their properties, you're ready to begin working with them. As we mentioned above, there are four basic operations you can perform on a number: addition, subtraction, multiplication, and division.

Order of Operations

When you work with numbers, you can't just perform the four operations in any way you please. Instead, there are some very specific rules to follow, which are commonly referred to as the **order of operations.**

It is absolutely necessary that you perform these operations in exactly the right order. In many cases, the correct order will be apparent from the way the problem is written. In cases in which the correct order is not apparent, you need to remember the following mnemonic.

Please **E**xcuse **M**y **D**ear **A**unt **S**ally, or **PEMDAS**.

What does PEMDAS stand for?

$$\begin{array}{c|c|c|c} P & E & MD & AS \\ & & \rightarrow & \rightarrow \end{array}$$

P stands for "parentheses." Solve anything in parentheses first.

E stands for "exponents." Solve exponents next. (We'll review exponents soon.)

M stands for "multiplication" and **D** stands for "division." The arrow indicates that you do all the multiplication and division together in the same step, going from left to right.

A stands for "addition" and **S** stands for "subtraction." Again, the arrow indicates that you do all the addition and subtraction together in one step, from left to right.

Let's look at an example:

$$12 + 4(2 + 1)^2 \div 6 - 7 =$$

Here's How to Crack It

Start by doing all the math inside the parentheses: $2 + 1 = 3$. Now the problem looks like this:

$$12 + 4(3)^2 \div 6 - 7 =$$

Next we have to apply the exponent: $3^2 = 9$. Now this is what we have:

$$12 + 4(9) \div 6 - 7 =$$

Now we do multiplication and division from left to right: $4 \times 9 = 36$, and $36 \div 6 = 6$, which gives us

$$12 + 6 - 7 =$$

Finally, we do the addition and subtraction from left to right: $12 + 6 = 18$, and $18 - 7 = 11$. Therefore,

$$12 + 4(2 + 1)^2 \div 6 - 7 = 11$$

Multiplication and Division

When multiplying or dividing, keep the following rules in mind:

- positive × positive = positive $2 \times 2 = 4$
- negative × negative = positive $-2 \times -2 = 4$
- positive × negative = negative $2 \times -2 = -4$
- positive ÷ positive = positive $8 \div 2 = 4$
- negative ÷ negative = positive $-8 \div -2 = 4$
- positive ÷ negative = negative $8 \div -2 = -4$

Before taking the GRE, you should have your times tables memorized from 1 through 15. It will be a tremendous advantage if you can quickly and confidently recall that $7 \times 12 = 84$, for example.

It seems like a small thing, but memorizing your times tables will really help you on test day.

FRACTIONS, DECIMALS, AND PERCENTAGES

One of the ways ETS tests your fundamental math abilities is through fractions, decimals, and percents. So let's expand our conversation on math fundamentals to include these concepts.

Fractions

A **fraction** expresses the number of parts out of a whole. In the fraction $\frac{2}{3}$, for instance, the top part, or **numerator**, tells us that we have 2 parts, while the bottom part of the fraction, the **denominator**, indicates that the whole, or total, consists of 3 parts. We use fractions whenever we're dealing with a quantity that's between two whole numbers.

Notice that the fraction bar is simply another way of expressing division. Thus, the fraction $\frac{2}{3}$ is just expressing the idea of "2 divided by 3."

> Fractions are important on the GRE. Make sure you're comfortable with them.

Reducing and Expanding Fractions

Fractions express a relationship between numbers, not actual amounts. For example, saying that you did $\frac{1}{2}$ of your homework expresses the same idea whether you had 10 pages of homework to do and you've done 5 pages, or you had 50 pages to do and you've done 25 pages. This concept is important because on the GRE you'll frequently have to reduce or expand fractions.

To reduce a fraction, express the numerator and denominator as the products of their factors. Then cross out, or "cancel," factors that are common to both the numerator and denominator. Here's an example:

$$\frac{16}{20} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 5} = \frac{\cancel{2} \times \cancel{2} \times 2 \times 2}{\cancel{2} \times \cancel{2} \times 5} = \frac{2 \times 2}{5} = \frac{4}{5}$$

You can achieve the same result by dividing the numerator and denominator by the factors that are common to both. In the example you just saw, you might realize that 4 is a factor of both the numerator and the denominator. That is, both the numerator and the denominator can be divided evenly (without a remainder) by 4. Doing this yields the much more manageable fraction $\frac{4}{5}$.

When you confront GRE math problems that involve fractions with great numbers, always reduce them before doing anything else.

Look at each of the following fractions:

$$\frac{1}{4} \qquad \frac{2}{8} \qquad \frac{6}{24} \qquad \frac{18}{72} \qquad \frac{90}{360} \qquad \frac{236}{944}$$

What do you notice about each of these fractions? They all express the same information! Each of these fractions expresses the relationship of "1 part out of 4 total parts."

Adding and Subtracting Fractions

Adding and subtracting fractions that have a common denominator is easy—you just add the numerators and put the sum over the common denominator. Here's an example:

$$\frac{1}{10}+\frac{2}{10}+\frac{4}{10}=$$

$$\frac{1+2+4}{10}=\frac{7}{10}$$

> Why Bother?
> You may be wondering why, if the GRE allows the use of a calculator, you should bother learning how to add or subtract fractions or to reduce them or even know

any of the topics covered in the next few pages. While it's true that you can use a calculator for these tasks, for many problems it's actually slower to do the math with the calculator than without. Scoring well on the GRE Math section requires a fairly strong grasp of the basic relationships among numbers, fractions, percents, and so on, so it's in your best interest to really understand these concepts rather than to rely on your calculator to get you through the question. In fact, if you put in the work now, you'll be surprised at how easy some of the problems become, especially when you don't have to refer constantly to the calculator to perform basic operations.

In order to add or subtract fractions that have different denominators, you need to start by finding a common denominator. You may remember your teachers from grade school imploring you to find the "lowest common denominator." Actually, any common denominator will do, so find whichever one you find most comfortable working with.

$$\frac{7}{8} - \frac{5}{12} = \frac{21}{24} - \frac{10}{24} = \frac{11}{24}$$

Here, we expanded the fraction $\frac{7}{8}$ into the equivalent fraction $\frac{21}{24}$ by multiplying both the numerator and denominator by 3. Similarly, we converted $\frac{5}{12}$ to $\frac{10}{24}$ by multiplying both denominator and numerator by 2. This left us with two fractions with the same denominator, which meant that we could simply subtract their numerators.

When adding and subtracting fractions, you can also use a technique we call the Bowtie. The Bowtie method accomplishes exactly what we just did in one fell swoop. To use the Bowtie, first multiply the denominators of each fraction. This gives you a common denominator. Then multiply the denominator of each fraction by the numerator of the other fraction. Take these numbers and add or subtract them—depending on what the question asks you to do—to get the numerator of the answer. Then reduce if necessary.

$$\frac{2}{3} + \frac{3}{4} =$$

$$\overset{8}{\frac{2}{3}} \times \overset{9}{\frac{3}{4}} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

and

$$\frac{2}{3} - \frac{3}{4} =$$

$$\overset{8}{\frac{2}{3}} \times \overset{9}{\frac{3}{4}} = \frac{8}{12} - \frac{9}{12} = -\frac{1}{12}$$

The Bowtie method is a convenient shortcut to use when you're adding and subtracting fractions.

Multiplying Fractions

Multiplying fractions is relatively straightforward when compared to addition or subtraction. To successfully multiply fractions, multiply the first numerator by the second numerator and the first denominator by the second denominator. Here's an example:

$$\frac{4}{5} \times \frac{10}{12} = \frac{40}{60} = \frac{2}{3}$$

When multiplying fractions, you can make your life easier by reducing before you multiply. Do this once again by dividing out common factors.

$$\frac{4}{5} \times \frac{10}{12} = \frac{4}{5} \times \frac{5}{6} = \frac{20}{30} = \frac{2}{3}$$

Multiplying fractions is a snap. Just multiply straight across, numerator times numerator and denominator times denominator.

Also remember that when you're multiplying fractions, you can even reduce diagonally; as long as you're working with a numerator and a denominator of opposite fractions, they don't have to be in the same fraction. So you end up with

$$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} = \frac{2}{1} \times \frac{1}{3} = \frac{2}{3}$$

Of course, you get the same answer no matter what method you use, so attack fractions in whatever fashion you find easiest. Or better yet, use one method to check your work on the other method.

Dividing Fractions

Dividing fractions is just like multiplying fractions, with one crucial difference: before you multiply, you have to find the reciprocal of the second fraction. To do this, all you need to do is flip the fraction upside down! Put the denominator on top of the numerator and then multiply just like before. In some cases, you can also reduce before you multiply. Here’s an example:

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{1}{3} \times \frac{5}{2} = \frac{5}{6}$$

ETS tests problems that involve fractions in which the numerators or denominators are themselves fractions. These problems might look intimidating, but if you’re careful, you won’t have any trouble with them. All you have to do is remember what we said about a fraction being shorthand for division. Always rewrite the expression horizontally. Here’s an example:

$$\frac{7}{\frac{1}{4}} = 7 \div \frac{1}{4} = \frac{7}{1} \times \frac{4}{1} = \frac{28}{1} = 28$$

Comparing Fractions

Sometimes ETS will test your ability to compare two fractions to decide which is greater. These are typically found on quant comp questions. There are a couple of ways to accomplish this. One is to find equivalent fractions that have a common denominator. If the fraction is fairly simple, this is a good strategy, but oftentimes the common denominator may be hard to find or work with.

If the denominator is hard to find or work with, you can use a variant of the Bowtie technique. In this variant, you don’t have to multiply the denominators together; instead, just multiply the denominators and the numerators. The fraction with the greater product in its numerator is the greater fraction. Let’s say we had to compare the following fractions:

$$\frac{3}{7} \qquad \frac{7}{12}$$

$$\overset{36}{\frac{3}{7}} \qquad \overset{49}{\frac{7}{12}}$$

Multiplying the first denominator by the second numerator yields 49. This means the numerator of the second fraction $\left(\frac{7}{12}\right)$ is 49. Multiplying the second denominator by the first numerator gives you 36, which means the first fraction has a numerator of 36. Since 49 is greater than 36, $\frac{7}{12}$ is greater than $\frac{3}{7}$.

> You can also use the calculator feature to change the fractions into decimals.

Comparing More Than Two Fractions

You may also be asked to compare more than two fractions. On these types of problems, don't waste time trying to find a common denominator for all of them. Simply use the Bowtie to compare two of the fractions at a time.

Here's an example:

Which of the following statements is true?

○ $\frac{3}{8} < \frac{2}{9} < \frac{4}{11}$

○ $\frac{2}{5} < \frac{3}{7} < \frac{4}{13}$

○ $\frac{4}{13} < \frac{2}{5} < \frac{3}{7}$

○ $\frac{3}{7} < \frac{3}{8} < \frac{2}{5}$

○ $\frac{2}{9} < \frac{3}{7} < \frac{3}{8}$

Here's How to Crack It

As you can see, it would be a nightmare to try to find common denominators for all these fractions, so instead we'll use the Bowtie method. Simply multiply the denominators and numerators of a pair of fractions and note the results. For example, to check (A), we first multiply 8 and 2, which gives us a numerator of 16 for the fraction $\frac{2}{9}$. But multiplying 9 and 3 gives us a numerator of 27 for the first fraction. This means that $\frac{3}{8}$ is greater than $\frac{2}{9}$, and we can eliminate (A), because the first part of it is wrong. Here's how the rest of the choices shape up:

○ $\frac{2}{5} < \frac{3}{7} < \frac{4}{13}$ Compare $\frac{3}{7}$ and $\frac{4}{13}$; $\frac{3}{7}$ is greater. Eliminate (B).

○ $\frac{4}{13} < \frac{2}{5} < \frac{3}{7}$ These fractions are in order.

○ $\frac{3}{7} < \frac{3}{8} < \frac{2}{5}$ $\frac{3}{7}$ is greater than $\frac{3}{8}$. Eliminate (D).

○ $\frac{2}{9} < \frac{3}{7} < \frac{3}{8}$ $\frac{3}{7}$ is greater than $\frac{3}{8}$. Eliminate (E).

The answer is (C).

Converting Mixed Numbers into Fractions

A **mixed number** is a number that is represented as an integer and a fraction, such as $2\frac{2}{3}$. In most cases on the GRE, you should get rid of mixed fractions by converting them to improper fractions. How do you do this? By multiplying the denominator of the fraction by the integer, then adding that result to the numerator, and then putting the whole thing over the denominator. In other words, for the fraction above, we would get $\frac{3 \times 2 + 2}{3}$, or $\frac{8}{3}$.

The result, $\frac{8}{3}$, is equivalent to $2\frac{2}{3}$. The only difference is that $\frac{8}{3}$ is easier to work with in math problems. Also, answer choices are usually not given in the form of mixed numbers.

> Improper fractions have a numerator that is greater than the denominator.
> When you convert mixed numbers, you'll get an improper fraction as the result.

Decimals

Decimals are just fractions in disguise. Basically, decimals and fractions are two different ways of expressing the same thing. Every decimal can be written as a fraction, and every fraction can be written as a decimal. For example, the decimal 0.35 can be written as the fraction $\frac{35}{100}$. Therefore, these two numbers, 0.35 and $\frac{35}{100}$, have the same value.

To turn a fraction into its decimal equivalent, all you have to do is divide the numerator by the denominator. Here, for example, is how you would find the decimal equivalent of $\frac{3}{4}$:

$$\frac{3}{4} = 3 \div 4 = 4\overset{\displaystyle 0.75}{\overline{\smash{)}3.00}}$$

Try this problem:

$$\frac{1}{3} + \frac{2}{5} = x$$

$$y = 3$$

Quantity A	**Quantity B**
$\frac{y}{x}$	4

◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

Here's How to Crack It

Begin this quant comp question by solving for *x*. The common denominator is easy to find, as it is 15, so adjust the fractions to have the denominator of 15.

$$\frac{1}{3}+\frac{2}{5}=\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$$

The problem gives the value of *y*, so now solve for Quantity A. Quantity A is $\frac{3}{\left(\frac{11}{15}\right)}$, which equals $3\times\frac{15}{11}=\frac{45}{11}$. Now compare this to Quantity B. Dividing 45 by 11 yields a result slightly greater than 4, which means that Quantity A is greater than Quantity B and the correct answer is (A).

Comparing Decimals

Which is greater: 0.00099 or 0.001? ETS loves this sort of problem. You'll never go wrong, though, if you follow these easy steps.

- Line up the numbers by their decimal points.
- Fill in the missing zeros.

Here's how to answer the question we just asked. First, line up the two numbers by their decimal points.

$$\begin{array}{l} 0.00099 \\ 0.001 \end{array}$$

Now fill in the missing zeros.

$$\begin{array}{l} 0.00099 \\ 0.00100 \end{array}$$

Can you tell which number is greater? Of course you can. 0.00100 is greater than 0.00099, because 100 is greater than 99.

Digits and Decimals

Sometimes ETS will ask you questions about digits that fall after the decimal point as well. Suppose you have the number 0.584.

- 0 is the units digit.
- 5 is the tenths digit.
- 8 is the hundredths digit.
- 4 is the thousandths digit.

Percentages

A **percentage** is just a special type of fraction, one that always has 100 as the denominator. Percent literally means "per 100" or "out of 100" or "divided by 100." If your best friend finds a dollar and gives you 50¢, your friend has given you 50¢ out of 100¢, or $\frac{50}{100}$ of a dollar, or 50 percent of the dollar. To convert fractions to percentages, just expand the fraction so it has a denominator of 100:

$$\frac{3}{5} = \frac{60}{100} = 60\%$$

Another way to convert a fraction into a percentage is to divide the numerator by the denominator and multiply the result by 100.

So, $\frac{3}{5} = 3 \div 5 = 0.6 \times 100 = 60\%$.

For the GRE, you should memorize the following percentage-decimal-fraction equivalents. Use these friendly fractions and percentages to eliminate answer choices.

$0.01 = \frac{1}{100} = 1\%$ $\qquad$ $0.333... = \frac{1}{3} = 33\frac{1}{3}\%$

$0.1 = \frac{1}{10} = 10\%$ $\qquad$ $0.4 = \frac{2}{5} = 40\%$

$0.2 = \frac{1}{5} = 20\%$ $\qquad$ $0.5 = \frac{1}{2} = 50\%$

$0.25 = \frac{1}{4} = 25\%$ $\qquad$ $0.6 = \frac{3}{5} = 60\%$

Converting Decimals to Percentages

To convert decimals to percentages, just move the decimal point two places to the right. For example, 0.8 turns into 80 percent, 0.25 into

25 percent, 0.5 into 50 percent, and 1 into 100 percent.

Translation

One of the best ways to handle percentages in word problems is to know how to translate them into an equation that you can manipulate. Use the following table to help you translate percentage word problems into equations you can work with.

Word	Equivalent Symbol
percent	$\frac{}{100}$
is	=
of, times, product	×
what (or any unknown value)	any variable (x, k, b)

These translations apply to any word problem, not just percent problems.

Here's an example:

56 is what percent of 80 ?

- ◯ 66%
- ◯ 70%
- ◯ 75%
- ◯ 80%
- ◯ 142%

Here's How to Crack It

To solve this problem, let's translate the question and then solve for the variable. So, "56 is what percent of 80," in math speak, is equal to

$$56 = \frac{x}{100}(80)$$

$$56 = \frac{80x}{100}$$

Don't forget to reduce the fraction: $56 = \frac{4}{5}x$.

Now multiply both sides of the equation by the reciprocal, $\frac{5}{4}$.

$$\frac{5}{4}(56) = x$$

$$\frac{56 \times 5}{4} = x$$

$$\frac{280}{4} = x = 70$$

The correct answer is (B), 70%.

Let's try a quant comp example.

5 is *r* percent of 25.
s is 25 percent of 60.

Quantity A	Quantity B
r	s

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here's How to Crack It

First translate the first statement.

$$5 = \frac{r}{100}(25)$$

$$5 = \frac{25r}{100}$$

$$5 = \frac{r}{4}$$

$$(4)(5) = \left(\frac{r}{4}\right)(4)$$

$$20 = r$$

That takes care of Quantity A. Now translate the second statement.

$$s = \frac{25}{100}(60)$$

$$s = \frac{1}{4}(60)$$

$$s = 15$$

So Quantity A is greater than Quantity B. The answer is (A).

Percentage Increase/Decrease

Rather than asking for percents, ETS typically will test your knowledge by asking for percent change. Percent change is the percentage by which something has increased or decreased. To find percent change, use the following formula.

$$\text{Percent Change} = \frac{\text{Difference}}{\text{Original}} \times 100$$

> On percent increase problems, the original is always the smaller number. On percent decrease problems, the original is the larger number.

When presented with a percent change problem, you will typically be given two numbers. The "difference" is the result when the lesser number is subtracted from the greater number. The "original" is whichever number you started with. If the question asks you to find a **percent increase**, then the original number is the **lesser number**. If the question asks you to find a **percent decrease**, then the original number is the **greater number**.

On the GRE, a percent change will not be stated as a negative number. Instead, the problem will ask for a percent decrease. So, if something declined by 50%, the problem will ask for a percent decrease and the answer will be stated as 50%. Note that when you use the percent change formula, you always subtract the lesser number from the greater number to find the difference. Doing so ensures that you get a positive result.

Here's an example.

During a certain three-month period, Vandelay Industries reported a $3,500 profit. If, over the next three-month period, Vandelay Industries reported $6,000 profit for those months, by approximately what percent did Vandelay Industries' profit increase?

- ◯ 25%
- ◯ 32%
- ◯ 42%
- ◯ 55%
- ◯ 70%

Here's How to Crack It

Let's use the percent change formula we just learned. The first step is to find the difference between the two numbers. The initial profit was $3,500 and the final profit is $6,000. The difference between these two numbers is 6,000 − 3,500 = 2,500. Next, we need to divide this number by the original, or starting, value.

One way to help you figure out what value to use as the original value is to check to see whether you're dealing with a percent increase or a percent decrease question. Remember that on a percent increase question, you should always use the lesser of the two numbers as the denominator and on a percent decrease question, you need to use the greater of the two numbers as the denominator. Because the question asks to find the percent increase, the number we want to use for our denominator is 3,500. The percent increase fraction looks like this: $\frac{2,500}{3,500}$. This can be reduced to $\frac{25}{35}$ by dividing by 100, and reduced even further by dividing by 5. The reduced fraction is $\frac{5}{7}$, which is approximately 70% (remember that the fraction bar means divide, so if you divide 5 by 7, you'll get 0.71). Thus, (E) is the correct answer.

Here's another question.

Model	Original Price	Sale Price
A	$12,000	$9,500
B	$16,000	$13,000
C	$10,000	$7,500
D	$17,500	$13,000
E	$20,000	$15,500
F	$22,000	$16,000

The table above shows the original price and the sale price for six different models of cars. For which car models is the percent decrease at least 25% ?

Indicate <u>all</u> such models.

☐ A

☐ B

☐ C

☐ D

☐ E

☐ F

Here's How to Crack It

The task of this question is to identify a 25% or greater percent decrease between the two prices for the different car models. Use the percent change formula for all of the models to solve this question. Start with model A. Using the calculator, subtract 9,500 from 12,000 to get 2,500. This is the difference. Divide it by the original, 12,000, to get about 0.2, which when multiplied by 100 is 20%. Since 20% is less than 25%, eliminate (A). Try the next one. 16,000 – 13,000 = 3,000. Divide 3,000 by 16,000. The result is less than 25%, so eliminate (B). Repeat this process for each of the answer choices. Choices (C), (D), and (F) are the correct answers.

A FEW LAWS

These two basic laws are not necessary for success on the GRE, so if you have trouble with them, don't worry too much. However, ETS likes to use these laws to make certain math problems more difficult. If you're comfortable with these two laws, you'll be able to simplify problems using them, so it's definitely worth it to use them.

Associative Laws

There are two associative laws—one for addition and one for multiplication. For the sake of simplicity, we've lumped them together.

Here's what you need to know:

When you are adding or multiplying a series of numbers, you can regroup the numbers in any way you'd like.

Here are some examples:

$$4 + (5 + 8) = (4 + 5) + 8 = (4 + 8) + 5$$
$$(a + b) + (c + d) = a + (b + c + d)$$
$$4 \times (5 \times 8) = (4 \times 5) \times 8 = (4 \times 8) \times 5$$
$$(ab)(cd) = a(bcd)$$

Distributive Law

This is often tested on the GRE. Here's what it looks like:

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

Here's an example:

$$12(66) + 12(24) = ?$$

Here's How to Crack It

This is in the same form as $ab + ac$. Using the distributive law, this must equal 12(66 + 24), or 12(90) = 1,080.

Math Fundamentals Drill

Click here to download a PDF of Math Fundamentals Drill.

Test your new skills and check your answers in Part V.

1 of 10

If a prime number, *p*, is squared and the result is added to the next prime number greater than *p*, which of the following integers could be the resulting sum?

Indicate all such integers.

☐ 3

☐ 4

☐ 7

☐ 14

☐ 58

☐ 60

☐ 65

☐ 69

2 of 10

A bookstore will only order books that come in complete cases. Each case has 150 books and costs $1,757.

Quantity A	**Quantity B**
The number of books that can be ordered for $10,550	The number of books that can be ordered for $12,290

◯ Quantity A is greater.

○ Quantity B is greater.

○ The two quantities are equal.

○ The relationship cannot be determined from the information given.

3 of 10

If the product of two distinct integers is 91, then which of the following could be the sum of the two integers?

Indicate <u>all</u> such sums.

☐ −92

☐ −91

☐ 7

☐ 13

☐ 20

4 of 10

Which of the following is the units digit for the sum of all of the distinct prime integers less than 20?

○ 4

○ 5

○ 6

○ 7

○ 8

5 of 10

During a sale, a store decreases the prices on all of its scarves by 25 to 50 percent. If all of the scarves in the store were originally priced at $20, which of the following prices could be the sale price of a scarf?

Indicate <u>all</u> such prices.

☐ \$8

☐ \$10

☐ \$12

☐ \$14

☐ \$16

6 of 10

$$-2, 3, -5, -2, 3, -5, -2, 3, -5, \ldots$$

In the sequence above, the first 3 terms repeat without end. What is the product of the 81st term through the 85th term?

7 of 10

Quantity A	**Quantity B**
$4\left(\frac{1}{2}x + 2y\right)$	$2x + 8y$

◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

8 of 10

Quantity A	**Quantity B**
The greatest number of consecutive nonnegative integers	6

which have a sum less
than 22

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

9 of 10

If x is the remainder when a multiple of 4 is divided by 6, and y is the remainder when a multiple of 2 is divided by 3, what is the greatest possible value of $x + y$?

- ○ 2
- ○ 3
- ○ 5
- ○ 6
- ○ 9

10 of 10

$$12 - \left(\frac{6}{3} - 4 \times 3\right) - 8 \times 3 =$$

- ○ −46
- ○ −30
- ○ −18
- ○ −6
- ○ −2

Summary

- Familiarity with the basic math concepts on the GRE is essential to achieving a great score.
- Digits are the numbers that make up other numbers, which are collections of digits, and those other numbers are determined by the place value of the digits.
- Integers are numbers with no fractional part (such as −6, −1, 1, 10, etc.) and can be positive or negative, and even or odd.
- Zero is an integer that is neither positive nor negative.
- Consecutive integers are integers listed from least to greatest.
- The absolute value of a number is that number's distance away from zero on a number line.
- A factor of a particular integer is an integer that divides evenly into that integer.
- A multiple of a number is a number that has the original number as a factor.
- Prime numbers have only two factors: 1 and the number itself.
- Divisibility is the ability for one number to be divided into another number with a result that is an integer. If a number divided by another number and the result is not an integer, the amount that is leftover is called the remainder.
- Always follow the order of operations when working a math problem.
- Fractions, decimals, and percents are all ways of expressing parts of integers and can be manipulated and compared.

- The associative and distributive laws are useful ways to group and regroup numbers.

Chapter 11
Algebra (And When to Use It)

The basics for math on the GRE are often used in the context of algebra. While comfort with algebraic operations is a good skill to have, plugging in numbers in lieu of doing the algebra is often a much faster way of getting the correct answer. This chapter provides an introduction to the strategies we call Plugging In and Plugging In the Answers. It also explains how to deal with exponents and square roots and how to manipulate equations, inequalities, quadratic equations, and simultaneous equations.

PLUGGING IN

Many of the hardest questions you might encounter on the GRE involve algebra. Algebra questions are generally difficult for two reasons. First, they are often complicated, multistep problems. Second, ETS studies the types of mistakes that people make when they solve questions using algebra. They generate wrong answers for the questions based on these common algebraic errors. So, if you aren't careful, you can make an algebraic mistake and still find your answer among the choices.

If you are one of the many students who take the GRE and struggle with algebra, you're in luck. Plugging In is a strategy that will turn even the hardest, messiest GRE algebra problem into an arithmetic problem.

Let's look at an example of how Plugging In can make a seemingly messy algebra problem much easier to work with.

> Why Plug In?
>
> Plugging In is a powerful tool that can greatly enhance your math score, but you may be wondering why you should Plug In when algebra works just fine. Here's why:
>
> Plugging In converts algebra problems into arithmetic problems. No matter how good you are at algebra, you're better at arithmetic. Why? Because you use arithmetic every day, every time you go to a store, balance your checkbook, or tip a waiter. Chances are you rarely use algebra in your day-to-day activities.
>
> Plugging In is oftentimes more accurate than algebra. When you Plug In real numbers, you make the

problems concrete rather than abstract. Once you're working with real numbers, it's easier to notice when and where you've messed up a calculation. It's much harder to see where you went wrong (or to even know you've done something wrong) when you're staring at a bunch of x's and y's.

The GRE allows the use of a calculator. A calculator can do arithmetic but it can't do algebra, so Plugging In allows you to take advantage of the calculator function.

ETS expects its students to attack the problems algebraically and many of the tricks and the traps built into the problem are designed to catch students who use algebra. By Plugging In, you'll avoid these pitfalls.

As you can see, there are a number of excellent reasons for Plugging In. Mastering this technique can have a significant impact on your score.

Dale gives Miranda x bottles of water. He gives Marcella two fewer bottles of water than he gives to Miranda and he gives Mary three more bottles of water than he gives to Marcella. How many bottles of water did Dale give to Miranda, Marcella, and Mary, in terms of x ?

- ◯ $3x - 1$
- ◯ $3x$
- ◯ $3x + 1$
- ◯ $3x + 2$
- ◯ $x - 2$

Here’s How to Crack It

This problem can definitely be solved using algebra. However, the use of Plugging In makes this problem much easier to solve. The problem has one variable in it, x, so start plugging in by picking a number for x. An easy number to use would be 10, so use your scratch paper and write down $x = 10$. Now read the problem again and follow the directions, only this time do the arithmetic instead of the algebra on the scratch paper. So, Miranda gets 10 bottles of water. The problem then states that Marcella gets two fewer bottles of water than Miranda, so Marcella gets 8 bottles. Next, Mary gets three more bottles than Marcella, so Mary gets 11 bottles. That’s a total of $10 + 8 + 11 = 29$ bottles of water. The problem asks how many bottles of water Dale gave to Miranda, Marcella, and Mary, so the answer to the question is 29 bottles of water. This is the target answer, which should always be circled on the scratch paper so you don’t forget it. Now Plug In 10 for the variable x in all the answer choices and see which answer choice equals 29. Be sure to check all five answer choices.

(A)	$3(10) - 1 = 29$	Looks good!
(B)	$3(10) = 30$	Nope
(C)	$3(10) + 1 = 31$	Nope
(D)	$3(10) + 2 = 32$	Nope
(E)	$10 - 2 = 8$	Nope

The correct answer to this question is (A), and if you successfully completed the algebra you would have gotten the same answer. Pretty easy compared to the algebra, huh?

As you can see, Plugging In turned this algebra problem into an arithmetic problem. The best news is that you can solve any problem with variables by using Plugging In.

Plugging In

This technique can be achieved by following these five simple steps. Plugging in numbers in place of variables can make algebra problems much easier to solve.

Here are the steps:

Step 1: Recognize the opportunity. See variables in the problem and answer choices? Get ready to Plug In. The minute you see variables in a question or answer choices, you should start thinking about opportunities to Plug In.

Step 2: Set up the scratch paper. Plugging In is designed to make your life easier. Why make it harder again by trying to solve problems in your head? You are not saving any notable amount of time by trying to work out all the math without writing it down, so use the scratch paper. Even if it seems like an easy question of translating a word problem into an algebraic equation, remember that there are trap answer choices. Whenever you recognize the opportunity to Plug In, set up the scratch paper by writing (A) through (E) down before you start to solve.

Step 3: Plug In. If the question asks for "x apples," come up with a number for x. The goal here is to make your life easier, so plugging in numbers such as 2, 3, 5, 10, 100 are all good strategies. However, for the first attempt at Plugging In on any given problem, avoid the numbers 1 or 0. These numbers can oftentimes create a situation in which more than one answer choice produces the target answer. If you Plug In a number and the math starts getting difficult (for example, you start getting fractions or negative numbers), don't be afraid to just change the number you Plug In.

Step 4: Solve for the Target. The Target is the value the problem asks you to solve for. Remember to always circle the Target so you don't forget what it is you are solving for.

Step 5: Check all of the answer choices. Anywhere you see a variable, Plug In the number you have written down for that variable and do the arithmetic. The correct answer is the one that matches the Target. If more than one answer matches the Target, just Plug In a different number for the variables and test the answer choice you were unable to eliminate with the original number.

Can I Just Plug In Anything?

You can Plug In any numbers you like, as long as they're consistent with any restrictions stated in the problem, but it's more effective if you use easy numbers. What makes a number easy? That depends on the problem, but in most cases, lesser numbers are easier to work with than greater numbers. Usually, it's best to start with a lesser number, such as 2 for example. Avoid the numbers 0 and 1; both 0 and 1 have special properties, which you'll hear more about later. You want to avoid these numbers because they will often make more than one answer choice match the target. For example, if we Plug In 0 for a variable such as x, then the answers $2x$, $3x$, and $5x$ would all equal 0. Also, try to avoid plugging in any numbers that are repeats of numbers that show up a lot in the question or answer choices. If you can avoid plugging in 0, 1, or repeat numbers, you can oftentimes avoid situations that may make you have to Plug In again.

Plug In numbers that make the calculations EASY.

Good Numbers Make Life Easier

However, numbers of lesser value aren't always the best choices for Plugging In. What makes a number good to work with depends on the context of the problem, so be on the lookout for clues to help choose the numbers you are going to use to Plug In. For instance, in a problem involving percentages the numbers 10 and 100 are good numbers to use. In a problem that involves minutes or seconds any multiple or factor of 60, such as 30 or 120 are often good choices.

Let's use the Plugging In steps from above to work through the following problem.

Plug In real numbers for variables to turn algebra into arithmetic!

Mara has six more than twice as many apples as Robert and half as many apples as Sheila. If Robert has x apples, then, in terms of x, how many apples do Mara, Robert, and Sheila have combined?

- ◯ $2x + 6$
- ◯ $2x + 9$
- ◯ $3x + 12$
- ◯ $4x + 9$
- ◯ $7x + 18$

On the GRE, Plugging In is often more accurate, and easier, than doing the algebra.

Here's How to Crack It

Step 1: Recognize the opportunity. Look at the question. There is the variable x in the question stem and the answer choices.

This is a clear indication to start thinking about Plugging In.

Step 2: Set up the scratch paper. Keep yourself organized by listing (A) through (E) on the scratch paper. Leave some space to work the problem.

Step 3: Plug In. Plug In a good number. The problem states that Robert has x apples, and doesn't indicate that the number of apples needs to be anything specific, so choose an easy number such as $x = 4$.

Step 4: Solve for the Target. Now use $x = 4$ to read the problem again and solve for the target. The problem states that "Mara has six more than twice as many apples as Robert." If Robert has 4 apples, then Mara must have 14. Next, the problem states that Mara has "half as many apples as Sheila." That means that Sheila must have 28 apples. The question asks for the number of apples that Robert, Sheila, and Mara have combined so add 4 + 14 + 28 = 46 apples. This is the target number, so circle it.

Step 5: Check all of the answer choices. Plug In $x = 4$ for all of the variables in the answer choices and use the scratch paper to solve them, eliminating any answer choice that does not equal 46.

(A) 2(4) + 6 = 14—This is not 46, so eliminate it.

(B) 2(4) + 9 = 17—Eliminate this too.

(C) 3(4) + 12 = 24—Also not 46, so eliminate this.

(D) 4(4) + 9 = 25—This is still not 46, so eliminate this as well.

(E) 7(4) + 18 = 46—Bingo! This is the correct answer.

> Always be on the lookout for variables, and if you see them, get ready to Plug In!

On the GRE, Plug In for variables in the question and answer choices. Remember to Plug In numbers that will be easy to work with based on the problem, as some numbers can end up causing more trouble than they are worth.

When Plugging In within the Math section of the GRE, follow these general rules (we'll get more specific later on):

1. Avoid plugging in 0 or 1. These numbers, while easy to work with, have special properties.
2. Avoid plugging in numbers that are already in the problem; this often leads to more than one answer matching your target.
3. Avoid plugging in the same number for multiple variables. For example, if a problem has x, y, and z in it, pick three different numbers to Plug In for the three variables.
4. Avoid plugging in conversion numbers. For example, don't use 60 for a problem involving hours, minutes, or seconds.

Finally, Plugging In is a powerful tool, but you **must remember to always check all five answer choices when you Plug In.** In certain cases, two answer choices can yield the same target. This doesn't necessarily mean you did anything wrong; you just hit some bad luck. When this happens, just Plug In different numbers, solve for a new target, and recheck the answer choices that worked the first time.

PLUGGING IN THE ANSWERS (PITA)

Some questions may not have variables in them but will try to tempt you into using algebra to solve them. We call these Plugging In the Answers questions, or PITA for short. These are almost always difficult problems but once you recognize the opportunity to PITA, these questions turn into simple arithmetic questions. In fact, the hardest part of these problems is often identifying them as opportunities for PITA. The beauty of these questions is that they take advantage of one of the inherent limitations of a multiple-choice test: the answers are given to you. ETS has actually given you the answers, and only one of them is correct. The essence of this technique is to systematically Plug In the Answers to see which answer choice works given the information in the problem.

Let's look at an example of a Plugging In the Answers question.

Strategy!

PITA (which has nothing to do with the delicious type of bread) is a tried-and-true Princeton Review strategy!

An office supply store sells binder clips that cost 14 cents each and binder clips that cost 16 cents each. If a customer purchases 85 binder clips from this store at a total cost of $13.10, how many 14-cent binder clips does the customer purchase?

◯ 16

◯ 25

◯ 30

◯ 35

◯ 65

> Are you tempted to try to set up an algebraic equation? Are there no quickly identifiable variables? Are the answer choices real numbers? Try Plugging In the Answers!

Here's How to Crack It

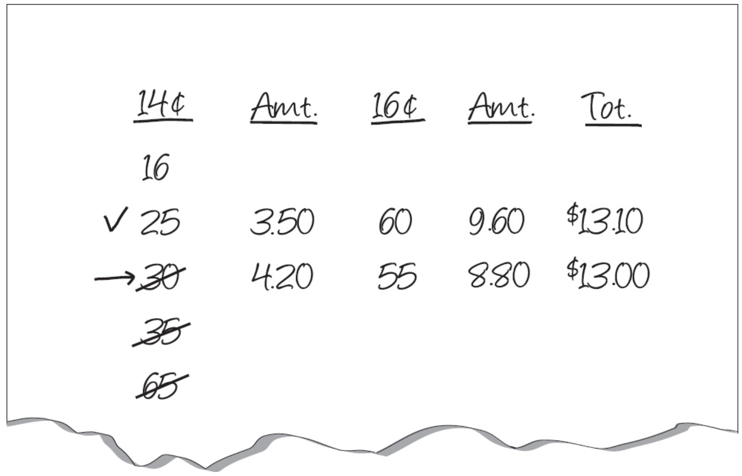
ETS would like you to solve this problem using algebra, and there is a good chance that you started to think about the variables you could use to set up some equations to solve this problem. That urge to do algebra is actually the first sign that you can solve this problem using Plugging In the Answers. Other signs that you can Plug In the Answers to solve this problem are that the question asks for a specific amount and that the numbers in the answer choices reflect that specific amount. With all these signs, it's definitely time to Plug In the Answers!

Start by setting up your scratch paper. To do so, just list the five answer choices in a column, with the actual numbers included. Since the problem is asking for the number of 14-cent binder clips purchased, these answer choices have to represent the number of 14-cent binder clips purchased. Label this column 14¢.

The answer choices will always be listed in either ascending or descending numerical order, so when you Plug In the Answers, start with (C). By determining whether or not (C) works, you can eliminate the other answer choices that are either greater or less than (C), based on the result of this answer choice. This effectively cuts the amount of work you need to do in half. So, start with the idea that the customer purchased 30 binder clips that cost 14 cents each. What can you figure out with this information? You'd know that the total spent on these binder clips is 30 × \$0.14 = \$4.20. So, make a column with the heading "amount spent" and write \$4.20 next to (C). Now, look for the next thing you'd know from this problem. If the customer purchased a total of 85 binder clips and 30 of them cost 14 cents each, that means that the customer purchased 55 binder clips at 16¢ each. Make another column with the heading "16¢" and write 55 in the row for (C). Next, make another column for the amount spent on 16-cent binder clips, label it "amount spent," and write 55 × \$0.16 = \$8.80 under this column in the row for (C). The next piece of information in the problem is that the customer spends a total of \$13.10 on the binder clips. This information allows you to determine if (C) is correct. All Plugging In the Answers questions contain a condition like this that lets you decide if the answer is correct. In this case, \$4.20 + \$8.80 = \$13.00, which is less than \$13.10, so eliminate (C). Since the total was not great enough, you can determine that to increase the total, the customer must have purchased more 16-cent binder clips. Since (D) and (E) would increase the number of 14-cent binder clips purchased, they cannot be correct. Eliminate (D) and (E) as well.

Now, do the same steps starting with (B). If the customer purchased 25 of the 14-cent binder clips, they cost \$3.50. The customer also purchased 60 of the 16-cent binder clips at a cost of \$9.60. The total amount spent is \$3.50 + \$9.60 = \$13.10. Since this matches the amount spent in the problem, (B) is correct.

Here's what your scratch paper should look like after this problem:



When you want to Plug In the Answers, here are the steps that you should follow.

Step 1: Recognize the opportunity. There are three ways to do this. The first indications are the phrases “how much...,” “how many...,” or “what is the value of....” When you see one of these phrases in a question, it’s a good indicator that you may be able to Plug In the Answers. The second tip-off is specific numbers in the answer choices in ascending or descending order. The last tip-off is your own inclination. If you find yourself tempted to write your own algebraic formulas and to invent your own variables to solve the problem, it’s a good sign that you can Plug In the Answers.

Step 2: Set up the scratch paper. The minute you recognize the opportunity, list the numbers in the answer choices in a column on the scratch paper.

Step 3: Label the first column. The question asks you to find a specific number of something so the answer choices must be options for that number. At the top of the column above the answer choices, write down what the numbers represent.

Step 4: Start with (C). Choice (C) will always be the number in the middle. This is the most efficient place to start because it will allow you to eliminate as many as three answer choices if it is wrong.

Step 5: Create your spreadsheet. Use (C) to work through the problem. It is always easier to understand the problem using a specific number. Work through the problem one step at a time, and every time you have to do something with the number, make a new column. Each column is a step in solving the problem that you may need to use again with a different answer choice, so don't leave anything out.

Step 6: Repeat with the other answer choices. On single-answer multiple-choice questions, only one answer choice can work. If (C) is correct, you are finished with the problem. If it is not correct, you may be able to determine if the value of the number is too great or too small. If it is too great, you can eliminate it and every answer choice that is greater. The same thing can be done if the value of the resulting answer is less than the value indicated by the problem. At this point, you have basically created your own little spreadsheet that is specifically designed to calculate the correct answer. Check the remaining answer choices by using the spreadsheet. As soon as you find an answer choice that works, you're finished.

On PITA questions, you don't need to check all five answer choices because only one of them can be correct. Once you have found an answer that works with the problem, select it and move on to the next problem. PITA is a great tool, but it requires a high level of organization, so make sure to keep track of everything that you do on the scratch paper.

PLUGGING IN ON QUANTITATIVE COMPARISON QUESTIONS

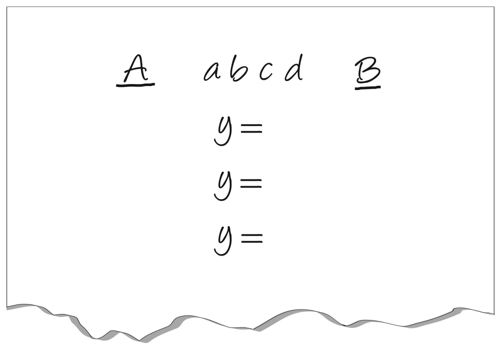
Quantitative comparison, or quant comp, questions with variables can be extremely tricky because the obvious answer is often wrong, whereas finding the correct answer may involve a scenario that is not commonly thought of. On the other hand, there is a simple setup and approach that you can use to help find the correct answers. As always, whenever you see variables, replace them with real numbers. On quant comp questions, however, it is crucial that you Plug In more than once and specifically that you Plug In different kinds of numbers that may not occur to you to think of initially. A good way to help you think of this is to always keep the nature of the answer choices in mind. Picking (A) means that you believe that the quantity in column A will *always* be greater than Quantity B—*no matter what number you Plug In*. Choice (B) means that the quantity in column B will *always* be greater than Quantity A—*no matter what number you Plug In*, and so forth. To prove that one of these statements is true you have to Plug In every possible number that could change the outcome. Don't worry. We have a simple process to help figure out what to Plug In and how to track your progress as you do.

> Quantitative comparison questions often test your knowledge of the properties of fractions, zero, one, negatives, and other weird numbers.

Here are the steps:

Step 1: Recognize the opportunity. The first seven or eight questions of any Math section will be quant comp. When a quant comp question appears and you see variables, you know that you can Plug In.

Step 2: Set up the scratch paper. The minute you see quant comp and variables, set up the scratch paper. The recommended setup should look something like the diagram below. Place Quantity A and B on either side. Quant comp questions have only four potential answer choices, so write (A), (B), (C), and (D) down as well, so you can eliminate answers as you go. Finally, leave space to write down the numbers that you Plug In for the variables in between the Quantities, so you can stay organized.



Step 3: Plug In and eliminate. Start with an easy number, just like outlined in the earlier Plugging In section, but make sure that you also follow any conditions in the problem. With the number you plugged in for the variable, solve for Quantity A and Quantity B and write the solutions down. If Quantity A is greater than Quantity B, eliminate (B) and (C). If Quantity B is greater than Quantity A, eliminate (A) and (C). If both quantities are the same, eliminate (A) and (B).

Step 4: Plug In again using FROZEN numbers. On quant comp questions with variables, you always need to Plug In more than once and the second time you do it, you need to use FROZEN numbers. FROZEN is an acronym that will be explained in the next section, as well the entire concept behind why to Plug In more than once, so keep reading!

Always Plug In More Than Once on Quant Comp Questions

Quant comp questions only have four options for answer choices but one of those options, (D), can be selected if the relationship between Quantity A and Quantity B cannot be determined based on the information given. After you Plug In the first time, you need to Plug In again, but this time you need to try to choose a number that will produce a different outcome for the question. While the first time you Plug In you can usually reliably eliminate two of answer choices (A), (B), or (C), you Plug In again to try to make sure that you can eliminate (D). Choice (D) can be eliminated only when you have a high level of confidence that no matter what number you Plug In, the answer will always remain the same. If even one of the numbers you choose creates a different answer, (D) should be selected.

On quant comp questions, Plug In easy numbers such as 2 or 5, and eliminate two choices. Then Plug In FROZEN numbers (Fractions, Repeats, One, Zero, Extremes, Negatives) to try to disprove your first answer. If different numbers give you different answers, you've proved that the answer is (D).

So, to eliminate (D), you need to choose a different number. But what makes a number different and what makes for a good number to choose that might create a different outcome for the problem? When you Plug In for the second (or sometimes third or fourth) time in a quant comp question, you should pick a FROZEN number. FROZEN is an acronym that highlights different types of numbers and it stands for

Fractions
Repeats
One
Zero
Extremes
Negatives

> **Pay Attention!**
> Note that on general GRE Math questions, conversion numbers (60 if a question is about minutes, hours, seconds) are a bad idea, but for quant comp questions, specifically, they are a great idea. This is because for quant comp Qs, we are seeking a number that will produce a different outcome for Quantity A and Quantity B, so Plugging In a repeat is great.

Fractions are numbers such as $\frac{1}{2}$ or $\frac{1}{4}$ that are great to use if the problem contains exponents or roots, as fractions respond to these two stimuli in a different way from whole numbers. Repeats are numbers that are found in the question stem, can be used in both Quantities, or numbers that are implied by the question stem (such as using the number 60 if the question is about seconds, minutes, or hours). The numbers 1 and 0 are special because they can result in two quantities being equal to each other, and each has a unique effect on other numbers. Extreme numbers are numbers such as 10 or 100 that should be used to see if the relationship between the quantities changes for numbers that are greater than the one that was initially chosen. Negative numbers, such as −2 or −3, are numbers that create different outcomes when plugged in for variables, as they can make Quantities negative or positive, which can alter the outcome.

FROZEN numbers can also be combined to create different numbers, such as −100, $-\frac{1}{2}$, or −1. ETS will often create a quant comp question that has a correct answer that depends on using these types of numbers. They do that because they know that most people will not think of these numbers, which is why it is important to Plug In more than once and, when you do, use FROZEN numbers.

Let's look at an example problem:

Quantity A	**Quantity B**
$2x^3$	$4x^2$

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here's How to Crack It

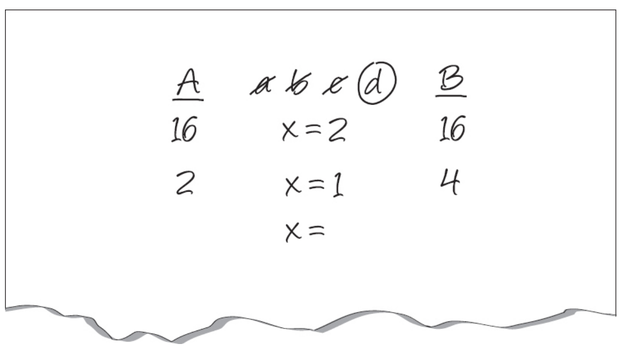
Step 1: Recognize the opportunity. This is a quant comp question with variables in the quantities, so this is a Plug In problem.

Step 2: Set up the scratch paper. Get yourself organized and ready to answer the problem by setting up the scratch paper.

Step 3: Plug In. Let's start with an easy number such as 2. Plug in 2 for x. When $x = 2$, Quantity A is $2 \times 2^3 = 16$, and Quantity B is $4 \times 2^2 = 16$ as well. Since both quantities are equal in this case, neither Quantity A nor Quantity B is always greater than the other, so eliminate (A) and (B).

Step 4: Plug In again using FROZEN numbers. Now look at the problem and try to decide on a FROZEN number for x that may create a different answer. Try One. If $x = 1$, then Quantity A is $2 \times 1^3 = 2$ and Quantity B is $4 \times 1^2 = 4$. Quantity B is now greater than Quantity A, which means that (C) is incorrect. Eliminate (C) and select (D), which is the correct answer.

If you chose to follow the recommended setup for the scratch paper, it should look like this:



You might also have noticed that choosing different FROZEN numbers, such as Fractions or Zero, would also yield a different result that would have allowed you to eliminate (C). This is not uncommon, as ETS is hoping you forget to use these FROZEN numbers when Plugging In. Make sure you use these numbers aggressively on quant comp problems because they can radically affect the relationship between the two Quantities.

PLUGGING IN ON FRACTION AND PERCENT PROBLEMS

Now that you've become familiar with fractions and percents, we'll show you a great method for solving many of these problems. When you come to regular multiple-choice questions, or multiple-choice, multiple-answer questions, that involve fractions or percents, you can simply Plug In a number and work through the problem using that number. This approach works even when the problem doesn't have variables in it. Why? Because, as you know, fractions and percents express only a relationship between numbers—the actual numbers don't matter. For example, look at the following problem:

> Plugging In on fraction and percent problems is a great way to make these problems much easier.

A recent survey of registered voters in City X found that $\frac{1}{3}$ of the respondents support the mayor's property tax plan. Of those who did not support the mayor's plan, $\frac{1}{8}$ indicated they would not vote to reelect the mayor if the plan were implemented. Of all the respondents, what fraction indicated that they would not vote for the mayor if the plan were enacted?

○ $\frac{1}{16}$

○ $\frac{1}{12}$

- $\frac{1}{6}$
- $\frac{1}{3}$
- $\frac{2}{3}$

What important information is missing from the problem?

Here's How to Crack It

Even though there are no variables in this problem, we can still Plug In. On fraction and percent problems, ETS will often leave out one key piece of information: the total. Plugging In for that missing value will make your life much easier. What crucial information did ETS leave out of this problem? The total number of respondents. So let's Plug In a value for it. Let's say that there were 24 respondents to the survey. 24 is a good number to use because we'll have to work with $\frac{1}{3}$ and $\frac{1}{8}$, so we want a number that's divisible by both those fractions. Working through the problem with our number, we see that $\frac{1}{3}$ of the respondents support the plan. $\frac{1}{3}$ of 24 is 8, so that means 16 people do not support the plan. Next, the problem says that $\frac{1}{8}$ of those who do not support the plan will not vote for the mayor. $\frac{1}{8}$ of 16 is 2, so 2 people won't vote for the mayor. Now we just have to answer the question: Of all respondents, what fraction will not vote for the mayor? Well, there were 24 total respondents and we figured out that 2 aren't voting. So that's $\frac{2}{24}$, or $\frac{1}{12}$. Choice (B) is the one we want.

ALGEBRA: OPERATIONS WITH VARIABLES

While Plugging In is a great strategy to make algebra problems easy on the GRE by turning them into arithmetic, in many cases being comfortable manipulating variables in an equation is necessary to answering a question. Plugging In will help you solve for a variable in a question, but sometimes the question requires only that you manipulate an equation to get the correct answer.

Dealing with Variables

The previous chapter familiarized you with number concepts and the previous section showed you how to turn algebra into arithmetic by using Plugging In. However, it's time to learn the basics of dealing with variables and manipulating equations to help make problems easier to work with and give you the best opportunity to optimize your score.

Manipulating Equations

When working with equations, you can do pretty much anything you want to them as long as you follow the golden rule:

> Whatever you do on one side of the equals sign you must also do on the other side.

Solving for One Variable

Let's begin the discussion of manipulating equations with one variable by solving for one variable. When presented with an equation with one variable, start by isolating the variable on one side of the equation and the numbers on the other side. You can do this by adding, subtracting, multiplying, or dividing both sides of the equation by the same number. Just remember that anything you do to one side of an equation, you must do to the other side. Let's look at a simple example:

$$3x - 4 = 5$$

> Don't assume you'll always need to solve for the variable on the GRE; sometimes you'll simply have to manipulate the equation to get the answer.

Here's How to Crack It

When presented with a problem like this, your goal is to isolate the variable on one side of the equation with all the real numbers, or constants, on the other. In the example above, begin manipulating this question by adding 4 to both sides of the equation. In general, you can eliminate negative numbers by adding them to both sides of the equation, just as you can eliminate positives by subtracting them from both sides of the equation.

$$\begin{array}{r} 3x - 4 = 5 \\ +\,4 = +\,4 \\ \hline 3x = 9 \end{array}$$

The variable is not quite isolated yet, as it is still being multiplied by 3. In the same way that you manipulated the equation earlier by moving the 4 to the other side of the equation, you must move the 3. Since the 3 is being multiplied to the variable, move it by doing the opposite operation, in this case division. This allows you to solve for x and finish the problem.

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

Let’s try another one:

$$5x - 13 = 12 - 20x$$

Here’s How to Crack It

Again, we want to get all the x values on the same side of the equation. This time, however, there is more than one instance of x so begin the question by combining the x values.

$$\begin{array}{r} 5x - 13 = 12 - 20x \\ \underline{+\ 20x \qquad\qquad\quad +\ 20x} \\ 25x - 13 = 12 \qquad\quad \end{array}$$

> As the problems get more involved, make sure to keep yourself organized by utilizing the scratch paper given to you.

Now that the values of x are combined, isolate the x by moving the negative 13 to the other side of the question.

$$\begin{array}{r} 25x - 13 = 12 \\ \underline{+\ 13 + 13} \\ 25x = 25 \end{array}$$

Solve for x by finishing the isolation by moving the 25 that it is being multiplied by.

$$25x = 25$$
$$\frac{25x}{25} = \frac{25}{25}$$
$$x = 1$$

Let's try one more that is slightly more complicated.

$$5x + \frac{3}{2} = 7x$$

Here's How to Crack It

The first thing you probably notice here is the fraction. Whenever you see an equation like this that contains a fraction, begin by "clearing" the fraction. To clear the fraction, multiply all the terms in the equation by the denominator of the fraction. In this case, multiply all the terms by 2.

$$10x + 3 = 14x$$

> You must always do the same thing to both sides of an equation.

Notice how all the terms have by multiplied by 2! This is very important, so don't forget to do it! Now, manipulate the equation to collect all the x's on the same side of the equation.

$$\begin{array}{rcl} 10x + 3 &=& 14x \\ -10x && -\ 10x \\ \hline 3 &=& 4x \end{array}$$

Now finish isolating the x by moving the 4.

$$3 = 4x$$
$$\frac{3}{4} = \frac{4x}{4}$$
$$\frac{3}{4} = x$$

WORKING WITH TWO VARIABLES

Many times, however, an equation on the GRE will involve two variables. An example of an equation with two variables looks like this:

$$3x + 10y = 64$$

Here's How to Crack It

The important thing to note about this situation is that we cannot solve this equation. Why, you ask? The problem is that since there are two variables, there are many possible solutions to this equation, all of which are equally valid. For example, plugging in the values $x = 8$ and $y = 4$ would satisfy the equation. But the equation would also be satisfied if you plugged in the values $x = 10$ and $y = 3.4$. Therefore, the GRE cannot test an equation with two variables without either providing a definitive way to solve for one of the variables, or providing a second equation. By giving two equations, you are able to find definitive values for the variables. So a more likely problem would look something like this:

$$3x + 10y = 64$$
$$6x - 10y = 8$$

You can't solve an equation with two variables unless you have a second equation.

Now there are 2 variables and 2 equations, which means we can solve for the variables. When two equations are given, look to combine them by adding or subtracting the entire equations. We do this so that we can cancel out one of the variables, leaving us with a simple equation with one variable. In this case, it's easier to add the two equations together, which will eliminate the *y* variable as seen below.

$$\begin{array}{rrl} & 3x + 10y & = 64 \\ + & 6x - 10y & = 8 \\ \hline & 9x & = 72 \end{array}$$

Add these two equations to get $9x = 72$. This is a simple equation, just like the ones discussed in the previous section, which we can solve to find $x = 8$. Once we've done that, we can solve for the other variable by inserting the value of x into one of the original equations. For example, if we substitute $x = 8$ into the first equation, we get $3(8) + 10y = 64$, and we can solve to find that $y = 4$.

The GRE will rarely give you two equations that line up as nicely as the above example does, though. You are more likely to find two equations with two variables and, while the variables match, the numbers associated with the variables are not equal. In this case, you will need to manipulate one equation so the numbers associated with a variable are equal. Doing this will allow the elimination of a variable when the two equations are added or subtracted. Try the next problem as an example.

$$4x + 7y = 41$$
$$2x + 3y = 19$$

Here's How to Crack It

Notice here that the numbers associated with the variables are not equal, which means that you cannot eliminate a variable. Adding the two equations yields $6x + 10y = 60$. That doesn't help; it's a single equation with two variables, which is impossible to solve. Subtracting the equations leaves $2x + 4y = 22$, which is also a single equation with two variables. To solve this system of equations, we need to make the coefficient for one of the variables in the first equation equal to the coefficient for that same variable in the second equation. In this case, try multiplying the second equation by 2.

$$2(2x + 3y) = 2(19)$$

This gives us the following:

$$4x + 6y = 38$$

Now we can subtract this equation from the first equation. Doing this operation yields $y = 3$. Now we can substitute $y = 3$ into either one of the original equations to find that $x = 5$.

Simultaneous Equations

Thus far, we have learned how to manipulate equations with one variable and two equations with two variables in order to solve for the variables. However, it is not uncommon for ETS to give you two equations and ask you to use them to find the value of a given expression. Much like manipulating two equations with two variables, all you need to do is add or subtract the two equations!

The only difference is this time you won't end up solving for an individual variable.

Here's an example:

If $5x + 4y = 6$ and $4x + 3y = 5$, then what does $x + y$ equal?

Here's How to Crack It

Remember that the problem is asking you to solve for $x + y$. This may appear like you need to solve for the variables individually, but try to add or subtract the equations first to see what they yield. First, try adding the two equations together.

$$\begin{array}{r} 5x+4y=6 \\ +\ 4x+3y=5 \\ \hline 9x+7y=11 \end{array}$$

Since the problem wants the value of $x + y$ and this gives us the value of $9x + 7y$, this is not useful. So try subtracting the two equations.

$$\begin{array}{r} 5x+4y=6 \\ -(4x+3y=5) \\ \hline x+\ \ y=1 \end{array}$$

Bingo. The value of the expression ($x + y$) is exactly what we're looking for. You could have tried to solve for each variable individually and solved the problem that way, but since the question is asking for the value of an expression, it was easier to manipulate the equations like this. So remember, on the GRE, you may see two equations written horizontally. Now you know that you don't need complicated math to solve them! Just rewrite the two equations, putting one on top of the other, and simply add or subtract them.

INEQUALITIES

The difference between an equation and an inequality is that in an equation one side always equals the other and in an inequality one side does *not* equal the other. Equations contain equal signs, while inequalities contain one of the following symbols:

$\neq$ is not equal to

$>$ is greater than

$<$ is less than

$\geq$ is greater than or equal to

$\leq$ is less than or equal to

The point of the inequality sign always points to the lesser value.

The good news is that inequalities are manipulated in the same way that you manipulated any of the equations in the previous sections of this chapter. However, there is one critical difference. When you multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol must change. So, if the inequality $x > y$ is multiplied by -1, the resulting inequality is $-x < -y$.

To see this rule in action, take a look at the following inequality:

$$12 - 6x > 0$$

Here's How to Crack It

There are two ways to solve this inequality. You could manipulate this inequality without ever multiplying or dividing by a negative number by just adding $6x$ to both sides and then dividing both sides of the inequality by the positive 6. In this case, the sign would not change, as seen below.

$$\begin{array}{r}
12 - 6x > 0 \\
\underline{\quad + 6x > + 6x} \\
12 > 6x
\end{array}$$

$$\frac{12}{6} > \frac{6x}{6}$$

$$2 > x$$

The other way to solve this inequality is to subtract 12 from both sides first. This will create a situation in which you need to divide both sides of the equation by -6, as shown below.

$$\begin{array}{r}
12 - 6x > 0 \\
\underline{-12 \qquad > -12} \\
-6x > -12
\end{array}$$

$$\frac{-6x}{-6} < \frac{-12}{-6}$$

$$x < 2$$

Notice that the sign flipped because you divided both sides by a negative number, but the answer for both methods of solving this inequality is the same thing. The first answer says that the number

2 is greater than x, and the second says that x is less than the number 2!

Flip the sign! When you multiply or divide both sides of an inequality by a negative number, the greater than/less than sign points the opposite way.

Inequalities show up on the GRE in a variety of ways. For instance, ETS may give you a range for two variables and then ask you to combine them in some way. This type of problem looks like the following question:

If $0 \le x \le 10$ and $-10 \le y \le -1$, then what is the range for $x - y$?

Here's How to Crack It

First, determine what the question is asking you to do. The question is asking you to solve for the range for the expressions $x - y$. To determine this, you need to consider all possible combinations of $x - y$. Since the inequalities are ranges themselves, find the greatest and least possible values of $x - y$ by calculating the largest x minus the largest y, the largest x minus the least y, the least x minus the largest y, and the least x minus the least y. The greatest value of x is 10 and the least value of x is 0. The greatest value of y is -1 and the least value is -10. Calculate these values and keep yourself organized by writing this all down on the scratch paper.

The calculations look as follows:

$$10 - (-1) = 11$$

$$10 - (-10) = 20$$

$$0 - (-1) = 1$$

$$0 - (-10) = 10$$

Since the least possible value of $x - y$ is $0 - (-1) = 1$ and the greatest possible value of $x - y$ is $10 - (-10) = 20$, the range is $1 \le x - y \le 20$.

QUADRATIC EQUATIONS

A quadratic equation is a special type of equation that generally shows up on the GRE in two forms. There is more to say about quadratic equations than simply stating that they are special types of equations, but that conversation is beyond the scope of this book. Instead, let's cover the two primary forms that quadratic equations take on the GRE and discuss how to respond to each of them.

On the GRE, quadratic equations are typically found as either full, expanded equations or fully factored equations.

Expanded Quadratic Equations

An expanded quadratic equation looks like this:

$$x^2 + 10x + 24 = 0$$

The hallmarks of this questions is a variable that is squared, a variable that multiplied by some coefficient, and then a coefficient. The operations in the equation are either addition or subtraction.

Occasionally, the GRE will try to disguise a fully expanded quadratic equation by moving terms around or including additional terms. But, if you can recognize the potential to create the more standard expanded quadratic, then you can treat these equations like any other expanded quadratic. For example, the GRE may try to disguise a standard expanded quadratic by writing it like this:

$$x^3 + 10x^2 + 24x = 0$$

While this may look similar to the previous instance of the equation, the exponents in the second instance of the equation are greater. But, this equation can be manipulated to look more like the standard quadratic by factoring out a value of x.

$$x(x^2 + 10x + 24) = 0$$

Now the information inside the parentheses looks exactly like the standard quadratic, and it can be manipulated by factoring the entire equation.

Factoring An Expanded Quadratic

The goal of factoring an expanded quadratic is to break the equation apart into values that would make the equation true. In the case of quadratics on the GRE, the way to make the equation true is to find the values of the variable that make the expression equal to 0. These are referred to as the roots of the equation. So, the roots of a quadratic equation are the values of the variable that make the expression equal to 0. For a standard expanded quadratic on the GRE, there are two correct roots for the equation.

To find these roots, you will need to factor the equation. Let's look at how to factor a quadratic and the steps that should be used to find the roots of the equation.

$$x^2 + 10x + 24 = 0$$

1. Begin by separating the x^2 into two values of x and placing them each inside of their own parentheses.

$$x^2 + 10x + 24 = 0$$

$$(x \quad)(x \quad) = 0$$

2. Find the factors of the third term that, when added or subtracted, yield the second term. In this case, the third term is 24 and the second term is 10. The factors of 24 are: 1 × 24, 2 × 12, 3 × 8, and 4 × 6. There are two pairs of factors that can be added or subtracted to produce a value of 10. Those factors are 2 × 12 and 4 × 6. So, you may need to try both pairs. Begin with 4 and 6 and place them inside the parentheses.

$$x^2 + 10x + 24 = 0$$
$$(x \quad 4)(x \quad 6) = 0$$

3. Determine the operations that correspond to each term. The combination of the signs and the term have to yield the second term of the original equation when added and the third term of the original equation when multiplied. In case this, because the second term of the original equation is 10 and the third term is 24, and both are positive, both operations are addition.

$$x^2 + 10x + 24 = 0$$
$$(x + 4)(x + 6) = 0$$

4. Finally, solve for the roots of the equation. Remember that the roots of the equation are the values of x that yield a value of 0 in the original equation. Therefore, to solve for the roots, take each element of the factored equation, set it equal to 0, and solve for x.

$$x^2 + 10x + 24 = 0$$
$$(x + 4)(x + 6) = 0$$
$$(x + 4) = 0 \text{ and } (x + 6) = 0$$
$$x = -4 \text{ and } x = -6$$

Therefore, the roots of the equation are −4 and −6. If these values are substituted for x in the equation, the equation is true.

Here's an example:

$$x^2 + 2x - 15 = 0$$

<u>**Quantity A**</u>	<u>**Quantity B**</u>
2	x

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here's How to Crack It

There is a fully expanded quadratic in the stem of the question, and Quantity B is the value of the variable in the quadratic, so solve for the roots of the equation. Do this by factoring the quadratic. First, set up the quadratic by placing the variables inside adjacent parentheses, $(x \quad)(x \quad)$. Next, determine the factors of −15 that, when added together, produce a sum of 2. The only factor pair of −15 that sums to 2 is 5 and −3, so place these values into the newly formed quadratic. This yields $(x + 5)(x - 3) = 0$. Now, solve for the roots of the equation, which are $x + 5 = 0$ and $x - 3 = 0$, so the roots are $x = -5$ and $x = 3$. Evaluate the quantities. If $x = 3$, then Quantity B is greater, so eliminate (A) and (C). However, if $x = -5$, then Quantity A is greater, so eliminate (B). The correct answer is (D).

Let's try another one:

If $x^2 + 8x + 16 = 0$, then what is the value of x ?

Here's How to Crack It

Let's factor the equation. Start with $(x \quad)(x \quad)$. Next, find the factors of 16 that have a sum of 8. Of the factors of 16, only 4 and 4 have a sum of 8. Thus, we end up with $(x + 4)(x + 4) = 0$. Now, we need to solve the equation. If $x + 4 = 0$, then $x = -4$.

Fully Factored Quadratic Equations

The fully factored quadratic equation is the other form that quadratics typically take on the GRE. You have already seen what this fully factored form looks like near the end of the previous section. The fully factored form of a quadratic equation looks like this:

$$(x + 4)(x - 7) = 0$$

In this form, the roots of the equation can be easily found by solving for each value of x, as done in the previous section. When working with full factored quadratics, usually the goal is to expand the quadratic back to the standard form. The completed standard form of the equation above is:

$$x^2 - 3x - 28 = 0$$

The goal of working with a fully factored quadratic will be to manipulate it into its expanded form. To do this, it's necessary to employ the FOIL tactic.

FOIL

FOIL stands for *First*, *Outside*, *Inside*, and *Last*. This acronym represents the order by which the elements of the factored quadratic will need to be multiplied by one another to manipulate a fully factored quadratic equation into its expanded form. The end result is to multiply each term in the first set of parentheses by each term in the second set of parentheses and then add the results.

The *First* step indicates to multiply the first term in the first set of parentheses by the first term in the second set of parentheses. In the fully factored equation provided above, the first term in each of the sets of parentheses is x, which produces a product of x^2.

$$(x + 4)(x - 7) = 0$$
$$x^2 = 0$$

The *Outside* step indicates to multiply the terms that are on the "outside" of the terms inside the parentheses. The terms that are on the outside of the terms inside the parentheses are the first term of the first set of parentheses and the second term of the second set of parentheses. In this case, those terms are x and -7, respectively, to produce a product of -7.

$$(x + 4)(x - 7) = 0$$
$$x^2 - 7x = 0$$

The Inside step indicates to multiply the terms that are on the "inside" of the terms inside the parentheses. The terms that are on the inside of the terms inside the parentheses are the second term of the first set of parentheses and the first term of the second set of parentheses. In this case, these terms are 4 and x, respectively, to product a product of $4x$.

$$(x + 4)(x - 7) = 0$$
$$x^2 - 7x + 4x = 0$$

Finally, the *Last* step indicates to multiply the terms that are “last” of the terms inside the parentheses. The terms that are last are the second term of the first set of parentheses and the second term of the second set of parentheses. In this case, those terms are 4 and −7, which produces a product of −28.

$$(x + 4)(x - 7) = 0$$
$$x^2 - 7x + 4x - 28 = 0$$

Combine the terms in the equation to reveal the expanded form of the quadratic, $x^2 - 3x - 28 = 0$.

Let’s look at a question that uses quadratics that you may see on the GRE.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$(4 + \sqrt{6})\,(4 - \sqrt{6})$	10

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here’s How to Crack It

First, eliminate (D) because there are only numbers in this question, so the answer can be determined. Quantity A is in the format to be solved for using FOIL. Begin by multiplying the first terms, $4 \times 4 = 16$. Next, multiply the outer terms to yield $-4\sqrt{6}$.

Multiply the inner terms to get $4\sqrt{6}$. Multiply the last terms to get -6. So, Quantity A is now $16 - 4\sqrt{6} + 4\sqrt{6} - 6$. The two inner terms cancel each other out and all that remains is $16 - 6$, or 10. Since Quantity B is also equal to 10, the two answer choices are equal and the correct answer is (C).

Common Quadratics

There are three types of quadratics that are considered common quadratics. They are considered common quadratics, because if you see a quadratic written in this exact form, it can always be factored out in the same way. Knowing these equations both in their factored and expanded forms may drastically save you time on these questions. These common quadratics are:

1. $x^2 - y^2 = (x + y)(x - y)$—this is sometimes referred to as the difference of squares.
2. $(x + y)^2 = x^2 + 2xy + y^2$
3. $(x - y)^2 = x^2 - 2xy + y^2$

You should devote some time to committing these common quadratics to memory.

Let's look at one more example.

If x and y are positive integers, and if $x^2 + 2xy + y^2 = 25$, then what is the value of $(x + y)^3$?

○ 5

- 15
- 50
- 75
- 125

Here's How to Crack It

While this problem may look like a lot of work, if you have committed the common quadratic equations from earlier in this section to memory, then the answer is easier to come by. The equation in this question is reflective of the common quadratic: $x^2 + 2xy + y^2 = (x + y)^2$. The question tells us that $x^2 + 2xy + y^2$ is equal to 25, which means that $(x + y)^2$ is also equal to 25. Think of $x + y$ as one unit that, when squared, is equal to 25. Since this question specifies that x and y are positive integers, what positive integer squared equals 25? The number 5 does, so $x + y = 5$. The question is asking for $(x + y)^3$. In other words, what's 5 cubed, or $5 \times 5 \times 5$? The answer is (E), 125.

EXPONENTS AND SQUARE ROOTS

Finally, the last section of this chapter is going to deal with exponents and square roots. Questions with exponents and square roots are common on the GRE and solving these questions often requires manipulating the exponents or roots. Here's the information you need to know in order to work with them.

What Are Exponents?

Exponents are a sort of mathematical shorthand for repeated multiplication. Instead of writing (2)(2)(2)(2), you can use an exponent and write 2^4. The little 4 is the **power** and the 2 is called the **base**. The power tells you how many times to multiply the base by itself. Knowing this terminology will be helpful in following the discussion in this section.

The Five Rules of Working with Exponents

For the GRE there are five major rules that apply when you work with exponents. The more comfortable you are with these rules, the more likely you will be to approach an exponent question with confidence and get the answer correct!

The first three rules deal with the combination and manipulation of exponents. Those three rules are represented by the acronym **MADSPM**, which stands for the following:

Multiply
Add

Divide
Subtract

Power
Multiply

These three rules will be explained in more detail shortly, but for now just remember the following conditions:

- When you see exponents with equal bases which are being **multiplied**, **add** the powers.
- When equal bases are **divided**, **subtract** the exponents.
- When an exponent is raised to a **power**, **multiply** the powers.

The fourth rule is the definition of a **negative exponent**. The fifth and final rule is the definition of a **zero exponent**.

The Multiply-Add Rule of Exponents

When two exponents with equal bases are multiplied, you must add the exponents. Consider the following example:

$$3^2 \times 3^3$$

As defined earlier, a power just tells you how many times to multiply a base by itself. So another way to write this expression is as follows:

$$3^2 \times 3^3 = (3 \times 3)(3 \times 3 \times 3) = 3^5$$

As you can see, the number of bases, which in this case is the integer 3, that are actually being multiplied together is five, as there are two 3's that are represented by 3^2 and three 3's that are represented by 3^3.

Now solve this question more quickly by using the multiply-add rule.

$$3^2 \times 3^3 = 3^{2+3} = 3^5$$

The Divide-Subtract Rule of Exponents

When two exponents with equal bases are divided, you must subtract the exponents. Consider the following example and expand the exponents to make it clear.

$$\frac{5^3}{5^2} = \frac{5 \times 5 \times 5}{5 \times 5} = \frac{5}{1} = 5$$

Now, instead of expanding the exponents, just apply the divide-subtract rule for the same problem.

$$\frac{5^3}{5^2} = 5^{3-2} = 5^1 = 5$$

The Power-Multiply Rule of Exponents

When an expression with an exponent is raised to another power, multiply the powers together. Consider the following example and expand the exponents to make it clear.

$$(6^2)^3 = (6^2)(6^2)(6^2) = (6 \times 6)(6 \times 6)(6 \times 6) = 6^6$$

Now, apply the power-multiply rule to solve the same problem.

$$(6^2)^3 = 6^{2 \times 3} = 6^6$$

For all of these rules, the bases must be the same. So, for example, you could not divide-subtract the expression $\frac{3^3}{2^2}$ because the bases are not the same.

Negative Exponents

A negative exponent is another way ETS uses exponents on the GRE. Consider the following example.

$$\frac{8^3}{8^5} = 8^{3-5} = 8^{-2} = \frac{1}{8^2} = \frac{1}{64}$$

So when you have a negative exponent, all that needs to be done is to put the entire expression in a fraction, with 1 in the numerator and the exponent in the denominator, and change the negative exponent to a positive. A term raised to a negative power is the reciprocal of that term raised to the positive power.

Zero Exponents

Sometimes ETS will give you an exponent question that, after you have successfully manipulated it, results in a base number raised to a power of 0. Any nonzero number raised to a power of 0 is equal to 1. Consider the following example.

$$4^3 \times 4^{-3} = 4^{3-3} = 4^0 = 1$$

Exponent Tips Beyond the Five Rules

Sometimes you will have an exponent problem for which none of the five rules discussed apply. If you reach this point, there are two tips to keep you moving forward.

Tip 1: Rewrite Terms Using Common Bases

ETS will always write questions that work out nicely, so if none of the bases in an exponent question seem to match up, see if you can find a way to rewrite the bases so that they match, and you will be able to use one of the five rules.

Tip 2: Look for a Way to Factor the Expression

Factoring the expression is often a way to reveal something about the exponent expression that you may not have noticed before. If you get stuck with an exponent question, try to factor the expression and see if there is a way to use one of the five rules.

It will be uncommon for ETS to test just one or two of these concepts on a GRE problem. Most times, two or more of these concepts will be combined to create a problem. Let's look at a couple of examples.

If $y \neq 0$, which of the following is equivalent to $\frac{y^9}{y(y^2)^3}$?

- ○ y
- ○ y^2
- ○ y^3
- ○ y^4
- ○ y^5

Here's How to Crack It

Begin by simplifying the denominator of the fraction. Use the power-multiply rule to combine $(y^2)^3$ into y^6. Since a number, or in this case a variable, by itself is the same thing as having that number or variable raised to a power of 1, use the add-multiply rule to combine $y(y^6)$ into y^7. Now use the divide-subtract rule to solve the problem; the correct answer is (B).

Let's look at another problem.

Quantity A	**Quantity B**
$15^{15} - 15^{14}$	$15^{14}(14) - 1$

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here's How to Crack It

The question wants you to compare the two quantities, but since none of the rules for exponents apply here, see if there is something else you can do to this problem. The expression in Quantity A can be factored, so begin there. Quantity A is now $15^{15} - 15^{14} = 15^{14}(15 - 1)$, which is the same thing as $15^{14}(14)$. Notice how this is the same as Quantity B, except Quantity B is 1 less than $15^{14}(14)$. Therefore, Quantity A is greater and the correct answer is (A).

Take a look at one more exponent problem.

If $x \neq 0$ and $64^3 = 8^x$, then what is the value of x^2 ?

- ◯ 5
- ◯ 6
- ◯ 25
- ◯ 36
- ◯ 64

Here's How to Crack It

Solve this question by determining the value for x. To solve for x in this equation, start by rewriting the exponent expressions using a common base. Since 64 can be written as 8^2, the equation can be rewritten as $(8^2)^3 = 8^x$. Since the bases are the same, for the equation to be equal the powers have to be the same as well. $(8^2)^3$ can be rewritten as 8^6 because of the power-multiply rule, so if $8^6 = 8^x$ then $x = 6$. Now plug that number into the value for x^2. This is now 6^2 which equals 36, so the correct answer is (D).

The Peculiar Behavior of Exponents

- Raising a number greater than 1 to a power greater than 1 results in a greater number. For example, $2^2 = 4$.
- Raising a fraction that's between 0 and 1 to a power greater than 1 results in a lesser number. For example, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.
- A negative number raised to an even power results in a positive number. For example, $(-2)^2 = 4$, because $(-2)(-2) = 4$.

- A negative number raised to an odd power results in a negative number. For example, $(-2)^3 = -8$, because $(-2)(-2)(-2) = -8$.
- A number raised to the first power ALWAYS results in the number itself. For example, $1{,}000^1 = 1{,}000$.

What Is a Square Root?

The radical sign $\sqrt{\ }$ indicates the **square root** of a number. For example, $\sqrt{4}$ means that some number times itself is equal to 4. In this case, since $2^2 = 4$, it can be determined that $\sqrt{4} = 2$. Think of square roots as the opposite of exponents. If you want to eliminate a square root in an equation, all you need to do is raise that square root to a power of 2. Just remember to do that for all of the elements in the equation!

Square roots can exist only on the GRE with nonnegative numbers. If the problem states that $x^2 = 16$, then $x = \pm 4$ as both a positive and a negative 4, when multiplied by itself, yields 16. However, when you find the square root of any number, the result will always be positive.

Rules for Square Roots

There are rules that dictate what you can and cannot do with square roots, just like there are rules about exponents.

> You can multiply and divide any square roots, but you can add or subtract roots only when the number under the radical sign is the same.

Adding and Subtracting Square Roots

You can add or subtract square roots only if the values under the radical sign are equal. So, for example, the expression $2\sqrt{5}+4\sqrt{5}$ can be simplified to $6\sqrt{5}$ because the value under the radical sign is equal. Conversely, the expression $3+2\sqrt{5}$ cannot be reduced any further because the values of the roots are not the same.

Rules for Adding and Subtracting Square Roots

$$a\sqrt{r}+b\sqrt{r}=(a+b)\sqrt{r}$$
$$a\sqrt{r}-b\sqrt{r}=(a-b)\sqrt{r}$$

Multiplying and Dividing Square Roots

Any square roots can be multiplied or divided. There aren't any restrictions on this, so keep an eye out for opportunities to combine roots by multiplying or dividing that could make a root easier to work with. For example, $\sqrt{3}\times\sqrt{12}=\sqrt{36}=6$. Roots can be divided as well; for example, $\sqrt{\frac{12}{3}}=\frac{\sqrt{12}}{\sqrt{3}}=\sqrt{4}=2$.

Rules for Multiplying and Dividing Square Roots

$$a\sqrt{r} \times b\sqrt{s} = (a \times b)\sqrt{rs}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Simplifying Square Roots

Oftentimes when you multiply square roots on the GRE, you will not get numbers under the radical sign that work out perfectly. When this happens, you will need to simplify the square root. You simplify a square root by looking for ways to factor the number under the root that results in at least one perfect square. A perfect square is an integer that when the square root of that integer is taken, the result is another integer. For example, 4 is a perfect square because $\sqrt{4} = 2$. Similarly, 9 and 25 are perfect squares because $\sqrt{9} = 3$ and $\sqrt{25} = 5$, respectively. Look at the following expression and try to simplify it.

$$\sqrt{2} \times \sqrt{10}$$

You can combine this expression to result in $\sqrt{20}$. However, this is not in the most simplified form. Look for ways to factor 20 in which one of the pairs of numbers is a perfect square. The factors of 20 are 1 and 20, 2 and 10, and 4 and 5. Since 4 is a perfect square, this can now be simplified even further.

$$\sqrt{2} \times \sqrt{10} = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

Now let's take a look at some examples of how ETS might test roots on the GRE.

What is the value of the expression $3\sqrt{80} - 2\sqrt{5}$?

- ◯ $4\sqrt{5}$
- ◯ $5\sqrt{3}$
- ◯ $10\sqrt{5}$
- ◯ $12\sqrt{3}$
- ◯ $20\sqrt{5}$

Here's How to Crack It

The problem is subtracting roots. Since roots cannot be subtracted unless the numbers under the radical sign are equal, look for a way to simplify the roots. Since 5 cannot be simplified any further, work with 80. The factors of 80 are 1 and 80, 2 and 40, 4 and 20, 5 and 16, and 8 and 10. Two of these pairs of factors contain a perfect square, but one contains a perfect square and a prime number. This is a good thing. This means that it could be reduced no further, so choose 5 and 16 and simplify to read $3\sqrt{80} = 3\sqrt{5 \times 16} = (3 \times 4)\sqrt{5} = 12\sqrt{5}$. Now that the bases are equal, subtract the expression to find that $12\sqrt{5} - 2\sqrt{5} = 10\sqrt{5}$, which is (C). The same answer would have been found if the numbers 4 and

20 had been chosen as the factors of 80, but there would have been another round of simplifying the root, as 20 would have needed to be reduced to 4 and 5 as factors.

Here's another problem.

$$z^2 = 144$$

Quantity A	**Quantity B**
z	$\sqrt{144}$

- ◯ Quantity A is greater.
- ◯ Quantity B is greater.
- ◯ The two quantities are equal.
- ◯ The relationship cannot be determined from the information given.

Here's How to Crack It

The trap answer here is (C). Remember that if $z^2 = 144$, then the value of z is either 12 or −12. However, when a value is under a radical sign—that is, when you're looking for the square root—it can only be positive. Therefore, Quantity A could be 12 or −12 and Quantity B can only be 12. Since there is no way to ensure that one is always greater than the other, the correct answer is (D).

Try one more problem:

What is the value of $\frac{\sqrt{75}}{\sqrt{27}}$?

◯ $\frac{5}{3}$

◯ $\frac{25}{9}$

◯ 3

◯ $3\sqrt{3}$

◯ $3\sqrt{5}$

To Simplify Roots

1. Rewrite the number as the product of two factors, one of which is a perfect square.
2. Use the multiplication rule for roots.

Here's How to Crack It

First, simplify each of these roots. $\sqrt{75}$ has a factor that is a perfect square—25, so it can be rewritten as $\sqrt{25 \times 3}$ and simplified to $5\sqrt{3}$. $\sqrt{27}$ has the perfect square 9 as a factor, so it can be written as $\sqrt{9 \times 3}$ and then simplified to $3\sqrt{3}$. This means that $\frac{\sqrt{75}}{\sqrt{27}}$ is equal to $\frac{5\sqrt{3}}{3\sqrt{3}}$; the $\sqrt{3}$ in the numerator and denominator cancel, leaving $\frac{5}{3}$. The correct answer is (A).

Algebra (And When to Use It) Drill

Click here to download a PDF of Algebra (And When to Use It) Drill.

Now it's time to try out what you have learned on some practice questions. Try the following problems and then check your answers in Part V.

1 of 10

The original selling price of an item at a store is 40 percent more than the cost of the item to the retailer. If the retailer reduces the price of the item by 15 percent of the original selling price, then the difference between the reduced price and the cost of the item to the retailer is what percent of the cost of the item to the retailer?

2 of 10

$$x^2 + 8x = -7$$

Quantity A	**Quantity B**
x	0

◯ Quantity A is greater.

◯ Quantity B is greater.

◯ The two quantities are equal.

◯ The relationship cannot be determined from the information given.

3 of 10

If $3^3 \times 9^{12} = 3^x$, what is the value of x ?

4 of 10

If $A = 2x - (y - 2c)$ and $B = (2x - y) - 2c$, then $A - B =$

- $-2y$
- $-4c$
- 0
- $2y$
- $4c$

5 of 10

A merchant sells three different sizes of canned tomatoes. A large can costs the same as 5 medium cans or 7 small cans. If a customer purchases an equal number of small and large cans of tomatoes for the same amount of money needed to buy 200 medium cans, how many small cans does she purchase?

- 35
- 45
- 72
- 199
- 208

6 of 10

If $6k - 5l = 27$ and $3l - 2k = -13$ and $5k - 5l = j$, what is the value of j?

7 of 10

If a is multiplied by 3 and the result is 4 less than 6 times b, what is the value of $a - 2b$?

- -12
- $-\frac{4}{3}$

- ○ $-\frac{3}{4}$
- ○ $\frac{4}{3}$
- ○ 12

8 of 10

Quantity A	Quantity B
$\frac{2^{-4}}{4^{-2}}$	$\frac{\sqrt{64}}{-2^{3}}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

9 of 10

$11x + 14y = 30$ and $3x + 4y = 12$

Quantity A	Quantity B
$x + y$	$(x + y)^{-2}$

- ○ Quantity A is greater.
- ○ Quantity B is greater.
- ○ The two quantities are equal.
- ○ The relationship cannot be determined from the information given.

10 of 10

If $x = 3a$ and $y = 9b$, then all of the following are equal to $2(x + y)$ EXCEPT

- ○ $3(2a + 6b)$
- ○ $6(a + 3b)$
- ○ $24(\frac{1}{4}a + \frac{3}{4}b)$
- ○ $\frac{1}{3}(18a + 54b)$
- ○ $12(\frac{1}{2}a + \frac{3}{4}b)$

Summary

- Plugging In converts algebra problems to arithmetic problems. Plug In by replacing variables in the question with real numbers or by working backward from the answer choices provided.
- Use easy numbers first when Plugging In for variables. If the need arises to Plug In again, use the FROZEN numbers to help eliminate tricky answer choices on math problems.
- The golden rule of equations: whatever you do to one side of the equation, you must do to the other.
- In order to solve an equation with two variables, you need two equations. Stack them up and add or subtract to cancel out one of the variables.
- Inequalities are manipulated the same way that equations are, with one notable difference: always remember to flip the sign when you multiply or divide by a negative number.
- Use the FOIL process to expand quadratics. To solve a quadratic equation, set it equal to zero, and factor.
- An exponent is shorthand for repeated multiplication. When in doubt on exponent problems, look to find common bases or ways to factor the expressions.
- Think of a square root as the opposite of an exponent. Square roots are always positive.

Chapter 12

Real-World Math

Real-world math is our title for the grab bag of math topics that will be heavily tested on the GRE. This chapter details a number of important math concepts, many of which you've probably used at one point or another in your daily adventures, even if you didn't recognize them. After completing this chapter, you'll have brushed up on important topics such as ratios, proportions, and averages. You'll also learn some important Princeton Review methods for organizing your work and efficiently and accurately answering questions on these topics.

EVERYDAY MATH

A few years ago when ETS reconfigured the GRE, they wanted the Math section to test more of what they call "real life" scenarios that a typical graduate student might see. You can therefore expect the math questions on the GRE to heavily test topics such as proportions, averages, and ratios—mathematical concepts that are theoretically part of your everyday life. Regardless of whether that's true of your daily life or not, you'll have to master these concepts in order to do well on the GRE Math section.

> The math on the GRE is supposed to reflect the math you use in your everyday activities.

RATIOS AND PROPORTIONS

If you're comfortable working with fractions and percentages, you'll be comfortable working with ratios and proportions, because ratios and proportions are simply special types of fractions. Don't let them make you nervous. Let's look at ratios first and then we'll deal with proportions.

What Is a Ratio?

Recall that a fraction expresses the relationship of a part to the whole. A **ratio** expresses a different relationship: part to part. Imagine yourself at a party with 8 women and 10 men in attendance. Remembering that a fraction expresses a part-to-whole relationship, what fraction of the partygoers are female? $\frac{8}{18}$, or 8 women out of a total of 18 people at the party. But what's the ratio, which expresses a part to part relationship, of women to men? $\frac{8}{10}$, or as ratios are more commonly expressed, 8:10. You can reduce this ratio to 4:5, just like you would a fraction.

> A ratio is just another type of fraction.

On the GRE, you may see ratios expressed in several different ways:

$x : y$
the ratio of x to y
x is to y

In each case, the ratio is telling us the relationship between parts of a whole.

Every Fraction Can Be a Ratio, and Vice Versa

Every ratio can be expressed as a fraction. A ratio of 1:2 means that the total of all the parts is either 3 or a multiple of 3. So, the ratio 1:2 can be expressed as the fraction $\frac{1}{2}$, or the *parts* of the ratio can be expressed as fractions *of the whole* as $\frac{1}{3}$ and $\frac{2}{3}$. Likewise, the fraction $\frac{1}{3}$ expresses the ratio 1:3. So if a question says "the ratio of x to $2y$ is $\frac{1}{3}$," then that would be expressed as $\frac{x}{2y} = \frac{1}{3}$.

Treat a Ratio Like a Fraction

Anything you can do to a fraction you can also do to a ratio. You can cross multiply, find common denominators, reduce, and so on.

Find the Total

The key to dealing with ratio questions is to find the whole, or the total. Remember, a ratio tells us only about the parts, not the total. In order to find the total, add the numbers in the ratio. A ratio of 2:1 means that there are three total parts. A ratio of 2:5 means that we're talking about a total of 7 parts. And a ratio of 2:5:7 means there are 14 total parts. Once you have a total you can start to do some fun things with ratios.

For example, let's say you have a handful of pennies and nickels. If you have 30 total coins and the pennies and nickels are in a 2:1 ratio, how many pennies do you have? The total for our ratio is 3, meaning that out of every 3 coins, there are 2 pennies and 1 nickel. So if there are 30 total coins, there must be 20 pennies and 10 nickels. Notice that $\frac{20}{10}$ is the same as $\frac{2}{1}$, is the same as 2:1!

> Like a fraction, a ratio expresses a relationship between numbers.

When you are working with ratios, there's an easy way not only to keep track of the numbers in the problem but also to quickly figure out the values in the problem. It's called a ratio box. Let's try the same question, but with some different numbers; if you have 24 coins in your pocket and the ratio of pennies to nickels is 2:1, how many pennies and nickels are there? The ratio box for this question is below, with all of the information we're given already filled in.

	Pennies	Nickels	Total
ratio	2	1	3
multiply by			
actual numbers			24

The minute you see the word "ratio," draw a Ratio Box on your scratch paper.

Remember that ratios are relationships between numbers, not actual numbers, so the real total is 24; that is, you have 24 actual coins in your pocket. The ratio total (the number you get when you add the number of parts in the ratio) is 3.

The middle row of the table is for the multiplier. How do you get from 3 to 24? You multiply by 8. Remember when we talked about finding equivalent fractions? All we did was multiply the numerator and denominator by the same value. That's exactly what we're going to do with ratios. This is what the ratio box looks like now:

	Pennies	Nickels	Total
ratio	2	1	3
multiply by	8	8	8
actual numbers			24

The multiplier is the key concept in working with ratios. Just remember that whatever you multiply one part by, you must multiply *every* part by.

Now let's finish filling in the box by multiplying everything else.