

**Assignment #2: Fundamentals of Digital Transmission, and  
Error Detection and Correction**

**Due date: 1387/2/12**

1. Suppose an uncompressed text file is 1 megabyte in size.
  - a) How long does it take to download the file over a 32 kilobit/second modem?
  - b) How long does it take to take to download the file over a 1 megabit/second modem?
  - c) Suppose data compression is applied to the text file. How much do the transmission times in parts (a) and (b) change?
2. Television channels are 6 MHz wide. How many bits/sec can be sent if four-level digital signals are used? Assume a noiseless channel.
3. If a binary signal is sent over a 3-kHz channel whose signal-to-noise ratio is 20 dB, what is the maximum achievable data rate?
4. Consider an analog repeater system in which the signal has power  $\sigma_x^2$  and each stage adds noise with power  $\sigma_n^2$ . For simplicity assume that each repeater recovers the original signal without distortion but that the noise accumulates. Find the SNR after  $n$  repeater links. Write the expression in decibels:  $\text{SNR dB} = 10\log_{10}\text{SNR}$ .
5. Suppose that a link between two telephone offices has 50 repeaters. Suppose that the probability that a repeater fails during a year is 0.01, and that repeaters fail independently of each other.
  - a) What is the probability that the link does not fail at all during one year?
  - b) Repeat (a) with 10 repeaters; with 1 repeater.
6. Most digital transmission systems are “self-clocking” in that they derive the bit synchronization from the signal itself. To do this the systems use the transitions between positive and negative voltage levels. These transitions help define the boundaries of the bit intervals.
  - a) The nonreturn-to-zero (NRZ) signaling method transmits a 0 with a +1 voltage of duration  $T$ , and a 1 with a -1 voltage of duration  $T$ . Plot the signal for the sequence  $n$  consecutive 1s followed by  $n$  consecutive 0s. Explain why this code has a synchronization problem.
  - b) In differential coding the sequence of 0s and 1s induces changes in the polarity of the signal; a binary 0 results in no change in polarity, and a binary 1 results in a change in polarity. Repeat part (a). Does this scheme have a synchronization problem?
  - c) The Manchester signaling method transmits a 0 as a +1 voltage for  $T/2$  seconds followed by a -1 for  $T/2$  seconds; a 1 is transmitted as a -1 voltage for  $T/2$  seconds followed by a +1 for  $T/2$  seconds. Repeat part (a) and explain how the synchronization problem has been addressed. What is the cost in bandwidth in going from NRZ to Manchester coding?
7. Draw the graph of the Manchester scheme using each of the following data streams, assuming that the last signal level has been positive. From the graphs, guess the bandwidth for this scheme using the average number of changes in the signal level.
  - a) 00000000
  - b) 11111111
  - c) 01010101
  - d) 00110011
8. We have a baseband channel with a 1-MHz bandwidth. What is the data rate for this channel if we use one of the following line coding schemes?
  - a) NRZ-L
  - b) Manchester
9. A twisted-wire pair has an attenuation of 0.7 dB/kilometer at 1 kHz.
  - a) How long can a link be if an attenuation of 20 dB can be tolerated?
  - b) A twisted pair with loading coils has an attenuation of 0.2 dB/kilometer at 1 kHz. How long can the link be if an attenuation of 20 dB can be tolerated?

10. Calculate the bandwidth of the range of light covering the range from 1200 nm to 1400 nm. Repeat for 1400 nm to 1600 nm. Keep in mind that the speed of light in fiber is approximately  $2 \times 10^8$  m/sec.
11. Suppose a transmission channel operates at 3 Mbps and that it has a bit error rate of  $10^{-3}$ . Bit errors occur at random and independent of each other. Suppose that the following code is used. To transmit a 1, the codeword 111 is sent; To transmit a 0, the codeword 000 is sent. The receiver takes the three received bits and decides which bit was sent by taking the majority vote of the three bits. Find the probability that the receiver makes a decoding error.
12. A block of bits with  $n$  rows and  $k$  columns uses horizontal and vertical parity bits for error detection. Suppose that exactly 4 bits are inverted due to transmission errors. Derive an expression for the probability that the error will be undetected.
13. A bit stream 10011101 is transmitted using the standard CRC method described in the text. The generator polynomial is  $x^3 + 1$ . Show the actual bit string transmitted. Suppose the third bit from the left is inverted during transmission. Show that this error is detected at the receiver's end.
14. Data link protocols almost always put the CRC in a trailer rather than in a header. Why?
15. An early code used in radio transmission involved using codewords that consist of binary bits and contain the same number of 1s. Thus, the 2-out-of-5 code only transmits blocks of 5 bits in which 2 bits are 1 and the others 0.
  - a) List the valid codewords.
  - b) Suppose that the code is used to transmit blocks of binary bits. How many bits can be transmitted per codeword?
  - c) What pattern does the receiver check to detect errors?
  - d) What is the minimum number of bit errors that cause a detection failure?
16. Find the probability of error-detection failure for the code in the previous problem for the following channels:
  - a) The random error vector channel.
  - b) The random bit error channel.
17. ATM uses an eight-bit CRC on the information contained in the header. The header has six fields:
  - First 4 bits: GFC field
  - Next 8 bits: VPI field
  - Next 16 bits: VCI field
  - Next 3 bits: Type field
  - Next 1 bit: CLP field
  - Next 8 bits: CRC
  - a) The CRC is calculated using the following generator polynomial:  $x^8 + x^2 + x + 1$ . Find the CRC bits if the GFC, VPI, Type, and CLP fields are all zero, and the VCI field is 00000000 00001111.
  - b) Can this code detect single errors? Explain why.
  - c) Draw the shift register division circuit for this generator polynomial.
18. Suppose a header consists of four 16-bit words: (11111111 11111111, 11111111 00000000, 11110000 11110000, 11000000 11000000). Find the internet checksum for this code.
19. Consider the  $m = 4$  Hamming code.
  - a) What is  $n$ , and what is  $k$  for this code?
  - b) Find parity check matrix for this code.
  - c) Give the set of linear equations for computing the check bits in terms of the information bits.
  - d) Write a program to find the set of all codewords. Do you notice anything peculiar about the weights of the codewords?
20. Suppose we take the (7,4) Hamming code and obtain an (8,4) code by adding an overall parity check bit.
  - a) Find the  $H$  matrix for this code.
  - b) What is the minimum distance?
  - c) Does the extra check bit increase the error correction capability? the error-detection capability?
21. Sixteen-bit messages are transmitted using a Hamming code. How many check bits are needed to ensure that the receiver can detect and correct single bit errors? Show the bit pattern transmitted for the message 1101001100110101. Assume that even parity is used in the Hamming code.
22. A 12-bit Hamming code whose hexadecimal value is 0xE4F arrives at a receiver. What was the original value in hexadecimal? Assume that not more than 1 bit is in error.