# NEURAL NETWORK WITH FORMED DYNAMICS OF ACTIVITY

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*The problem of developing a neural network with a given pattern of the state sequence is considered. A neural network structure and an algorithm of forming its bond matrix which lead to an approximate but robust solution of the problem are proposed and discussed. Limiting characteristics of the serviceability of the proposed structure are studied. Various methods of visualizing dynamic processes in a neural network are compared. Possible applications of the results obtained for interpretation of neurophysiological data and in neuroinformatics systems are discussed.*

## 1. EQUATIONS OF THE DYNAMICS OF NEURAL NETWORK ACTIVITY

The problem of the calculation and qualitative analysis of processes in neural networks has attracted the attention of many scientists since the end of the forties. Numerous analytical results and empirical observations in the field have been published in many papers on computer simulation modeling. Some of these observations must still be explained and analyzed mathematically. The computer modeling of dynamic processes in neural networks is related to the development and improvement of special means for mapping and analyzing of these processes. This paper is devoted to the simulation study of dynamic processes in a particular group of neural networks. The study requires an improvement of the available methods of visualization of neural network activity. The results obtained can be used for other applied and theoretical problems.

We limit our analysis to neural networks with a relatively high excitation duration and refractoriness. Such networks allow us to fill in the "gap" between the pure "spin" one-cycle models [1, 2] and complicated physiologically plausible networks [3]. The models studied are based on the following network equation [4-6]:

$$U_i(t) = \sum_{j=1}^{n} \mathbf{G}_{ij} \mathcal{V}_j, \tag{1}$$

where $t$ is the discrete time, $U_i(t)$ is the membrane potential, $\mathcal{V}_j$ is the excitation of a $j$th neuron, $\mathcal{V}_j(t) = \mathcal{V}_j(\tau_j(t))$, $\tau_i$ is the phase of the $i$th neuron. The phase is determined by the following relationship:

$$\tau_i(t) = \begin{cases} \tau_i(t-1)+1, & \text{if } t \neq t_i^*(t)+1 \\ 1, & \text{if } t = t_i^*(t)+1, \end{cases} \tag{2}$$

where $t_i^*$ is the neuron excitation time immediately before $t$ such that the following statement is valid: $t_i^*$ is the excitation time if

$$U_i(t^*) \geq \pi_i \quad \text{and} \quad \tau_i(t^*) > \mathcal{R}_i, \tag{3}$$

where $\pi_i$ is the threshold and $\mathcal{R}_i$ is the refractoriness. The excitation $\mathcal{V}_i$ is determined from the relationship

$$\mathcal{V}_i = \begin{cases} 1, & \text{if } \tau \leq \mathcal{W} \\ 0, & \text{if } \tau > \mathcal{W}, \end{cases} \tag{4}$$

where $\mathcal{W}$ is the excitation duration. The significant (as compared with the time-cycle) duration of excitation corresponds to the fact that the duration of the post-synaptic potentials of real neurons is usually much greater than

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that of the action potentials and synaptic delays. $\mathbf{G}_{ij}$ is the bond matrix, $i, j = 1 \ldots n$, which is determined by the relationship

$$\mathbf{G}_{ij} = \sigma_i^+ \mathbf{G}_{ij}^+ + \sigma_i^- \mathbf{G}_{ij}^-, \quad \mathbf{G}_{ij} \ni \{0, 1\}, \tag{5}$$

where $\sigma_i^+$, $\sigma_i^-$ are the integer coefficients, and $\mathbf{G}_{ij}^+$, $\mathbf{G}_{ij}^-$ are the matrices of zeros and units. Therefore, the bond matrix is determined by $2 \times N^2$ bits of information plus a number of bits used to encode the coefficients in Eq. (5). Such a choice is a reasonable compromise between the requirement of a sufficient complexity of the network, on the one hand, and the calculation speed and memory used, on the other hand.

## 2. NETWORKS WITH RANDOM BONDS

We consider a network with a constant threshold, constant excitation duration, and with random interneural bonds. The elements of a matrix of interneural bonds were generated with the help of a random-number generator independently for each element, but with fixed and identical particular value probabilities for all the elements.

Neural networks with random bonds possess properties that are close to dynamic chaos. This is literally impossible. On the strength of the fact that the number of states of a network is finite and its behavior is deterministic, the general form of the neural network trajectory in the configuration space has the form of a line with a closed loop at the end since the activity of a neural network necessarily enters a limiting cycle after a certain transition process. It will be assumed, however, that the network has stochastic dynamics if the time for the neural network to reach the periodic regime or the duration of the limiting cycle are sufficiently large, i.e., commensurable with the volume of the configuration space.

Even in the first papers on the modeling of neural networks, a network with "random bonds" was shown to possess "random" activity. The result was so natural that no questions were posed. However, only recently have these results been understood rigorously [7]. We have found another property of neural networks with a random bond matrix, and this property seems to have a universal meaning. The results of the simulation modeling given below lead to a hypothesis about the properties of the activity dynamics of a "random" neural network, which will be presented after the description of the modeling results.

Figure 1 shows the data of the simulation modeling of a neural network with random bonds. Qualitatively, the result was the same for many calculations. Its essence is as follows. Figure 1 shows the time development of the network activity. The state of each neuron is given by a point whose abscissa corresponds to a time instant, and whose ordinate corresponds to a neuron number. It can be easily seen that, despite the obvious random character of the activity, some neurons are prone to spend more time in the active state, whereas this time is smaller for other neurons. This property becomes especially evident if at a certain time the neuron numbering is ordered in accordance with a time interval (from a relatively large time interval $T$) during which the neuron is excited. After that, the picture of the neural network activity resembles "curling smoke." The activity does not repeat itself for a long time, and the frequently working neurons remain the frequently working neurons.

This observation allows us to formulate the following hypothesis about the properties of neural networks with random bonds.

**Hypothesis 1.** Let the matrix of interneural bonds be chosen in line with the above rules. Then, there exists a value of a neuron threshold such that the network activity is that of the "curling smoke" type at large time intervals for almost all initial conditions. That is, for almost all subsequent time instants $t_1$, $t_2$, and $t_3$ such that

$$|t_2 - t_1|, \ \ |t_3 - t_2| \geq C(N) * N, \tag{6}$$

where $C(N) \to \infty$ for $N \to \infty$, for almost all pairs $i, j$ ($i, j = 1 \ldots N$), the equations

$$\frac{1}{t_2 - t_1} \Big| \sum_{t_1 < t < t_2} \mathcal{V}_i(t) - \sum_{t_1 < t < t_2} \mathcal{V}_j(t) \Big| >$$

$$> \Big| \frac{1}{t_2 - t_1} \sum_{t_1 < t < t_2} \mathcal{V}_i(t) - \frac{1}{t_3 - t_2} \sum_{t_2 < t < t_3} \mathcal{V}_i(t) \Big| \tag{7}$$

hold. It seems that the proof of the hypothesis can be obtained by comparing the dispersion of an individual neuron activity at various time intervals with that for the activities of various neurons at the same time interval.

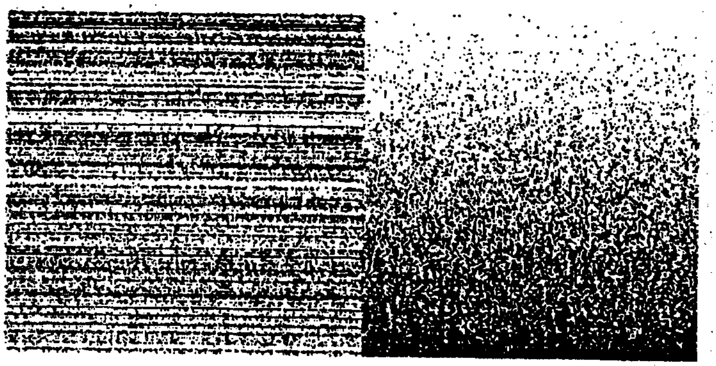

Fig. 1. Work of a random neural network. The work of neurons is shown vertically, whereas time is shown horizontally. In the right part of the figure, the neurons are ordered in accordance with the frequency of their activation, starting from the time the network began to work till the time of its ordering.

It must be noted that a fact similar to the one in Hypothesis 1 was also observed for networks with symmetric bonds. The latter have been called Hopfield networks in recent years.

The Hopfield theory refers to the networks with symmetric bonds where the interaction of neurons is the same for any pair of neurons. In this case, for a set of equations that describe the activity dynamics in a neural network, there exists a Lyapunov function whose values do not increase along the system trajectories. Therefore, the neural network rapidly comes to a stationary state from any initial state. As a matter of fact, this formulation refers to a somewhat complicated and, actually, nonphysical model of neurons. In the case of a network described by Eqs. (1)-(5), it can be shown that a network with symmetric bonds enters a certain limiting cycle with length $\mathcal{W}+1$ (speaking generally, different limiting cycles for different initial states) from any initial state. When the matrix of interneuron bonds is random, i.e., is randomly chosen from a matrix ensemble in which the function of distribution of values of the bond strength is the same for all neuron pairs, the number of possible limiting cycles with length $\mathcal{W}+1$ into which the network activity can "roll down" is exponentially (by the number of network neurons, $N$) great.

For the network with symmetric bonds, a fact similar to that described in Hypothesis 1 is as follows. Figure 2 shows the activity of a neural network with random bonds so that the network bonds are made symmetric at a time instant that corresponds to Fig. 2 in the abscissa axis. The character of the network activity differs drastically. If now, allowing for the new conditions, the neurons are again ordered by the time interval during which they are active, the activity pattern can be called a "condensed liquid" within the frames of the "curling smoke" analogy. If the initial conditions are changed, then those neurons remain active which were active under other initial conditions.

In Hopfield's theory, the neural networks whose bond matrix is the Hebb's matrix are of special interest from the viewpoint of theoretical physics (the theory of spin systems).

**Definition 1.** A matrix $\mathbf{A}_{ij}$ is the Hebb's matrix if a set of vectors $\mathbf{V}_k$, $k = 1 \ldots n$, exists such that

$$\mathbf{A}_{ij} = \sum_{k=1}^{n} \mathbf{V}_{ik}\mathbf{V}_{jk}. \tag{8}$$

The set $\{\mathbf{V}_k\}$ is called reference or basic for the matrix $\mathbf{A}_{ij}$. These definitions are motivated by the physiological ideas set forth by Hebb [8] and formulated by Hopfield [1, 2]. From the viewpoint of the theory of spin glasses, the networks with the Hebb matrix are remarkable in that the presence of a great number of isolated spin states is possible in them.

The theory and technology of neuron-like systems that use options of the "Hebb's synapses" (an example of which are Hebb's matrices) are the main subjects of today's boom in the field. It must be noted that the main ideas here actually date back to Pavlov's conditional reflexes [9]. The Hebb's synapse [8] can, in fact, be treated

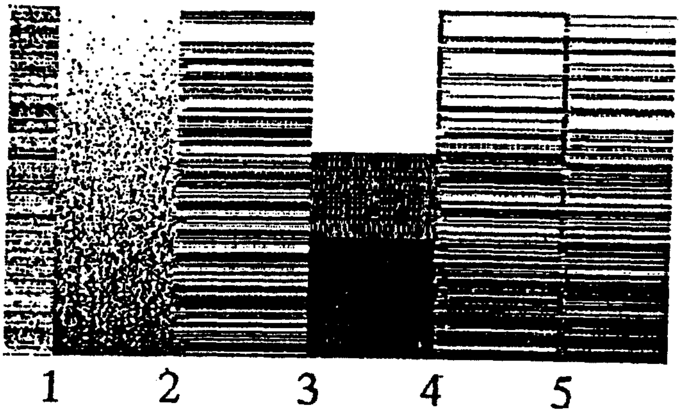


Fig. 2. Work of a neural network with random symmetric bonds. The numbers denote the following: 1) the ordering; 2) the symmetrization of interneuron bonds; 3) the ordering; 4) the random external perturbation; 5) the random external perturbation.

as an elementary conditional reflex. Pavlov, indeed, predicted that all behavior can consist of a great number of elementary conditional reflexes. Even the early papers on simulation and technology [10, 12] showed that he was not far from the truth. Neural network systems have been attracting the attention of many specialists in theoretical physics and technology since 1982, owing to Hopfield's papers [1, 2]. A law of bond formation similar to (8) will be used below to solve the problem of analysis and synthesis of neural network constructions, which is very important from the theoretical and applied viewpoints.

## 3. NETWORKS WITH FORMED DYNAMICS

Networks with formed dynamics are networks that must reproduce a given set of patterns. This problem, one of the first formulations of which was given in [4], has been considered recently with great attention [13, 21].

A general formulation of the problem is very simple. A neural network must be developed whose states take the given values subsequently in time. In other words, there is a set that has $n$ vectors with dimension $N$. A neural network must be developed whose states run through a given set of values in time. We solved this problem allowing for the physical restrictions formulated in [4] where, namely, it was noted that, in practice, the neural network activity is always ordinary, i.e., only one neuron can be excited at each time instant. If the excitation duration is finite, then a similar rule also holds for the neuron excitation stoppage. We restricted our consideration to the case of homogeneous neural networks that have no clear-cut layers or hierarchical structures. It is natural under such restrictions to consider the restrictions which should also be satisfied by the stored sequences to allow the network to satisfactorily reproduce them. As the stored sequences, we considered codes of the type "snake in a box," i.e., vector sets that satisfy the following conditions:

**Definition 2.** The "snake in a box" is a set of vectors $\mathbf{E}_1 \dots \mathbf{E}_n$, that satisfy the following conditions:

$$\rho_{ij} = \begin{cases} |i-j|, & \text{if } |i-j| \le \delta, \\ \ge \delta, & \text{if } |i-j| > \delta, \end{cases} \tag{9}$$

where $\rho_{ij}$ is the Hamming distance between $\mathbf{i}$th and $\mathbf{j}$th vectors; $\delta > 0$ is an integer, the snake's "thickness." A closed snake whose "head" joins the "tail" (all the differences in Eq. (9) should be understood as the differences in modulus $n$) is called a ring. We shall consider complete rings—rings during which each neuron of the network is excited exactly once (and remains excited during $\mathcal{W}$ cycles), and hyper-rings—rings during which the neurons can be excited more than once. Therefore, a hyper-ring is a closed, "continuous" curve in the configuration space with non-neighboring points of the curve located sufficiently far from each other. We considered rings and hyper-rings with variable and constant activity levels. For the variable activity, a used neuron switches off at the $i$th step, and

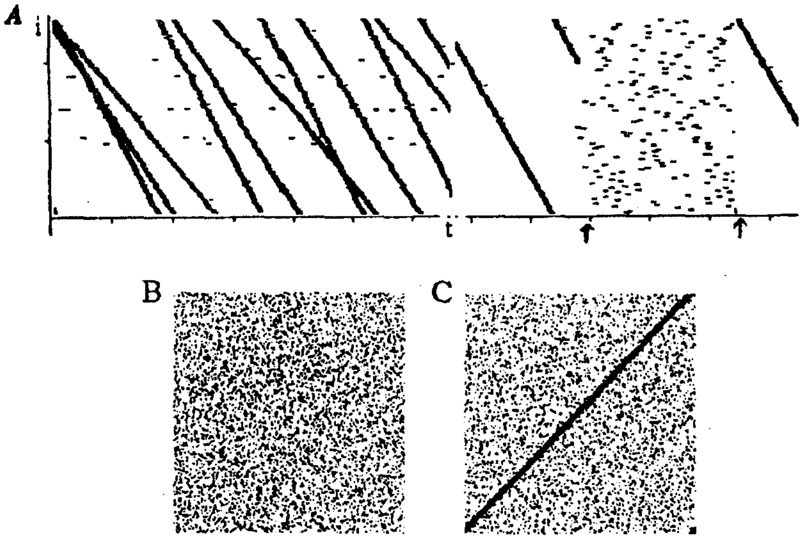

Fig. 3. Operation of a network in which five independently and randomly chosen complete rings are written. A) The left half—superposition of several cycles of activity propagation through the ring for equal threshold values (0, 3, 6); the right half—at the first moment of discontinuity of the writing (indicated by an arrow at the abscissa axis) the signals that correspond to ring $\mathcal{N}$ 5 are sent on the network whose activity propagates through ring $\mathcal{N}$ 1 for 10 time cycles; at the second moment of discontinuity, the neuron activity is ordered by the phase demonstrating that activity again propagates through the complete ring. Obviously, this is ring $\mathcal{N}$ 5 (not shown in the figure). B) The matrix of interneuron bonds; the neuron numbering is randomized. C) The same matrix of interneuron bonds; the neurons are ordered by the phase values at the time when activity in the network corresponds to propagation through ring $N$ 5.

the activity decreases by a unity, while a new neuron switches on at the $(i+1)$st step and the activity increases by a unity. For the constant activity, a used neuron switches off and a new neuron switches on at each step, and in this case Eq. (9) takes the form

$$\rho_{ij} = \begin{cases} |i-j| * 2, & \text{if } |i-j| * 2 \leq \delta, \\ \geq \delta, & \text{if } |i-j| * 2 > \delta. \end{cases} \tag{10}$$

The length of a complete ring coincides with the excitation vector dimension ($N = 256$) for the constant activity and is equal to the doubled dimension of the excitation vector ($N = 512$) for the variable activity.

The problem of construction of a network with formed dynamics is reduced to the following:

(a) construction of a ring (hyper-ring);

(b) construction of a matrix of interneuron bonds such that for the initial conditions satisfying the reference ring the network operation would resemble a part of the "snake" for $0 < t < T$, where $T$ is the period of the snake.

The law of the interneuron bond formation proposed in [1] was used. The algorithm used is as follows. In the initial state of an interneuron matrix, all the neurons are connected by braking bonds, i.e., $\mathbf{G}_{ij}^{-} = 1$ for all $i$ and $j$, whereas $\mathbf{G}_{ij}^{+} = 0$. Then the activity that should be learned is applied to the network. The neuron activation mechanisms (Eqs. (1)-(6)) are inactive in this case. Matrix elements of interneuron bonds are formed in the network at each cycle of information representation. The elements $\mathbf{G}_{ij}^{+}$ that correspond to the synapses for the neurons involved in the given time cycle from the neurons that continue to remain excited at a given time cycle are transferred to state 1. The matrix elements $\mathbf{G}_{ij}^{-}$ that have the same name are simultaneously transferred to state 0. After the ring is "rotated" in this way, it is considered to be "written" in the matrix of interneuron bonds.

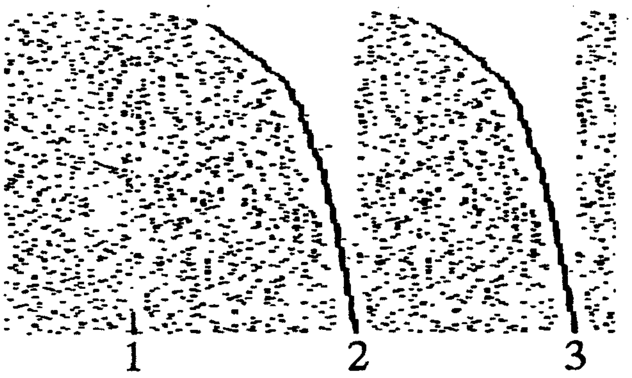


Fig. 4. Work of a neural network in which a hyper-ring is written. 1) Phase-ordered neurons; 2, 3) times of accomplishment of the complete activity cycle.

Naturally, the writing procedure indicated in this section is also possible when the reference matrix already contains some written information. It can be readily seen that the given writing procedure is, in fact, a modification of the rule (9) that prevents us from reproducing a sequence of activity states.

The reading of the written information was performed by one of two slightly different methods. In the first instance, a sequence of signals that correspond to the written ones was applied to a network as an external signal. In the second instance, in accordance with Eq. (2), the phase value was brought to zero for neurons in accordance with the neuron switch-on times in the written sequence.

Figure 3 shows the network activity where five different complete rings were written. These rings were obtained by choosing several different random orders of neuron enumeration. For a small number of complete rings, such a choice provides a large distance between different complete rings. After an activity is "started" in a neural network with the help of one of the complete rings, the activity in the network repeats the written sequence, however, with some deviations. Changing the neuron threshold, one can increase or decrease the velocity of activity propagation in a ring. If the network activity reproduces one of the written complete rings, then, once the activity corresponding to the other ring is applied to this network, we can make the network switch over to the reproduction of the activity of that ring. Sometimes, a spontaneous transition of activity from one ring to another can be observed in networks.

One of the essential problems is the determination of the resemblance of the network activity to a written pattern. This problem is easily solved for complete rings. If, after a sufficiently long operation of a network, the mapping of neurons on the screen is ordered in accordance with their phase values at a certain time, then, for the case where a network reproduces one of the written complete rings, the neuron numbering will correspond to the ring reproduced. It is typical of such a numbering that the matrix of interneuron bonds looks like a regulated matrix with a strip of nonzero elements near the main diagonal and a low density of the other nonzero elements.

For large-length hyper-rings, the ordering by the phases also allows us to perform the diagnostics of coincidence (or, rather, resemblance) of the network activity with the written pattern. The neuron activity develops linearly in time near the phase of the network activity where neurons were ordered by the phases (Fig. 4). If, for the given numbering, the same linear development of the activity is observed in the representation of the written hyper-ring, then a pronounced resemblance between the written and reproduced information takes place. Using this criterion, we managed to write hyper-rings with lengths up to 1800 time cycles in a network of 256 neurons with a satisfactory quality of reproduction.

## 4. CONCLUSION

The simulation modeling of some classes of "random" and "random"-like neural networks performed in this paper revealed a number of practically important and qualitatively special regimes of operation of these networks. The very fact of the qualitatively special regimes and evaluation of parameters in which they are possible needs to be analyzed from the viewpoint of the theory of nonlinear dynamic networks. From this point of view, the material proposed is a particular piece of "information to be considered." The main argument in favor of the further theoretical analysis of the given experimental material is the indisputable importance of the regimes discussed for applications of the neural networks.

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