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Hebb-Hopfield neural networks based on one-dimensional sets of neuron states

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Abstract. Neural Networks (NN), which interconnection matrix is the Hebb matrix of Hopfield (HH) [2,3] are considered. Quasi-continuos sets of neuron states are being used for network matrix production. It is shown, that in this case minima of Hopfield energy are at the bottom of deep ditches, corresponding to the basic set of network activity states for the HH NN. The corresponding states can be made to be stable states of the network. When neuron threshold fatigue is introduced, depending of its recent activity state, the network activity becomes cyclic, moving with a constant rate in one of the two possible directions in the ring, depending on the initial conditions. The phenomena described present novel robust types of NN behavior, which have a high probability to be encountered in living neural systems.

1. Introduction

Many neural network (NN) paradigms are using specially selected sets of neural network activity states as basic sets for obtaining network interconnection matrices. For example in [1] a set of many thousands of randomly chosen NN activity states have been used for network connections modification. Conditions for a recall of the recorded events in a collateral network have been explored by D.Marr [1]. A very like paradigm have been used in Hopfield's formulations of Hebb type neural networks [2,3]. In later works some special types of basic sets for Hopfield type networks have been explored [4]. We are presenting here some results in computer experiments with neural networks, basic sets for which matrices are in a certain sense continuos and one-dimensional.

2. Neural network dynamics

We are using a practically standard model of neural networks. The main peculiarities of our model are: (i) using of long duration, W, of excitation states of neurons (about 10 units of time); (ii) introduction of neuron *refractory period*, R, during which a neuron cannot be excited; (iii) extensive using for updating the states of the network of neuron *activity phase*, i.e. time duration between last firing of a neuron and current moment; and (iv) a two-bit elements of interconnection matrices - for accelerating computing time. Details of our models are described elsewhere [5-7].

3. One-dimensional sets of network activity states

We can define a one-dimensional set of NN states, $X = \{V^1, ..., V^n\}$, if in this set any NN state, with a possible exception of two marginal states, has exactly two neighbors (states, which distance to a given one is equal to some minimal value (e.g. 1 or 2) and the whole set of states is (simply) connected. A natural metric for sets of NN states is a Hemming distance. The well-known example of one-dimensional sets of NN states is presented by the *snake-in-a-box* codes [6]. In this work we are dealing with simplest sets, having the pointed properties: the ring sets of NN states.

For defining the *ring set* we suppose, that all neurons are considered to constitute a ring, so as the first neuron is connected to the second, the second to the third, etc., and the N-th is connected to the first. This ring, evidently, depends on the neuron enumeration. This fact will be used later. Let us further consider the following process in time of the type, reminding excitation propagation in a ring of excitable tissue [8]. Let the first neuron of a ring be switched from the silent to excited state at $t = 1$ and remain be excited for W units of time at $t = 2$ the second neuron is being excited and keeps be excited W units of time, etc., at $t = N + 1$ the first neuron is excited again (we suppose, that W (*excitation duration*) is substantially less, than N). The set of neural network states, which are activated, by the postulated process we will denote as a *ring of states* in a configuration space of N neurons. Quite natural *multi-rings sets* of

NN activity can be defined, should we consider several independent randomly chosen enumerations of neurons. For each enumeration we define a ring of network states, as described. A set of states, consisting of several rings of network states, we denote as a the *multi-ring*. So a *k-multi-ring* is defined by a set of *neuron enumerations* $E_1,\ldots, E_k$ and excitation duration, W; each enumerations, E_i, is a permutation of numbers $\{1,\ldots, N\}$.

4. Hebb-Hopfield networks with multi-rings basic sets

We say, that a NN is of the Hebb-Hopfield (HH) type, when its interconnection matrix can be presented in a form [2, 3]:

$$T_{i,j} = \sum_{k=1}^{n} \left(V_i^{(k)} - \overline{V}\right)\left(V_j^{(k)} - \overline{V}\right), \tag{1}$$

where $\overline{V}$ is a mean value of $V_j^{(k)}$, $i,j = 1\ldots N$, $\{V^{(k)}\}$ is a set of n activity states of the neural network, which we denote as a basic set for the matrix $T_{i,j}$. Below we describe activity patterns, which could be observed in HH neural networks, which basic sets are (multi-)rings. To reduce the necessary computational power, we transform the matrix elements from integer form of (1) to two bit representation [6,7].

The fairly anticipated feature of HH neural networks with basic sets, having one-dimensional continuos topology, i.e. a possible stability of any NN state, belonging to the NN basic set, consisting of two full rings, is illustrated in figures 1, 2.

For testing NN properties we are feeding the NN with signals, corresponding to activity propagation, as stated above, in one of the rings of the NN multi-ring bawic set. This external uignal can be stopped and activity dynamics in the network be watched, due to so established initial conditions. The outlook of activity pattern, as we ave used to demonstrate it (see figures 1 and 2), depends on *screen enumeration* of neurons. We can tune this enumeration in different ways (e. g. in accordance to values of some parameters of individual neurones, such as *activity phase* or others) for demonstration of different NN features. The figure 1 demonstrates, that the NN activity remaines stable, when external signal disappears. This is true for the both rings, which constitute the NN basic set.

At the same figure the NN interconnection matrix is presented before (figure 1b) and after (figure 1c) neurons reordering, i.e. making screen enumeration of neurons to correspond to the first of the rings of NN basic sets. At the figure 1b the whole matrix is filled with randomly scattered elements; at the figure 1c the reordered matrix has the diagonal strip of non-zero elements and numerous scattered non-diagonal non-zero elements. One can readily understand, that scattered elements present in fact the other strip of non-zero matrix elements, which can be seen, when elements are reordered in accordance with the second ring set of network states.

These strips couldn't be seen simultaneously, because of their *randomness* in relation to each other. At figure 1a before demonstration of the network activity with stable steady states, display of the excitation propagation along the corresponding ring is presented; the interconnection matrix during this display is tuned for the linear presentation of the first ring.

The capacity of the NN connections for the volume of the basic sets, which can be effectively stored in the network is evidently limited. Excitation patterns in the network, which basic set consists of three full rings are much less stable, than in networks, based on two or one full rings.

5. Activity propagation along rings of the basic set

Another NN activity paradigm arise in the explored networks, when we introduce neuron threshold, $\pi_i(t)$ dependence on neuron activity state:

$$\pi_i(t) = \pi_O + \lambda \int_0^t V_i(t) \bullet h(t-\tau)d\tau, \tag{2}$$

where $h(\theta)$ is a unite function of the duration W and π_0 and λ are constants.

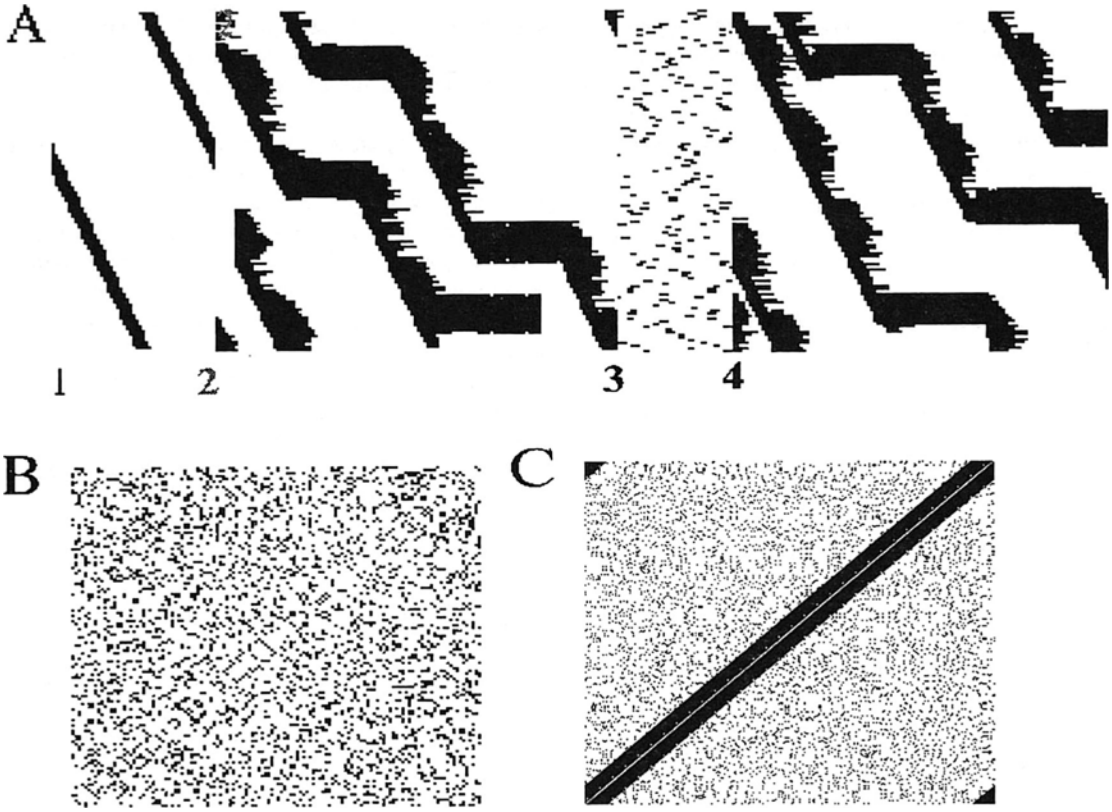

Fig. 1. Activity dynamics and matrix representation for a NN, which basic set consists of two full rings. A - diagram of neuron activity; between moments, pointed with 1 and 2 and with 3 and 4, the activity, which corresponds to excitation propagation in the first and the second rings is presented, with the matrix, tuned to the first ring. Activity between moments 2 and 3 corresponds to interference to the work of the network with activity, corresponding to the first full ring of the NN basic sets; activity after the moment 4 corresponds to network activation with signals, corresponding to the second basic set; several activity patterns are superimposed. After the moment 4 matrix is rearranged for demonstration of the second ring and experiment like those between moments 2 and 3 are demonstrated. B, C - representation of interconnection matrix before and after neurons reordering in accordance with values of neuron phase after the reproducing by the network of excitation propagation along the first ring.

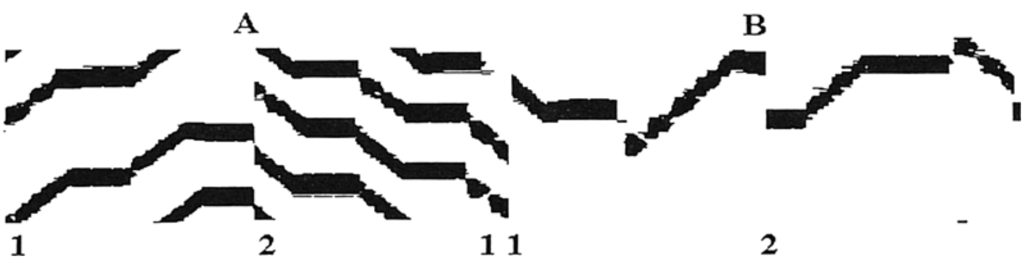

Fig. 2. Influence of neurons threshold fatigue on NN activity dynamics. The NN basic set consists of two rings, matrix representation is adjusted for display of the first ring. Activity changes are observed, when threshold fatigue is switched on. Activity propagation in both direction of the ring can be observed, depending on initial conditions; A and B - two different simulation runs; in A several simulations are superimposed.

In this case the network activity becomes cyclic, propagating with a constant rate, depending on λ, in one of the possible directions in the ring, depending on the initial conditions. The phenomena are illustrated at the figure 2. Here one can see, that the activity becomes propagating, as soon as dependence of the neuron threshold, π, on recent activity is *switched on.* The activity propagation in standard direction is represented here as wide non-smooth strips of activity with negative slope. A turning off the threshold dependence on the recent activity results in horizontal strips, which correspond, as in figure 1, to stable states of NN activity. Activity propagation in opposite direction is represented with the alike strips, having, however a positive slope. At the figure 2 A several examples of activity propagation initiations are shown, some of them resulting in standard direction of propagation (left part of the figure 2 A) and other in the opposite (right part of the figure 2 A). Figure 2 B illustrates a case, when original propagation of activity in one direction is stopped with turning off the threshold fatigue and after turning on the fatigue the direction of activity propagation is reversed.

6. Conclusion

We have described at length activity dynamics in NN, which matrices have been formed with making use of some *continuous* one-dimensional basic sets with procedures, which present modified HH procedure for learning. A type of neuron pattern generators is described as a result of these studies. It is possible in NN with symmetric connections, making use of a dependence of neurons threshold on their recent activity. This type of the NN activity dynamics have been predicted earlier [6]. Computer simulation of the *random-looking* (cf. figure 1b) neural networks in the present work have revealed some important for applications modes of work of such networks. These experimental observations may give rise to further analytical and theoretical works.

7. Acknowledgements

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References

[1] D. Marr. Simple memory: A theory for archicortex, *Phil. Trans. Roy. Soc.*, vol. B-262, pp. 23-81, 1971.

[2] J.J. Hopfield. Neural networks and physical systems with emergent collective computational abilities, *Proc. Nal. Acad. Sci.*, vol. 79, pp. 2554-2558, 1982.

[3] J.J. Hopfield. Neurons with graded response have collective computational properties like those of two-state neurons, *Proc. Nat. Acad. Sci.*, vol. 81, pp. 3088-3092, 1984.

[4] V.S. Dotsenko, L.B. Ioffe, M.V. Feigelman, M.V. Tsodyks. Statisticheskie modeli nejronnykh setej, - In A. Vedenov, editor, *Fizicheskie i matematicheskie modeli nejronnych setej*, Moscow, VINITI, vol. 1, 1990 (in Russian).

[5] W.L. Dunin-Barkowski. Konfiguracionnye generatory nejronnoj ritmiki, *Biofizika*, vol. 29, no. 5. pp. 899-902,1984 (in Russian).

[6] E.W. Dunina-Barkowska, W.L. Dunin-Barkowski. Analog/digital controversies in neural computations. *Neural Network World*, vol. 3, no. 4, p. 361-372, 1993.

[7] W.L. Dunin-Barkowski, N.B Osovets. Neural network with a predetermined activity dynamics. - In: *Optical Memory and Neural Networks '94: Optical Neural Networks*, A.L. Mikaelyan, ed., Proc. SPIE, vol. 2430, pp. 118-127, 1994.

[8] Y.I. Arshavskij, M.B. Berkinblit, W.L.Dunin-Barkowski. Rasprostranenie impulsov v koltse vozbudimoj tkani. *Biofizika*, vol. 10, no. 6, pp. 1048-1053, 1965 (in Russian).