Prepare metabolic graph

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Introduction

IN THIS TUTORIAL we are going to work with graphs. Graphs, or networks are usually object consisting of several kind of nodes (or vertices) and links (or edges) connecting them. Analysis of network usually splitting into following parts

- 1. Loading network
- 2. Configuration of the network, structure changing
- 3. Estimation of network metrics
- 4. Analysis and visualisation
- 5. Extension, modification of the network

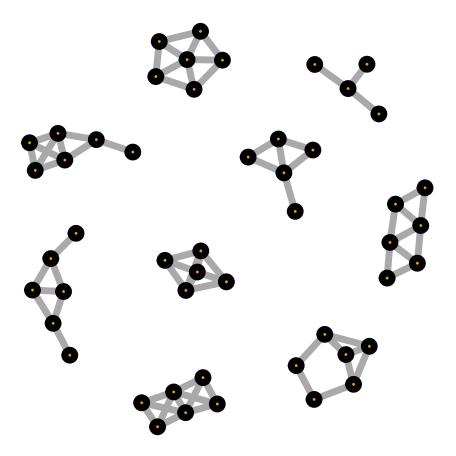
plot(g,vertex.label=NA,vertex.size=5)

In general network is loaded only once, while other steps could be repeated in arbitrary order in line with analysis purpoces. In this tutorial we are going to consider steps in the order shown above.

igraph

igraph – one of the most widely used network analysis package. It supports most of standard graph metrix, set of layout algorithms and rich visualisation framework. You have installed igraph already, but for the record for its installatition the following command should be executed:

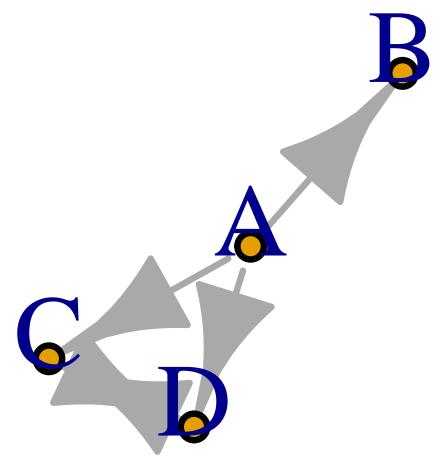
```
install.packages('igraph', dependencies = c("Depends", "Imports", "Suggests"))
  Let's test that installation is successfull:
library(igraph)
library(ggplot2)
g<-make_graph('Nonline')</pre>
```



Making a graph

As I said already graph is a combination of two sets: G = (V, E), where V is set of all vertices, and E is a set of all edges linking those vertices. Visually vertises are drown as points or shapes, while edges as connecting lines or arrows. There are two types of graphs: directed and undirected. In undirected graph edge A - > B is the same as edge B->A, while in directed graph they are considered different. Let's make simple directed graph in igrap:

```
graph <- make_graph( ~ A-+B:C:D,C+-+D)</pre>
plot(graph,vertex.label.dist=2,vertex.label.degree=-pi/2)
```



By using this way of graph assembly, which is called "symbolic", we explicitly define names of vertices and edges between them. Note sign ~ in the begining of definition string. That symbol marks all following as formula. Formulas are used in R widely, for example in regression and hypothesis testing, but their discussion is outside the scope of this tutorial.

Vertices in the previous example are named by letters from A to D, but any string could be used as a vertex name. The only restriction is that if name contains spaces or special symbols it should be quoted:

```
make_graph("this is" +- "a silly" -+ "graph here" )
```

Directed edges are represented by string -+ and +-, bidirectional link in that case will be shown as +-+. Undirected edges are represented by string - or -- or even -----. The number of - letters is meaningless, but such representation is useful to make code more readable.

Function V returns list of graph vertices, and function E – list of all edges.

V(graph)

Practical task: make your own graph with 5 nodes and plot it. We will discuss parameters of plot function later.

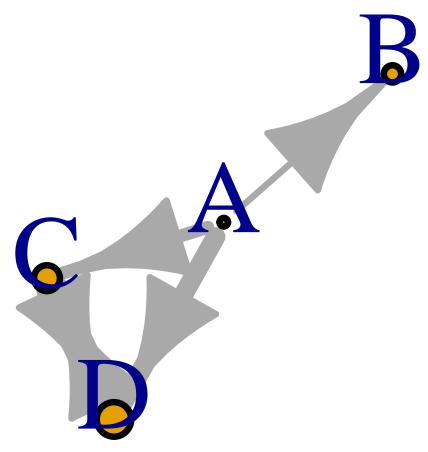
```
## + 4/4 vertices, named, from 7f0958f:
## [1] A B C D
E(graph)
## + 5/5 edges from 7f0958f (vertex names):
## [1] A->B A->C A->D C->D D->C
```

All three kind of objects: graphs, vertices and edges could have attributes, required for analysis. For example, node could have name, reference id in some database, etc. Some of this attributes could influence plotting of the graph.

To get access to attributes standard $R \$ notation is used V(g) att_name. To demonstrate we will change size of the vertex and width of the edge:

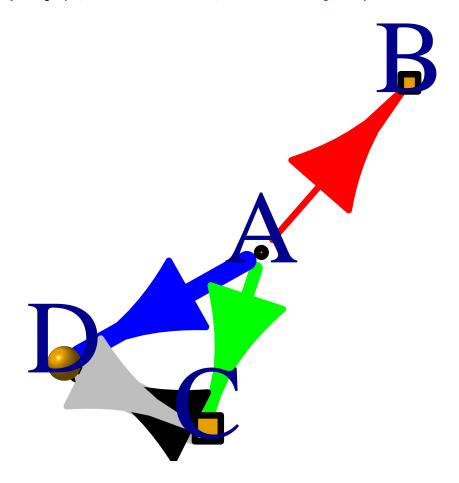
List of plotting attributes could be found in documentation: ?igraph.plotting The most important we will show

```
V(graph)$size<-1:4*5
E(graph) $width<-1:5
plot(graph,vertex.label.dist=2,vertex.label.degree=-pi/2)
```



In the code above we have changed size of vertices and edge of edges. Let's assign different shapes to our graph nodes:

E(graph)\$color <- c('red','green','blue','black','gray')</pre> V(graph)\$shape<-c('circle', 'square', 'csquare', 'sphere')</pre> plot(graph,vertex.label.dist=2,vertex.label.degree=-pi/2)



Simple visualisation

Brilliant tutorial on igraph visualisation is available at http:// kateto.net/network-visualization. We will consider most basic functions leaving color and font properties aside.

Graphical parameters

The most frequently used parameters to specify visualisation is shown below.

For vertices:

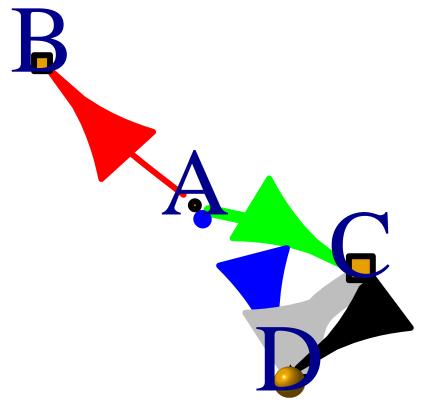
Attribute Value vertex.color Fill color vertex.frame.color Line color Shape - "none", "circle", "square", "csquare", vertex.shape "rectangle", "crectangle", "vrectangle", "pie", "raster", or "sphere" Vertex size, the default is 15, for large networks 1-3 vertex.size vertex.label Label, to hide all labels place NA vertex.label.dist The distance between vertex and label vertex.label.degree Angular alignment of the label: o on the right, "pi" on the left, "pi/2" under, and "-pi/2" above

For edges

	Attribute Value
edge.color	Color
edge.width	Line width, by default is 1
edge.arrow.size	Arrow size by default is 1, better use 0.2-0.4
edge.lty	line type o or "blank" – no line,
	1 or "solid" solid line,
	2 or "dashed" dashed line,
	3 or "dotted" dotted line,
	4 or "dotdash" dot-dash line,
	5 or "longdash" long dash line,
	6 or "twodash" double dash line
edge.label	Edge label
edge.curved	Edge curvature, in a range o-1 (o – strait line)

Some of this attribute we have used already. Dealing with real life networks you should use vertex.size=1,vertex.label=NA to prevent hiding graph structure behind non necessary labels. You can specify these parameters at plot function call or set them as attributes to nodes and edges. In the latter case individual values will be used. If in the plot function parameter have length 1 than this value is applied to all elements of the graph, if length of the value is equal to the number of edges or nodes, than parameters applied individually:

```
plot(graph,
     vertex.label.dist=2,
     vertex.label.degree=-pi/2,
     edge.lty=1:5,
     edge.curved=1:5/10)
```

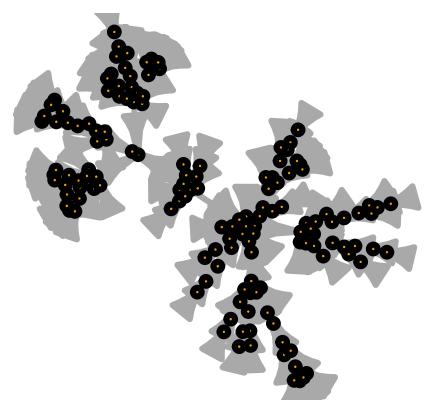


On the plot above each edge has its own curvature and line type.

Layout

The most diffucult as in theory of graphs as in practical applications is the task of laying graph on the plane with minimal vertex overlapping and edge crossing. The layout task. To demonstrate various layout algorithms we will create small scale-free graph:

```
bg<-barabasi.game(150,0.7)</pre>
plot(bg,vertex.size=5,vertex.label=NA,edge.arrow.size=0.3)
```

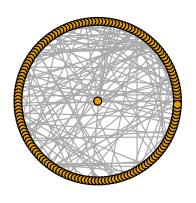


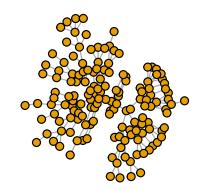
and layout our graph with various algorithms:

```
layouts <- grep("^layout_", ls("package:igraph"), value=TRUE)[-1]</pre>
layouts <- layouts[!grepl("bipartite|merge|norm|sugiyama|tree", layouts)]</pre>
par(mfrow=c(1,2), mar=c(1,1,1,1))
for (layout in layouts) {
  print(layout)
  l <- do.call(layout, list(bg))</pre>
  plot(bg,vertex.size=10,vertex.label=NA, edge.arrow.mode=0, layout=1, main=layout) }
## [1] "layout_as_star"
## [1] "layout_components"
```

layout_as_star

layout_components

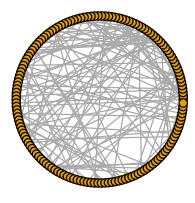


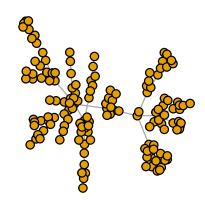


- ## [1] "layout_in_circle"
- ## [1] "layout_nicely"

layout_in_circle

layout_nicely

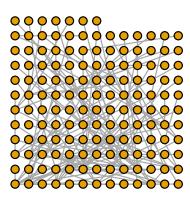


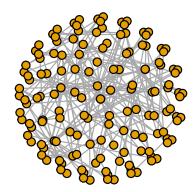


- ## [1] "layout_on_grid"
- ## [1] "layout_on_sphere"

layout_on_grid

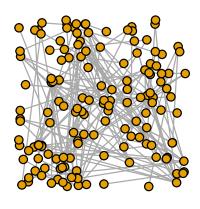
layout_on_sphere

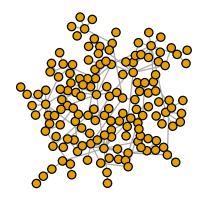




- ## [1] "layout_randomly"
- ## [1] "layout_with_dh"

layout_randomly layout_with_dh

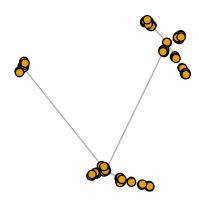


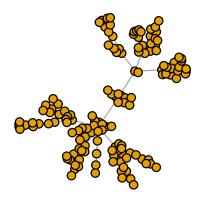


- ## [1] "layout_with_drl"
- ## [1] "layout_with_fr"

layout_with_drl

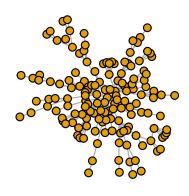
layout_with_fr

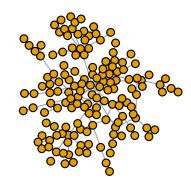




- ## [1] "layout_with_gem"
- ## [1] "layout_with_graphopt"

layout_with_gem layout_with_graphopt



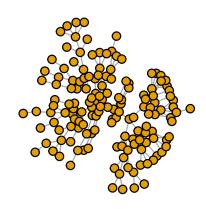


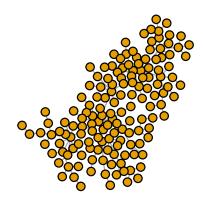
[1] "layout_with_kk"

[1] "layout_with_lgl"

layout_with_kk

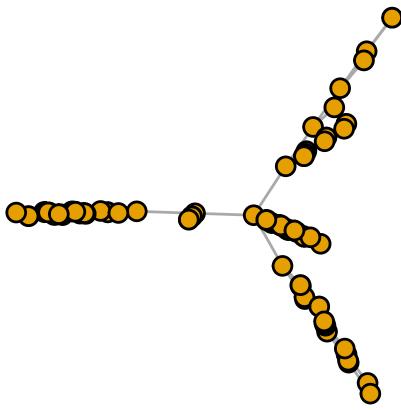
layout_with_lgl





[1] "layout_with_mds"

layout_with_mds



igraph has function layout_nicely, which uses various heuristics to choose the best layout, but you should try manually. The *_layout functions returns coordinates of vertices in matrix with 2 (and for 3D layout 3) columns, so it is possible to create your own layout if needed:

pander(head(l))

1.649	-1.135
1.005	0.05993
1.294	-0.05375
-0.3106	0.1059
1.288	-0.05184
-0.3784	0.03147

Loading data from external resources

In real life we would never create graphs by hand. There are packages to load data from a standard formats: for example NetPath-Miner is able to load data from SBML, BioPAX etc. For educational purposes we will load graph from SIF (simple interaction format) file. That format was developed by Cytoscape and used quite widely. SIF file contains three columns separated byt tabulation: initial vertex, edge type, source vertex. We can load this data as data.frame:

```
sif.df<-read.table('BINDhuman.sif',</pre>
                     header=FALSE,
                     sep='\t',
                     quote = '')
names(sif.df)<-c('A','type','B')</pre>
```

Parameters to read.table function are following: header = FALSE tells that first line is not name of the column and have to be considered as data, sep='\t' defines tabulation as separation symbol.

Let's check structure of loaded data:

```
summary(sif.df)
```

```
##
             Α
                                   type
              : 2582
##
   Unknown
                       interactsWith:10235
   HNF4-alpha: 2314
##
                                     :12740
   TAF1
              : 1776
                                     :15740
##
                       pp
##
  c-Myc
              : 1640
   TAFII250 : 1159
##
   Max
              : 933
##
   (Other)
              :28311
##
          В
   F2
              291
##
##
   Unknown:
              249
   Insulin:
##
              207
##
   CDK2
              123
  HLA-A :
              101
##
   AFP
##
               96
    (Other):37648
pander(head(sif.df))
```

A	type	В
LAT	pp	Grb2
LAT	pp	PI3K_p85-
		alpha

A	type	В
LAT	pp	PLC-gamma
LAT	pp	Grap
LAT	pp	Gads
SLP-76	pp	Vav

It is interesting that most frequent initial node is Unknown. Let's build graph with data.frame obtained:

Task: remove from graph nodes with names Unknown and "" (empty string). Count number of edges deleted. Repeat analysis with clean network.

```
g<-graph_from_data_frame(sif.df[,c(1,3,2)],directed = FALSE)</pre>
## IGRAPH c2a5487 UN-- 19906 38715 --
## + attr: name (v/c), type (e/c)
## + edges from c2a5487 (vertex names):
              --Grb2
   [1] LAT
   [2] LAT
              --PI3K_p85-alpha
##
   [3] LAT
              --PLC-gamma
##
   [4] LAT
              --Grap
   [5] LAT
              --Gads
##
   [6] SLP-76--Vav
  [7] SLP-76--Nck
## [8] SLP-76--SLAP-130
## + ... omitted several edges
```

First line shows that graph is undirected (UN) and consists of 19K vertices and 37K edges. Size of graph and its type could be obtained explicitly:

```
vcount(g)
## [1] 19906
ecount(g)
## [1] 38715
is.directed(g)
## [1] FALSE
```

Most graph algorithms assumes that there is no parallel edges and loops, so we will use finction simplify to remove unnecessary edges:

```
agg<-function(.x)toString(unique(.x))</pre>
sg<-simplify(g,</pre>
               remove.multiple = TRUE,
```

```
remove.loops = TRUE,
             edge.attr.comb = agg)
sg
## IGRAPH 2eea7d7 UN-- 19906 30243 --
## + attr: name (v/c), type (e/c)
## + edges from 2eea7d7 (vertex names):
## [1] LAT
              --PI3K_p85-alpha
  [2] LAT
              --Grb2
##
##
  [3] LAT
              --ITK
## [4] LAT
              -- Unknown
## [5] LAT
              --Grap
              --PLC-gamma
  [6] LAT
## [7] LAT
              --Gads
## [8] LAT
              --PLC-gamma-1
## + ... omitted several edges
  we can see that about 8K edges were removed
table(E(g)$type)
##
## interactsWith
                            pd
                                           pp
           10235
                         12740
##
                                        15740
table(E(sg)$type)
##
##
       interactsWith
                                     pd
##
                6972
                                 12452
##
                  pp pp, interactsWith
```

edges interactsWith were most abundant in parallel edges, and pp were least abundant.

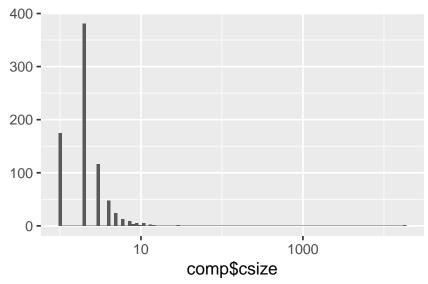
1

Connectivity is the most important property of the network. Usually biological network contains one or two large cluster and many small ones. It is often enough to analyse properties of the main componet. So let's see how many components are in our network:

```
comp<-components(sg)</pre>
max(comp$no)
## [1] 785
qplot(comp$csize,log='x',bins=100)
```

10818

##



We can see that there is 785 components and most of them consists of one to three verties only. Largest component contains 17963 vertices (90.2391239% of 19906 total graph nodes). Let's extract main component from the graph:

```
i<-which.max(comp$csize)</pre>
vg<-groups(comp)
csg<-induced_subgraph(sg,vg[[i]])</pre>
c(vcount(csg),max(comp$csize))
## [1] 17963 17963
table(E(csg)$type)
##
       interactsWith
##
                                        pd
##
                 6586
                                    12354
##
                   pp pp, interactsWith
##
                10056
```

Function groups split vertices into corresponding component lists, while induced_subgraph create subgraph from those lists. Edge type frequency analysis shows that most of small components contains edges of type pp.

Basic graph metrix

Vertex degree and scale free networks

Number of incident edges for the vertex is called degree:

```
d<-igraph::degree(csg)</pre>
summary(d)
                         Median
##
       Min.
              1st Qu.
                                     Mean 3rd Qu.
##
      1.000
                1.000
                          1.000
                                    3.228
                                              2.000
##
       Max.
## 2314.000
```

It could be seen that half of nodes have degree 1 and about 75% degree 2. At the same time the highest degree is 2314. Let's look at this vertex:

```
topD<-which.max(d)
topD
## HNF4-alpha
##
         1535
V(csg)[topD]
## + 1/17963 vertex, named, from 4ace70d:
## [1] HNF4-alpha
table(incident_edges(csg,topD)[[1]]$type)
##
##
     pd
## 2314
```

all its edges have type pd.

Scale-free networks

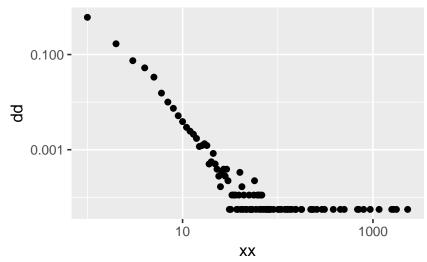
The probability to find a vertex with degree *k* in scale free network have power law with α between 2 and 3:

$$P(x=k) = \frac{x^{-\alpha}}{\zeta(\alpha)}, \quad 2 \le \alpha \le 3$$

The naive approach to check if the network is scale-free would be log-log plot of the degree distribution:

```
dd<-degree_distribution(csg)</pre>
xx<-1: length (dd) -1
ind<-which(dd>0)
dd<-dd[ind]
xx<-xx[ind]
qplot(xx,dd,log='xy')
```

Task: find position in the degree list for the node which have 50% of pp edges.

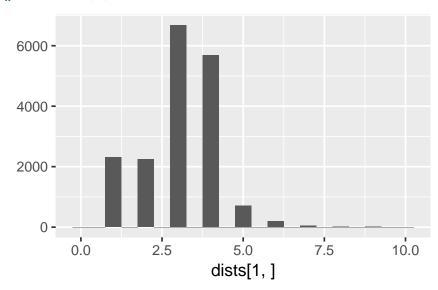


It is seen that the plot have wide hail and the slope of linear function is defined by the nodes with smallest degree. At the same time α parameter of the distribution is mainly influenced by "hubs", vertices with highest degree. It should be also noted that due to accuracy of experimental data in most real networks power law starts working at some threshold value k_0 (A. Clauset, C. R. Shalizi, and M. Newman, "Power-law distributions in empirical data," SIAM review, 2009.). The igraph package allow us to estimate α , k_0 and the p-value of hypothesis that the network is scale-free:

```
fit1 <- fit_power_law(d)</pre>
fit1
## $continuous
## [1] FALSE
##
## $alpha
## [1] 2.464437
##
## $xmin
## [1] 6
##
## $logLik
## [1] -3574.683
##
## $KS.stat
## [1] 0.02843181
##
## $KS.p
## [1] 0.2930298
```

Average path length, diameter and centrality

```
distances(csg,v=topD)->dists
qplot(dists[1,],binwidth=0.5)
```



```
table(dists)
```

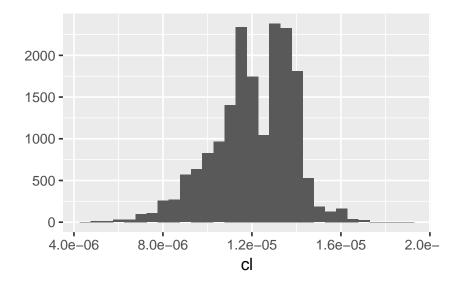
```
## dists
                 2
                                 5
                                            7
                                                 8
##
           1
                      3
                                       6
##
      1 2314 2246 6690 5696 712 200
                                           59
                                                26
##
      9
          10
     18
           1
##
```

summary(dists[1])

```
##
      Min. 1st Qu.
                      Median
                                 Mean 3rd Qu.
##
         3
                  3
                           3
                                    3
##
      Max.
##
         3
```

it could be seen that most vertices are located in HNF4-alpha steps from the main hub and average shortest path length is also 3, the diameter of the graph – 18 also shows that the main hub located in the center of the graph. Lets calculate centrality measure for the graph:

```
cl<-closeness(csg)</pre>
topCl<-which.max(cl)</pre>
qplot(cl)
## 'stat_bin()' using 'bins = 30'. Pick
## better value with 'binwidth'.
```



summary(cl)

```
Min.
               1st Qu.
                          Median
## 4.631e-06 1.106e-05 1.212e-05 1.209e-05
    3rd Qu.
                  Max.
## 1.367e-05 1.917e-05
```

The bimodal distribution of the centrality could indicate presence of several communities in the graph.

```
cl[order(cl,decreasing = TRUE)[1:7]]
```

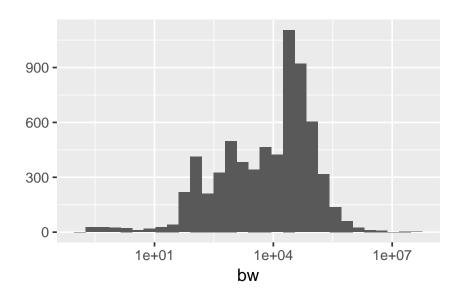
```
##
        Unknown
                         Tat
                               HNF4-alpha
## 1.916517e-05 1.833853e-05 1.811168e-05
           E2F4
                         p53
## 1.733553e-05 1.721585e-05 1.718449e-05
           CDK2
## 1.718449e-05
```

d[order(cl,decreasing = TRUE)[1:7]]

##	Unknown	Tat HM	NF4-alpha	E2F4
##	1575	709	2314	688
##	p53	TBP	CDK2	
##	155	43	81	

Betweenness

```
bw<-betweenness(csg,directed = FALSE)</pre>
qplot(bw, log = 'x')
## 'stat_bin()' using 'bins = 30'. Pick
## better value with 'binwidth'.
```



summary(bw)

```
##
       Min. 1st Qu.
                       Median
                                  Mean 3rd Qu.
                   0
                            0
                                 33464
                                           1921
##
##
       Max.
## 51644173
```

bw[order(bw,decreasing = TRUE)[1:7]]

```
Unknown HNF4-alpha
                                          TAF1
##
                                Tat
##
     51644173
                50973366
                           28886614
                                      27695538
##
        c-Myc
                    E2F4
                                Sp1
##
     18949027
               14879454
                            7121987
```

d[order(bw,decreasing = TRUE)[1:7]]

##	Unknown	HNF4-alpha	Tat	TAF1
##	1575	2314	709	1763
##	c-Myc	E2F4	Sp1	
##	1650	688	383	

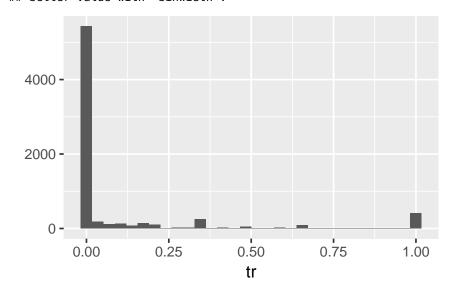
cl[order(bw,decreasing = TRUE)[1:7]]

```
Unknown
                 HNF4-alpha
## 1.916517e-05 1.811168e-05 1.833853e-05
##
          TAF1
                      c-Myc
                                    E2F4
## 1.667473e-05 1.480538e-05 1.733553e-05
##
            Sp1
## 1.613320e-05
```

Cluster coefficient

In the igraph package cluster coefficient is known by its other name transitivity:

```
tr<-transitivity(csg,type = 'local', vids = V(csg))</pre>
qplot(tr)
## 'stat_bin()' using 'bins = 30'. Pick
## better value with 'binwidth'.
```



summary(tr)

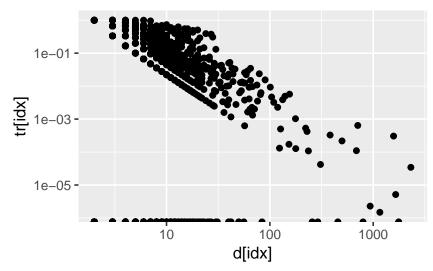
```
##
      Min. 1st Qu. Median
                               Mean 3rd Qu.
     0.000
             0.000
                     0.000
                              0.101
                                      0.000
##
##
     Max.
              NA's
     1.000
             10877
```

The cluster coefficient for the main hub

```
tr[order(d,decreasing = TRUE)[1:7]]
## [1] 3.437786e-05 0.000000e+00 5.145451e-06
## [4] 3.049555e-04 1.487608e-06 2.290190e-06
## [7] 0.00000e+00
tr[order(cl,decreasing = TRUE)[1:7]]
## [1] 3.049555e-04 6.414700e-04 3.437786e-05
## [4] 1.100166e-04 5.697528e-03 3.543743e-02
## [7] 5.55556e-03
```

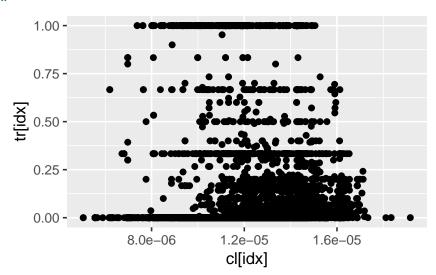
Cluster coefficient vs degree

```
idx<-which(!is.na(tr))</pre>
qplot(d[idx],tr[idx],log='xy')
```



Cluster coefficient vs centrality

idx<-which(!is.na(tr))</pre> qplot(cl[idx],tr[idx])



Assortativity degree

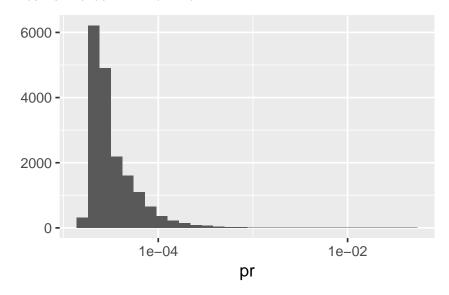
assortativity_degree(csg,directed = FALSE)

[1] -0.257744

negative value shows that hubs are repel each other.

Page rank

```
pr<-page_rank(csg,directed = FALSE)$vector</pre>
qplot(pr, log = 'x')
## 'stat_bin()' using 'bins = 30'. Pick
## better value with 'binwidth'.
```



summary(pr)

```
##
        Min.
               1st Qu.
                           Median
                                       Mean
## 1.391e-05 2.282e-05 2.660e-05 5.567e-05
     3rd Qu.
                  Max.
## 3.971e-05 4.208e-02
d[order(pr,decreasing = TRUE)[1:7]]
## HNF4-alpha
                    TAF1
                             Unknown
                                           c-Myc
##
         2314
                    1763
                                1575
                                            1650
##
     TAFII250
                     Tat
                                 Max
```

709

Clusterisation

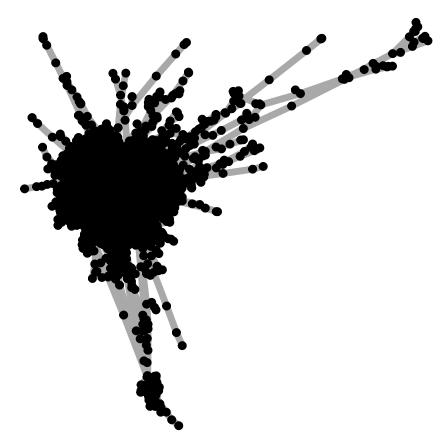
##

1160

Let's keep only pp edges and extract the main connected component:

935

```
pprm<-which(E(csg)$type!='pp')</pre>
t<-delete_edges(csg,pprm)</pre>
ct<-components(t)</pre>
ppg<-induced_subgraph(t,groups(ct)[[1]])</pre>
plot(ppg,vertex.size=1,vertex.label=NA,layout=layout.fruchterman.reingold)
```



Most vertices are located in the dense core of the graph. Let's try to split graph into modules in such a way that there will be more edges within module that between modules:

$$Q = \frac{1}{2m} \sum_{i,j} \frac{A_{ij} - k_i \cdot k_j}{2m} \delta(c_i, c_j)$$

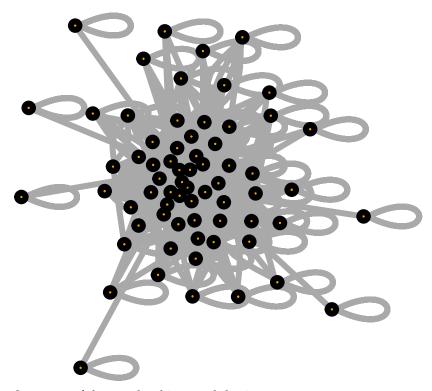
, where m – number of edges, i and j vertex indices, k_i degree of vertex i, A_{ij} element of the connectivity matrix, c_i index of the module.

We will remove edges with highest betweenness untill graph will be split into disconnected components9

ebcl<-cluster_edge_betweenness(ppg)

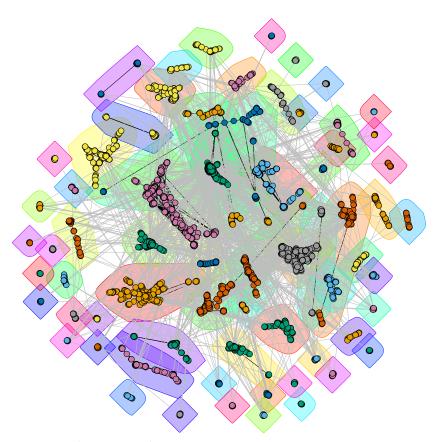
As a result we have got 67 (length(unique(ebcl\$membership))) modules by deletion of 5992 (length(ebcl\$bridges)) intermodules edges, the modularity value is – 0.8130693. The structure of the graph in modules:

eb.comm.graph <- contract.vertices(ppg, ebcl\$membership, vertex.attr.comb=list(size="sum", "ignore"))</pre> plot(eb.comm.graph,vertex.size=5,vertex.label=NA,layout=layout.fruchterman.reingold)



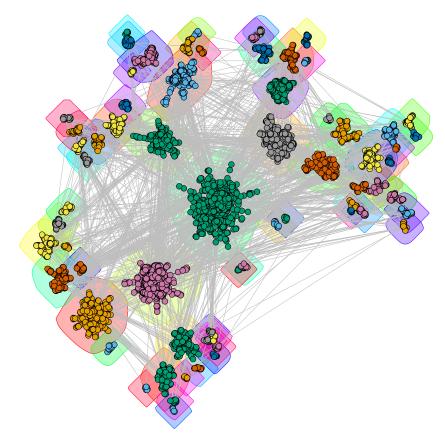
Structure of the graph taking modules into account:

```
cg<-delete_edges(ppg,which(crossing(ebcl,ppg)))</pre>
l<-layout_nicely(cg)</pre>
plot(ebcl,ppg,
     vertex.size=3,
     vertex.label=NA,
     layout=l,
     edge.color=c("black", "gray")[crossing(ebcl,ppg) + 1])
```

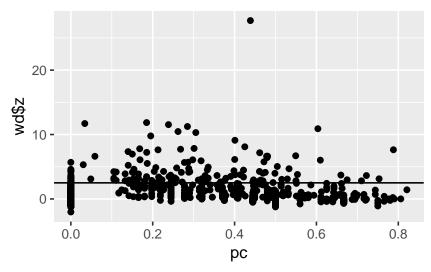


We can make our own layout

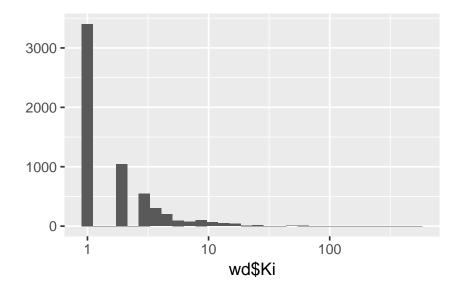
```
l<-memb_layout(ppg,ebcl)</pre>
plot(ebcl,ppg,
     vertex.size=3,
     vertex.label=NA,
     layout=l,
     edge.color=c("black", "gray")[crossing(ebcl,ppg) + 1])
```



pc<-part.coeff(ppg,ebcl\$membership)</pre> wd<-within_module_deg_z_score(ppg,ebcl\$membership)</pre> qplot(pc,wd\$z)+geom_hline(yintercept = 2.5)

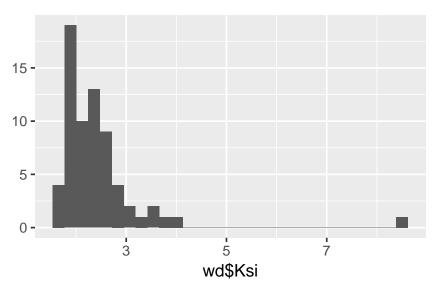


qplot(wd\$Ki,log='x') ## 'stat_bin()' using 'bins = 30'. Pick ## better value with 'binwidth'.



qplot(wd\$Ksi)

'stat_bin()' using 'bins = 30'. Pick ## better value with 'binwidth'.



R and Cytoscape

To work with graphs interactively it is easier to use Cytoscape rathe then igrap package. To set up communication between R and Cytoscape we will need to install RCy3 package:

```
source("https://bioconductor.org/biocLite.R")
biocLite("RCy3")
```

Appendix

Custom Functions

```
# @author Christopher G. Watson, \email{cgwatson@@bu.edu}
# @references Guimera, R. and Amaral, L.A.N. (2005) Cartography of complex
# networks: modules and universal roles, Journal of Statistical Mechanics:
# Theory and Experiment, 02, P02001.
part.coeff <- function(g, memb) {</pre>
  i <- NULL
  if ('degree' %in% vertex_attr_names(g)) {
    degs <- V(g)$degree</pre>
  } else {
    degs <- degree(g)</pre>
  }
  es <- E(q)
  vs \leftarrow which(degs > 0)
  PC <- rep(0, length(degs))
  for (i in vs) {
    Kis <- vapply(seq_len(max(memb)), function(x)</pre>
      sum(neighbors(g, i) %in% which(memb == x)),
      integer(1))
    Ki <- degs[i]</pre>
    PC[i] \leftarrow 1 - sum((Kis/Ki)^2)
  }
  return(PC)
}
# @author Christopher G. Watson, \email{cgwatson@@bu.edu}
# @references Guimera, R. and Amaral, L.A.N. (2005) Cartography of complex
# networks: modules and universal roles, Journal of Statistical Mechanics:
# Theory and Experiment, 02, P02001.
within_module_deg_z_score <- function(g, memb) {</pre>
  i <- NULL
  stopifnot(is_igraph(g))
  if ('degree' %in% vertex_attr_names(g)) {
    degs <- V(g)$degree</pre>
  } else {
    degs <- degree(g)</pre>
  }
```

```
vs \leftarrow which(degs > 0)
  es <- E(g)
  z <- Ki <- rep(0, length(degs))</pre>
  for (i in vs) {
    Ki[i] <- length(es[i %--% which(memb == memb[i])])</pre>
  }
  di <- lapply(seq_len(max(memb)), function(x) Ki[memb == x])</pre>
  Ksi <- vapply(di, mean, numeric(1))</pre>
  sigKsi <- vapply(di, sd, numeric(1))</pre>
  z[vs] <- (Ki[vs] - Ksi[memb[vs]]) / sigKsi[memb[vs]]</pre>
  z <- ifelse(!is.finite(z), 0, z)</pre>
  return(list(z=z,Ki=Ki,Ksi=Ksi,sigKsi=sigKsi))
}
memb_layout<-function(graph,comm){</pre>
  graphs<-list()</pre>
  layouts<-list()</pre>
  for(i in unique(membership(comm))){
    gi<-induced_subgraph(graph,which(membership(comm)==i))</pre>
    li<-layout.fruchterman.reingold(gi)</pre>
    graphs[[i]]<-gi</pre>
    layouts[[i]]<-li</pre>
  }
  g <- disjoint_union(graphs)</pre>
  lay <- merge_coords(graphs, layouts)</pre>
  l<-lay[match(V(graph)$name,V(g)$name),]</pre>
  return(l)
}
Document version
Tue Jul 31 18:15:21 2018
Session Info
Platform
                            name value
                            R version 3.5.1 (2018-07-02)
               - version
                            x86_64, darwin15.6.0
               - system
               - ui
                            X_{11}
```

	name value
- language	(EN)
- collate	en_US.UTF-8
- tz	Asia/Tokyo
- date	2018-07-31

Packages

pack	age	* ver	sion date	source
assertthat		0.2.0	2017-04-11	CRAN (R 3.5.0)
backports		1.1.2	2017-12-13	CRAN (R 3.5.0)
base	*	3.5.1	2018-07-05	local
bindr		0.1.1	2018-03-13	CRAN (R 3.5.0)
bindrcpp		0.2.2	2018-03-29	CRAN (R 3.5.0)
Biobase		2.40.0	2018-05-01	Bioconductor
BiocGenerics		0.26.0	2018-05-01	Bioconductor
biomformat	*	1.8.0	2018-05-01	Bioconductor
Biostrings		2.48.0	2018-05-01	Bioconductor
bitops		1.0-6	2013-08-17	CRAN (R 3.5.0)
codetools		0.2-15	2016-10-05	CRAN (R 3.5.1)
colorspace		1.3-2	2016-12-14	CRAN (R 3.5.0)
compiler		3.5.1	2018-07-05	local
crayon		1.3.4	2017-09-16	CRAN (R 3.5.0)
data.table		1.11.4	2018-05-27	CRAN (R 3.5.0)
datasets	*	3.5.1	2018-07-05	local
devtools		1.13.6	2018-06-27	CRAN (R 3.5.0)
digest		0.6.15	2018-01-28	CRAN (R 3.5.0)
dplyr		0.7.6	2018-06-29	CRAN (R 3.5.1)
evaluate		0.11	2018-07-17	CRAN (R 3.5.0)
flexmix		2.3-14	2017-04-28	CRAN (R 3.5.0)
formatR		1.5	2017-04-25	CRAN (R 3.5.0)
ggplot2		3.0.0	2018-07-03	CRAN (R 3.5.0)
glue		1.3.0	2018-07-17	CRAN (R 3.5.0)
graphics	*	3.5.1	2018-07-05	local
grDevices	*	3.5.1	2018-07-05	local
grid		3.5.1	2018-07-05	local
gtable		0.2.0	2016-02-26	CRAN (R 3.5.0)
htmltools		0.3.6	2017-04-28	CRAN (R 3.5.0)
httr		1.3.1	2017-08-20	CRAN (R 3.5.0)
igraph	*	1.2.1	2018-03-10	CRAN (R 3.5.0)
IRanges		2.14.10	2018-05-16	Bioconductor
jsonlite		1.5	2017-06-01	CRAN (R 3.5.0)
KEGGREST		1.20.1	2018-06-27	Bioconductor

	package	* ver	sion	date	source
		VCI			
knitr	*	1.20		02-20	, , ,
lattice		0.20-35		03-25	
lazyeval		0.2.1	-	10-29	
magrittr		1.5	-	11-22	
Matrix		1.2-14		04-13	
memoise		1.1.0	_	04-21	
methods	* 3.5.1		8-07-05		
mmnet	* 1.15.0		8-07-31		oconductor
modeltools	0.2-22		8-07-16		RAN (R 3.5.0)
munsell	0.5.0	201	8-06-12		RAN (R 3.5.0)
nnet	7.3-12	2 201	6-02-02	2 CR	RAN (R 3.5.1)
pander	* 0.6.2	201	8-07-13	3 Git	thub (Rapporter/pander@843907d)
parallel	3.5.1	201	8-07-05	5 loc	cal
pillar	1.3.0	201	8-07-14	t CR	RAN (R 3.5.0)
pkgconfig	2.0.1	201	7-03-21	ı CR	RAN (R 3.5.0)
plyr	1.8.4	201	6-06-08		RAN (R 3.5.0)
png	0.1-7	201	3-12-03	3 CR	RAN (R 3.5.0)
purrr	0.2.5	201	8-05-29) CR	RAN (R 3.5.0)
R6	2.2.2	201	7-06-17	7 CR	RAN (R 3.5.0)
Rcpp	0.12.1	.8 201	8-07-23	3 CR	RAN (R 3.5.0)
RCurl	1.95-4	ļ.11 201	8-07-15	5 CR	RAN (R 3.5.0)
rhdf5	2.24.0	201	8-05-01	ı Bic	oconductor
Rhdf5lib	1.2.1	201	8-05-17	7 Bio	oconductor
RJSONIO	* 1.3-0	201	4-07-28	3 CR	RAN (R 3.5.0)
rlang	0.2.1	201	8-05-30	CR	RAN (R 3.5.0)
rmarkdown	1.10	201	8-06-11	ı CR	RAN (R 3.5.0)
rprojroot	1.3-2	201	8-01-03	CR	RAN (R 3.5.0)
rstudioapi	0.7	201	7-09-07	7 CR	RAN (R 3.5.0)
S ₄ Vectors	0.18.3	201	8-06-08	Bic Bic	oconductor
scales	0.5.0	201	7-08-24	4 CR	RAN (R 3.5.0)
stats	* 3.5.1	201	8-07-05	5 loc	cal
stats4	3.5.1	201	8-07-05	5 loc	cal
stringi	1.2.4	201	8-07-20	CR	RAN (R 3.5.0)
stringr	1.3.1	201	8-05-10	CR	RAN (R 3.5.0)
tibble	1.4.2	201	8-01-22	2 CR	RAN (R 3.5.0)
tidyselect	0.2.4	201	8-02-26	6 CR	RAN (R 3.5.0)
tools	3.5.1	201	8-07-05	5 loc	cal
tufte	* 0.4	201	8-07-15	5 CR	RAN (R 3.5.0)
utils	* 3.5.1	201	8-07-05	5 loc	cal
withr	2.1.2	201	8-03-15	5 CR	RAN (R 3.5.0)
XML	3.98-1	1.12 201	8-07-15	5 CR	RAN (R 3.5.0)
XVector	0.20.0	201	8-05-01	ı Bic	oconductor

	package *	version date source
yaml	2.2.0	2018-07-25 CRAN (R 3.5.0
zlibbioc	1.26.0	2018-05-01 Bioconductor