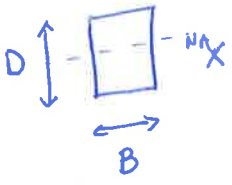
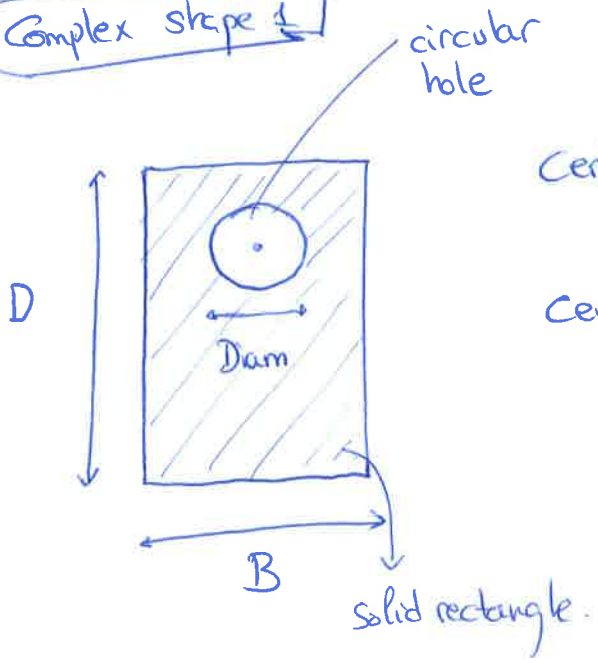
Centroid and 2nd moment of Area – Pilar Garcia

Basic shape	2nd moment of area	centroid
	$I_x = \frac{B \cdot D^3}{12}$ $I_y = \frac{D \cdot B^3}{12}$	$x = B/2$ $y = D/2$
	$I_x = I_y = \frac{\pi D^4}{64}$	$x = y = D/2$

Complex shape 1



Centroid for the rectangle = $(\bar{X}_R, \bar{y}_R)$

Centroid for circle = $(\bar{x}_c, \bar{y}_c)$

Centroid of the full shape:

$$\bar{x}_F = \frac{A_R \cdot \bar{x}_R - A_c \cdot \bar{x}_c}{A_R - A_c}$$

$$\bar{y}_F = \frac{A_R \cdot \bar{y}_R - A_c \cdot \bar{y}_c}{A_R - A_c}$$

where

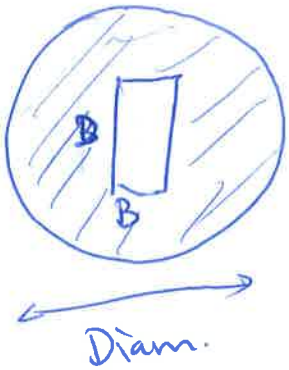
$$\begin{cases} A_R = B \cdot D \\ A_c = \frac{\pi \cdot (Diam)^2}{4} \end{cases}$$

And the second moment of area of the full shape is:

$$I_x = \frac{B \cdot D^3}{12} + A_R \cdot (\bar{y}_F - \bar{y}_R)^2 - \frac{\pi \cdot (Diam)^4}{64} + A_C \cdot (\bar{y}_F - \bar{y}_C)^2$$

$$I_y = \frac{D \cdot B^3}{12} + A_R (\bar{x}_F - \bar{x}_R)^2 - \frac{\pi \cdot (Diam)^4}{64} - A_C \cdot (\bar{x}_F - \bar{x}_C)^2$$

Complex shape 2



Same as before → $(\bar{x}_R, \bar{y}_R)$, $(\bar{x}_C, \bar{y}_C)$, A_R, A_C

Centroid of the full shape:

$$\bar{x}_F = \frac{A_C \cdot \bar{x}_C - A_R \cdot \bar{x}_R}{A_C - A_R}$$

$$\bar{y}_F = \frac{A_C \cdot \bar{y}_C - A_R \cdot \bar{y}_R}{A_C - A_R}$$

Second moment of area:

$$I_x = \frac{\pi (Diam)^4}{64} + A_C \cdot (\bar{y}_F - \bar{y}_C) - \frac{B \cdot D^3}{12} - A_R \cdot (\bar{y}_F - \bar{y}_R)^2$$

$$I_y = \frac{\pi \cdot (Diam)^4}{64} + A_C \cdot (\bar{x}_F - \bar{x}_C) - \frac{D B^3}{12} - A_R \cdot (\bar{x}_F - \bar{x}_R)^2$$