E439 - Modelling and Analysis in Engineering
Tutorial Sheet A5 - Partial Differential Equations

Tutorial on Tuesday 19th of March at 12:00.

1. The oscillations of a flexible elastic string of length L fixed at two end points is governed by

$$\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$$

The string is plucked at two points in such a way that the initial string displacement is given by:

$$y(x,o) = \begin{cases} \frac{3}{L}x & ,o < x \le L/3 \\ 1 & ,L/3 < x \le 2L/3 \\ 1 - \frac{1}{L}(3x - 2L), & 2L/3 < x \le L \end{cases}$$

If the initial velocity in the string is zero, find an expression for the ensuing motion of the string.

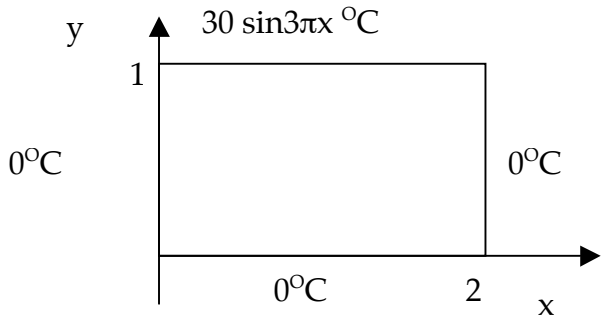
2. (a) Sketch the function $f(x) = L - |x|$ for $-2L<x<2L$, where $f(x+4L) = f(x)$. Express f as a Fourier series in x.

   (b) A bar of length L lies along the x axis from x = o to x = L. Initially the temperature in the bar varies linearly from zero at x = o to To at x = L.

      (i) What is the steady temperature distribution in the bar if the end at x = o is insulated and that x = L is held at To?

      (ii) Find the temperature distribution at intermediate times using the series obtained in (a).

3. The edges of a plate are held at temperatures shown below. Determine the steady state temperature distribution.



(NB This solution to this problem can be obtained without applying the superposition principle).

Mar 2013

[MZ/MKH/1115]