# Issue Questions

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## Question 1

Why haven't you included $\Delta t$ (time difference) in your prediction step?

**Answer.** I made some assumptions while solving the assignment. Since we are given synchronized data of RADAR and LiDAR sensors. The first assumption that I made was that the time difference between each observation i.e. $\Delta t = 1 sec$.

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Time stamp of measurement  1
Making update using RADAR measurement:  [1.81208901 0.04748265 2.30494084 0.06039697]
Prediction is:  [1.37955223 0.03614877 2.32913082 0.06103083]
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Time stamp of measurement  2
Making update using LiDAR measurement:  [ 3.890927  -0.1341657]
Prediction is:  [ 3.85623192 -0.09012285  2.54009775 -0.20677618]
*************************************************
Time stamp of measurement  3
Making update using RADAR measurement:  [6.43926395 0.47287945 2.43191718 0.17859241]
Prediction is:  [6.40817861 0.33530086 2.46012449 0.24470209]
*************************************************
Time stamp of measurement  4
Making update using LiDAR measurement:  [9.077331  0.5932112]
Prediction is:  [9.03864283 0.59076653 2.69192291 0.25934919]
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Time stamp of measurement  5
Making update using RADAR measurement:  [11.44471     1.42957874  2.48170895  0.30999461]
Prediction is:  [11.48308545  1.28372375  2.46956919  0.40547424]
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Figure 1: First few observations to estimate time between observations.

## Question 2, 4

Are you sure your state transition matrix is true?
Can you provide some comments on the noise covariance matrix?

**Answer.** In the prediction step we use the following system dynamic equations to transform the state for the next time step:

$$x_{t+1} = \underbrace{x_t + v_t \times \Delta t}_{\mathrm{I}} + \underbrace{\frac{1}{2} a_t \times \Delta t^2}_{\mathrm{II}} \tag{1}$$

where, $x_{t+1}$ is the new position, $x_t$ is the position information, $v_t$ is the velocity and $a_t$ is the acceleration information at time $t$. In matrix form this can be given as:

$$\hat{x}_{t+1} = \mathbf{A} \times x_t,$$
$$\mathbf{P}_{t+1} = \mathbf{A} \times \mathbf{P}_t \times \mathbf{A}^T + \mathbf{Q}.$$

Here, the matrix $\mathbf{A}$ is the state transition matrix of size $4 \times 4$. In the expanded form the above equation can be written as:

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{v}_x \\ \hat{v}_y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix} \tag{2}$$

Here in this equation, the acceleration term is not added since it is an unknown and we can add it to the noise component. The acceleration term can be represented as vector $r$ which has zero mean and a covariance matrix $\mathbf{Q}$:

$$r = \begin{bmatrix} \frac{a_x \Delta t^2}{2} \\ \frac{a_y \Delta t^2}{2} \\ a_x \Delta t \\ a_y \Delta t \end{bmatrix} \tag{3}$$

Using partial derivative w.r.t to $a_x$ and $a_y$ in each of the rows, the above equation can be converted to matrix form as:

$$r = \underbrace{\begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix}}_{\mathbf{G}} \underbrace{\begin{pmatrix} a_x \\ a_y \end{pmatrix}}_{a} = \mathbf{G}a \tag{4}$$

Now, by definition the covariance matrix of a vector $\mathbf{X}$ is given as:

$$\mathrm{var}(\mathbf{X}) = \mathrm{E}\left[(\mathbf{X} - \mathrm{E}[\mathbf{X}])(\mathbf{X} - \mathrm{E}[\mathbf{X}])^{\mathrm{T}}\right] \tag{5}$$

In our case, this can be reduced to:

$$\mathbf{Q} = \mathrm{E}\left[rr^T\right] \tag{6}$$

Adding the value of $r$ from equation 4 we get:

$$\mathbf{Q} = \mathrm{E}\left[\mathbf{G}aa^T\mathbf{G}^T\right] \tag{7}$$

Since matrix $G$ does not contain any part of the random variable it comes out. Additionally, the expectation of the random variable vector and its transpose $aa^T$ is the correlation matrix which is given as:

$$\mathbf{Q} = \mathbf{G} \begin{pmatrix} \sigma_{ax}^2 & \sigma_{axy} \\ \sigma_{axy} & \sigma_{ay}^2 \end{pmatrix} \mathbf{G}^T. \tag{8}$$

Here, we consider the components $a_x$ and $a_y$ uncorrelated. Thus values of the cross diagonal elements are 0. In the end, on multiplying the matrix $\mathbf{Q}$ can be given as:

$$\mathbf{Q} = \begin{pmatrix} \frac{\Delta t^4}{4}\sigma_{ax}^2 & 0 & \frac{\Delta t^3}{2}\sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^4}{4}\sigma_{ay}^2 & 0 & \frac{\Delta t^3}{2}\sigma_{ay}^2 \\ \frac{\Delta t^3}{2}\sigma_{ax}^2 & 0 & \Delta t^2\sigma_{ax}^2 & 0 \\ 0 & \frac{\Delta t^3}{2}\sigma_{ay}^2 & 0 & \Delta t^2\sigma_{ay}^2 \end{pmatrix} \tag{9}$$

which is the one that we used in our formulation. Where we set the values $\sigma_{ax}$ and $\sigma_{ax}$ maually. Here again, the value of $\Delta t$ is 1.

## Question 3

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| You have included radar measurements to initialize the filter, and not after that in the entire process. Why is that? |

**Answer.** In short, I was trying to implement this algorithm differently at the beginning where I use the velocity in the radar measurement as control commands but it was not giving appropriate results. And later I forgot to check that. But now I use the LiDAR and RADAR measurements alternately.